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Inventory stocks management under the limited capital conditions — nonlinear analysis

Zarządzanie zapasami towarów w warunkach ograniczonego kapitału — analiza nieliniowa

W artykule przedstawiono możliwość wykorzystania mnożników Lagrangea do zarządzania zapasami w warunkach ograniczonego kapitału. Zaproponowano weryfikację ww. metody w oparciu o programowanie nieliniowe i aplikację Solver.

Słowa kluczowe:
mnożniki Lagrangea, ograniczony kapitał, programowanie nieliniowe, zarządzanie zapasami.

The article presents the use of Lagrange multipliers functions to inventory stocks management under the limited capital conditions. The example verified the method based on the process analysis, the nonlinear programming and the Solver application.

Key words:
lagrange multipliers, limited capital, nonlinear programming, inventory stocks Management.

Introduction

In the literature (Harrison 2010, p. 260) many classical and statistical methods are presented as they provide an efficient stocks management on each stage of the stream supply flow in the supply chain. They concern deterministic, stochastic and under the discontinuous market conditions cases. Mostly they are connected with the manipulating of the homogeneous stock. Although the deterministic case is commonly used in the order process optimization, its abilities are limited. It is a result of the fact that it is a case in which a market and the restocking cycles are the constant values in the specified time horizon. The next limitations are connected with the lack of risk of the stock deficiency and singular range delivery amount. Despite the information technologies which provide very exact and fast solution of the complex optimization problem, determining economical amounts of deliveries for each stock management case, the contemporary trends in stock management in supply chains determine other priorities and they are connected with two strategies, so called the agile and lean supply chain. The first one that is consumer-oriented and meeting the highest standards of its service concerns:

- Business processes integration adding a value in the supply chain.
- Use of the modern information technologies in the stream demand of the supply chains.
- Flexibility.
- Short time of reaction.

The second one, the priority is the economy of scale and low logistics costs concerns:

- Small batches of production.
- Minimum volumes of deliveries.
- Elimination of waste (*muda*).

The key to success is then: flexibility, reaction rate, customer orientation, small batches of production, minimum volumes of orders and susceptibility to changes (Harrison 2010, p. 307). In business practice, in the stocks management there often occur the limited capital case for the medium value of the stocks (the financial condition of the company, avoiding freezing of the capital in the inventory amounts bigger than the specified limit). In the next part of the article there is an empirical analysis of the problem and a solution of the practical case by using the non-linear programming and the SOLVER application.

The research problems are then:

1. How to manage the inventory of particular stocks, to maintain the limited capital conditions for the average value of inventory of stocks in the company?
2. How to minimise the inventory of particular stocks costs (so their average value does not exceed a determined amount) by using non-linear programming and the SOLVER application?

To solve research problems following methods and research tools are needed:

1. Lagrange Multipliers Method — nonlinear programming.
2. SOLVER Application.

Assumptions and methodology of the stocks manipulating under the limited capital conditions

■ Assumptions

Decision problem of kind:

$$f(x) \to \max;\ f(x) \to \min$$

$$g_i(X) \geq \mathbf{0};\ g_i(X) \leq \mathbf{0}\ (i = 1, \ldots, m)$$

$$\boldsymbol{h}_i(X) = \mathbf{0};\ \boldsymbol{h}_i(X) = \mathbf{0}\ (i = m + 1, \ldots, r)$$

is a nonlinear programming task, if the objective function $f(x)$ or one of the constraints has a nonlinear form.

Let us consider a nonlinear programming case under the constraints.

$$f(x) \to min$$

$$\boldsymbol{h}_i(X) = \mathbf{0} \quad (i = 1, \ldots, r)$$

If f and h are differentiable in the R^n set then the nonlinear programming problem with the constraints solves itself by creating so-called Lagrange's auxiliary function (1).

$$L(x, \lambda) = f(x) + \sum_{i=1}^{r} \lambda_i \cdot \boldsymbol{h}_i(x) \qquad (1)$$

Minimization topic determined with the formula (1) is called the Lagrange's problem whereas λ_i is called Lagrange multipliers (Sikora, 2008, p. 118).

■ Methodology

Methodology of the problem seeks to determine the optimum for which function f reaches minimum and consists the following stages:

1. Determination of the objective function.
2. Determination of the reduction of capital.
3. Introducing the objective function of the Lagrange multiplier.
4. Calculation of the objective function's minimum (partial derivatives).
5. Solution of equations.
6. Optimal quantity calculation.

Solution of the nonlinear task — practical use

■ Input data

Trading company has decided to manipulate the inventory of various stocks, so their average value of inventory did not exceed predetermined amount, that is 1300$ and so its total cost was minimum (tab. 1).

Total costs of the stocks are the sum of the total costs of order and stocks maintenance in the considered period of time.

Order costs *cpo* are divided into:

Fixed:

- Costs of delivery department (salaries, room usage, energy usage, office supplies).

Variable:

- Costs of ordering,
- Special costs (e.g. reception of the delivery into the warehouse, laboratory tests).

Practically, one cost of the order is calculated as quotient of the costs incurred by the department of supplies in the year scale by the number of orders generated in this period of time, whereas the annual order cost as a ratio of the amount of deliveries during the year and the unit order cost (2):

$$acpo = n \cdot cpo = \frac{P}{Q} \cdot cpo \qquad (2)$$

Costs of supplies maintenance are also divided into the fixed and the variable:

The fixed are as following:

- Depreciation of storage buildings, also the equipment,
- Cost of warehouse staff.

The variable ones are as following:

- Capital,
- Service,
- Warehouse,
- Risk.

Practically the supply maintenance unit cost is calculated from the formula (3):

$$cmi = c \cdot u_r \tag{3}$$

On the other hand the supply maintenance cost is determined with the formula (4):

$$acmi = \frac{Q}{2} \cdot cmi \tag{4}$$

Where:
c — purchase price of the goods,
u_r — ratio of the annual cost of maintaining inventory,
$\frac{Q}{2}$ — the average size of the storage,
P — annual demand,
n — numer of deliveries,
Q — unit delivery volume.

◆ Step 1 — Determination of the objective function

The objective function is an equation of the total cost of inventory and occurs in the following form (5, 5'):

$$f(x) = cpo \sum_{i=1}^{k} \frac{P_i}{Q_i} + u_r \sum_{i=1}^{k} \frac{Q_i \cdot c_i}{2} \rightarrow min \tag{5}$$

$$f(x) = 50 \sum_{i=1}^{6} \frac{P_i}{Q_i} + 0,1 \sum_{i=1}^{6} \frac{Q_i \cdot c_i}{2} \rightarrow min \tag{5'}$$

◆ Step 2 — Determination of the capital limits (6):

$$\sum_{i=1}^{6} \frac{Q_i \cdot c_i}{2} = 1300 \tag{6}$$

◆ Step 3 — The inclusion of the objective function Lagrange multiplier (7):

$$f(x) = 50 \sum_{i=1}^{6} \frac{P_i}{Q_i} + 0,1 \sum_{i=1}^{6} \frac{Q_i \cdot c_i}{2} + \lambda \left(\sum_{i=1}^{6} \frac{Q_i \cdot c_i}{2} - 1300 \right) \rightarrow min \tag{7}$$

◆ Step 4 — The calculation of the minimum of the objective function (partial derivatives) for the variables Q and? (8):

$$\begin{cases} \frac{df(x)}{dQ_i} = 0 \\ \frac{df(x)}{d\lambda} = 0 \end{cases} \tag{8}$$

◆ Step 5 — Solution of the system of equations (8):

Table 1
Input parameters of case analysis

INVENTORY MANAGEMENT UNDER THE LIMITED CAPITAL CONDITIONS – CASE ANALYSIS

Product	Demand	Price	Ur	cpo
A	4613	1,36	0,1	50
B	168	0,75		
C	1136	1,31		
D	427	0,7		
E	1529	1,33		
F	557	1,32		

Source: Own elaboration.

As a result of solving the system of equations we obtain the following expressions

$$Q_i = \sqrt{\frac{100 \cdot P_i}{(0{,}1 + \lambda) \cdot c_i}} \qquad (9)$$

- Step 6 — Calculation of the optimal size of supply.

In order to calculate the optimal size of the supply it should be substituted into the expression Q_i (9) the expression for the multiplier λ (10). The final result to solve the problem presents the table 2 (Sarjusz-Wolski, 2000, pp. 158–160).

Inventory management implementation in the Solver application

Solving the same problem using Solver a non-linear optimization program Generalized Reduced Gradient (GRG2) is used, which was developed at the University of Leon Lasdon from the University of Texas in Austin and Allan Waren from the State University of Cleveland. After entering all the data and formulas into a spreadsheet, run the Solver (fig. 1). On the screen, a dialog box presents „Solver Parameters", where in another part of the address field of the objective function, ie. Equation of the total cost of inventory (5'), the type of opti-

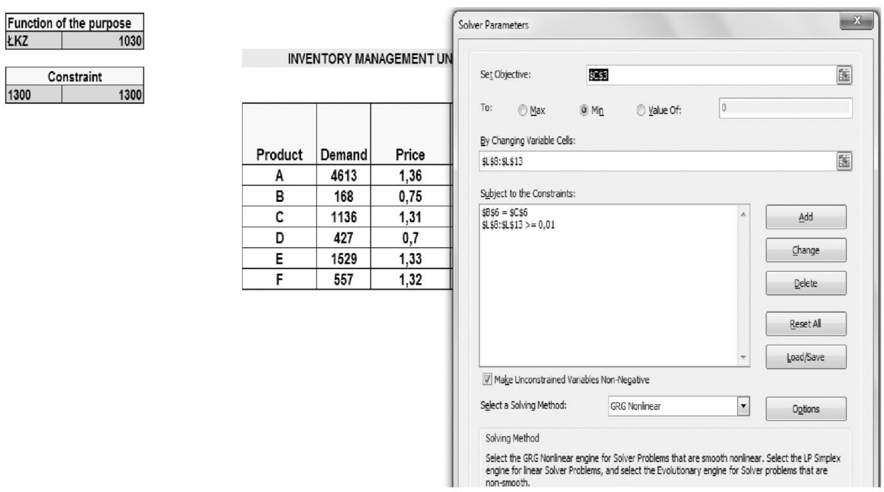
Table 2
Optimal supply volumes with the number of purchases of individual assortments

INVENTORY MANAGEMENT UNDER THE LIMITED CAPITAL CONDITIONS – CASE ANALYSIS

Product	Demand	Price	Value of demand	Number of deliveries	Delivery volume	Ur	cpo	Average value of inventory
A	4613	1,36	6273,68	7	659	0,1	50	448
B	168	0,75	126	1	168			44
C	1136	1,31	1488,16	3	379			241
D	427	0,7	298,9	1	427			112
E	1529	1,33	2033,57	4	382			252
F	557	1,32	735,24	2	279			204
							Total	1300,0

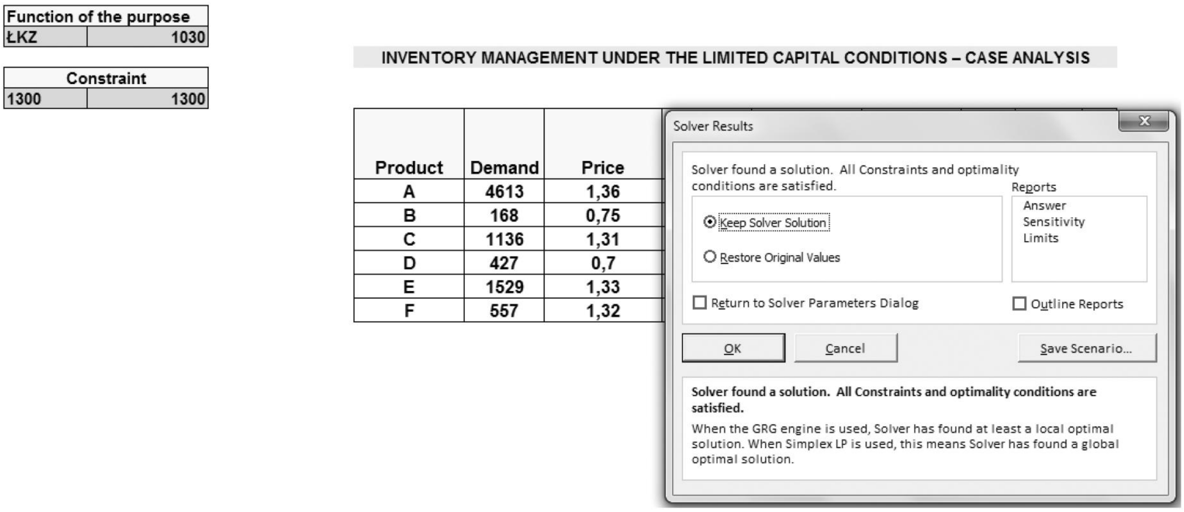
Source: Own elaboration.

Figure 1
Dialog application tasks nonlinear optimization Solver



Source: Own elaboration.

Figure 2
The Solver application results



Source: Own elaboration.

mization addresses the decision variables and constraints, these are reduction of capital (6) and volumes of supplies $Q_i \geq 1$.

Pressing the „Solve" button in the dialog box solves the task showing the optimal size of the order and the objective function, which determines the minimum cost inventory at a specific restriction. The message appearing in the „Solver Results" informs about the correctness of the selection of input parameters and the optimal solution task (fig. 2).

Conclusions

As a result of the six-step analysis and verification based on non-linear programming applications using the SOLVER application we obtained the optimum volume of deliveries (tab. 2), the minimum total cost of inventories amounting to $1030 and the assumption constraints in the form of capital stock of the average value of $1300. The reader can check that solving the problem on the basis of Economic Order Quantity-EOQ (EOQ is a characteristic size of the order, at which the total cost of ordering and maintaining inventories are minimal. The metod of EOQ is associated with the model R.H. Wilson $EOQ = \sqrt{\frac{2 \cdot P \cdot cpo}{cmi}}$) of the average value of inventory delivery will be approx. $3500. Thus, the saving of approx. $2200 may be directed to other more profitable targets and compensate for slightly higher total cost of inventory. Spreadsheet application Excel and Solver allow solving complex optimization tasks in the processes of economic organization. Due to the ease of use, availability, and costs it can be used to support small and medium-sized enterprises.

Literature

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