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EVALUATING ORGANIZATIONAL ANTIFRAGILITY VIA FUZZY LOGIC. THE CASE OF AN IRANIAN COMPANY PRODUCING BANKNOTES AND SECURITY PAPER

The concept of antifragility has received much attention from researchers in recent years. Contrary to fragile systems which fail when exposed to stressors, antifragile systems prosper and improve in response to unpredictability, volatility, randomness, chaos and disturbance. The implications of antifragility goes beyond resilience or robustness. A resilient system resists stress and remains the same; while an antifragile system improves. Taleb argues that antifragility is required for dealing with events that he called black swans or X-events, which are scarce, unpredictable, and extreme events. Such events come as a surprise and have major consequences. The concept of antifragility was developed by Taleb in a socioeconomic context, not in industrial production. However, the authors think that this concept may have its greatest practical utilization when applied to industrial environments. Thus, they focused on this concept in the article aiming to investigate the level of antifragility in an organization. In order to perform this, the authors used a case study based on an Iranian manufacturer of banknotes and security paper (TAKAB). Firstly, a questionnaire was designed based on 7 criteria related to antifragility using the five-point Likert scale and a triangular fuzzy number for each linguistic term is defined. In the next phase, the weight of each component was obtained using the entropy technique. In the final stage, the Euclidean distance between the aggregated fuzzy antifragility index (*FAI*) and each linguistic term used during this case study was calculated. Finally, based on these results, the level of the organization's antifragility was assessed as satisfactorily antifragile, based on the minimum Euclidean distance.

Keywords: *antifragility, X-events, triangular fuzzy number, entropy technique, linguistic terms, Euclidean distance*

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1. Introduction

Systems can vary in their ability to endure unexpected events (X-events or black swans). This ability varies on a continuum that ranges from fragile (harmed by a crisis), to robust (unchanged by a crisis), to antifragile (progressing as a result of a crisis) [1].

Analytical frameworks are needed to measure this ability in systems, as well as their tendency to produce X-events. The reason for concentrating on such a framework is that the capacity to suitably measure antifragility is a necessary precondition for improving an organization's strategic goal to become less fragile or more antifragile. Organizations that recognize their level of fragility can develop structures, systems, processes and cultures that enable them to not only survive but also thrive. It is far easier to assess and enhance antifragility than forecast events that would cause damage [1].The implications of the concept of antifragility have developed considerably during the few years that have passed since its emergence.

Despite the rapid growth of electronic trading and use of other methods of electronic payment like credit cards, the banknote is still considered the most reliable means of payment. The increasing growth in the use of ATMs is an indicator of this. There is high demand for secure and extra secure documents, and the TAKAB complex, with its expert staff, advanced machinery and equipment can supply the entire national demand. Utilizing a wide range of threads and security parameters, the complex can manufacture all types of watermarked papers with multi-tone quality and here we are going to present a case study about how this organization reacts to unexpected events.

A framework is offered for analysing and measuring antifragility through a case study on the Iranian manufacturer of security paper, TAKAB. The authors' motivation is to measure antifragility. The data were gathered via a questionnaire using the five-point Likert scale with a range from strongly disagree (1) to strongly agree (5). Based on this, a triangular fuzzy number was assigned to the linguistic variables. Next, the weight of each criterion was computed using the entropy method. Finally, the minimum Euclidean distance between the fuzzy aggregation rating of the criteria of antifragility and each linguistic term indicated that the degree of antifragility in this organization is medium level.

The paper is organized as follows: In Section 2, the authors make a brief review of theoretical foundations and criticisms of antifragility. The authors' method for measuring antifragility based on calculating entropy and Euclidean distance is explained in Section 3. Section 4 provides suggestions to make the organisation more antifragile. Finally, Section 5 concludes the paper, presents the results and limitations of the research, as well as the scope for future research.

2. Theoretical foundations

When systems are performing effectively, they are in a predetermined condition and conversely when they are not functioning correctly, they are in an unintended state. An unintended condition can be known or unknown. Stressors are forces that threaten to transfer a system from an intended to an unintended condition [2, 3].

Unanticipated extreme events are those stresses that have catastrophic outcomes. Their intensity and frequency cannot usually be inferred from historical data. These are the so-called black swans or X-events [4, 5].

X-events have the features described below:

- X-events are too scarce for their frequency or incidence to be predicted with any accuracy.
- Their impact is extreme.
- Such events are only “retrospectively predictable” [6].

The solution is to thrive on variation and uncertainty and also defects, to some degree [7]. Regular exposure to small amounts of stress can strengthen a system and, sometimes, some systems not only develop the ability to withstand stress, but they even improve when they encounter X-events.

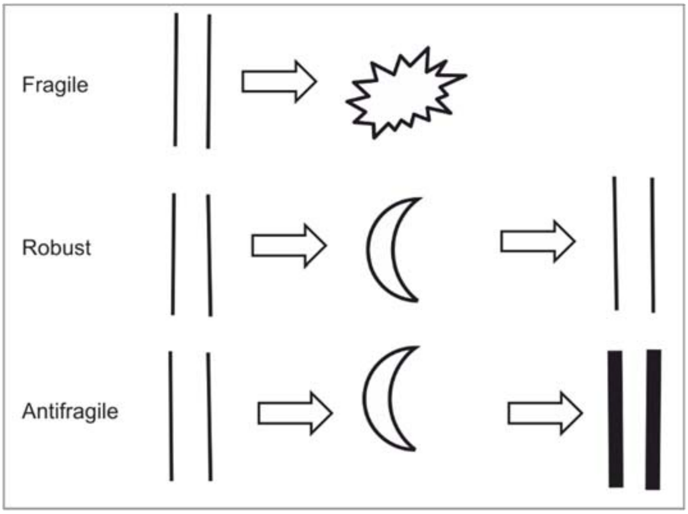

Fig. 1. Fragile, robust and antifragile systems. Source [7]

Since fragility is an unpleasant feature, it makes sense to define its antonym. Contrary to common assumption, the opposite of fragile is not robustness or resilience; it is described as antifragility [8]. The concept of antifragility is a scheme for living in a black swan world, where unexpected extreme events may occur.

Antifragile systems are capable of tolerating stress and even reacting positively to it. The human body, whose immune system becomes stronger when exposed to sickness, is a good example of an antifragile system that adapts in response to stress and shock [5].

How systems change when an X-event occurs, can be described as follows:

A fragile system fails and breaks, a robust system remains unchanged, and an antifragile system gets stronger and grows (Figs. 1, 2).

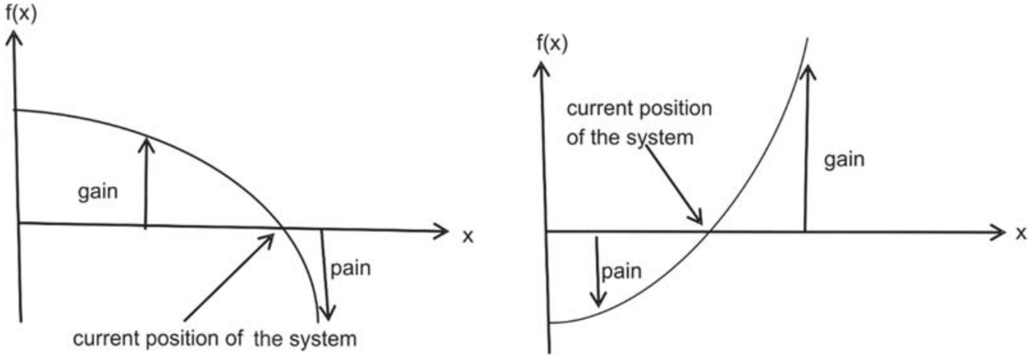

Fig. 2. When volatility increases fragile system loses (concave gain function)and an antifragile system gains (the *x*-axis indicates the level of volatility, increasing towards the right). Source [5]

Considering the movement towards sustainability, antifragility is a better goal than resilience. The reason for this is that resilience depends on knowing what types of stress the system will be exposed to and reinforcing one's defences against such stress, while antifragility does not require knowledge of what types of stress may appear, because the system is expected to become stronger and improve through stress. Therefore, the added advantage of antifragility is as follows:

- It is not necessary to predict all the types of stress a system will be exposed to in order to prepare for them.
- The system will be improved by stress and so there is no reason to be afraid of its occurrence.

Fragility is measurable but risk cannot be measured. This is one reason why it is far easier to discover whether a system is fragile than to forecast the occurrence of an event that can harm it. This provides a solution to the impossibility of measuring the risks of highly significant, scarce events and forecasting their occurrence (which Taleb called the black swan problem). Assessing an organisation's sensitivity to X-events is more tractable than predicting these events. Therefore, we suggest redirecting our current approaches to evaluating antifragility, instead of predicting events that would cause harm. The next phase, after evaluating the level of antifragility, is to suggest rules for moving from being fragile toward being antifragile, via the reduction of fragility. Fragility and antifragility are degrees on a spectrum. Antifragility cannot be increased without decreasing fragility, just as light cannot be made more intense without reducing darkness, or wealth raised without diminishing poverty. Hence, we can assess antifragility and fragility through a test of asymmetry: any system that responds negatively to unexpected events or certain types of stress is fragile; the reverse situation corresponds to antifragility. Accordingly, we can specify the fragility of a system and decrease it, in order to increase antifragility.

Table 1. Criteria used to analyse antifragility

Absorption	The ability of a system to absorb shocks will increase as a result of designing margins to increase the magnitude and duration of shocks that the system can withstand during a crisis, to ensure that it continues being in an intended state. The greater the robustness, the higher the level of antifragility.
Redundancy	Having multiple criteria that can carry out a function, or multiple ways of meeting the same purpose, creates excess system capacity and is an effective form of defence when X-events happen. Redundancy tends to enhance robustness and make systems less fragile and more antifragile.
Introduction of low level stress	Eliminating stress from systems or attempting to reduce uncertainty in them can lead to weakness, fragility, and expose them to serious X-events. Regularly exerting low levels of stress on a system increases its robustness and can result in antifragility when the system learns from these controlled levels of stress.
Non-monotonicity	Learning from failures and negative consequences can be an effective defence against stressors and create new information. As new data become available, it overcomes previous thinking[10, 11], which can lead to new approaches, improve the system and increase antifragility.
Requisite variety	There are regulators in the system attempting to control the consequences of the behaviour of agents. When the number of regulators is inadequate, the behaviour of the system cannot be anticipated precisely and black swan events emerge. Thus the more regulators, the greater antifragility.
Emergence	When there is little or no traceability between the micro and macro level, output is said to be emergent, the frequency of unintended states increases and X-events appear. Conversely, when there are cause-effect relationships between the activity of criteria at micro level and outcomes at macro level, the system output is said to be resultant which results in greater antifragility [12].
Uncoupling	Failures can reverberate through firmly coupled systems increasing in amplitude and potentially leading to disaster. The lower the degree of coupling between systems and system criteria, the more antifragile the system becomes.

Jaaron and Backhouse [18] described applying a systems approach to the design of service delivery in order to build an “antifragile” organization that can face disruptions. They used a case study based on in-depth interviews with key informants. The results of their research reveal that clarity of the whole service system, effects of the working structure and employees’ engagement and readiness to learn are the main contributing factors to antifragility in service organizations. Taleb also argues that systems need stress, volatility, and abnormality to function well and even improve and progress but only up to a specific point, because too much stress will eliminate an antifragile system, just as it will wipe out a resilient system.

Casti and Taleb discussed criteria of a system's antifragility according to theories of a system's responses to black swans in their books *X-events: The Collapse of Everything* and *Antifragile: Things that Gain from Disorder* [19, 20]. Jackson and Ferris suggested a list of attributes based on domain analysis by an expert of 10 case studies on system processes intended to improve a system's ability to survive a challenge [9]. In addition, some researchers tried to propose a method for the mathematical detection of fragility, robustness, and antifragility using a single "fast-and-frugal", probability free heuristic that also considers exposure to errors in formulating the model [22]. Seven of these criteria are used in this paper. These criteria and their definitions are summarized in Table 1.

3. Methodology

This section gives an overall overview of the research methodology implemented by the authors. Valuable information has been gathered in this research in order to justify the reliability and validity of the findings. Data were collected from the TAKAB complex. This paper describes questionnaire research based on a field survey. The respondents were asked to answer each question using a five-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree). During data collection, we were very careful to make sure that the respondents' privacy would be kept strictly secret. In addition, the survey did not include any question that could have made respondents vulnerable to risk.

The study population consists of about 70 employees who are familiar with strategic and quality management. According to the Cochran approximation for determining the required sample size from a finite population [17], we obtain:

$$n=\frac{Nz_{\alpha/2}^{2}p(1-p)}{\varepsilon^{2}(N-1)+z_{\alpha/2}^{2}p(1-p)}=\frac{70\times(1.96)^{2}\times0.5\times0.5}{(0.06)^{2}\times69\times(1.96)^{2}\times0.5\times0.5}=55.0$$

70 questionnaires were distributed to the staff of the TAKAB complex. Fifteen questionnaires were not filled in completely and thus excluded from this study, so the remaining 55 responses were used for the analysis. Using standard statistical procedures described by Cronbach's alpha calculated using SPSS software, the overall reliability of 31 items was assessed as $\propto = 92\%$, and the reliability of each individual criteria was derived as follows (Table 2):

Table 2. Cronbach's alpha for the antifragility study

Item	Criteria	No. of items	Cronbach's alpha
1	Absorption	5	0.7
2	Redundancy	5	0.7
3	Introduction of stressors	3	0.681
4	Non-monotonicity	5	0.821
5	Requisite variety	5	0.719
6	Emergence	5	0.674
7	Uncoupling	3	0.754

In order to investigate the validity of the questionnaires, the validity of both content and structure were used. The validity of the questionnaires was determined by academic and executive experts in the field under question[13]. The experts were satisfied with the apparent similarity of the items to the topic of research (validity of the structure). Moreover, the experts agreed on the ability of the results of the questionnaire based on these items to describe the criteria of antifragility (validity content).

3.1. Assigning fuzzy numbers to linguistic variables

In some cases, it cannot be asserted that a phenomenon is well-understood until it can be defined in quantitative terms [14]. In the real world, data for decision-making processes cannot be measured precisely, so crisp values are inappropriate. Many evaluation criteria are clearly imperfect and factors of uncertainty probably exist. In fact, human judgment about preferences is often unclear and hard to estimate using exact numerical values. Thus, here fuzzy numbers are allocated to linguistic terms. Among various types of fuzzy number used, triangular fuzzy numbers (TFN) are the most popular.

A triangular fuzzy number is defined by a triplet $A = (l, m, u)$ where l and u are the lower and upper bounds. We can also define it as $A = (\alpha, m, \beta)$ where α, β are the left--hand and right-hand spreads. In both cases, the point m, with a membership grade of 1, is called the mean value.

The membership function is defined as:

$$\mu_A(x) = \begin{cases} \dfrac{x-l}{m-l} & l < x < m \\ 1 & x = m \\ \dfrac{u-x}{u-m} & m < x < l \\ 0 & \text{otherwise} \end{cases}$$

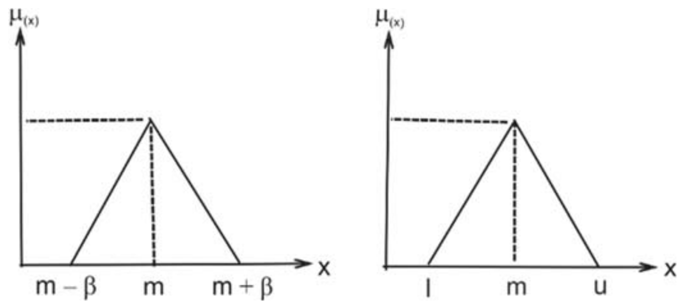

Fig. 3. Triangular fuzzy numbers $A = (\alpha, m, \beta)$, $A = (l, m, u)$. Source [14]

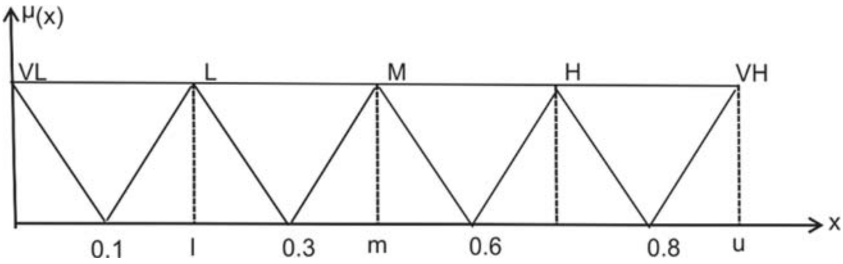

Fig. 4. Membership functions of linguistic terms. Source [21]

To transform a linguistic term into a triangular fuzzy number (TFN), a group of experts were requested to assign a sub-interval of [0, 1] to each of the points of the Likert scale: strongly disagree, disagree, neutral, agree, strongly agree, where 0 represents complete disagreement and 1 complete agreement. The triangular fuzzy numbers computed according to the experts' opinions are depicted in Table 3 and Fig. 4 [21].

Table 3. Linguistic terms and their corresponding triangular fuzzy numbers [21].

VL	*L*	*M*	*H*	*VH*
(0.0, 0.0, 0.1)	(0.1, 0.2, 0.3)	(0.3, 0.45, 0.6)	(0.6, 0.7, 0.8)	(0.8, 0.8, 1.0)

3.2. Determination of the weight of indicators using the entropy technique

As regards to each component having a different meaning, it cannot be supposed that all of them have identical weights. Hence, finding the appropriate weight for each component is one of the most important points in multiple attribute decision-making (MADM). There are various techniques for determining the weights of criteria, which can be classified into two groups: subjective and objective. Subjective methods such as the analytic hierarchy process (AHP) method, weighted least squares method, and Delphi method, are defined according to the decision makers' preferences. Objective methods, such as the entropy method, multiple objective programming, and principal element analysis, do not consider the decision maker's preferences and obtain weights according to a mathematical model. In this research, Shannon's entropy method was used to obtain weights.

The average of the answers of each respondent to the questions related to each component are shown in Table 4. For instance, five questions were designed for the first component (absorption). These answers were changed into fuzzy numbers. For example, the first respondent (P1) gave the following answers to questions 1–5:

$$(0.6, 0.7, 0.8), (0.3, 0.45, 0.6), (0.3, 0.45, 0.6), (0.1, 0.2, 0.3), (0.6, 0.7, 0.8)$$

Hence, the average is:

$$\mathrm{Avg}\,(P_1, P_2, ..., P_5)$$

$$= \frac{(0.6, 0.7, 0.8) + (0.3, 0.45, 0.6) + (0.3, 0.45, 0.6) + (0.1, 0.2, 0.3) + (0.6, 0.7, 0.8)}{5}$$

$$= (0.38, 0.5, 0.62)$$

This average is taken to be the assessment made by the first respondent (p_1) of the first component (c_1).

Table 4. The fuzzy data set for 7 criteria and 33 respondents

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
P_1	(0.38, 0.5, 0.62)	(0.52, 0.66, 0.76)	(0.5, 0.6166, 0.7333)	(0.52, 0.66, 0.76)	(0.56, 0.68, 0.74)	(0.46, 0.61, 0.72)	(0.4, 0.533, 0.66)
P_2	(0.3, 0.41, 0.54)	(0.46, 0.57, 0.66)	(0.2, 0.3, 0.4333)	(0.3, 0.4, 0.5)	(0.18, 0.3, 0.42)	(0.28, 0.4, 0.52)	(0.3, 0.38, 0.5)
P_3	(0.62, 0.77, 0.84)	(0.22, 0.29, 0.38)	(0.3, 0.3833, 0.8)	(0.48, 0.6, 0.72)	(0.2, 0.27, 0.38)	(0.28, 0.4, 0.52)	(0.0333, 0.0666, 0.1666)
P_4	(0.18, 0.3, 0.42)	(0.18, 0.23, 0.52)	(0.1333, 0.2166, 0.3333)	(0.76, 0.94, 0.96)	(0.52, 0.68, 0.72)	(0.66, 0.83, 0.88)	(0.0333, 0.0666, 0.1666)
P_5	(0.22, 0.31, 0.42)	(0.34, 0.47, 0.58)	(0.1333, 0.2166, 0.3333)	(0.8, 1, 1)	(0.38, 0.49, 0.56)	(0.6, 0.78, 0.84)	(0.1, 0.15, 0.2666)
P_6	(0.36, 0.47, 0.56)	(0.14, 0.18, 0.28)	(0.2666, 0.3333, 0.4)	(0.52, 0.62, 0.7)	(0.2, 0.27, 0.380)	(0.42, 0.52, 0.6)	(0.0333, 0.0666, 0.1666)

Table 4. The fuzzy data set for 7 criteria and 33 respondents

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
P_7	(0.24, 0.35, 0.46)	(0.32, 0.41, 0.52)	(0.0666, 0.1333, 0.2333)	(0.72, 0.88, 0.92)	(0.48, 0.61, 0.7)	(0.3, 0.38, 0.64)	(0.3333, 0.45, 0.5666)
P_8	(0.24, 0.36, 0.5)	(0.4, 0.56, 0.68)	(0.2, 0.3, 0.4333)	(0.42, 0.55, 0.68)	(0.36, 0.5, 0.64)	(0.36, 0.5, 0.64)	(0.3, 0.45, 0.6)
P_9	(0.38, 0.5, 0.62)	(0.1, 0.17, 0.28)	(0.3333, 0.45, 0.5666)	(0.72, 0.88, 0.92)	(0.44, 0.56, 0.64)	(0.44, 0.58, 0.66)	(0.0666, 0.1333, 0.2333)
P_{10}	(0.28, 0.4, 52)	(0.14, 0.25, 0.36)	(0.1666, 0.2833, 0.4)	(0.44, 0.55, 0.66)	(0.38, 0.46, 0.56	(0.28, 0.4, 0.52)	(0.2666, 0.3666, 0.4666)
P_{11}	(0.34, 0.45, 0.56)	(0.16, 0.26, 0.38)	(0.2333, 0.3666, 0.5)	(0.48, 0.6, 0.72)	(0.42, 0.51, 0.62)	(0.28, 0.4, 0.52)	(0.3333, 0.45, 0.5666)
P_{12}	(0.58, 0.71, 0.8)	(0.68, 0.82, 0.88)	(0.6333, 0.8166, 0.8666)	(0.8, 1, 1)	(0.72, 0.88, 0.92)	(0.8, 1, 1)	(0.6666, 0.8, 0.8666)
P_{13}	(0.2, 0.31, 0.44)	(0.2, 0.27, 0.38)	(0.1333, 0.2166, 0.3333)	(0.12, 0.21, 0.32)	(0.06, 0.12, 0.22)	(0.18, 0.24, 0.32)	(0.0333, 0.0666, 0.1666)
P_{14}	(0.3, 0.41, 0.54)	(0.38, 0.51, 0.6)	(0.2333, 0.3666, 0.5)	(0.32, 0.45, 0.58)	(0.14, 0.25, 0.36)	(0.2, 0.31, 0.44)	(0.1666, 0.2833, 0.4)
P_{15}	(0.1, 0.2, 0.3)	(0.26, 0.36, 0.48)	0.1666, 0.2833, 0.4)	(0.54, 0.65, 0.76)	0.32, 0.41, 0.52)	0.44, 0.55, 0.66)	(0.2333, 0.3, 0.4)
P_{16}	(0.54, 0.65, 0.76)	(0.48, 0.6, 0.76)	(0.4, 0.5333, 0.6666)	(0.72, 0.88, 0.92)	(0.52, 0.66, 0.76)	(0.66, 0.83, 0.88)	(0.4, 0.5333, 0.6666)
P_{17}	(0.64, 0.76, 0.84)	(0.24, 0.36, 0.5)	(0.4, 0.5333, 0.6666)	(0.6, 0.7, 0.8)	(0.48, 0.6, 0.72)	(0.48, 0.6, 0.72)	(0.3, 0.45, 0.6)
P_{18}	(0.22, 0.31, 0.42)	(0.14, 0.18, 0.28)	(0, 0, 0.1)	(0.32, 0.45, 0.58)	(0.18, 0.27, 0.4)	(0.14, 0.22, 0.34)	(0.0333, 0.0666, 0.1666)
P_{19}	(0.12, 0.21, 0.32)	(0.12, 0.14, 0.24)	(0, 0, 0.1)	(0.36, 0.51, 0.62)	(0.18, 0.27, 0.4)	(0.14, 0.22, 0.34)	(0.0333, 0.0666, 0.1666)
P_{20}	(0.22, 0.35, 0.48)	(0.14, 0.22, 0.34)	(0.2333, 0.3666, 0.5)	(0.36, 0.51, 0.62)	(0.2, 0.31, 0.44)	(0.26, 0.36, 0.48)	(0.0666, 0.1333, 0.2333)

Table 4. The fuzzy data set for 7 criteria and 33 respondents

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
P_{21}	(0.18, 0.27, 0.4)	(0.68, 0.82, 0.88)	(0.5666, 0.7166, 0.8)	(0.8, 1, 1)	(0.64, 0.8, 0.82)	(0.76, 0.94, 0.96)	(0.6666, 0.8, 0.8666)
P_{22}	(0.6, 0.7, 0.8)	(0.3, 0.4, 0.5)	(0.2666, 0.3666, 0.4666)	(0.54, 0.65, 0.76)	(0.38, 0.46, 0.56)	(0.44, 0.55, 0.66)	(0.2666, 0.3666, 0.4666)
P_{23}	(0.3, 0.45, 0.6)	(0.46, 0.61, 0.72)	(0.3, 0.45, 0.6)	(0.54, 0.65, 0.76)	(0.48, 0.56, 0.66)	(0.36, 0.46, 0.58)	(0.4, 0.5333, 0.6666)
P_{24}	(0.36, 0.5, 0.64)	(0.54, 0.65, 0.76)	(0.4, 0.5333, 0.6666)	(0.36, 0.51, 0.62)	(0.22, 0.35, 0.48)	(0.38, 0.5, 0.62)	(0.3333, 0.45, 0.5666)
P_{25}	(0.44, 0.55, 0.66)	(0.3, 0.38, 0.46)	(0.1333, 0.2166, 0.3333)	(0.76, 0.94, 0.96)	(0.4, 0.53, 0.6)	(0.62, 0.78, 0.82)	(0.0333, 0.0666, 0.1666)
P_{26}	(0.48, 0.61, 0.7)	(0.42, 0.52, 0.6)	(0.5, 0.6333, 0.7)	(0.66, 0.83, 0.88)	(0.4, 0.53, 0.6)	(0.54, 0.69, 0.74)	(0.0666, 0.1333, 0.2333)
P_{27}	(0.48, 0.6, 0.72)	(0.42, 0.55, 0.68)	(0.4, 0.5333, 0.6666)	(0.68, 0.82, 0.88)	(0.34, 0.45, 0.56)	(0.5, 0.63, 0.7)	(0.2333, 0.3666, 0.5)
P_{28}	(0.58, 0.71, 0.8)	(0.54, 0.66, 0.74)	(0.6666, 0.8, 0.8666)	(0.68, 0.82, 0.88)	(0.64, 0.76, 0.84)	(0.68, 0.82, 0.88)	(0.4333, 0.5333, 0.6333)
P_{29}	(0.64, 0.76, 0.84)	(0.58, 0.72, 0.78)	(0.6666, 0.8, 0.8666)	(0.54, 0.66, 0.74)	(0.5, 0.6, 0.7)	(0.68, 0.82, 0.88)	(0.4333, 0.5333, 0.6333)
P_{30}	(0.28, 0.4, 0.52)	(0.36, 0.46, 0.58)	(0.1333, 0.2166, 0.3333)	(0.42, 0.56, 0.66)	(0.32, 0.45, 0.58)	(0.22, 0.32, 0.4)	(0.4, 0.5333, 0.6666)
P_{31}	(0.38, 0.5, 0.62)	(0.16, 0.26, 0.38)	(0.3333, 0.45, 0.5666)	(0.68, 0.82, 0.88)	(0.44, 0.56, 0.64)	(0.4, 0.52, 0.62)	(0.0666, 0.1333, 0.2333)
P_{32}	(0.38, 0.5, 0.62)	(0.1, 0.17, 0.28)	(0.3333, 0.45, 0.5666)	(0.72, 0.88, 0.92)	(0.54, 0.66, 0.74)	(0.44, 0.58, 0.66)	(0.2333, 0.3, 0.4)
P_{33}	(0.44, 0.56, 0.64)	(0.18, 0.27, 0.4)	(0.1333, 0.2166, 0.3333)	(0.32, 0.45, 0.58)	(0.14, 0.25, 0.36)	(0.32, 0.45, 0.58)	(0.0333, 0.0666, 0.1666)

Shannon's entropy method consists of the following series of steps [15]:

Step 1. The data are normalized to remove abnormalities and obtain unit-free measurements of various criteria, to allow us to compare them. Suppose $(a_{ij}^{l}, a_{ij}^{m}, a_{ij}^{u})$ are the fuzzy data for the *i*th respondent's assessment of the *j*th component.

The normalized values of the decision matrix are obtained as below (Table 5):

$$(A_{ij}^{l}, A_{ij}^{m}, A_{ij}^{u}) = \left(\frac{a_{ij}^{l}}{\sum_{i=1}^{p} a_{ij}^{u}}, \frac{a_{ij}^{m}}{\sum_{i=1}^{p} a_{ij}^{m}}, \frac{a_{ij}^{u}}{\sum_{i=1}^{p} a_{ij}^{l}} \right), \; i = 1, 2, ..., n$$

Table 5. The normalized data

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
P_1	(0.0226, 0.0315, 0.0369)	(0.0295, 0.0472, 0.0431)	(0.0292, 0.0470, 0.0429)	(0.0205, 0.0291, 0.0300)	(0.0291, 0.0423, 0.0384)	(0.0215, 0.0335, 0.0337)	(0.0281, 0.0501, 0.0468)
P_2	(0.0178, 0.0258, 0.0322)	(0.0261, 0.0407, 0.0374)	(0.0117, 0.0229, 0.0253)	(0.0118, 0.0176, 0.0197)	(0.0093, 0.0186, 0.0218)	(0.0131, 0.0219, 0.0243)	(0.0210, 0.0360, 0.0351)
P_3	(0.0369, 0.0486, 0.0500)	(0.0124, 0.0207, 0.0215)	(0.0175, 0.0292, 0.0468)	(0.0190, 0.0265, 0.0285)	(0.0103, 0.0168, 0.0197)	(0.0131, 0.0219, 0.0243)	(0.0023, 0.0062, 0.0117)
P_4	(0.0107, 0.0189, 0.0453)	(0.0102, 0.0164, 0.0295)	(0.0078, 0.0165, 0.0195)	(0.0300, 0.0415, 0.0380)	(0.0270, 0.0423, 0.0374)	(0.0309, 0.0456, 0.0412)	(0.0023, 0.0062, 0.0117)
P_5	(0.0131, 0.0195, 0.0369)	(0.0192, 0.0336, 0.0329)	(0.0078, 0.0165, 0.0195)	(0.0316, 0.0441, 0.0395)	(0.0197, 0.0305, 0.0291)	(0.0281, 0.0428, 0.0393)	(0.0070, 0.0141, 0.0187)
P_6	(0.0214, 0.0296, 0.0369)	(0.0079, 0.0128, 0.0158)	(0.0156, 0.0254, 0.0234)	(0.0205, 0.0273, 0.0277)	(0.0103, 0.0168, 0.0197)	(0.0196, 0.0285, 0.0281)	(0.0023, 0.0062, 0.0117)
P_7	(0.0143, 0.0220, 0.0310)	(0.0181, 0.0293, 0.0295)	(0.0039, 0.0101, 0.0136)	(0.0285, 0.0388, 0.03640)	(0.0249, 0.0379, 0.0363)	(0.0140, 0.0208, 0.0299)	(0.0234, 0.0423, 0.0398)
P_8	(0.0143, 0.0227, 0.0226)	(0.0227, 0.040, 0.0385)	(0.0117, 0.0229, 0.0253)	(0.0166, 0.0243, 0.0269)	(0.0187, 0.0311, 0.0332)	(0.0168, 0.0274, 0.0299)	(0.0210, 0.0423, 0.0421)
P_9	(0.0226, 0.0315, 0.0250)	(0.0056, 0.0121, 0.0158)	(0.0195, 0.0343, 0.0332)	(0.0285, 0.0388, 0.0364)	(0.0228, 0.0348, 0.0332)	(0.0206, 0.0318, 0.0309)	(0.0046, 0.0125, 0.0163)

Table 5. The normalized data

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
P_{10}	(0.0166, 0.0252, 0.0357)	(0.0079, 0.0178, 0.0204)	(0.0097, 0.0216, 0.0234)	(0.0174, 0.0243, 0.0261)	(0.0197, 0.0286, 0.0291)	(0.0131, 0.0219, 0.0243)	(0.0187, 0.0344, 0.0327)
P_{11}	(0.0202, 0.0284, 0.0310)	(0.0090, 0.0185, 0.0215)	(0.0136, 0.0279, 0.0292)	(0.0190, 0.0265, 0.0285)	(0.0218, 0.0317, 0.0322)	(0.0131, 0.0219, 0.0243)	(0.0234, 0.0423, 0.0398)
P_{12}	(0.0345, 0.0448, 0.0393)	(0.0385, 0.0586, 0.0499)	(0.0371, 0.0623, 0.0507)	(0.0316, 0.0441, 0.0395)	(0.0374, 0.0547, 0.0478)	(0.0374, 0.0549, 0.0468)	(0.0468, 0.0752, 0.0608)
P_{13}	(0.0119, 0.0195, 0.0477)	(0.0113, 0.0193, 0.0215)	(0.0078, 0.0165, 0.0195)	(0.0047, 0.0092, 0.0126)	(0.0031, 0.0074, 0.0114)	(0.0084, 0.0131, 0.0149)	(0.0023, 0.0062, 0.0117)
P_{14}	(0.0178, 0.0258, 0.0453)	(0.0215, 0.0364, 0.0340)	(0.0136, 0.0279, 0.0292)	(0.0126, 0.0198, 0.0229)	(0.0072, 0.0155, 0.0187)	(0.0093, 0.0170, 0.0206)	(0.0117, 0.0266, 0.0281)
P_{15}	(0.0059, 0.0126, 0.0405)	(0.0147, 0.0257, 0.0272)	(0.0097, 0.0216, 0.0234)	(0.0213, 0.0287, 0.0300)	(0.0166, 0.0255, 0.0270)	(0.0206, 0.0302, 0.0309)	(0.0163, 0.0282, 0.0281)
P_{16}	(0.0322, 0.0410, 0.0429)	(0.0272, 0.0429, 0.0431)	(0.0234, 0.0407, 0.0390)	(0.0285, 0.0388, 0.0364)	(0.0270, 0.0410, 0.0395)	(0.0309, 0.0456, 0.0412)	(0.0281, 0.0501, 0.0468)
P_{17}	(0.0381, 0.0479, 0.0429)	(0.0136, 0.0257, 0.0283)	(0.0234, 0.0407, 0.0390)	(0.0237, 0.0309, 0.0316)	(0.0249, 0.0373, 0.0374)	(0.0224, 0.0329, 0.0337)	(0.0210, 0.0423, 0.0421)
P_{18}	(0.0131, 0.0195, 0.0405)	(0.0079, 0.0128, 0.0158)	(0, 0, 0.0058)	(0.0126, 0.0198, 0.0229)	(0.0093, 0.0168, 0.0207)	(0.0065, 0.0120, 0.0159)	(0.0023, 0.0062, 0.0117)
P_{19}	(0.0071, 0.0132, 0.0345)	(0.0068, 0.010, 0.0136)	(0, 0, 0.0058)	(0.0142, 0.0225, 0.0245)	(0.0093, 0.0168, 0.0207)	(0.0065, 0.0120, 0.0159)	(0.0023, 0.0062, 0.0117)
P_{20}	(0.0131, 0.0220, 0.031)	(0.0079, 0.0157, 0.0192)	(0.0136, 0.0279, 0.0292)	(0.0142, 0.0225, 0.0245)	(0.0103, 0.0193, 0.0228)	(0.0121, 0.0197, 0.0224)	(0.0046, 0.0125, 0.0163)
P_{21}	(0.0107, 0.0170, 0.0226)	(0.0385, 0.0586, 0.049)	(0.0332, 0.0547, 0.0468)	(0.0316, 0.0441, 0.0395)	(0.0332, 0.0498, 0.0426)	(0.0356, 0.0516, 0.0449)	(0.0468, 0.0752, 0.0608)
P_{22}	(0.0357, 0.0441, 0.0202)	(0.017, 0.0286, 0.0283)	(0.0156, 0.0279, 0.0273)	(0.0213, 0.0287, 0.0300)	(0.0197, 0.0286, 0.0291)	(0.0206, 0.0302, 0.0309)	(0.0187, 0.0344, 0.0327)
P_{23}	(0.0178, 0.0284, 0.02027)	(0.0261, 0.0436, 0.0408)	(0.0175, 0.0343, 0.0351)	(0.0213, 0.0287, 0.0300)	(0.0249, 0.0348, 0.0343)	(0.0168, 0.0252, 0.0271)	(0.0281, 0.0501, 0.0468)

Table 5. The normalized data

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
P_{24}	(0.0214, 0.0315, 0.0226)	(0.0306, 0.0464, 0.0431)	(0.0234, 0.0407, 0.0390)	(0.0142, 0.0225, 0.0245)	(0.0114, 0.0217, 0.0249)	(0.0178, 0.0274, 0.0290)	(0.0234, 0.0423, 0.0398)
P_{25}	(0.0262, 0.0347, 0.0286)	(0.0170, 0.0271, 0.0261)	(0.0078, 0.0165, 0.0195)	(0.0300, 0.0415, 0.0380)	(0.0207, 0.0330, 0.0311)	(0.0290, 0.0428, 0.0384)	(0.00234, 0.0062, 0.0117)
P_{26}	(0.0286, 0.0385, 0.0310)	(0.0238, 0.0371, 0.0340)	(0.0292, 0.0483, 0.0410)	(0.0261, 0.0366, 0.0348)	(0.0207, 0.0330, 0.0311)	(0.0253, 0.0379, 0.0346)	(0.0046, 0.0125, 0.0163)
P_{27}	(0.0286, 0.0378, 0.0250)	(0.0238, 0.0393, 0.0385)	(0.0234, 0.0407, 0.0390)	(0.0269, 0.0362, 0.0348)	(0.0176, 0.0280, 0.0291)	(0.0234, 0.0346, 0.0328)	(0.0163, 0.0344, 0.0351)
P_{28}	(0.03459, 0.0448, 0.0214)	(0.0306, 0.0472, 0.0419)	(0.0390, 0.0610, 0.0507)	(0.0269, 0.0362, 0.0348)	(0.0332, 0.0473, 0.0436)	(0.0318, 0.0450, 0.0412)	(0.0304, 0.0501, 0.0444)
P_{29}	(0.0381, 0.0479, 0.019)	(0.0329, 0.0515, 0.0442)	(0.0390, 0.0610, 0.0507)	(0.0213, 0.0291, 0.0292)	(0.0259, 0.0373, 0.0363)	(0.0318, 0.0450, 0.0412)	(0.0304, 0.0501, 0.0444)
P_{30}	(0.0166, 0.0252, 0.0119)	(0.0204, 0.0329, 0.0329)	(0.0078, 0.0165, 0.0195)	(0.0166, 0.0247, 0.0261)	(0.0166, 0.0280, 0.0301)	(0.0103, 0.0175, 0.0187)	(0.0281, 0.0501, 0.0468)
P_{31}	(0.0226, 0.0315, 0.0099)	(0.00908, 0.0185, 0.0215)	(0.0195, 0.0343, 0.0332)	(0.0269, 0.0362, 0.0348)	(0.0228, 0.0348, 0.0332)	(0.0187, 0.0285, 0.0290)	(0.0046, 0.0125, 0.0163)
P_{32}	(0.0226, 0.0315, 0.0119)	(0.0056, 0.0121, 0.0158)	(0.0195, 0.0343, 0.0332)	(0.0285, 0.0388, 0.0364)	(0.0280, 0.0410, 0.0384)	(0.0206, 0.0318, 0.0309)	(0.0163, 0.0282, 0.0281)
P_{33}	(0.0262, 0.0353, 0.0059)	(0.0102, 0.0193, 0.0227)	(0.0078, 0.0165, 0.0195)	(0.0126, 0.0198, 0.0229)	(0.0072, 0.0155, 0.0187)	(0.0149, 0.0247, 0.0271)	(0.0023, 0.0062, 0.0117)

Step 2. Let $E_j = \left(E_j^l, E_j^m, E_j^u\right)$ denote the entropy of the jth component. It is calculated using the following formulas to obtain Table 6, where k is the entropy constant and is equal to $-1/\ln p$.

If $A_{ij} = 0$, then $A_{ij}\ln A_{ij}$ is defined to be equal to 0.

$$E_j^l = \min\left\{-k\sum_{i=1}^{p} A_{ij}^l \ln A_{ij}^l, -k\sum_{i=1}^{p} A_{ij}^u \ln A_{ij}^u\right\}, i = 1, 2, ..., p, \; j = 1, 2, ..., n$$

$$E_j^m = -k\sum_{i=1}^{m} A_{ij}^m \ln A_{ij}^m, i = 1, 2, ..., p, \; j = 1, 2, ..., n$$

$$E_j^u = \max\left\{-k\sum_{i=1}^{p} A_{ij}^l \ln A_{ij}^l, -k\sum_{i=1}^{p} A_{ij}^u \ln A_{ij}^u\right\}, i = 1, 2, ..., p, \; j = 1, 2, ..., n$$

Table 6. Calculated values of $A_{ij}\ln A_{ij}$

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
P_1	(–0.0858, –0.1090, –0.1219)	(–0.1039, –0.1441, –0.1355)	(–0.1034, –0.1438, –0.1352)	(–0.0799, –0.1030, –0.1054)	(–0.1029, –0.1338, –0.1253)	(–0.0827, –0.1138, –0.1143)	(–0.1003, –0.1500, –0.1433)
P_2	(–0.0719, –0.0945, –0.1106)	(–0.0951, –0.1304, –0.1230)	(–0.0521, –0.0864, –0.0932)	(–0.0526, –0.0713, –0.0776)	(–0.0437, –0.0743, –0.0834)	(–0.0568, –0.0839, –0.0905)	(–0.0813, –0.1197, –0.1176)
P_3	(–0.1219, –0.1469, –0.1499)	(–0.0547, –0.0803, –0.0827)	(–0.0710, –0.1033, –0.1434)	(–0.0753, –0.0962, –0.1014)	(–0.0474, –0.0686, –0.0775)	(–0.0568, –0.0839, –0.0905)	(–0.0141, –0.0317, –0.0520)
P_4	(–0.0486, –0.0751, –0.1402)	(–0.0468, –0.0675, –0.1039)	(–0.0379, –0.0678, –0.0768)	(–0.1054, –0.1321, –0.1242)	(–0.0975, –0.1338, –0.1229)	(–0.1075, –0.1408, –0.1314)	(–0.0141, –0.0317, –0.0520)
P_5	(–0.0568, –0.0769, –0.1219)	(–0.0761, –0.1140, –0.1123)	(–0.0379, –0.0678, –0.0768)	(–0.1093, –0.1378, –0.1278)	(–0.0775, –0.1064, –0.1029)	(–0.1004, –0.1350, –0.1273)	(–0.0348, –0.0601, –0.0745)
P_6	(–0.0824, –0.1043, –0.1219)	(–0.0384, –0.0560, –0.0658)	(–0.0649, –0.0934, –0.0879)	(–0.0799, –0.0985, –0.0993)	(–0.0474, –0.0686, –0.0775)	(–0.0773, –0.1016, –0.1004)	(–0.0141, –0.0317, –0.0520)
P_7	(–0.0607, –0.0842, –0.1077)	(–0.0727, –0.1035, –0.1039)	(–0.0216, –0.0466, –0.0586)	(–0.1014, –0.1262, –0.1206)	(–0.0920, –0.1242, –0.1205)	(–0.0599, –0.0808, –0.1051)	(–0.0879, –0.1338, –0.1283)
P_8	(–0.0607, –0.0860, –0.0858)	(–0.0859, –0.1288, –0.1256)	(–0.0521, –0.0864, –0.0932)	(–0.0681, –0.0903, –0.0973)	(–0.0744, –0.1080, –0.1132)	(–0.0688, –0.0987, –0.1051)	(–0.0813, –0.1338, –0.1334)
P_9	(–0.0858, –0.1090, –0.0923)	(–0.0293, –0.0536, –0.0658)	(–0.0768, –0.1158, –0.1130)	(–0.1014, –0.1262, –0.1206)	(–0.0863, –0.1170, –0.1132)	(–0.0800, –0.1098, –0.1075)	(–0.0251, –0.0549, –0.0673)
P_{10}	(–0.0683, –0.0928, –0.1191)	(–0.0384, –0.0719, –0.0794)	(–0.0452, –0.0829, –0.0879)	(–0.0705, –0.0903, –0.0952)	(–0.0775, –0.1017, –0.1029)	(–0.0568, –0.0839, –0.0905)	(–0.0745, –0.1161, –0.1120)
P_{11}	(–0.0790, –0.1011, –0.1077)	(–0.0426, –0.0741, –0.0827)	(–0.0586, –0.1000, –0.1034)	(–0.0753, –0.0962, –0.1014)	(–0.0834, –0.1095, –0.1106)	(–0.0568, –0.0839, –0.0905)	(–0.0879, –0.1338, –0.1283)
P_{12}	9–0.1163, –0.1391, –0.1273)	(–0.1256, –0.1663, –0.1496)	(–0.1222, –0.1730, –0.1513)	(–0.1093, –0.1378, –0.1278)	(–0.1229, –0.1591, –0.1453)	(–0.1231, –0.1594, –0.1434)	(–0.1433, –0.1946, –0.1704)
P_{13}	(–0.0528, –0.0769, –0.1451)	(–0.0508, –0.0762, –0.0827)	(–0.0379, –0.0678, –0.0768)	(–0.0254, –0.0434, –0.0553)	(–0.0179, –0.0365, –0.0511)	(–0.0402, –0.0571, –0.0629)	(–0.0141, –0.0317, –0.0520)
P_{14}	(–0.0719, –0.0945, –0.1402)	(–0.0827, –0.1207, –0.1150)	(–0.0586, –0.1000, –0.1034)	(–0.0553, –0.0779, –0.0866)	(–0.0358, –0.0647, –0.0744)	(–0.0437, –0.0693, –0.0800)	(–0.0520, –0.0965, –0.1003)

Table 6. Calculated values of $A_{ij} \ln A_{ij}$

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
P_{15}	(–0.0305, –0.0552, –0.1299)	(–0.0622, –0.0942, –0.0981)	(–0.0452, –0.0829, –0.0879)	(–0.0822, –0.1019, –0.1054)	(–0.0681, –0.0936, –0.0975)	(–0.0800, –0.1057, –0.1075)	(–0.0673, –0.1006, –0.1003)
P_{16}	(–0.1106, –0.1310, –0.1351)	(–0.0981, –0.1351, –0.1355)	(–0.0879, –0.1303, –0.1266)	(–0.1014, –0.1262, –0.1206)	(–0.0975, –0.1311, –0.1276)	(–0.1075, –0.1408, –0.1314)	(–0.1003, –0.1500, –0.1433)
P_{17}	(–0.1246, –0.1457, –0.1351)	(–0.0585, –0.0942, –0.1010)	(–0.0879, –0.1303, –0.1266)	(–0.0888, –0.1075, –0.1093)	(–0.0920, –0.1228, –0.1229)	(–0.0853, –0.1125, –0.1143)	(–0.0813, –0.1338, –0.1334)
P_{18}	(–0.0568, –0.0769, –0.1299)	(–0.0384, –0.0560, –0.0658)	(0, 0, –0.0301)	(–0.0553, –0.0779, –0.0866)	(–0.0437, –0.0686, –0.0805)	(–0.0329, –0.0533, –0.0659)	(–0.0141, –0.0317, –0.0520)
P_{19}	(–0.0353, –0.0573, –0.1163)	(–0.0339, –0.0461, –0.0585)	(0, 0, –0.0301)	(–0.0605, –0.0854, –0.0909)	(–0.0437, –0.0686, –0.0805)	(–0.0329, –0.0533, –0.0659)	(–0.0141, –0.0317, –0.0520)
P_{20}	(–0.0568, –0.0842, –0.1077)	(–0.0384, –0.0653, –0.0761)	(–0.0586, –0.1000, –0.1034)	(–0.0605, –0.0854, –0.0909)	(–0.0474, –0.0761, –0.0863)	(–0.0537, –0.0776, –0.0853)	(–0.0251, –0.0549, –0.0673)
P_{21}	(–0.0486, –0.0694, –0.0858)	(–0.1256, –0.1663, –0.1496)	(–0.1130, –0.1589, –0.1434)	(–0.1093, –0.1378, –0.1278)	(–0.1132, –0.1494, –0.1344)	(–0.1187, –0.1531, –0.1395)	(–0.1433, –0.1946, –0.1704)
P_{22}	(–0.1191, –0.1378, –0.0790)	(–0.0693, –0.1016, –0.1010)	(–0.0649, –0.1000, –0.0984)	(–0.0822, –0.1019, –0.1054)	(–0.0775, –0.1017, –0.1029)	(–0.0800, –0.1057, –0.1075)	(–0.0745, –0.1161, –0.1120)
P_{23}	(–0.0719, –0.1011, –0.0790)	(–0.0951, –0.1366, –0.1306)	(–0.0710, –0.1158, –0.1177)	(–0.0822, –0.1019, –0.1054)	(–0.0920, –0.1170, –0.1156)	(–0.0688, –0.0929, –0.0979)	(–0.1003, –0.1500, –0.1433)
P_{24}	(–0.0824, –0.1090, –0.0858)	(–0.1068, –0.1426, –0.1355)	(–0.0879, –0.1303, –0.1266)	(–0.0605, –0.0854, –0.0909)	(–0.0511, –0.0833, –0.0920)	(–0.0717, –0.0987, –0.1028)	(–0.0879, –0.1338, –0.1283)
P_{25}	(–0.0955, –0.1166, –0.1017)	(–0.0693, –0.0979, –0.0951)	(–0.0379, –0.0678, –0.0768)	(–0.1054, –0.1321, –0.1242)	(–0.0805, –0.1125, –0.1081)	(–0.1028, –0.1350, –0.1252)	(–0.0141, –0.0317, –0.0520)
P_{26}	(–0.1017, –0.1254, –0.1077)	(–0.0890, –0.1224, –0.1150)	(–0.1034, –0.1464, –0.1309)	(–0.0952, –0.1212, –0.1169)	(–0.0805, –0.1125, –0.1081)	(–0.0930, –0.1241, –0.1165)	(–0.0251, –0.0549, –0.0673)
P_{27}	(–0.1017, –0.1239, –0.0923)	(–0.0890, –0.1272, –0.1256)	(–0.0879, –0.1303, –0.1266)	(–0.0973, –0.1202, –0.1169)	(–0.0713, –0.1001, –0.1029)	(–0.0879, –0.1164, –0.1120)	(–0.0673, –0.1161, –0.1176)
P_{28}	(–0.1163, –0.1391, –0.0824)	(–0.1068, –0.1441, –0.1331)	(–0.1266, –0.1707, –0.1513)	(–0.0973, –0.1202, –0.1169)	(–0.1132, –0.1443, –0.1367)	(–0.1098, –0.1397, –0.1314)	(–0.1063, –0.1500, –0.1384)

Table 6. Calculated values of $A_{ij} \ln A_{ij}$

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
P_{29}	(–0.1654, –0.1457, –0.0662)	(–0.1591, –0.1527, –0.1747)	(–0.1676, –0.1707, –0.1792)	(–0.0908, –0.1030, –0.1086)	(–0.1186, –0.1228, –0.1374)	(–0.1404, –0.1397, –0.1564)	(–0.1632, –0.1500, –0.1829)
P_{30}	(–0.0683, –0.0928, –0.0528)	(–0.0794, –0.1123, –0.1123)	(–0.0379, –0.0678, –0.0768)	(–0.0681, –0.0915, –0.0952)	(–0.0681, –0.1001, –0.1055)	(–0.0471, –0.0710, –0.0745)	(–0.1003, –0.1500, –0.1433)
P_{31}	(–0.0858, –0.1090, –0.0458)	(–0.0426, –0.0741, –0.0827)	(–0.0768, –0.1158, –0.1130)	(–0.0973, –0.1202, –0.1169)	(–0.0863, –0.1170, –0.1132)	(–0.0745, –0.1016, –0.1028)	(–0.0251, –0.0549, –0.0673)
P_{32}	(–0.0858, –0.1090, –0.0528)	(–0.0293, –0.0536, –0.0658)	(–0.0768, –0.1158, –0.1130)	(–0.1014, –0.1262, –0.1206)	(–0.1002, –0.1311, –0.1253)	(–0.0800, –0.1098, –0.1075)	(–0.0673, –0.1006, –0.1003)
P_{33}	(–0.0955, –0.1181, –0.0305)	(–0.0468, –0.0762, –0.0859)	(–0.0379, –0.0678, –0.0768)	(–0.0553, –0.0779, –0.0866)	(–0.0358, –0.0647, –0.0744)	(–0.0629, –0.0915, –0.0979)	(–0.0141, –0.0317, –0.0520)

Step 3. The degree of diversification $d_j = \left(d_j^l, d_j^m, d_j^u\right)$ is obtained using the following equations:

$$d_j^l = 1 - E_j^u,\ j = 1, 2, ..., n$$

$$d_j^m = 1 - E_j^m,\ j = 1, 2, ..., n$$

$$d_j^u = 1 - E_j^l,\ j = 1, 2, ..., n$$

Step 4. The fuzzy weight of the *j*th component $w_j = \left(w_j^l, w_j^m, w_j^u\right)$ is computed as (Table 7):

$$w_j^l = \frac{d_j^l}{\sum_{i=1}^{n} d_j^u},\ \ j = 1, 2, ..., n$$

$$w_j^m = \frac{d_j^m}{\sum_{i=1}^{n} d_j^m},\ \ j = 1, 2, ..., n$$

$$w_j^u = \frac{d_j^u}{\sum_{i=1}^{n} d_j^u},\ j = 1, 2, ..., n$$

Table 7. Entropy, the degree of diversification and weight of the criteria

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
E_j	(0.777, 0.9836, 0.9749)	(0.68128, 0.9688, 0.9928)	(0.63228, 0.9546, 0.9832)	(0.7725, 0.9874, 0.9946)	(0.7118, 0.9796, 0.9935)	(0.7270, 0.9798, 0.9961)	(0.6056, 0.9321, 0.9749)
d_j	(0.025, 0.0163, 0.2228)	(0.0071, 0.03119, 0.3187)	(0.0167, 0.0453, 0.3677)	(0.0053, 0.0125, 0.2274)	(0.0064, 0.0203, 0.2881)	(0.0038, 0.02015, 0.2729)	(0.02504, 0.0678, 0.4410)
W_j	(0.0117, 0.0763, 2.489)	(0.0033, 0.1459, 3.557)	(0.00736, 0.21214, 4.107)	(0.00247, 0.0588, 2.5408)	(0.003, 0.095, 3.2187)	(0.0018, 0.09428, 3.0486)	(0.0119, 0.3174, 4.9258)

The aggregated fuzzy rating for a component of antifragility is computed using the data from Table 3 and $R_j = (l_j, m_j, u_j) = (R_{1j} + R_{2j} + ... + R_{nj})/n.$ These ratings are presented in Table 8.

Table 8. Fuzzy average weight and fuzzy average rating for each component of antifragility

Component	Fuzzy average rating	Fuzzy average weight
Absorption	(0.3636, 0.48, 0.5080)	(0.0117, 0.0763,2.4891)
Redundancy	(0.3230, 0.4236, 0.5339)	(0.0033, 0.1459, 3.557)
Introduction of stressors	(0.2898, 0.3969, 0.5171)	(0.00736, 0.21214, 4.107)
Non-monotonicity	(0.5454, 0.6857, 0.7654)	(0.00247, 0.0588, 2.5408)
Requisite variety	(0.3745, 0.4866, 0.5830)	(0.003, 0.095, 3.2187)
Emergence	(0.4242, 0.5512, 0.6466)	(0.0018, 0.09428, 3.0486)
Uncoupling	(0.2343, 0.3222, 0.4313)	(0.0119, 0.3174, 4.9258)

3.3. Obtaining the fuzzy antifragility index (FAI) [16]

The average fuzzy weights and the average fuzzy ratings were used to calculate a fuzzy antifragility index (*FAI*) according to the equation below:

$$FAI = \frac{\sum_{j=1}^{n} w_j R_j}{\sum_{j=1}^{n} w_j} = \frac{N}{D}$$

where

$$N = (0.3636,\ 0.48,\ 0.5080) \times (0.0117,\ 0.0763,\ 2.4891)$$
$$+\ (0.3230,\ 0.4236,\ 0.5339) \times (0.0033,\ 0.1459,\ 3.557)$$
$$+\ (0.2898,\ 0.3969,\ 0.5171) \times (0.00736,\ 0.21214,\ 4.107)$$
$$+\ (0.5454,\ 0.6857,\ 0.7654) \times (0.00247,\ 0.0588,\ 2.5408)$$
$$+\ (0.3745,\ 0.4866,\ 0.5830) \times (0.003,\ 0.095,\ 3.2187)$$
$$+\ (0.4242,\ 0.5512,\ 0.6466) \times (0.0018,\ 0.09428,\ 3.0486)$$
$$+\ (0.2343,\ 0.3222,\ 0.4313) \times (0.0119,\ 0.3174,\ 4.9258)$$

$$D = (0.0117,\ 0.0763,\ 2.4891) + (0.0033,\ 0.1459,\ 3.557)$$
$$+ (0.00736,\ 0.21214,\ 4.107) + (0.00247,\ 0.0588,\ 2.5408)$$
$$+ (0.003,\ 0.095,\ 3.2187) + (0.0018,\ 0.09428,\ 3.0486)$$
$$+ (0.0119,\ 0.3174,\ 4.9258)$$

Therefore, we have: *FAI* = (0.3244, 0.4235, 0.5528).

3.4. Computing a Euclidean distance to compare the *FAI* with the linguistic terms

When the *FAI* has been calculated, it can be compared with the linguistic terms. Here, minimisation of the Euclidean distance was applied for this purpose, because it is the most intuitive technique to understand proximity.

The linguistic terms that are used in this research are described in Table 9, together with the corresponding fuzzy numbers:

Table 9. The linguistic terms describing the level of antifragility and the corresponding triangular fuzzy numbers[16]

Slightly antifragile	Fairly antifragile	Satisfactorily antifragile	Very antifragile	Extremely antifragile
(0.0, 0.1, 0.1)	(0.1, 0.2, 0.3)	(0.3, 0.45, 0.6)	(0.6, 0.7, 0.8)	(0.8, 1.0, 1.0)

The Euclidean distance is calculated as described below:

$$D\left(FAI, AL_i\right) = \sqrt{\sum\left(f_{FAI}\left(x\right) - f_{AL_i}\left(x\right)\right)^2}$$

$$D\left(FAI, SA\right) = \sqrt{\left(0.3244 - 0\right)^2 + \left(0.4235 - 0\right)^2 + \left(0.5528 - 0.1\right)^2} = 0.6908$$

$$D(FAI, SA) = \sqrt{\left(0.3244 - 0\right)^2 + \left(0.4235 - 0\right)^2 + \left(0.5528 - 0.1\right)^2} = 0.6998$$

$$D(FAI, FA) = \sqrt{\left(0.3244 - 0.1\right)^2 + \left(0.4235 - 0.2\right)^2 + \left(0.5528 - 0.3\right)^2} = 0.4053$$

$$D(FAI, A) = \sqrt{\left(0.3244 - 0.3\right)^2 + \left(0.4235 - 0.45\right)^2 + \left(0.5528 - 0.6\right)^2} = 0.0594$$

$$D(FAI, VA) = \sqrt{\left(0.3244 - 0.6\right)^2 + \left(0.4235 - 0.7\right)^2 + \left(0.5528 - 0.8\right)^2} = 0.4620$$

$$D(FAI, EA) = \sqrt{\left(0.3244 - 0.8\right)^2 + \left(0.4235 - 1\right)^2 + \left(0.5528 - 1\right)^2} = 0.8709$$

As can be seen, the minimum Euclidean distance corresponds to the linguistic term satisfactorily antifragile. Therefore, it is concluded that the organisation's degree of antifragility is medium level.

4. Conclusions

This paper has contributed a model for assessing antifragility based on fuzzy logic. A case study was carried out in the Iranian manufacturer of security paper, TAKAB.

An impressive trait of this model for measuring antifragility is that it is combined with the approach of fuzzy logic, which allows the use of linguistic terms to assess criteria of antifragility.

According to our research findings, it seems that the degree of antifragility in the Iranian firm studied is medium level. Although this might be considered sufficient, structures, systems, processes, and cultures could be developed to make the organization more antifragile. The authors are investigating an approach using linear regression to indicate whether the system is moving in the right direction to increase antifragility.

Because of time constraints, our model for assessing antifragility was only applied to a single organization. In future, many case studies could be performed in different organizations to improve the framework for analysing antifragility. More work is required to determine standards for selecting the assessment criteria, the role that particular criteria play in specific industries and techniques for aggregating the outcomes of assessment. In addition, more criteria could be added, so that the measurements provide more information. By more criteria, we mean that some criteria of antifragility could be considered in addition to the seven criteria that we applied here, depending on the type of industry/company considered. For example, when the company surveyed is a food plant, a component related to food spoilage could be considered.

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Appendix 1. Questionnaire of antifragility

To what extent do you agree or disagree with the following statements for your organization?
(strongly disagree – 1, disagree – 2, neither agree nor disagree – 3, agree – 4, strongly agree – 5)

Absorption	1	2	3	4	5
Our organization readily responds to changes in our business environment.					
Our staff have the information and knowledge they need to respond to unexpected problems.					
We have planned for what support we could provide to the community in a crisis.					
Our organization maintains sufficient resources to absorb some unexpected change.					
We are able to shift rapidly from business-as-usual to respond to crises.					
Redundancy	1	2	3	4	5
In a crisis, we have agreements with other organizations to access resources from them.					
If key people were unavailable, always others could fill their role.					
When a problem occurs, it is easier to get approval for additional resources to get job done.					
Critical information is available by different means and from different locations.					

If you are building or work area, were inaccessible due to physical damage or a hazard where would you replace? (Please check the option closest to the arrangements that your organization has):

We would not relocate.
We would arrange for our staff to work from home although we have not planned or practiced this.
We have plans (that have already been tested) for our staff to work from home.
A temporary building or office that we would arrange when needed.
A temporary building or office that we have already arranged.

Induced small stressors	1	2	3	4	5
Staff can take time from their day-to-day roles to practice how to respond in a crisis.					
Our organization has some plans to create small crisis at regular intervals to be ready to deal with major crisis.					
In our organization's culture, small problems are considered as the vaccine that stop major crisis from becoming disaster.					
Non-monotonicity	1	2	3	4	5
We learn lessons from past projects and make sure those lessons are carried through to future projects.					
In a crisis, we seek opportunities for our organization.					
We tend to be optimistic and find positives from most situations.					
Whenever our organization suffers a narrow escape, we use it for self-evaluation rather than confirmation of our success.					

How would you feel about change?

I t scares me because I do not have complete information.
I find it frustrating because I cannot control it.
It is inevitable, but I wish I could avoid it.
It is inevitable, so I just deal it.
I enjoy challenge.

Requisite variety	1	2	3	4	5
Problems should occur; Staff have direct access to someone with authority to make decisions.					
In our organization, the most qualified people make decisions, regardless of seniority.					
We readily obtain expert assistance when there is a problem.					
We plan our strategy carefully before taking action.					
Emergence	1	2	3	4	5
Government's actions would affect our ability to respond in a crisis.					
There are few barriers stopping us from working well with each other and with other organizations.					
We build relationships with organizations we might have to work with in a crisis.					
The results of one area reflects on the total result of the organization.					
When a phenomenon influences on your industry, to what extent can impress on your organization? (1 = very low, 2 = low, medium, 3 = high, 4 = very high)					
Uncoupling	1	2	3	4	5
The success or failure of one area of our organization strongly depends on the success or failure of another.					
A crisis in our organization would extremely influence others.					

Please rate how severe your organization's most recent crisis was for your department.

We dealt it as part of business-as-usual.
It challenged us but was not overly disruptive.
It definitely challenged us and was moderately disruptive.
It definitely challenged us and was very disruptive.
It could have shut us down permanently.

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