Tradeoff between Equity and Efficiency in Revenue Sharing Contracts

Bogumił Kamiński,* Maciej Łatek†

Abstract

We investigate the problem of setting revenue sharing rules in a team production environment with a principal and two agents. We assume that the project output is binary and that the principal can observe the level of agents' actual effort, but does not know the production function. Identifying conditions that ensure the efficiency of the revenue sharing rule, we show that the rule of equal percentage markups can lead to inflation of project costs. This result provides an explanation for project cost overruns other than untruthful cost reporting.

Keywords: Moral Hazard, Team Production, Cost Inflation, Project Management

JEL Classification: C72, D86, L24, M52.

---

*Warsaw School of Economics, e-mail: bkamins@sgh.waw.pl
†George Mason University, e-mail: mlatek@gmu.edu

1 Introduction

While revenue sharing in team production has been extensively studied since Holmstrom's seminal work (1982), setting revenue sharing rules for *project team* production has received less research attention. Project teams differ from other types of production teams in one key respect: they either succeed and "create a unique product, service or result" (PMBOK 2004) or fail and yield nothing. The *binary* production function of project teams is in stark contrast with one in Holmstrom (1982) that is strictly increasing and concave in team members' efforts. Whereas Holmstrom's assumptions can explain effort undersupply by production team members, empirical evidence suggests that project teams are often plagued with cost overruns (Flyvbjerg, Holm, Buhl 2002; Yeo 2002). This phenomenon is explained by overstatement of actual effort by team members (Gensemer and Kanagaretnam 2004) or inefficiency of project execution.

In this paper we show how setting revenue sharing rules in projects influences the efficiency of project execution. We argue that inappropriate incentive schemes motivate project team members to not only overstate but actually exert higher effort than the minimal required for the project to succeed. Such a behavior is called squandering by Courtney and Marschak (2006) who analyze it in a non-project team production setting.

Our framework consists of one principal and two team members as agents. This setting replicates a business scenario where a two-division firm is engaged in a project with the firm's CEO acting as the principal and the two divisions — the agents. The principal is interested in maximizing total profits and sets a rule for sharing the project revenue between the agents. Each agent wants to maximize its profit and decides how much effort to contribute to the project. This scenario implies that the principal sets a revenue sharing rule before any project is formulated. Central and a priori setting of revenue sharing rules enjoys a twofold rationale. First, setting a fixed sharing rule eliminates the need for inter-team negotiation every time a new project is undertaken, reducing contractual costs. Second, it ascertains procedural justice (Robbins 1998) in the organization; therefore, ensuring the long term stability of the firm. Additionally, we measure effort as project expenses of each division and assume that the firm can compel both its divisions to report project costs truthfully in order to focus on the oversupply of effort rather than overstatement of effort by project team members.

The project succeeds if both agents provide some predetermined minimal effort that is possibly different for each agent. This assumption is valid in environments with strong complementarity in agents' skills where project objectives can be achieved with certainty. For example, a firm needs two types of complementary expertise to implement a new IT system: business skills to analyze processes, provide end-user training and manage the organizational change, and technical skills to implement the IT solution and guarantee its functionality, scalability and security; see Soejarto, Goldman, Adams (2007). These two types of complementary expertise are usually

supplied by different project team members. The second condition — that project objectives are certainly achievable — is plausible under most circumstances (Yeo 2002). However, the probability of the successful execution of a project also depends on the industry and the task at hand. For instance, the risk of project failure is high in cutting-edge research in the pharmaceutical industry. In such a setting a stochastic framework with binary production functions is more appropriate (Kobayashi 2008; Williams and Radner 1988).

We assume that the minimal effort thresholds are known to the agents but not to the principal who can only observe the agents' efforts and the final project output. The information asymmetry inherent in our approach offers a new perspective for analyzing production functions, compared with the standard contract theory in which the principal knows the production function and is able to observe the project result, but is unable to verify the agents' efforts (Salanie 2005). A situation when only agents know the minimal effort thresholds is encountered when (1) the principal (for example, a CFO or CMO managing an IT project or an investor overseeing a construction project) cannot assess the thresholds from past experiences and (2) the principal cannot estimate threshold efforts from the project value, because the project revenue is driven by business benefit, not by production cost. This happens when a project is assigned to a team without a tender and in public sector projects when the level of funding is decided by an election or when the project budget is assembled by obtaining financing from many independent sources; see Flyvbjerg, Holm, Buhl (2002).

We consider only budget-balancing revenue sharing rules that guarantee that the agents' revenue shares add up to 1. This assumption translates into the rule that all project revenues must be allocated to lines of business. The principal wishes to choose such a budget-balancing sharing rules that discourage agents from inflating project costs, further called *efficient rules*. However, he also wishes to ensure equity in dividing project revenues; see Ray (2007), Gensemer and Kanagaretnam (2004), Fehr and Schmidt (1999), Fehr, Klein, Schmidt (2005). The most prevalent rule for sharing revenues among project teams that produce joint output is called *stand-alone revenue-allocation* according to which project revenues should be allocated to teams in proportion to each team's incurred cost; see Horngren, Datar, Foster (2005). Supported by the equity theory (Robbins 1998), stand-alone revenue-allocation leads to the equality of percentage markups among agents or *relative equity*. Equity can also be shared in equal absolute markups or *absolute equity* (Gensemer and Kanagaretnam 2004).

The paper is organized as follows. In Section 2 we introduce a formal model of revenue sharing in project teams. In Section 3 we identify the conditions for the efficiency of revenue sharing rules, analyze the properties of relative and absolute equity sharing rules and prove that it is impossible to simultaneously achieve relative equity and efficiency. Section 4 relaxes some of the model assumptions and analyzes two alternatives. First, we search for an efficient sharing rule that best approximates relative equity. Second, we allow agents to do pre-project signaling. In Section 5 we

summarize the results and provide practical recommendations for project managers. Appendix A contains all proofs.

2   The model of project team production

Consider agents 1 and 2 who maximize their respective profits Z_1 and Z_2 and a principal who maximizes total profit $Z_1 + Z_2$. This setting reflects the fact that agents belong to the organization managed by the principal. Therefore in this case principal maximizes his revenue net of costs generated by agents when sum of profits yielded by agents is maximized.

Assume that both the principal and the agents have profit reservation levels of zero. The agents are engaged in a project that will either succeed and generate positive revenue, normalized to 1, or fail and produce no revenue. In order for the project to succeed, each agent's *actual* effort n_i, $i = 1, 2$, must be greater than the *minimal* effort $p_i > 0$. Project revenue is $[n_1 \geq p_1][n_2 \geq p_2]$, where $[A] = 1$ if A is true and 0 if it is false.

The game is played in two stages. First the principal sets a revenue sharing function $s_i(n_1, n_2)$ for each agent i without being able to observe any of the agent's minimal costs p_i. Next i-th agent learns both minimal effort thresholds p_1 and p_2 and chooses its effort level n_i. In the second step both agents play simultaneously.

The value of the function $s_i(n_1, n_2)$ indicates the share of the project revenue allocated to i-th agent, whose profit can be calculated as:

$$Z_i = [n_1 \geq p_1][n_2 \geq p_2]s_i(n_1, n_2) - n_i \tag{1}$$

and the principal's profit is

$$Z_1 + Z_2 = [n_1 \geq p_1][n_2 \geq p_2] - n_1 - n_2. \tag{2}$$

We assume that any revenue sharing functions $s_1(\cdot)$ and $s_2(\cdot)$ meet three conditions:

A1) $s_i(\cdot)$ is invariant to permutation of players: $s_1(a, b) = s_2(b, a)$;

A2) all project revenue is shared: $s_1(a, b) + s_2(a, b) = 1$;

A3) $s_i(\cdot)$ guarantees profitability: $s_1(a, b) \geq a \wedge s_2(a, b) \geq b$.

We are interested in functions $s_i(\cdot)$ that guarantee that the principal's profit $Z_1 + Z_2$ is maximized and we will call such functions *efficient*.

Corollary 1. *A revenue sharing rule is efficient iff in equilibrium:*

$$(n_1, n_2) = \begin{cases} (p_1, p_2) & for \quad p_1 + p_2 \leq 1 \\ (0, 0) & for \quad p_1 + p_2 > 1 \end{cases}. \tag{3}$$

First notice that $(n_1, n_2) = (0,0)$ is always an equilibrium in the game. If one of the players decides not to put any effort then the other player's best response is also to set his effort level to 0. Therefore subsequently we will analyze only equilibria that are different from $(0,0)$.

Notice however, that under any budget-balancing sharing rule, i.e. meeting condition A2, if $p_1 + p_2 > 1$ then $(0,0)$ is the only equilibrium. To see this assume that $(n_1, n_2) \neq (0,0)$ is an equilibrium of the game. If $n_1 < p_1$ or $n_2 < p_2$ then project would fail and at least one player would incur a loss, thus he would prefer to switch to no-effort option. However if we assume that $n_1 \geq p_1$ and $n_2 \geq p_2$ then $n_1 + n_2 > 1$. But this means that, due to budget balance restriction at least one Z_i would be negative, so again this player would switch to no effort.

Following Corollary 1 we will call a revenue sharing rule *inefficient* if it is not efficient. Additionally when only one agent chooses minimal cost in the equilibrium (i.e. if for some i we have $n_i = p_i$ and $n_{-i} \neq p_{-i}$) we will call such a rule *semi-efficient*.

The notion of sharing rule efficiency leads us to the rationale behind assumption A3. It implies that the point $(n_1, n_2) = (p_1, p_2)$ *can be* an equilibrium in the game (later, we show that it does *not guarantee* that it is an equilibrium). Taking into account the efficiency condition stated above, condition A3 must be met only for $a + b \leq 1$. However, as we have shown if $a + b > 1$ then the only equilibrium in the game is $(0,0)$, so without loss of generality the condition $a + b \leq 1$ is omitted in A3.

Under A1-A3, we show that in fact the principal chooses a single revenue sharing function $s(n_i, n_{-i})$ for both agents. The value of $s(n_i, n_{-i})$ indicates the share of the project revenue allocated to i-th agent if his actual cost was n_i and the other agent's actual cost was n_{-i}.

Theorem 1. *Under the assumptions A1-A3 there exists a function $s : [0;1]^2 \to [0;1]$ such that $s_i(n_1, n_2) = s(n_i, n_{-i})$. Function $s(\cdot)$ has the following properties:*

C1) $s(a,b) + s(b,a) = 1$;

C2) $s(a,b) \geq a$;

C3) $s(a,a) = \frac{1}{2}$;

C4) $s(a, 1-a) = a$;

C5) *Its domain is the triangle* $\{(a,b) : a, b > 0 \land a + b \leq 1\}$.

Under assumption of $s(\cdot)$ efficiency we consider two equity criteria:

° Absolute equity of profits Z_i;

° Relative equity of percentage markups Z_i/n_i.

Equity is the principal's secondary objective; he strives to achieve it only after he has reached efficiency. Hence, in equilibrium $Z_1 + Z_2$ remains fixed. Without loss

of generality, absolute and relative equity objectives can be defined as maximizing $\min\{Z_1, Z_2\}$ and $\min\{Z_1/n_1, Z_2/n_2\}$, respectively.

We analyze two additional properties of revenue sharing functions. A revenue sharing function is *sequence-proof* if it yields the same equilibrium even when one agent moves first. For example, Strausz (1996) shows that Holmstrom's (1982) results directly depend on the assumption that agents act simultaneously. Similarly, a revenue sharing function is *knowledge-proof* if an agent's optimal strategy does not depend on knowing either the minimal cost or the effort level of the other agent. Knowledge-proof revenue sharing functions are sequence-proof; the opposite is not true.

3   Efficiency of equity-assuring sharing rules

Let us define the conditions for $s(\cdot)$ to guarantee efficiency.

Theorem 2. *A sharing function $s(\cdot)$ guarantees efficiency for all admissible values of p_1, p_2 iff*

$$\forall n_1 \in\,]p_1; 1 - p_2] : \frac{s(n_1, p_2) - s(p_1, p_2)}{n_1 - p_1} \leq 1 \tag{4}$$

$$\forall n_2 \in\,]p_2; 1 - p_1] : \frac{s(p_1, n_2) - s(p_1, p_2)}{n_2 - p_2} \geq -1 \tag{5}$$

where one condition implies the other. Such a revenue sharing function is knowledge-proof.

By Theorem 2, if a revenue sharing function is efficient then the assumption about symmetric information between agents is unnecessary. Therefore, an agent does not need to know the other agent's minimal costs and we do not need to assume simultaneous acting for the subsequent results about efficient revenue sharing rules to hold. However, as shown later in Theorem 5, an agent's knowledge of the other agent's p_{-i} influences his strategy under inefficient sharing functions.

Theorem 2 directly leads to the following conclusion:

Corollary 2. *Differentiable sharing functions are efficient iff*

$$\frac{\partial s}{\partial n_i} \leq 1 \ and \ \frac{\partial s}{\partial n_{-i}} \geq -1.$$

Additionally:

Theorem 3. *If a sharing function $s(\cdot)$ guarantees efficiency for all admissible values of p_1, p_2 then:*

$$n_1 \geq n_2 \Rightarrow s(n_1, n_2) \leq \frac{1}{2} + n_1 - n_2. \tag{6}$$

Applying the symmetry of sharing functions to Theorem 3 yields that if $n_1 \leq n_2$ then $s(n_1, n_2) \geq \frac{1}{2} + n_1 - n_2$. This result forms the necessary condition for testing the efficiency of revenue sharing function.

The properties of revenue sharing rules that guarantee either absolute or relative equity are summarized in the following theorems:

Theorem 4. *Only $s(n_i, n_{-i}) = \frac{1}{2} - (n_{-i} - n_i)/2$ guarantees absolute equity. It is an efficient revenue sharing function, is knowledge-proof and is the only sharing function linear with respect to its arguments.*

Theorem 5. *Only $s(n_i, n_{-i}) = n_i/(n_i + n_{-i})$ guarantees relative equity. It leads to the following equilibria in regions E, S_1, S_2 and I on Figure 1:*

Efficient (E): $\forall i : n_i = p_i$ when $\forall i : p_i \geq \sqrt{p_{-i}} - p_{-i}$;
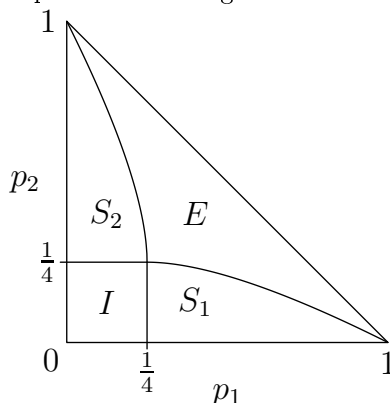
Semi-efficient (S_i): $n_i = p_i \wedge n_{-i} = \sqrt{p_i} - p_i$ when $p_i > \frac{1}{4} \wedge p_{-i} < \sqrt{p_i} - p_i$;

Inefficient (I): $\forall i : n_i = \frac{1}{4}$ when $\forall i : p_i \leq \frac{1}{4}$.

It is sequence-proof, but not knowledge-proof.

We have shown that the absolute equity assumption guarantees efficient outcomes. Conversely, the relative equity sharing rule can lead to inefficiency when minimal cost of at least one agent is less than $\frac{1}{4}$ of the project revenue. In the next section, we will identify efficient sharing rules that minimize deviations from the relative equity requirement. We will also verify if changing the basic setup of the model can lead to efficiency of relative equity sharing rules.

Figure 1: Types of equilibria resulting from relative equity sharing rule

Note: p_1 and p_2 denote minimal efforts of agents, E denotes region of efficiency of the sharing rule, S_1 and S_2 denote regions of semi-efficiency and I denotes region of inefficiency.

4 Relative equity approximation

Using Theorem 5 we can identify a simple rule guaranteeing efficiency of relative equity sharing rule:

Corollary 3. *If $p_i \geq \frac{1}{4}$ then relative equity revenue sharing function is efficient and knowledge-proof.*

This finding has an important implication for cost allocation regimes in firms, especially for consulting firms that achieve relatively high markups in their projects. In order to avoid the inflation of efforts, the direct costs of the project should be augmented with all possible indirect project costs. For example, project teams should pay not only for wages of the project members, but also for the use of office space and general and administrative expenses. Such a policy increases individual p_i, reducing the risk of inefficiency.

Reversing the problem, an optimal efficient sharing rule that minimizes deviations from relative equity also exists:

Theorem 6. *An efficient sharing function that ensures relative equity on area E and maximizes relative equity outside of E has the following form:*

$$s(n_1, n_2) = \begin{cases} \frac{n_1}{n_1 + n_2} & \text{for} \quad (n_1, n_2) \in E \\ 2\sqrt{n_1} - n_1 - n_2 & \text{for} \quad (n_1, n_2) \in S_1 \\ 1 + n_1 - 2\sqrt{n_2} + n_2 & \text{for} \quad (n_1, n_2) \in S_2 \\ \frac{1}{2} + n_1 - n_2 & \text{for} \quad (n_1, n_2) \in I \end{cases} \tag{7}$$

This function is knowledge-proof.

We have proven that within the basic setup for relative equity no efficient sharing function exists. Let's abandon the basic setup and verify if changing assumptions on players' information sets and game dynamics can guarantee efficiency of a relative equity sharing rule. If the principal knows p_i and abandons assumption A1, the problem of relative equity can be readily solved.

Theorem 7. *A revenue sharing function $s_i(n_1, n_2)$ that guarantees relative equity and efficiency for all admissible values of p_1, p_2 assuming that the principal knows p_i has the form:*

$$s_i(n_1, n_2) = \frac{p_i}{p_1 + p_2}(1 - n_1 - n_2) + n_i. \tag{8}$$

It does not meet assumption A1.

Notice that the above sharing function preserves budget balance even if agents would not choose equilibrium actions.

A natural question arises if the principal can force agents to reveal their p_i. Assume that the agents play the game in two stages. In the first stage they are allowed to

send a signal chosen from their communication set. For example they could inform the principal about their expected costs. In this case their signal is a real number and communication set is a set of nonnegative numbers. In the second stage they incur the cost. Further we will denote the communication set of agent i is A_i (the set is not restricted to any predefined type of signals).

The following impossibility theorem holds for a *generalized revenue sharing function* $g(a_i, a_{-i}, n_i, n_{-i})$, where $a_i \in A_i$, that fulfills the budget balancing condition:

Theorem 8. *No efficient, budget-balancing generalized revenue sharing function $g(\cdot)$ exists that guarantees relative equity in equilibrium for all admissible values of p_1, p_2.*

This implies that the principal cannot extract any information about p_i from agents if the budget balancing condition holds. In other words signals a_i have no credibility. Nevertheless, note that the required revenue sharing functions exist when budget breaking is allowed:

Theorem 9. *The generalized revenue sharing function $g(a_i, a_{-i}, n_i, n_{-i}) = \dfrac{n_i}{n_1 + n_2}[a_i = n_{-i}][a_{-i} = n_i], a_i \in [0; 1]$ guarantees relative equity and efficiency.*

The result from Theorem 9 implies that agents audit each other. Unfortunately, this sharing function is very sensitive to audit errors. Even a minute misestimation of the other agent's p_{-i} will directly lead to zero revenues for both agents.

5 Concluding remarks

We have investigated the problem of revenue sharing in project teams in a principal-multiagent setting where the principal can observe the agents' effort, but does not know minimal costs that agents have to incur in order for a project to succeed. This is an alternative approach to the standard contract theory where production function is assumed known, but agents' efforts are unobservable. Theorem 2 provides the necessary and sufficient conditions for a revenue sharing rule to motivate agents to provide socially optimal efforts. We found that under efficient revenue sharing functions, it is irrelevant whether agents act sequentially or whether they know each other's minimal costs. Theorem 4 characterizes unique sharing function that guarantees absolute equity. It is efficient and knowledge-proof. According to Theorem 5 there also exists unique sharing function that guarantees relative equity. It is sequence-proof but it is not efficient.

Inefficient revenue sharing rules can lead to inflation of project costs. Such inefficiencies in project execution complement untruthful cost reporting as explanations for project costs overruns. We have shown that absolute equity guarantees efficiency. On the other hand, relative equity can lead to cost overruns if at least one of the agents has a low effort threshold. Theorem 6 identifies an efficient revenue sharing function that best approximates the relative equity rule. If the principal knows the production

function, he can guarantee both relative equity and efficiency. Nevertheless, no pre-project signaling scheme enables the principal to learn the production function from agents so long as budget balancing and relative equity are assumed. If the budget balancing requirement is removed, such a solution exists and can yield relative equity and efficiency at the same time.

Our research has following managerial implications. First, it shows how inadequate motivation schemes can contribute to cost overruns in projects. Second, for managers who desire relative equity, high-margin projects all indirect costs should be added to the direct cost of the project. This increases minimal effort thresholds and reduces the incentive to squander. Last, as we show that contractual only arrangements are in general insufficient, managers should try to approximate the project production function by gaining expertise in the project field, by using independent auditors or by running tenders.

Appendix A

Proof of Theorem 1. By definition, $s(a,b) = s_1(a,b)$ and $s(a,b) = s_2(b,a)$. Using property A1, we get $s_1(a,b) = s_2(b,a)$, so $s(\cdot)$ is properly defined. Let us now verify the properties C1-C5:

C1) $s(a,b) + s(b,a) = s_1(a,b) + s_2(a,b) = 1$;

C2) $s(a,b) = s_1(a,b) \geq a$;

C3) $s(a,a) = \frac{s(a,a)+s(a,a)}{2} = \frac{1}{2}$;

C4) Combining $s(a,1-a) = 1 - s(1-a,a) \leq a$ with the condition $s(a,1-a) \geq a$, we get $s(a,1-a) = a$;

C5) Combining $1 = s(a,b) + s(b,a) \geq a + b$ with the assumption that $a,b > 0$, we get the required domain;

$\square$

Proof of Theorem 2. First, we will show that it is a necessary condition. Assume that for some p_1, n_1, p_2 meeting the criteria from the theorem, the inequality does not hold. Assume that the first player has minimal cost p_1 and second player cost p_2. Consider that first player compares option to either show cost p_1 or some higher cost n_1. Because $n_1 > p_1$ and by assumption $s(\cdot)$ is efficient, we must have $s(p_1,p_2) - p_1 \geq s(n_1,p_2) - n_1$. By rearranging terms we get $\frac{s(n_1,p_2)-s(p_1,p_2)}{n_1-p_1} \leq 1$. A contradiction to the assumption. So the condition from the theorem is necessary.

Similar argument works for showing the sufficiency. Assume that the first player has minimal cost p_1 and second player cost p_2. Consider any $n_1 > p_1$. By the assumption $\frac{s(n_1,p_2)-s(p_1,p_2)}{n_1-p_1} \leq 1$, hence $s(p_1,p_2) - p_1 \geq s(n_1,p_2) - n_1$ and the first player shall

choose p_1 in this game. The condition is sufficient.

By substituting $s(a, b) = 1 - s(b, a)$ we directly get the second equation.

Notice that in the derivation of the equilibrium, no assumption on the values of p_2 or n_2 was made. Therefore, such a revenue sharing function is knowledge-proof. $\square$

Proof of Theorem 3. We have $s(n_2, n_2) = \frac{1}{2}$. Substitute this into the inequality in the Theorem 2, getting $\frac{s(n_1, n_2) - \frac{1}{2}}{n_1 - n_2} \leq 1$. After rearrangement, this yields the required condition. $\square$

Proof of Theorem 4. Let us check if it is a proper revenue sharing function:

C1) $s(n_1, n_2) + s(n_2, n_1) = \frac{1}{2} - \frac{n_2 - n_1}{2} + \frac{1}{2} - \frac{n_1 - n_2}{2} = 1$;

C2) $1 \geq n_1 + n_2 \Leftrightarrow \frac{1}{2} - \frac{n_2 - n_1}{2} \geq n_1 \Leftrightarrow s(n_1, n_2) \geq n_1$;

C3) $s(n_1, n_1) = \frac{1}{2} - \frac{n_1 - n_1}{2} = \frac{1}{2}$;

C4) $s(n_1, 1 - n_1) = \frac{1}{2} - \frac{1 - n_1 - n_1}{2} = n_1$.

The total profit from the project is $1 - (n_1 + n_2)$ and the profit of i-th player is:

$$s(n_i, n_{-i}) - n_i = \left(\frac{1}{2} - \frac{n_{-i} - n_i}{2} \right) - n_i = \frac{1}{2} - \frac{n_{-i} + n_i}{2} \tag{9}$$

it is exactly a half of total profit. Observe that:

$$\frac{s(n_1, p_2) - s(p_1, p_2)}{n_1 - p_1} = \frac{\frac{1}{2} - \frac{p_2 - n_1}{2} - \frac{1}{2} + \frac{p_2 - p_1}{2}}{n_1 - p_1} = \frac{1}{2} \leq 1 \tag{10}$$

thus the revenue sharing function is efficient and knowledge-proof, by the Theorem 2. Lastly, we demonstrate that this is the only revenue sharing function linear with respect to its arguments.

Consider a revenue sharing function $s(a, b) = \alpha + \beta_1 a + \beta_2 b$. Because $s(a, a) = \frac{1}{2}$ we get $\alpha + (\beta_1 + \beta_2)a = \frac{1}{2}$. Rearranging, we get $\beta_2 = -\beta_1$ and $\alpha = \frac{1}{2}$. This yields $s(a, b) = \frac{1}{2} + \beta_1(a - b)$ and $s(a, 1 - a) = \frac{1}{2} + \beta_1(a - (1 - a)) = a$. Hence, $\frac{1}{2} - \beta_1 = a(1 - 2\beta_1)$. Which yields desired result $\beta_1 = \frac{1}{2}$. $\square$

Proof of Theorem 5. Let us check that it is a proper revenue sharing function:

C1) $s(n_1, n_2) + s(n_2, n_1) = \frac{n_1}{n_1 + n_2} + \frac{n_2}{n_1 + n_2} = 1$;

C2) $n_1 + n_2 \leq 1 \Rightarrow s(n_i, n_{-i}) = \frac{n_i}{n_1 + n_2} \geq n_i$;

C3) $s(n_i, n_i) = \frac{n_i}{n_i + n_i} = \frac{1}{2}$;

C4) $s(n_i, 1 - n_i) = \frac{n_i}{n_i + 1 - n_i} = n_i$.

The margin Z_i/n_i is equal for each player:

$$\frac{Z_i}{n_i} = \frac{s(n_i, n_{-i}) - n_i}{n_i} = \frac{\frac{n_i}{n_i + n_{-i}} - n_i}{n_i} = \frac{1}{n_i + n_{-i}} - 1. \tag{11}$$

Consider the optimization problem of the i-th player:

$$\begin{aligned} Z_i = \frac{n_i}{n_i + n_{-i}} - n_i &\to \max \\ \text{subject to} & \\ n_i \geq p_i & \end{aligned} \tag{12}$$

The FOC in this problem is $n_{-i} = (n_i + n_{-i})^2$. Solving this, we get that $n_i = \sqrt{n_{-i}} - n_{-i}$ (notice that it is a maximum). Taking into account the budget balancing constraint, we get $n_i = \max\{p_i; \sqrt{n_{-i}} - n_{-i}\}$. Notice that $\sqrt{n_{-i}} - n_{-i} \leq \frac{1}{4}$.
In the equilibrium we get:

$$\begin{cases} n_1 = \max\{p_1; \sqrt{n_2} - n_2\} \\ n_2 = \max\{p_2; \sqrt{n_1} - n_1\} \end{cases} \tag{13}$$

If $p_i \geq \frac{1}{4}$ then:

$$\begin{cases} n_i = p_i \\ n_{-i} = \max\{p_{-i}; \sqrt{p_i} - p_i\} \end{cases} \tag{14}$$

Thus the solution is unique. It is efficient if and only if $p_{-i} \geq \sqrt{p_i} - p_i$.
Now assume that $\max\{p_1, p_2\} < \frac{1}{4}$. Under this assumption $\sqrt{p_i} - p_i > p_i$. First we will show that $n_i > p_i$. Assume the converse, ie. $n_i = p_i$. Then we get that $p_i \geq \sqrt{n_{-i}} - n_{-i}$. But $n_{-i} = \max\{p_{-i}, \sqrt{p_i} - p_i\} \geq \max\{p_{-i}, p_i\} \geq p_i$, so $p_i \geq \sqrt{p_i} - p_i$. A contradiction. Thus $n_i > p_i$. This leaves us with the equations:

$$\begin{cases} n_1 = \sqrt{n_2} - n_2 \\ n_2 = \sqrt{n_1} - n_1 \end{cases} \tag{15}$$

By adding them we get $n_1 = n_2$, so $n_i = \frac{1}{4}$.
The strategy of the i-th player depends on the knowledge of the minimum cost of player $-i$. Therefore, this revenue sharing function is not knowledge proof. It is sequence proof, though.
Without loss of generality, we can assume that the agent 1 moves first. From the earlier analysis, we see that the agent 2's response will be $n_2 = \max\{p_2, \sqrt{n_1} - n_1\}$. Therefore, agent 1 will face following optimization problem:

$$\begin{aligned} \frac{n_1}{n_1 + \max\{p_2, \sqrt{n_1} - n_1\}} - n_1 &\to \max \\ \text{subject to} & \\ n_1 \geq p_1 & \end{aligned} \tag{16}$$

If $p_2 \geq \frac{1}{4}$ we get $n_2 = p_2$. In this case, we get exactly the same result as in the simultaneous game. If $p_2 < \frac{1}{4}$ and $p_1 \geq \frac{1}{4}$ we get $n_1 = p_1$. This leads to the same equilibrium. Lastly, assume that $p_i < \frac{1}{4}$:

$$\frac{n_1}{n_1 + \max\{p_2, \sqrt{n_1} - n_1\}} - n_1 \leq \frac{n_1}{n_1 + \sqrt{n_1} - n_1} - n_1 = \sqrt{n_1} - n_1 \leq \frac{1}{4}.$$

The equality above holds only for $n_1 = \frac{1}{4}$. Hence, in equilibrium $n_1 = n_2 = \frac{1}{4}$, which is the same solution as in simultaneous game. $\qquad\square$

Proof of Theorem 6. In the region E relative equity must be ensured, so $\forall (n_1, n_2) \in E : s(n_1, n_2) = \frac{n_1}{n_1 + n_2}$.

The proof will be done in two steps. Firstly, we will separately find revenue sharing rules that maximize relative equity and meet necessary conditions for efficiency in regions I and S_i. Secondly, we will show that taken together, they form a revenue sharing rule that guarantees efficiency.

Let us analyze possible values revenue sharing function in I with assumption $n_1 \geq n_2$. By Theorem 3, we get the that necessary condition for efficiency, $s(n_1, n_2) \leq \frac{1}{2} + n_1 - n_2$, must be met. Take $\alpha \geq 0$:

$$\min\left\{\frac{Z_1}{n_1}, \frac{Z_2}{n_2}\right\} = \min\left\{\frac{\frac{1}{2} - n_2 - \alpha}{n_1}, \frac{\frac{1}{2} - n_1 + \alpha}{n_2}\right\} =$$

$$= \frac{\frac{1}{2} - n_2 - \alpha}{n_1} + \min\left\{0, \frac{(n_1 - n_2)(\frac{1}{2} - n_1 - n_2) + \alpha(n_1 + n_2)}{n_1 n_2}\right\}.$$

In region I, we have $n_1 + n_2 \leq \frac{1}{2}$ and the relative equality is maximized when $\alpha = 0$. Therefore, in this area $s(n_1, n_2) = \frac{1}{2} + n_1 - n_2$ minimizes inequality. For area I, with assumption $n_1 \leq n_2$, we have $s(n_1, n_2) = 1 - s(n_2, n_1)$, such that $s(n_1, n_2) = \frac{1}{2} + n_1 - n_2$.

Now take an arbitrary point $(n_1, n_2) \in S_1$. It must meet conditions $n_1 > \frac{1}{4}$ and $n_2 < \frac{1}{4}$. The point $(n_1, \hat{n}_2) = (n_1, \sqrt{n_1} - n_1) \in E$ lies on the boundary of region S_1. By Theorem 2, we get:

$$\frac{s(n_1, \hat{n}_2) - s(n_1, n_2)}{\hat{n}_2 - n_2} \geq -1 \Rightarrow s(n_1, n_2) \leq 2\sqrt{n_1} - n_1 - n_2.$$

For $\alpha \geq 0$:

$$\min\left\{\frac{Z_1}{n_1}, \frac{Z_2}{n_2}\right\} = \min\left\{\frac{2\sqrt{n_1} - 2n_1 - n_2 - \alpha}{n_1}, \frac{1 - 2\sqrt{n_1} + n_1 + \alpha}{n_2}\right\} =$$

$$= \frac{2\sqrt{n_1} - 2n_1 - n_2 - \alpha}{n_1} + \min\left\{0, \frac{(n_1 + n_2 - \sqrt{n_1})^2 + \alpha(n_1 + n_2)}{n_1 n_2}\right\}.$$

The relative equity is maximized when $\alpha = 0$. Thus in region S_1, the relative equity maximizing revenue sharing function has the form $s(n_1, n_2) = 2\sqrt{n_1} - n_1 - n_2$.

Next, notice that if $(n_1, n_2) \in S_2$ then $(n_2, n_1) \in S_1$ and $s(n_1, n_2) = 1 - s(n_2, n_1) = 1 + n_1 - 2\sqrt{n_2} + n_2$.

We have identified the formulas for the revenue sharing function in regions E, I, S_1 and S_2. Let us show, that the resulting function is an efficient revenue sharing function.

The verification of conditions C1, C2 for regions E and I, C3, C4 and C5 is immediate. In order to finish the proving, that the proposed function is a proper revenue sharing function, let us notice that condition C2 is met for region S_1:

$$2\sqrt{n_1} - n_1 - n_2 \geq 2\sqrt{n_1} - n_1 - (\sqrt{n_1} - n_1) = \sqrt{n_1} \geq n_1$$

and S_2:

$$1 + n_1 - 2\sqrt{n_2} + n_2 \geq 1 - \sqrt{n_2} + n_1 \geq n_1.$$

By Corollary 2, in order to show that the function is efficient, it is enough to check two conditions: continuity and derivative with respect to n_1. Continuity of the function on junction between E and S_i regions is guaranteed by the construction. Continuity on junction between regions I and S_1 is readily verified as both formulas yield $\frac{3}{4} - n_2$ there. Similarly, on I and S_2 junction we get $\frac{1}{4} + n_1$.

To finish the proof, we have to verify that the derivative over n_1 in each region is less or equal to 1. We get:

$$\frac{\partial s(n_1, n_2)}{\partial n_1} = \begin{cases} \frac{n_2}{(n_1 + n_2)^2} & \text{for} \quad (n_1, n_2) \in E \\ \frac{1}{\sqrt{n_1}} - 1 & \text{for} \quad (n_1, n_2) \in S_1 \\ 1 & \text{for} \quad (n_1, n_2) \in S_2 \\ 1 & \text{for} \quad (n_1, n_2) \in I \end{cases}.$$

In regions I and S_2 the condition is trivially met. In region S_1 the derivative is less than $\frac{1}{\sqrt{\frac{1}{4}}} - 1 = 1$. In region E we have $n_1 \geq \sqrt{n_2} - n_2$ thus:

$$\frac{n_2}{(n_1 + n_2)^2} \leq \frac{n_2}{(\sqrt{n_2} - n_2 + n_2)^2} = 1.$$

By Theorem 2, the function is knowledge-proof. $\square$

Proof of Theorem 7. Notice that $\frac{\partial s_i}{\partial n_i} = 1 - \frac{p_1}{p_1 + p_2} \leq 1$ and the function is efficient. In the equilibrium, we get $s_i(p_1, p_2) = \frac{p_1}{p_1 + p_2}(1 - (p_1 + p_2)) + p_1 = \frac{p_1}{p_1 + p_2}$ and the relative equity is assured.

However, in general $s_1(n_1, n_2) \neq s_2(n_2, n_1)$, so assumption A1 is not met. $\square$

Proof of Theorem 8. Assume that such a function exists. Consider two (p_1, p_2) parameter settings: $(0.1, 0.3)$ and $(0.2, 0.3)$.

Denote the equilibrium for the first pair as $(\vartheta_1, \vartheta_2, 0.1, 0.3)$ and for the second pair as $(\sigma_1, \sigma_2, 0.2, 0.3)$. Because of the relative equity assumption, we get: $Z_1(\vartheta_1, \vartheta_2, 0.1, 0.3) = 0.15$ and $Z_2(\sigma_1, \sigma_2, 0.2, 0.3) = 0.3$.

As $(\sigma_1, \sigma_2, 0.2, 0.3)$ is an equilibrium, we must have $\forall \gamma \in A_2 : Z_2(\sigma_1, \gamma, 0.2, 0.3) \leq 0.3$. By budget-balancing condition, we get $\forall \gamma \in A_2 : Z_1(\sigma_1, \gamma, 0.2, 0.3) \geq 0.2$. This implies that by choosing σ_1 player 1 guarantees himself $Z_1 \geq 0.2$ no matter what player 2 does and no matter what his p_1 is. However, then $(\vartheta_1, \vartheta_2, 0.1, 0.3)$ cannot be an equilibrium. A contradiction. $\qquad\square$

Proof of Theorem 9. First analyze a subgame for $a_1 \geq p_2$ and $a_2 \geq p_1$ fixed. We get that the optimal play is $n_i = a_{-i}$. Notice that $\frac{a_{-i}}{a_i + a_{-i}}$ is maximized when $a_i = p_{-i}$, so in equilibrium $n_i = a_{-i} = p_i$. $\qquad\square$

Acknowledgements

References

[1] Courtney D., T. Marschak (2006), Shirking and Squandering in Sharing Games, *Topics in Theoretical Economics*, Vol. 6 (1), pp. 1-33

[2] Gensemer S., K. Kanagaretnam (2004), Alliances and Cost Declaration, *Managerial and Decision Economics*, Vol. 25 (3), pp. 121-136

[3] Fehr E., K.M. Schmidt (1999), A Theory of Fairness, Competition, and Cooperation, *The Quarterly Journal of Economics*, Vol. 114 (3), pp. 817-868

[4] Fehr E., A. Klein, K.M. Schmidt (2005), Fairness and Contract Design, *GESY Discussion Paper* No. 67

[5] Flyvbjerg B., M.S. Holm, S. Buhl (2002), Underestimating Costs in Public Works Projects, Error or Lie, *Journal of the American Planning Association*, Vol. 68 (2), pp. 279-295

[6] Holmstrom B. (1982), Moral Hazard in Teams, *The Bell Journal of Economics*, Vol. 13 (2), pp. 324-340

[7] Horngren C.T., S.M. Datar, G. Foster (2005), *Cost Accounting: A Managerial Emphasis*, Pearson Prentice Hall

[8] Kobayashi H., K. Ohta, T. Sekiguchi (2008), Optimal Sharing Rules in Repeated Partnerships, *Discussion Paper* No. 650, Kyoto Institute of Economic Research

[9] PMBOK (2004), *A guide to the project management body of knowledge: PMBOK guide*, 3rd ed., Project Management Institute Inc.

[10] Ray D. (2007), Inequality and Inefficiency in Joint Projects, *The Economic Journal*, Vol. 117 (522), pp. 922-935

[11] Robbins S.P. (1998), *Organizational behavior: concepts, controversies, applications*, 8th ed., Prentice-Hall

[12] Soejarto A., M. Goldman, C. Adams (2007), Key Issues for Consulting and Solution Implementation Services, Gartner Inc., ID Number: G00147425

[13] Salanie B. (2005), *The Economics of Contracts*, The MIT Press

[14] Strausz R. (1996), Moral Hazard in Sequential Teams, *Discussion paper* No. 1996/27, FU-Berlin

[15] Williams S.R., R. Radner (1988), Efficiency in Partnership when the Joint Output is Uncertain, *AT&T Labs Discussion Paper* No. 760

[16] Yeo K.T. (2002), Critical failure factors in information system projects, *International Journal of Project Management*, Vol. 20 (3), pp. 241-246