Fig. 1 The pyramid of Neferirkare, view towards the south-west (photo Y. Kawae)

Abusir 3D survey 2015

Yukinori Kawae – Yoshihiro Yasumuro – Ichiroh Kanaya – Hiroshige Dan – Fumito Chiba

Current studies on Old Kingdom pyramids are being carried out from multiple perspectives. Various archaeological data that betters our understanding of these massive construction projects is now available from both textual information and excavations of adjacent settlements and tombs, as well as from investigations of interactive commerce with the Levant and Nubia. In comparison, however, data related to the pyramids themselves has rarely been updated since the time of the architectural investigations of Vito Maragioglio and Celeste Rinaldi in the 1960s to the 1970s (Maragioglio – Rinaldi 1963–1977).

Furthermore, conventional line drawings of plans and elevations/sections focus on main architectural features such as the dimensions of a pyramid's footprint, inner rooms, and corridors (such as Petrie 1883; Borchardt 1909; Maragioglio – Rinaldi 1963–1977; Dormion 2004). Such an approach places very little emphasis on the recording of the core masonry structure with its mortar and debris.

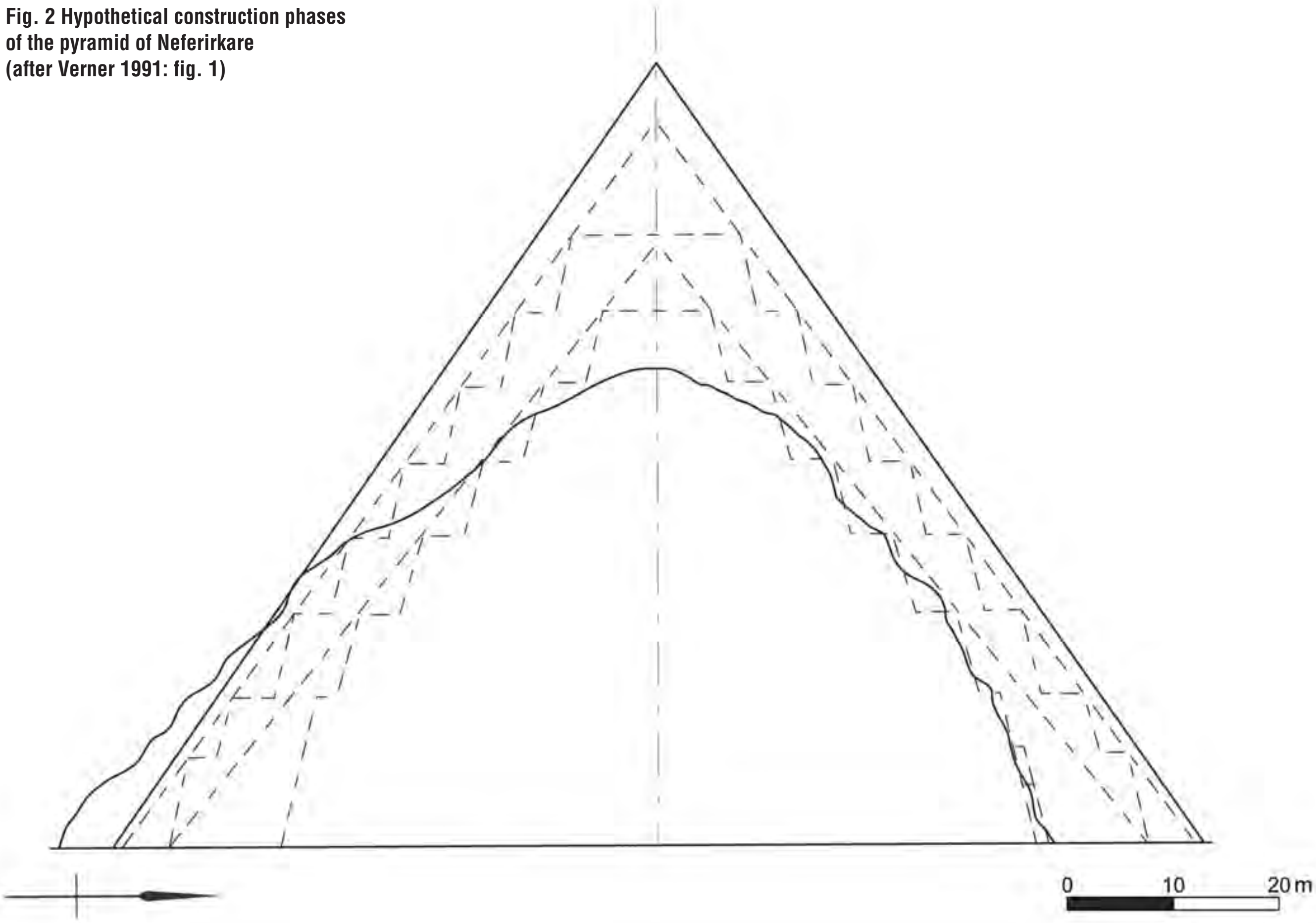
In 2015, as a sub-project of the Abusir archaeological explorations conducted by the Czech Institute of Egyptology, we, a Japanese consortium, iniciated the Abusir 3D Survey (A-3DS) for the 3D documentation of the site's pyramids.

Aims of the survey

Academic research using 3D digitizing technology has become popular in the field of Egyptian archaeology in the last ten years. Although this new technology allows for a more detailed documentation than conventional hand mapping, it has yet to be fully integrated into archaeological research.

There seem to be several reasons for this situation. Firstly, there still continues to be a commonly held yet misguided belief that a laser scanner will gather all shape information of a subject, with no gaps in the data. Secondly, in some cases, unfamiliarity with 3D scanning technology

Fig. 2 Hypothetical construction phases of the pyramid of Neferirkare (after Verner 1991: fig. 1)



can lead to ambiguous instructions for the survey team and wasted field time and costs, resulting in 3D surveyors having no other option than to scan a subject as thoroughly as possible in hopes of gathering whatever data might be considered relevant to a project's archeological goals – in essence, "over scanning" of a subject. This results in a large and unnecessary volume of 3D data that is not easily handled without expensive software and professional expertise. Furthermore, even though the original data contains 3D information, point cloud images – currently the standard way of disseminating 3D information – by their very nature can only represent 2D information on paper. For these reasons, 3D data is often still considered difficult for researchers to use.

In offering a solution to these problems, we promote an interactive, interdisciplinary research program conducted by archaeologists, engineers, computer scientists, as well as a software engineer and an applied mathematician, in order to properly document monuments. The aims of this project are to develop new uses for the archaeological interpretations of the royal constructions at Abusir by developing a strategic, mathematical 3D survey, planning appropriate step-by-step 3D documentation methods that suit archaeological needs, and producing a new display method for the resultant 3D data.

The first season of the project focused on the exterior of Neferirkare's pyramid, the largest pyramid at Abusir (fig. 1). The pyramid has a unique appearance and is thought to have originally been constructed using the step pyramid style. It was later altered to a true pyramid for unknown reasons, with a base length of 105 m rising to a height of 72 m (Krejčí 2010; fig. 2).

Goals:

- Re-measurement of the exterior of Neferirkare's pyramid using 3D data;
- Observation of the pyramid using 3D data to identify structurally and archaeologically important areas such as inner masonry structure and architecture phases;
- Assessment of the usefulness of our newly developed mathematical 3D survey plan.

Methodology

From our previous scanning experience (Kawae – Kamei *et al.* 2009; Kawae – Sato *et al.* 2009; Kawae *et al.* 2013 and *forthcoming*), we know that a combination of laser scanners and the use of both a structure from motion (SFM) technique and multi-view stereo (MVS) reconstruction algorithms as a photogrammetric technique can produce the most favorable results.

Terrestrial time-of-flight laser scanners are suitable for the main sections of a monument. The scanner uses infrared beams to gather the coordinates and elevations of points on a monument, collecting data at an exceedingly fast rate of 11,000 points per second (*e.g.* a Riegl LMS-Z420i). This produces high-density point clouds of the subject.

However, when we scan a pyramid from the ground using terrestrial time-of-flight laser scanners, the laser beams will not reach the top sides of the stones, and so each course is left partly in shadow. To avoid having these unscanned areas (or shadows), the usage of a digital camera/video camera and SFM and MVS techniques is appropriate. SFM is a process that estimates the three-dimensional structure of the geometrical relation between the subject and the camera's position by tracking "feature points" in a two-dimensional image (digital photo) sequence. Then, MVS algorithms are employed to reconstruct the 3D geometry of the subject. MVS takes the estimated camera positions and sparse point clouds created by SFM as input images, then breaks those input images into a set of image clusters of manageable size. The final product created using the SFM and MVS techniques is the point cloud of the subject, which can be integrated into point clouds of laser scanners.

In order to adopt these techniques in a strategic way based on a pre-designed method, we implemented a "mathematical optimization" system for 3D record planning. Unlike an experience-based approach, this method mathematically creates an effective recording plan.

3D data allows researchers to more intuitively understand structures. However, orthophotographic images produced from 3D data do not reveal the undulations in the structure – the ridges and valleys – important for an archaeological analysis. Furthermore, even though the original data contains 3D information, point cloud images can only represent two-dimensional information. To address these issues, our newly developed image-processing technology, called PEAKIT, creates a new way to display 3D data. PEAKIT represents 3D data by selectively overlaying multiple images: a "positive openness map", a "negative openness map", a "shaded relief map", and a "colored distance map". One of the main characteristics of PEAKIT is "openness", which is a concept formulated in topography, in which the degree of the prospect from the viewpoint within an arbitrary distance *L* is digitized according to the terrain line-of-sight principle. Openness is classified into "positive" and "negative" (Yokoyama 2002). A shaded relief map represents an image of the unevenness of the terrain by the contrast between light and shadow. A colored distance map is a digital image whose color expresses quantized distances. These are created by calculating Digital Elevation Models (Chiba *et al.*, *forthcoming*).

3D models and PEAKIT images allow researchers to focus on purely archaeological interpretations such as line drawings and observation recordings on site. In other words, these engineering techniques allow us to separate an archaeological analysis of the shape of the subjects from the geometric data, themselves. Eventually, archaeologists will be able to share the geometric data and openly study the pyramids.

The preliminary survey for a strategic planning of 3D recording

In this project, we attempted to apply a quantitatively modeled strategy (mathematical optimization) instead of relying exclusively on the experience of 3D surveyors. For this purpose, however, 3D information of a subject is needed beforehand. Thus, we were faced with the dilemma that a 3D model is required in order to appropriately plan a 3D scanning survey with mathematical programming. We chose an SFM technique to produce a rough 3D model of our study target. The effectiveness of this technique was already proven in a small area of Khufu's pyramid (Kawae *et al.* 2013 and *forthcoming*) and other archaeological sites such as Giza (http://www.aeraweb.org/news/the-pedestal-puzzle/; accessed August 8, 2016), Saqqara (Yasumuro 2015) and Wad Ben Naga (Gatzsche 2013), but this is the first time it has been applied to an entire pyramid.

It took only a few hours to obtain digital images and video footage of the pyramid using a Nikon COOLPIX L820

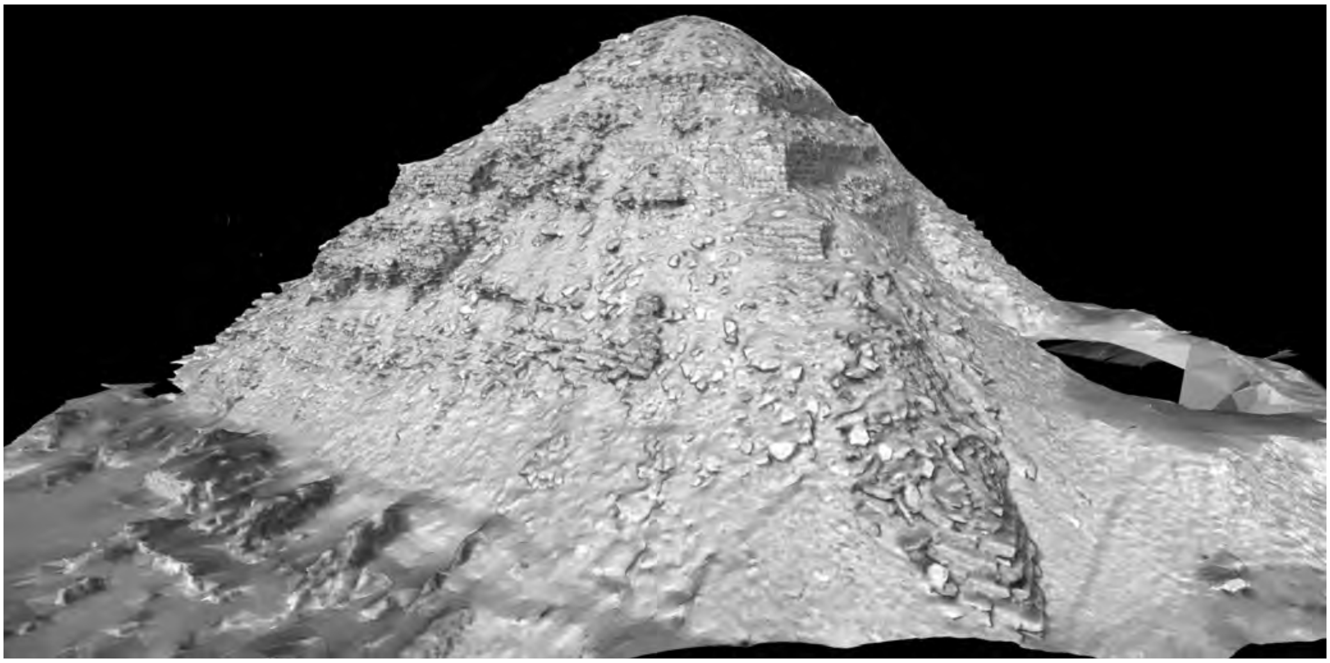


Fig. 3 The point cloud data of the eastern side of Neferirkare's pyramid produced by visual SFM and PMVS/CMVS (Patch-based multi-view stereo/ Clustering views for multi-view stereo)

camera (22.5 mm wide angle, 35 mm photography equivalent), a Sony Nex 5 camera with SEL1820 lens (18 mm wide angle, 35 mm photography equivalent) and a Canon 5D Mark II camera with a Canon EF24-105mm F4L IS USM lens (24 mm wide angle). The data were then analyzed using Visual SFM and CMP-MVS software (Wu 2013; Jancosek – Pajdla 2011). We produced a 1.69 million point cloud data set (fig. 3) from 624 digital images (165 still images photographed by a Nikon COOLPIX, 331 movie frames filmed by a Nikon COOLPIX, 119 still images photographed by a Sony Nex5, and 9 still images photographed by a Canon PowerShot) employed in an SFM process. The result clearly indicates that the SFM and MVS techniques are effective in recording large monuments and appropriate to the creation of the 3D triangulated mesh model for our mathematical optimization process.

Mathematical programming formulation for scanning plan

Dan *et al.* (2010 and 2011) formulated two 0-1 integer optimization problems for an efficient scanning plan. We used the same formulation in the A-3DS project. In this formulation, we assume that the surfaces of the target region are covered by a triangulated mesh virtually. Under this assumption, we can compute some parameters of the optimization problems by using this model. As explained above, we first constructed a 3D triangulated mesh model of the site. Then we were able to use this optimization approach. The following symbols are used in the mathematical optimization problems:

[Set and Index]
$i \in I$: candidate scanner positions,
$j \in J$: triangles on the surfaces of target objects.

[Variable]

$$x_i := \begin{cases} 0, & \text{a candidate scanner position } i \text{ is unadopted as a scanner position,} \\ 1, & \text{a candidate scanner position } i \text{ is adopted as a scanner position.} \end{cases}$$

[Parameter]
$r :=$ the upper bound of the number of scans,

$$d_{ij} := \begin{cases} 0, & \text{a triangle } j \text{ is not scanned from a candidate scanner position } i, \\ 1, & \text{a triangle } j \text{ is scanned from a candidate scanner position } i, \end{cases}$$

$$a_{ij} := \begin{cases} 0, & d_{ij} = 0, \\ \text{the amount of scanned data on } j \text{ from } i, & d_{ij} = 1. \end{cases}$$

Note that we can calculate the values of parameters d_{ij} and a_{ij} from the 3D model of the target area, which is the product of the preliminary survey. The details of this calculation are in Dan *et al.* (2011).

In this project, we use the following two mathematical optimization problems:

$$\begin{aligned} \text{Minimize} \quad & \textstyle\sum_{i \in I} x_i \\ \text{subject to} \quad & \textstyle\sum_{i \in I} d_{ij} x_i \geq 1 \ (\forall\, j \in J), \\ & x_i \in \{0, 1\}. \end{aligned} \tag{1}$$

$$\begin{aligned} \text{Minimize} \quad & \textstyle\sum_{i \in I,\, j \in J} a_{ij} x_i \\ \text{subject to} \quad & \textstyle\sum_{i \in I} d_{ij} x_i \geq 1 \ (\forall\, j \in J),\ \sum_{i \in I} x_i \leq r, \\ & x_i \in \{0, 1\}. \end{aligned} \tag{2}$$

The objective function of (1) is to minimize the number of scanner positions. Also, the first constraint of (1) means that all the triangles must be scanned from at least one scanner position. Consequently, we can find the least number of scanner positions from which to scan the target by solving (1).

On the other hand, the objective function of (2) is to maximize the sum of the (expected) amount of scanned data. In addition, the second constraint of (2) is to restrict the number of scans less than or equal to *r*. Typically, equals the optimal value of problem (1), that is, the least number of scanner positions necessary to measure all the surfaces of the targets. Consequently, by solving (2), we can find the optimal layout of *r's* scanner positions in order to obtain as much scanned data of the target as possible.

Problems (1) and (2) are categorized in the 0-1 integer optimization problem. It is well known that various optimization solvers can be applied for this problem. As we explain later, we employed a certain solver to answer these problems in this project.

Work flow

For choosing an optimal scanning plan, we must first determine the appropriate number of scans and their positions. In our mathematical programming framework, the input information necessary to solve the problem requires a 3D model consisting of the scanning target's meshes, the meshes of other surroundings objects, and the candidate scanner positions. This process is shown in fig. 4 (left). As part of a preliminary on-site survey, collecting photos by means of a digital camera or video is necessary. The processes of SFM and multi-view stereo result in a total 3D mesh of the subject. In many cases, the mesh models generated by the MVS process contain too many triangles (by the order of millions or more) to compute the parameters of the optimization problems. Thus we need to down-sample the mesh data by reducing the number of triangles down to an order of thousands before we can apply our optimization method.

The right-hand diagram in fig. 4 shows the process flow used to create an optimized scanning plan based on the 3D mesh model prepared by a preliminary survey. The first step is to parameterize the d_{ij} and a_{ij} in (1). This is mainly done by checking the visibility from each candidate scanner position (i) toward the target triangle (j), considering the solid angles of the triangles on the target surfaces viewed from the candidate scanner positions as well as the self and mutual occlusions of the triangles in the whole model of the site. Solving the problem expressed by (1) yields the minimum number of scans required to scan the whole target mesh. The problem expressed by (2) provides the best layout of the scanner positions using the least number of scans.

There is flexibility in the transition between the process to solve (1) and the process to solve (2). In the middle of executing the scanning task according to the prepared plan, there might be a change necessitated in the scanning

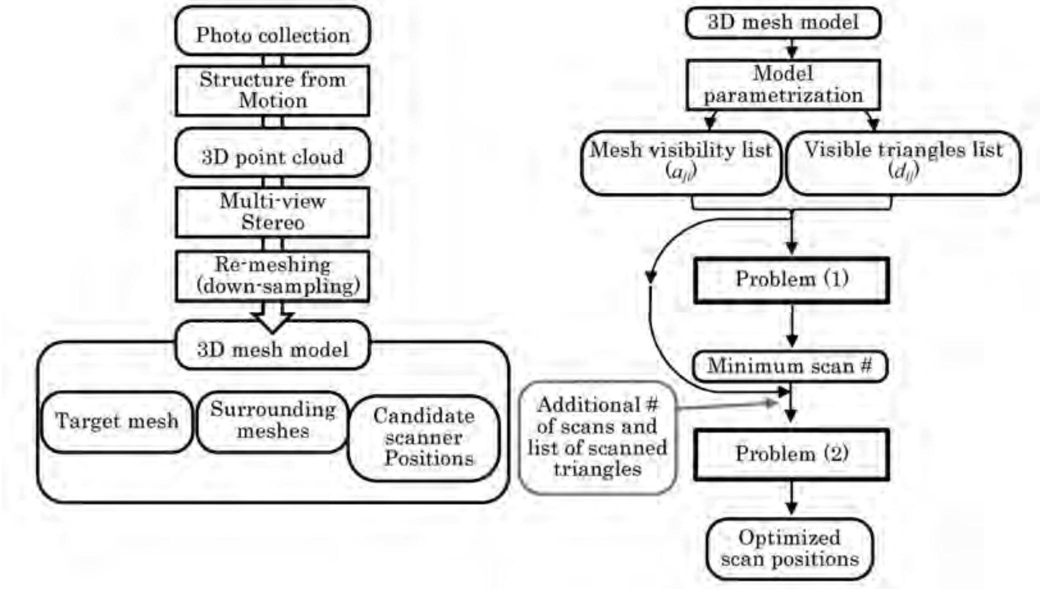


Fig. 4 Process flow: 3D modeling for preliminary survey (left) and scan planning with mathematical programming framework (right)

plan for some reason, such as time schedule changes, the physical unavailability of planned scan positions, and so on. In order to complete the rest of the scanning task, it is important to recalculate for optimizing the arrangement of the additional scan positions, making the most use of already scanned data. This recalculation can be handled by modifying the parameters for solving (2), so as to make a list of the scanner positions already finished, the remainder of the available scanner positions, and the amount of scans already done.

The optimal scan plan for Abusir

In this project, we prepared the values of the parameters in the optimization problems expressed by (1) and (2), and found the optimal layout for scanning Neferirkare's pyramid.

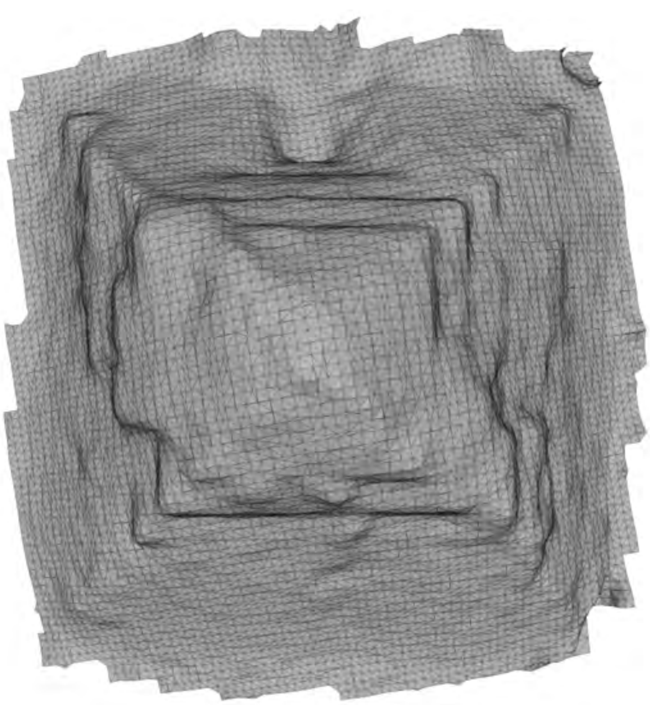


Fig. 5 Simplified triangle mesh model of Neferirkare's pyramid used for our optimized scanning plan

First, we downsized the obtained mesh of the target area and made a simplified model from 5,076 triangles, as shown in fig. 5. We also identified camera positions for 624 photos, which were used during the SFM process. These camera positions were used as candidate scanner positions. Through this procedure, we prepared the data for our optimization models.

After that, we solved our optimization problems by using IBM ILOG CPLEX (IBM) – one of the fastest optimization solver software programs – and obtained the optimal 3D scan plan. In this case, we needed seven scans to measure all the faces of Neferirkare's pyramid, and pl. 5 shows the optimal layout of the seven resulting scanner positions. The white dots are the candidate scanner positions and the red dots depict the resultant optimal scanner positions. We also visualized the effectiveness of the derived scanning plan. In pl. 5, the brighter colors depict the target surfaces that will receive higher numbers of scanning laser beams from those seven positions.

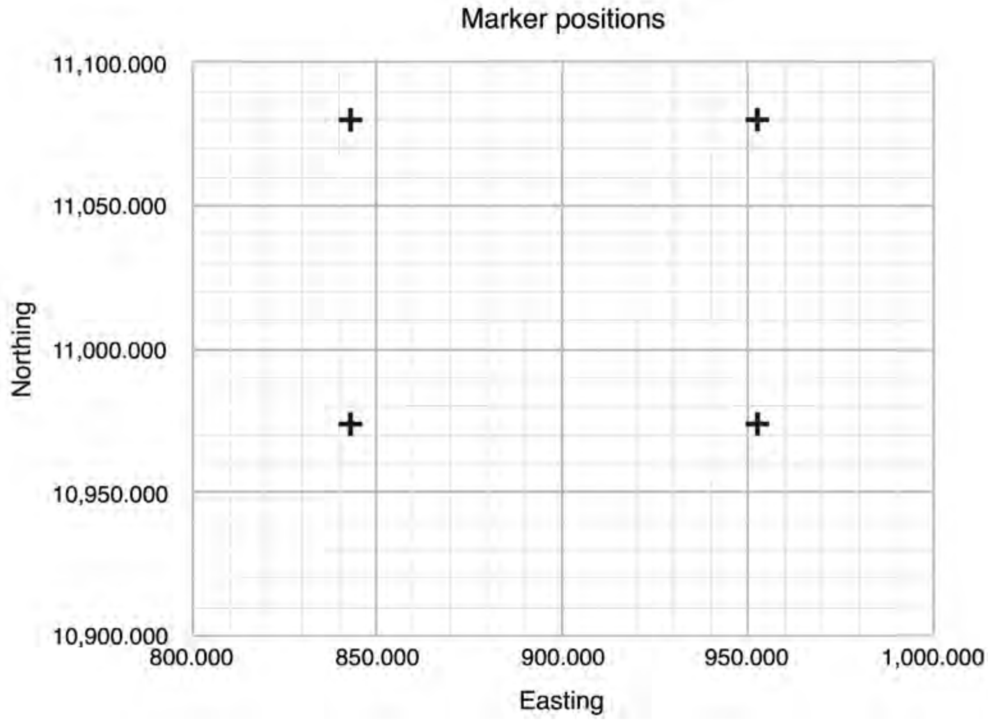
Main survey

Transformation from the machine coordinate system to the Czech mission's coordinate system

A terrestrial time-of-flight laser scanner has its own coordinate system, known as a machine coordinate system. The axes of the scanner are generally not set to north-south or east-west. Thus, it may be necessary to move the scanned data from the machine coordinate system into a global coordinate system, or another coordinate system such as the one used by the Czech mission's coordinate system.

In 2015, we used four reference points that we surveyed around the pyramid using a total station in accordance with the Czech mission's coordinates. Using the Kabsch (1978) algorithm, we computed the transformation matrix that changed the machine coordinate system into the Czech mission's coordinate system. This algorithm finds a transformation matrix such that the root mean square (RMS), or quadratic mean, of errors of the transformation

Fig. 6 Positions of the reference markers



is minimal. The RMS of the transformation error was 0.218 m.

The four reference points (named NE, NW, SW and SE) were distributed around the pyramid, whose positions on the machine coordinate systems were (NE) –250.593, –55.319, 3.606; (NW) –165.256, –124.162, –0.159; (SW) –98.707, –41.551, –2.462; and (SE) –184.107, 27.313, –0.104. The corresponding representations of the Czech mission's coordinate system were (NE) 11080.174, 952.526, 54.496; (NW) 11080.135, 842.847, 52.094; (SW) 10973.952, 842.804, 58.383; and (SE) 10973.966, 952.571, 54.336. Fig. 6 illustrates the positions of the reference points.

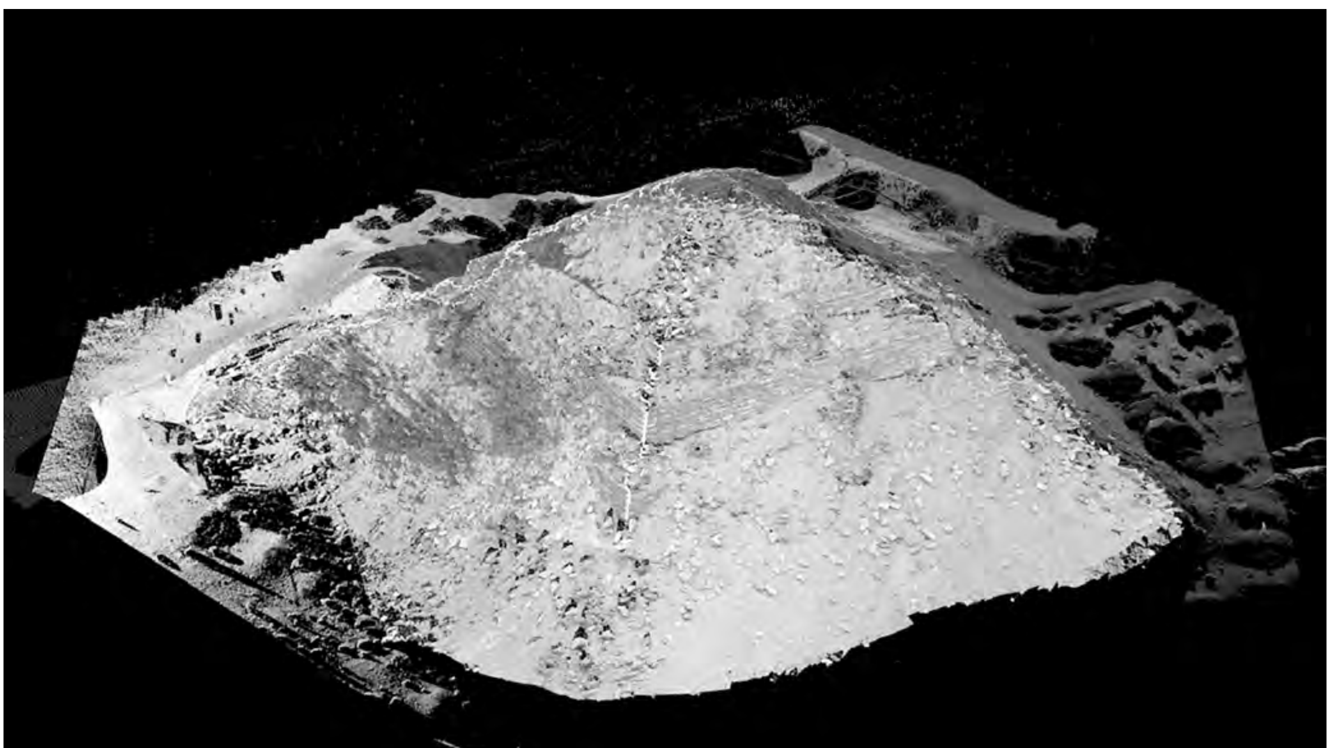


Fig. 7 A 3D scan data of Neferirkare's pyramid

Laser scanning
In accordance with the mathematical optimization model, we conducted our laser scanning using a RIEGL LMS-Z420i scanner over a period of five days, capturing the surface of Neferirkare's pyramid from seven positions (fig. 7). The angular divisions of the scanning laser resolution are from 0.006° to 0.03°. We used the cylindrical type of reflection markers (with sizes of a 10 cm width and radius) for registration during post-processing. The standard deviation of the residual positioning errors in registration was about 17–27 mm.

PEAKIT post-processing
As discussed above, 3D data allows researchers to intuitively understand the structures. However, the orthophotographic images produced from 3D data do not reveal the undulations in the structure that are important for a proper archaeological analysis. Furthermore, even though the original data contains 3D information, by their nature, point cloud images can only represent 2D information. PEAKIT, our newly developed image-processing technology, is appropriate for displaying 3D data more effectively (Chiba *et al.*, forthcoming). The PEAKIT image we produced for Neferirkare's pyramid is shown in pl. 6.

Results

3D data
We set out to obtain 3D target data in the form of a point cloud with additional color information using the Riegl LMS-Z420i laser scanner. This data was collected and then placed in the local coordinate system set up by the Czech Institute of Egyptology. The specifications of this data are as follows:

- Number of points: 37,785,829;
- *X*-direction range: 161.881 m wide (from 817.701 m to 979.582 m on the local coordinate system);
- *Y*-direction range: 146.028 m wide (from 10957.806 m to 11103.834 m on the local coordinate system);
- *Z*-direction range: 46.920 m wide (from 48.306 m to 95.226 m on the local coordinate system);
- File size: About 1.8 GB.

Dimensions of Neferirkare's pyramid in its present condition
We obtained the following measurements from the orthophotographic plan and elevations produced by our PEAKIT process.

The height of the pyramid in its current condition (see pl. 6).

- The measurement of the orthophotographic elevation of the northern face of the pyramid from the north-western corner to the top: 41.63 m;
- The measurement of the orthophotographic elevation of the eastern face of the pyramid from the south-eastern corner to the top: 41.53 m;
- The measurement of the orthophotographic elevation of the southern face of the pyramid from the south-western corner to the top: 41.32 m;
- The measurement of the orthophotographic elevation of the western face of the pyramid from the north-western corner to the top: 41.63 m.

The average height of the pyramid in its current condition: approximately 41.5 m.

Notes: The measurements obtained were the distances from the bottom of the well-preserved corner to the top of the pyramid. However, the base around the pyramid is still covered by sand and debris such that the bottom has not been ascertained yet.

The base length of the pyramid in its current condition (see pl. 6).

- Northern side: 91.98 m;
- Southern side: 92.00 m;
- Eastern side: 92.20 m;
- Western side: 92.14 m.

The average base length of the pyramid in its current condition: approximately 92.00 m.

Notes: Again, since the base around the pyramid is still covered by sand and debris, the measurements obtained were the distances connecting to the well-preserved corners.

BIM Guide for 3D imaging
After we acquired the 3D data of the study area, we then visualized the effectiveness of the data. In pl. 7, the brighter colors on the target surfaces represent a higher number of scanning laser beams (that is, the warmer the color, the higher the number of beams that scanned that surface). According to the US General Services Administration (GSA) (GSA BIM Guide Series 03 2015), Building Information Modeling (BIM) Guide Series 03 (3D Laser Scanning), the general quality levels of scanned 3D point data should be described based on point density or point cloud resolution because the levels of detail (or resolution) of the data has a trade-off relative to the distance between the target and the scanner. Tab. 1 shows a "project definition matrix" that can be used to identify how 3D imaging data can be used to support project objectives. According to this matrix, the resultant scan data can be categorized into four levels based on the density of the point cloud data acquired by the scanner.

The colors in pl. 7 (blue, cyan, green, yellow, orange, and red) are shown in ascending order. The gray area represents data that measures under the standards of the GSA BIM Guide Series, Level 1 includes blue and cyan, Level 2 includes green and yellow, and Levels 3 and 4 are the same density and thus include both orange and red regions. Comparing the optimal layout illustrated in pl. 5 with that of pl. 7, it is clear that almost the same scanning distribution was successfully achieved.

Levels of detail	Category	Resolution mm × mm (in × in)
Level 1	Point cloud	152 × 152 (6 × 6)
Level 2	Plan elevation surface model point cloud	25 × 25 (1 × 1)
	Elevation surface model point cloud	25 × 25 (1 × 1)
	Elevation point cloud	13 × 13 (½ × ½)
Level 3	Plan elevation point cloud	13 × 13 (½ × ½)
	Elevation point cloud	13 × 13 (½ × ½)
Level 4	Surface model point cloud	13 × 13 (½ × ½)

Tab. 1 Levels of detail and point cloud resolution (excerpt from GSA BIM Guide Series)

Conclusions

When a 3D survey of a megalithic structure is conducted using only terrestrial time-of-flight laser scanners, it can be difficult to capture the entire surface of the monument. Therefore, in this first season of the A-3DS project, we were able to identify in advance, using a mathematical optimization approach, areas where a scanner's laser beam could not reach, thus making the measurement of the dimensions of the pyramid in a cost-effective way a primary goal. This resulted in a Level 1 point cloud resolution by the standards of the GSA BIM Guide Series, meaning that the usefulness of the newly developed mathematical 3D survey plan becomes very clear in the archaeological survey.

Our aim for next season is to target structurally important areas in the pyramid by setting Regions of Interest (ROI) in the PEAKIT image data and scanning them in a cost-effective way. Overall, we will use a small, portable laser scanner (*e.g.* a Focus3D X 330) and digital cameras with SFM for covering unscanned areas of the 3D data we obtained this season.

We also plan to produce line drawings of every construction phase of the pyramid from the integrated 3D data. It is important to note that any line drawings will be archaeological interpretations, and should be viewed separately from the current 3D documentation. They are different levels of archaeological study.

Bibliography:

Borchardt, Ludwig

1909 *Das Grabdenkmal des Königs Nefer-ir-ka-re[c]*, Leipzig: Hinrichs'sche Buchhandlung [Ausgrabungen der Deutschen Orient-Gesellschaft in Abusir 1902–1908. Bd. 5].

Chiba, Fumito – Kawae, Yukinori – Yokoyama, Shin – Kanaya, Ichiroh – Kamei, Hiroyuki

forthcoming "An Introduction of "PEAKIT" to 3D Data of the Tomb of Khentkawes [I] at Giza", in: Lehner, Mark (ed.). *Giza Plateau Mapping Project. Season 2010. Preliminary Report*, Boston, MA: Ancient Egypt Research Associates [Giza Occasional Papers 6].

Dan, Hiroshige – Yasumuro, Yoshihiro – Ishigaki, Taisuke – Nishigata, Tatsuaki

2010 "Shape Measurement Planning of Outdoor Constructions with Mathematical Programming and Its Practical Applications". Paper presented at the *10th International Conference on Construction Applications of Virtual Reality*, Miyagi.

2011 "Measurement Planning of Three-dimensional Shape by Mathematical Programming". Paper presented at the *11th International Conference on Construction Applications of Virtual Reality*, Weimar.

Dormion, Gilles

2004 *La chambre de Chéops. Analyse architecturale*, Paris: Fayard.

Gatzsche, Alexander

2013 "Case study of an open source application for 3D acquisition of archaeological structures at the archaeological site Wad Ben Naga", Der Antike Sudan. Mitteilungen der Sudanarchäologischen Gesellschaft zu Berlin 24, pp. 125–134.

Kabsch, Wolfgang

1978 "A discussion of the solution for the best rotation to relate two sets of vectors", Acta Crystallographica A34, pp. 827–828.

Kawae, Yukinori – Kamei, Hiroyuki – Tsukamoto, Toshio – Kanaya, Ichiroh – Okamoto, Atsushi

2009 "Giza Laser Scanning Survey 2006", in: Lehner, Mark – Kamel, Mohsen – Tavares, Ana (eds.). *Giza Plateau Mapping Project. Seasons 2006–2007. Preliminary Report*, Boston: Ancient Egypt Research Associates [Giza Occasional Papers 3], pp. 166–175, Pl. 43–47.

Kawae, Yukinori – Sato, Kousuke – Kamei, Hiroyuki – Nakano, Tomoaki – Kanaya, Ichiroh

2009 "Saqqara Laser Scanning Survey 2008", in: Lehner, Mark – Kamel, Mohsen – Tavares, Ana (eds.). *Giza Plateau Mapping Project. Season 2008. Preliminary Report*, Boston: Ancient Egypt Research Associates [Giza Occasional Papers 4], pp. 63–71, Pl. 7.1–8.2.

Kawae, Yukinori – Yasumuro, Yoshihiro – Kanaya, Ichiroh – Chiba, Fumito

2013 "3D Reconstruction of the "Cave" of the Great Pyramid from Video Footage", Paper presented at the *Digital Heritage International Congress*, Marseille.

forthcoming "3D Reconstruction and its Interpretation of the "Cave" of the Great Pyramid: An Inductive Approach", in: Kuraszkiewicz, Kamil Omar – Kopp, Edyta – Popiełska-Grzybowska, Joanna (eds.). *Old Kingdom Art and Archaeology in Warsaw*, Warsaw.

Krejčí, Jaromir

2010 *The Royal Necropolis in Abusir*, Prague: Charles University in Prague, Faculty of Arts [Abusir XVIII].

Maragioglio, Vito – Rinaldi, Celeste

1963–1977 *L'architettura delle piramidi menfite*, Torino: Tip. Artale.

Jancosek, Michal – Pajdla, Tomas

2011 "Multi-View Reconstruction Preserving Weakly-Supported Surfaces". Paper presented at *EEE Computer Vision and Pattern Recognition (CVPR)*, Colorado Springs.

Petrie, W. M. Flinders

1883 *The Pyramids and Temples of Gizeh*, London: Field & Tuer.

Verner, Miroslav
1991 "Remarks on the Pyramid of Neferirkare", Mitteilungen des Deutschen Archäologischen Instituts Abteilung Kairo 47, pp. 411–418.
Wu, Changchang
2013 "Towards linear-time incremental structure from motion". Paper presented at the *International Conference on 3DTV*, Seattle, Washington.
Yasumuro, Yoshihiro – Matsushita, Ryosuke – Higo, Tokihisa – Suita, Hiroshi
2015 "On-site Augmented Reality system for the Mastaba Idout Project", The Journal of Center for the Global Study of Cultural Heritage and Culture 3, pp. 1–8.
Yokoyama, Ryuzo – Shirasawa, Michio – Pike, Richard J.
2002 "Visualizing topography by Openness: A new application of image processing to digital elevation models", Photogrammetric Engineering and Remote Sensing 63/3, pp. 257–265.

Internet resources:
http://www-03.ibm.com/software/products/en/ibmilogcpleno
http://www.gsa.gov/portal/mediaId/226819/fileName/GSA_BIM_Guide_Series_03.action
http://www.aeraweb.org/news/the-pedestal-puzzle/

Abstract:

In 2015, in collaboration with the Czech Institute of Egyptology, we, a Japanese consortium, initiated the Abusir 3D Survey (A-3DS) for the 3D documentation of the site's pyramids, which have not been updated since the time of the architectural investigations of Vito Maragioglio and Celeste Rinaldi in the 1960s to the 1970s. The first season of our project focused on the exterior of Neferirkare's pyramid, the largest pyramid at Abusir. By developing a strategic mathematical 3D survey plan, step-by-step 3D documentation to suit specific archaeological needs, and producing a new display method for the 3D data, we successfully measured the dimensions of the pyramid in a cost-effective way.

Abusir – Old Kingdom – Fifth Dynasty – Neferirkare – pyramid – 3D survey – SFM/MVS

Yukinori Kawae (*yukinegy@gmail.com*)
Research Center for Cultural Heritage and Texts, Graduate School of Letters, Nagoya University
Yoshihiro Yasumuro (*yasumuro@kansai-u.ac.jp*)
Faculty of Environmental and Urban Engineering, Kansai University
Ichiroh Kanaya (*kanaya@pineapple.cc*)
School of Information Systems, the University of Nagasaki
Hiroshige Dan (*dan@kansai-u.ac.jp*)
Faculty of Environmental and Urban Engineering, Kansai University
Fumito Chiba (*f-chiba@lang-co.jp*)
LANG CO., LTD