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Simulating the propagation of rumours with spreaders of distinct characters

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Abstract

The occurrence of a disaster and the appearance of a new product are items of news that can reach many people rapidly. Decision-makers would like to know how many individuals will obtain such information. In this paper, we study the case in which there are spreaders who think that the news is true and others that claim it is false. We simulate the propagation of a rumour and propose an algorithm for estimating the parameters of the resulting dynamical system. The relation between the parameters defining the simulation and the dynamical system is also studied. We conclude that the simulation describes the phenomenon well, in particular when the method proposed in this paper is used to estimate the parameters.

Keywords: *propagation, rumour, computer simulation, dynamical system, parameter estimation*

1. Introduction

The study of the rumour propagation phenomenon in social media is very important. In fact, platforms like Twitter or Facebook generate a lot of news that may reach many people, via friends, friends of friends, and so on. In the case of fake news, see some examples in [6, 8, 14], the consequences may be extremely dangerous. So, it is important to study the diffusion of a rumour.

Since 1965, see [3], many papers have been devoted to modelling this problem. As reported in [10], the number of papers on this topic has an exponential growth since 2009. Many variants have been considered: the existence of two populations, two rumours, the influence of social media, etc. [1, 5, 11]. In [2, 7] a generic social network is considered. Its population is divided into three classes: people that do not know about the rumour (ignorants), those who spread it (spreaders), and the individuals who know the rumour but do not spread it (stiflers). In this class, the positive or negative character concerning the rumour is taken into account. Indeed, people are more willing to propagate the rumour if they get it from an individual who believes it is true. Spreaders are more active than stiflers. So, different dynamics can

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be introduced if spreaders are differentiated. That is why in this article we divide this class into two types: the positive spreaders, i.e., those individuals who spread the information believing it is true, and the negative spreaders or those who share the information but express clearly that they think it is false.

This work aims to simulate the propagation of the rumour considering four classes of individuals: ignorants, positive spreaders, negative spreaders and stiflers. Given a network and a fixed interval of time, a number of encounters are randomly generated between individuals chosen according to the Poisson law. After a particular encounter takes place, the status of the involved individuals is updated using the rates that define the probabilities of changing from one class to another. As output, the simulation provides the number of ignorants, positive spreaders, negative spreaders and stiflers at the end of each period. Using this data as observations of the phenomenon, the parameters of the dynamical system are fitted. The auxiliary ordinary differential equations systems will be solved using a Runge-Kutta method as in [13]. Different examples show the out-coming differences when rates vary.

The paper is organized as follows. Section 2 describes the dynamics of the rumour and presents the corresponding differential equations system. Then the simulation model is described. Section 4 introduces the parameter fitting approach. The numerical experience is presented in Section 5. The paper ends with some final remarks.

2. Rumour dynamics

In this part, we will present the dynamics of the rumour in a closed population,i.e., where individuals do not leave or enter the population but can change their character during the analysed period. As already mentioned, the population will be divided into four groups:

- I – set of individuals who do not know the rumour or are ignorant,
- S^+ – set of individuals who know and spread the rumour and states it is true,
- S^- – set of individuals who know and divulge the rumour and give negative criteria about its veracity,
- R – set those individuals who know the rumour but are not interested in spreading it.

The respective densities are I_0, S_0^+, S_0^- and R_0.

We assume that an individual cannot change their character alone. A meeting between two people is needed. We suppose that only the following possible combinations lead to a change in the character of the person. The corresponding rates will be:

α – rate at which an ignorant becomes a positive spreader after meeting a positive spreader,

β – rate at which an ignorant becomes a negative spreader after meeting a negative spreader,

γ_1 – rate at which an ignorant becomes a stifler after meeting a positive spreader,

γ_2 – rate at which an ignorant becomes a stifler after meeting a negative spreader,

θ – rate at which a positive spreader becomes a stifler after a meeting,

φ – rate at which a negative spreader becomes a stifler after a meeting,

ϕ – rate at which a positive spreader becomes a negative spreader after a meeting.

Figure 1 shows possible state transitions after an encounter. For each case, a double arrow represents the meeting of two individuals. The transitions are represented by directed arcs whose origins are the initial states of the individuals and their destiny, the final one. The letter associated with these arcs is the rate at which this transition takes place.

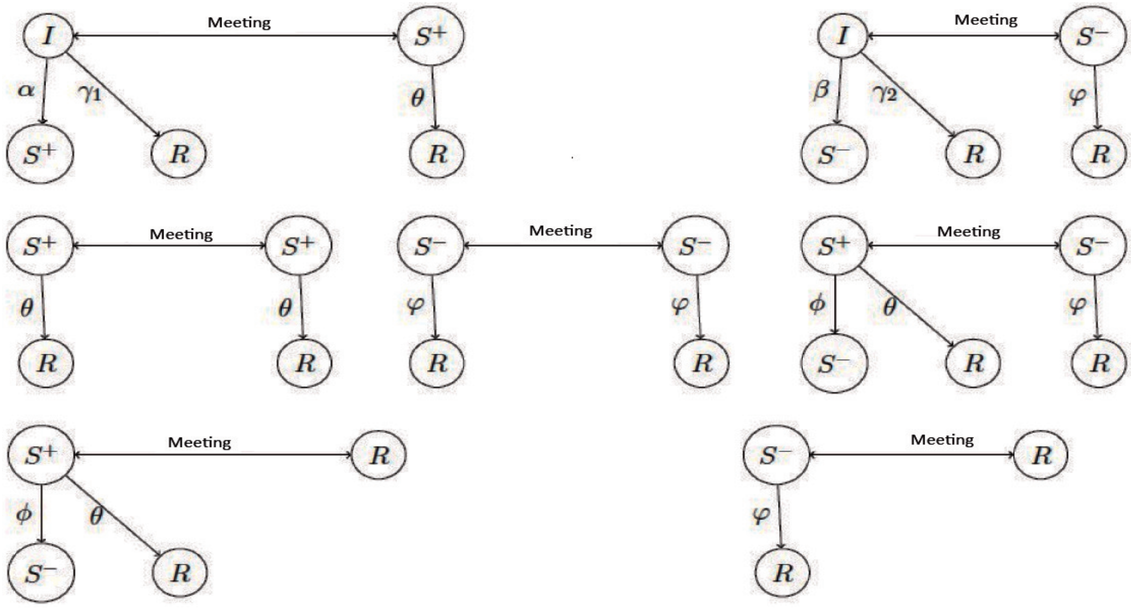

Figure 1. Transitions

Note that both γ_1 and γ_2 represent changes of the same nature, from ignorant to indifferent, but we consider different rates. Indeed, it is expected that after meeting a positive spreader, the individual will be more eager to spread the information than to remain indifferent. On the other hand, if the information is received from a negative spreader, the probability of becoming a stifler should be larger. So, we expect that in applications $\gamma_1 < \gamma_2$.

Considering the densities and taking into account that

$$I_0(t) + S_0^+(t) + S_0^-(t) + R_0(t) = 1$$

the dynamic system describing the problem is:

$$\begin{aligned}
\dot{I}_0(t) &= -\left(\gamma_1 + \alpha(t)\right) I_0(t)S_0^+(t) - \left(\gamma_2 + \beta(t)\right) I_0(t)S_0^-(t) \\
\dot{S}_0^+(t) &= \alpha(t)I_0(t)S_0^+(t) - \phi(t)S_0^+(t)S_0^-(t) - \phi(t)S_0^+(t)R_0(t) - \theta(t)S_0^+ \\
\dot{S}_0^-(t) &= \beta(t)I_0(t)S_0^-(t) + \phi(t)S_0^+(t)S_0^-(t) + \phi(t)S_0^+(t)R_0(t) - \varphi(t)S_0^- \\
\dot{R}_0(t) &= \gamma_1(t)I_0(t)S_0^+ + \gamma_2(t)I_0(t)S^-(t) + \theta(t)S_0^+(t) + \varphi(t)S_0^-(t)
\end{aligned} \tag{1}$$

For more details see [9]

3. Simulation of the rumour

In this Section, we will present a probabilistic simulation model that describes the propagation of the rumour. We assume we have d days of length T/d and a population of N individuals. The mean of the number of daily encounters (λ) and the transition probabilities are also given. We included the case in which the transition probabilities are given as $pr = \begin{cases} pr_0 & \text{if } i < d_0 \\ pr_1 & \text{otherwise} \end{cases}$, that is during the first $d_0 - 1$ days the rumour will be propagated with certain probabilities and then it changes. This includes news which is trendy at the beginning and after some days they lose popularity.

Before presenting the simulation, we fix the following notation. For each $i = 1, \ldots, d$, $k = 1, \ldots, N$, we say that $P_i^k = 0$ (respectively 1, 2, 3) if individual k is an ignorant (respectively a positive spreader, a negative spreader or a stifler) at the end of the day $i,\ i = 1, \ldots, d$. So, P_i is an N-dimensional vector whose components are in $\{0, 1, 2, 3\}$. The number of times that 0 appears in P_i represents the number of ignorants at the end of the day $i,\ i = 1, \ldots, d$. We will denote it as $\mathcal{I}$. Analogously, for the number of positive spreaders (respectively, negative spreaders and stiflers), the notation $\mathcal{S}_i^+$, (respectively, $\mathcal{S}_i^-$, and $\mathcal{R}_i$) is used. $\mathcal{P}_i \in \mathbb{Z}^4$, denote the state of the population at the end of the day $i,\ i = 1, \ldots, d$, i.e., $\mathcal{P}_1 = (\mathcal{I}_1, \mathcal{S}_1^+, \mathcal{S}_1^-, \mathcal{R}_1), \ldots, \mathcal{P}_d = (\mathcal{I}_d, \mathcal{S}_d^+, \mathcal{S}_d^-, \mathcal{R}_d)$.

For the simulation, the following functions were implemented.

Function *Graf_pro*. It is the main function. The input data are the previously defined parameters $(d, \lambda, d_0, pr_0, pr_1)$ and M, the times the simulation will be run. With these data, $Simula\_pro$ is called. Its numerical output is used by function $Stat\_pro$ to generate the graphics corresponding to the statistical analysis.

Function *Simula_pro*. For each $m = 1, \ldots M$, *Simula_pro* calls function *Enc_final* and collects its result, i.e., vectors $(P_i)^m \in \{0,\ 1, 2,\ 3\}^N$, for $i = 1, \ldots, d$. From $(P_i)^m$, $(\mathcal{P}_i)^m$ is computed.

Function *Enc_final*. First, this function generates a network connecting individuals. In one case, we assume all combinations are possible. The second option is a random graph and the last one is a small world network, for more details see [4]. Then, the initial population is generated.

Using an integer uniform distribution in $[1, N]$, an individual is randomly chosen as the initial positive spreader, while the others are ignorants. For instance, if the $\hat{k}\mathrm{th}$ individual is chosen, the initial population will be given by the vector $P_0 = \{p\}_{i=\overline{1,N}}$, where $P_0^k = 0\ \ \forall k \neq \hat{k}$, and $P_0^{\hat{k}} = 1$.

Then, *Enc_final* calls *genera_enc* for generating v_i, the number of encounters vector at day i.

Taking as input the initial population $\mathcal{P}_0$, the encounters vector $(v_1, v_2, \ldots, v_{d_0-1})$, pr_0 and $d_0 - 1$, the number of days, function *Enc2_pro* is called. As output, $(P_1, \ldots, P_{d_0-1})$ is obtained. Using P_{d_0-1} as the initial population of the second period, the encounters vector $(v_{d_0}, v_{d_0+1}, \ldots, v_d)$, the second set of rates pr_1 and $d - \overline{d} + 1$, the number of remaining days, *Enc_final* calls *Enc2_pro* again. The resulting output $(P_{d_0}, \ldots, P_d)$, completes the population for the d days of the simulation.

Function *genera_enc*. The input variables of this function are d and λ, which represent the number of days and the mean of the number of encounters that will occur in each day, respectively. The output is the vector $V = \{v_1, v_2, \ldots, v_d\}$, where $v_i,\ i = \overline{1, d}$ stands for the number of encounters on day i. It is generated according to the Poisson law with parameter λ. Of course, an encounter between individuals i and j is considered if the network generated in *Enc_final* includes arc (i, j).

Function *Enc2_pro*. The input variables are the initial population P_0, the encounters vector $(v_1, \ldots, v_d\}$, the transition probabilities pr and d, the number of days.

From this data, for every day $i,\ i = 1, \ldots, d$ and given P_{i-1}, the population at the beginning of day

i, the function generates a random matrix $\Phi_i = \{\phi\}_{lj}$, $l = 1, \ldots, v_l$, $j = 1, 2,$. Here the lth encounter of day i will be held between the individual ϕ_{11} of type $P_{i-1}^{\phi_{l1}}$ and ϕ_{l2} of type $P_{i-1}^{\phi_{l2}}$. According to the dynamics shown in Figure 1, the type of the individual changes according to the probability given by the corresponding component of pr. So, population P_i is generated. Roughly speaking, for each day, the resulting population is a consequence of the simulated encounters and the possible new status of the involved individuals. $(P_1, \ldots, P_d)$ is the final output of this function.

Function *Stat_pro.* It performs a basic statistical analysis of the results. Consider for each $i = 1, \ldots, d$, the observations $\mathcal{P}_i^1, \ldots, \mathcal{P}_i^M \in \mathbb{Z}^4$. The mean value, the standard deviation and the 99% confidence interval of the mean of each component are computed and represented graphically. As usual the confidence interval of X is

$$CI_X = [\overline{X} - H_X,\ \overline{X} + H_X] \tag{2}$$

where $\overline{X}$ and s are the estimations of the mean value and the standard deviation, respectively, and $H_X = t_{(K-1)_{(0.001)}} \dfrac{s}{\sqrt{K}}$, where $t_{(K-1)_{(0.001)}}$ is the 0.001 percentile of the t-student with $K-1$ degrees of freedom. The sequence of steps of the function is shown in the graphic depicted in Figure 2.

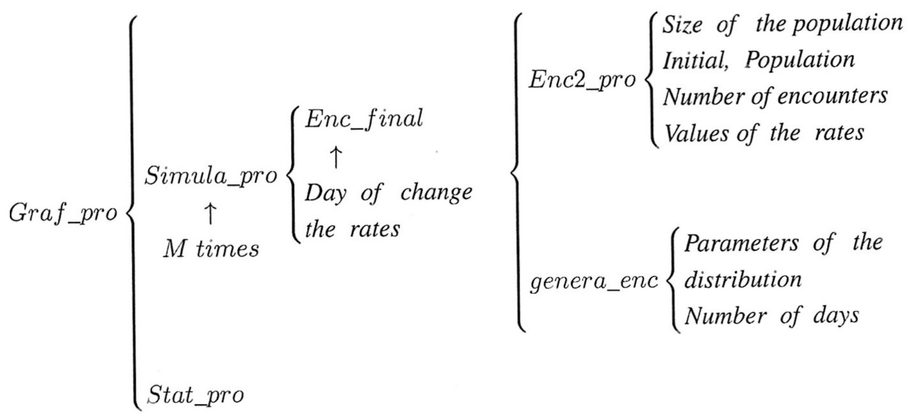

Figure 2. Used Functions

Note, that due to the random character of the simulation, an individual may not participate at a meeting during a "day". Furthermore, he/she can be involved in more than one encounter. In this case, the first meeting is simulated. As a result, the individual will be in a certain class. Then, the second meeting's simulation is performed.

4. Comparison between the dynamical system and the simulation

Since the system of differential equations and the simulation describes the same phenomenon, it is important to find possible relations between the rates defining both of them. Let $\mathcal{P}_y(t_i)$ be the $\mathbb{Z}^4$ vector that denotes how many individuals of the population are at each class at the moment t_i. Here $t_i = iT/d,\ i = 0, 1, \ldots, d$ and $y = S$, ODE stands for the way the number of individuals at each class is computed. Note that, for the dynamical system, this means $\mathcal{P}_{\mathrm{ODE}}(t_i) = N(I_0(t_i), S_0^+(t_i), S_0^-(t_i), R_0(t_i))$, where N is the number of individuals in the population. For solving this fitness problem, given the rates, we run the simulation and compute the mean value of the simulation (MS) component-wise. The resulting solution will be denoted by $\mathcal{P}_{\mathrm{MS}}(t_i)$. Due to its random character, we consider that the simulation is well fitted by the ODE system if $\mathcal{P}_{\mathrm{ODE}}(t_i)$ belongs to the 99% confidence interval of the mean for all $t_i \in [0, T]$. The lower and the upper limits of these intervals will be denoted by $\mathcal{P}_{\mathrm{LMS}}(t_i)$ and $\mathcal{P}_{\mathrm{UMS}}(t_i)$ respectively. As we assume the mean of the simulation describes a good approximation of reality, the parameters of the ODE system are fitted by solving the problem:

$$\min_{\alpha,\beta,\gamma,\theta,\phi,\psi} \left\| \begin{matrix} (\mathcal{P}_{\mathrm{MS}}(t_1) - \mathcal{P}_{\mathrm{ODE}}(t_1)(\alpha,\, \beta,\, \gamma,\, \theta,\, \phi,\, \psi) \\ \vdots \\ \mathcal{P}_{\mathrm{MS}}(t_d) - \mathcal{P}_{\mathrm{ODE}}(t_d)(\alpha,\, \beta,\, \gamma,\, \theta,\, \phi,\, \psi) \end{matrix} \right\|_\infty \tag{3}$$

where $t_i = (iT)/d$ and $\mathcal{P}_{\mathrm{ODE}}(t_i)(\alpha,\, \beta,\, \gamma,\, \theta,\, \phi,\, \psi)$ is the evaluation of the ODE system solution at t_i with rates $(\alpha,\, \beta,\, \gamma,\, \theta,\, \phi,\, \psi)$.

Due to the complexity of problem (3), we will solve a simplified version based on fixing $(\gamma,\, \theta,\, \phi,\, \psi)$ and fitting (α, β)

$$P(\gamma,\, \theta,\, \phi,\, \psi) \quad \min_{\alpha,\beta} \left\| \begin{matrix} (\mathcal{P}_{\mathrm{MS}}(t_1) - \mathcal{P}_{\mathrm{ODE}}(t_1)(\alpha,\, \beta,\, \gamma,\, \theta,\, \phi,\, \psi) \\ \vdots \\ \mathcal{P}_{\mathrm{MS}}(t_d) - \mathcal{P}_{\mathrm{ODE}}(t_d)(\alpha,\, \beta,\, \gamma,\, \theta,\, \phi,\, \psi) \end{matrix} \right\|_\infty \tag{4}$$

We denote by $\mathcal{P}_{\mathrm{FIT}} = N(I_0(t_i), S_0^+(t_i), S_0^-(t_i), R_0(t_i))$, the number of individuals in each class at the solution of the ODE system with the fitted parameters. The solution of the optimization problem (4) was obtained using the interior points algorithm implemented in the MATLAB R2018a function *fmincon*. At each evaluation of the objective function of (4), the resulting dynamical system is solved with the *ode45* function. As the rates are piecewise functions, this function is called for the first interval with the corresponding rates and, given the solution and the new rates, called for the second interval.

5. Numerical results

In this part, we illustrate the proposed approach. The experiments were performed on a Laptop with an AMD Ryzen 3 2200U processor, 8.00 GB RAM, and Windows 10 Home as the operating system. All the functions were implemented in MATLAB R2018a.

We consider the ODE system given in (1) with $T = 60$, the initial conditions $I_0(0) = 0.99$, $S_0^+(0) = 0.01$, $S_0^-(0) = 0$, $R_0(0) = 0$. Consequently, for the simulation, we consider a period of 60 days and a population size of 100 individuals such that 99 of them are ignorants, and one is a positive spreader. On average, 5000 meetings will take place per day, at the beginning. In all cases, we take $(\gamma_1, \gamma_2, \theta, \phi, \psi)$ as $(0.095, 0.105, 0.3, 0.1, 0.1)$, respectively. 100 simulations are performed. In all instances, we first present the result of the simulation: $\mathcal{P}_y(t), t = 0, 1, \ldots, 60,\ y = \mathrm{MS}, \mathrm{LMS}, \mathrm{UMS}$ and $\mathcal{P}_{\mathrm{ODE}}(t)$, the solution of the ODE system with the rates used in the simulation. As already remarked, problem (4) is numerically solved using the *fmincon* function of MatLab with a maximum of 500 iterations. The obtained solution is denoted by $\mathcal{P}_{\mathrm{FIT}}(t)$ and compared with the other results.

This section is divided into two parts. In Subsection 5.1, assuming that all encounters are possible, different combinations of α and β are considered. The second subsection illustrates the performance of the approach for random graphs and small world networks.

5.1. Different parameters

In Experiment 1, we assume that α and β are constant. Experiment 2 considers one of the rates to be constant and the other piecewise constant. In Experiment 3, both rates are piecewise constant.

Experiment 1

In this experiment, we consider the following cases:

$$\alpha > \beta, \alpha = 0.7, \beta = 0.1 \text{ (a), and } \alpha < \beta, \alpha = 0.1 \quad \beta = 0.7 \text{ (b)}$$

Figure 3a shows how in the first case, as α is larger than β, the population of positive spreaders grows more than in the second case. As positive spreaders may turn into negative ones, $\mathcal{S}^-$ is also larger. Note that by day 20, $\mathcal{I}_{\mathrm{UMS}} \leq 15$ and $\mathcal{S}^+_{\mathrm{UMS}} \leq 5$. So, with a probability of 0.99, around the 90% of the population knows the rumour. The chances of a meeting between such a small group of ignorants and positive spreaders are almost zero. In the case of an encounter between 10 ignorants and a negative spreader, the probability is larger. However, as β is small, in most cases the ignorant will become a stifler. As the negative spreaders will mostly meet individuals from the (large) population of stiflers, they will enter too in this class.

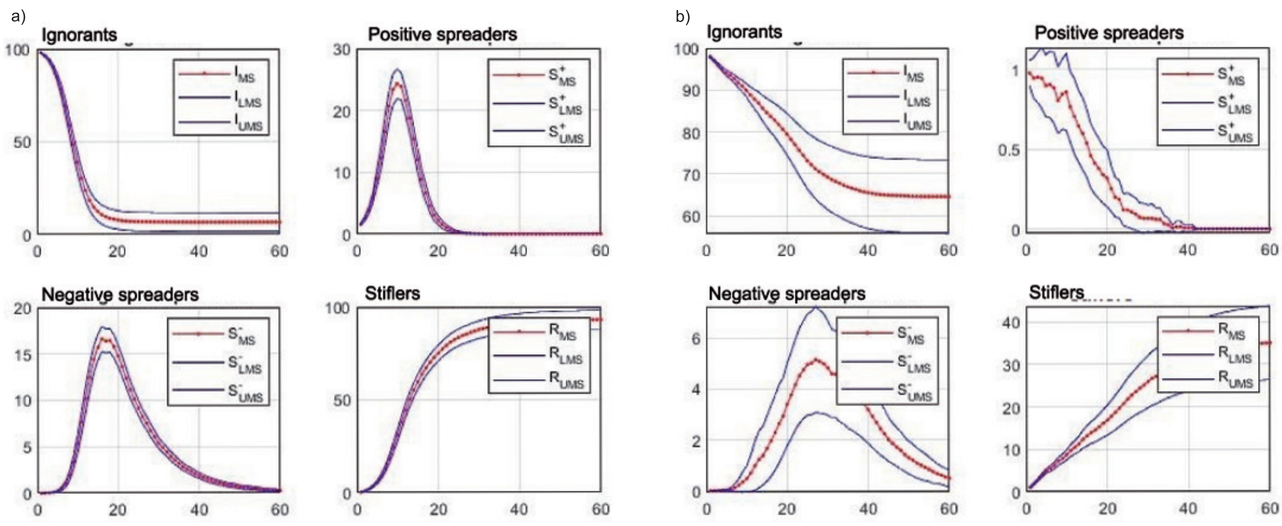

Figure 3. Solution of the simulation. Experiment 1: a) $\alpha = 0.7, \beta = 0.1$, b) $\alpha = 0.1, \beta = 0.7$

Figure 3 confirms some facts that are consequences of the dynamics of the problem: the ignorant population decreases and the number of stiflers increases in time, spreaders populations tend to zero, and the rumour propagation ends. Indeed, when t is large enough, the information is not so important and those individuals knowing the rumour will stop to spread it and become stiflers. However as the behaviour of the spreaders is different in the two cases, the spreading process is different.

On the other hand, if α is small (Figure 3b), as $\mathcal{S}^+_{\mathrm{UMS}} \leq 1$ in most cases, i.e., the number of positive spreaders remains small. Most of the ignorants become stiflers and the positive spreaders become negative. Actually, the rumour is mostly spread by negative spreaders. As a consequence, on average the rumour only reaches 35% of the population.

We shall remark that the confidence intervals of $\mathcal{S}^+$ and $\mathcal{S}^-$ are narrow, their length is less than 5 in all cases. So, the mean provides a good approximation of the number of spreaders. For ignorants and stiflers, the intervals are larger. In the first case (Figure 3a) larger confidence intervals are observed around day 20, the moment the rumour spreading stops. In the second one (Figure 3b), this variability appears almost since the very beginning. More simulations would provide a better approximation to the mean value of the number of ignorants and stiflers.

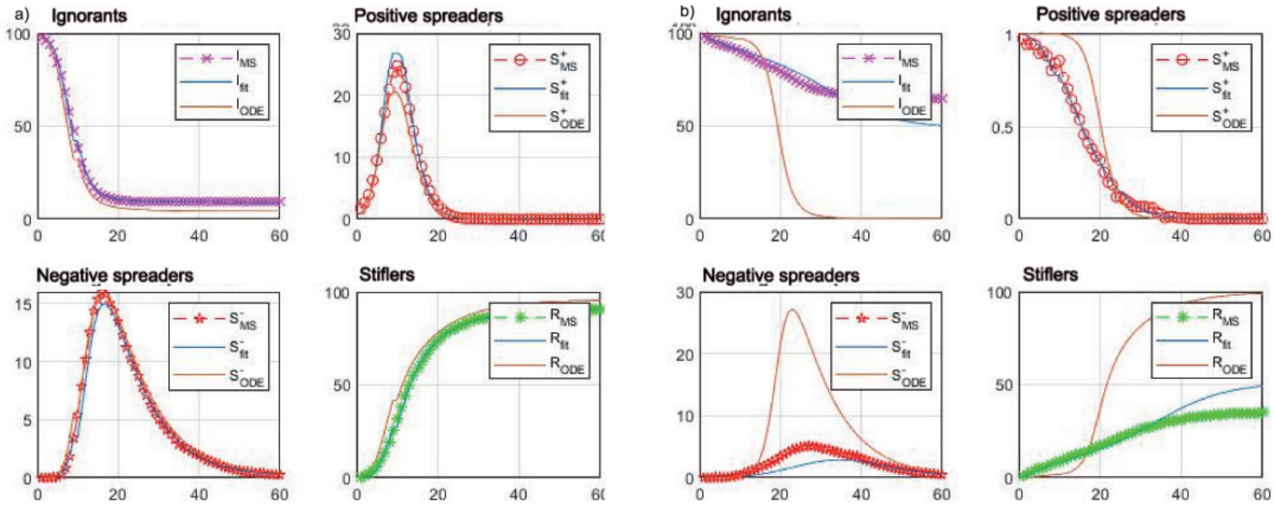

Figure 4. Simulation, ODE and fitted solution. Experiment 1: a) $\alpha = 0.7, \beta = 0.1$, b) $\alpha = 0.1, \beta = 0.7$

With respect to the comparison between $\mathcal{P}_{\text{MS}}$ and $\mathcal{P}_{\text{ODE}}$, again we observe differences. In the first case (Figure 4a), they are very similar, actually the absolute value of $\mathcal{P}_{\text{MS}} - \mathcal{P}_{\text{ODE}}$ is smaller than 5. We want to point out that the fitted model $\mathcal{P}_{\text{FIT}}$ provides a better approximation to the simulated population. However, in the second case (Figure 4b), the difference is too large, for instance, the number of ignorants on day 40 differs by more than 50, and the difference between the number of negative spreaders on day 20 is larger than 12. Furthermore, at the end of the simulation, the dynamical system predicts the end of the propagation because everyone is stifler, hence knowing the rumour while for the simulation there are still 60 ignorants. The fitted model is much better (Figure 4). For all t, the difference between ignorants and stifflers predicted by the two models is not higher than 15. For the spreaders, this bound is 3. Moreover, except for the stiflers for $t > 40$, all the fitted model lies within the confidence interval obtained by the simulation approach. This shows that fitting the parameters provides a better approximation.

Experiment 2

In this part we again consider the following two cases:

$$\alpha = \begin{cases} 0.7, & t \in [0,\,10] \\ 0.1, & t \in [10,\,T] \end{cases} \quad \beta = 0.1,\ \forall t\ \text{(a), and}\ \beta = \begin{cases} 0.7, & t \in [0,\,10] \\ 0.1, & t \in [10,\,T] \end{cases} \quad \alpha = 0.1,\ \forall t\ \text{(b)}$$

In the first case, the rate an ignorant becomes a positive spreader is a piecewise constant function and in the second case, β has this characterization. It can be seen that the results are similar to those of

Experiment 1 (Figure 5). During the first 10 days, they coincide. Then the number of spreaders decreases more rapidly. As there are fewer spreaders, the number of ignorants at the end of the simulation is larger than that in Experiment 2. With respect to the ODE model, for Case a, the difference with the mean of the simulation is larger than 20 (15,12 and 10) for ignorants at $t = 40$ (respectively, for positive spreaders at t=10, negative spreader at $t = 20$ and stifler at $t = 60$). For Case b, these numbers are larger than those reported in Experiment 1b. As can be seen in Figure 6, the difference between the mean of the simulation and the fitted model is smaller than 3, a huge improvement given by the proposed approach.

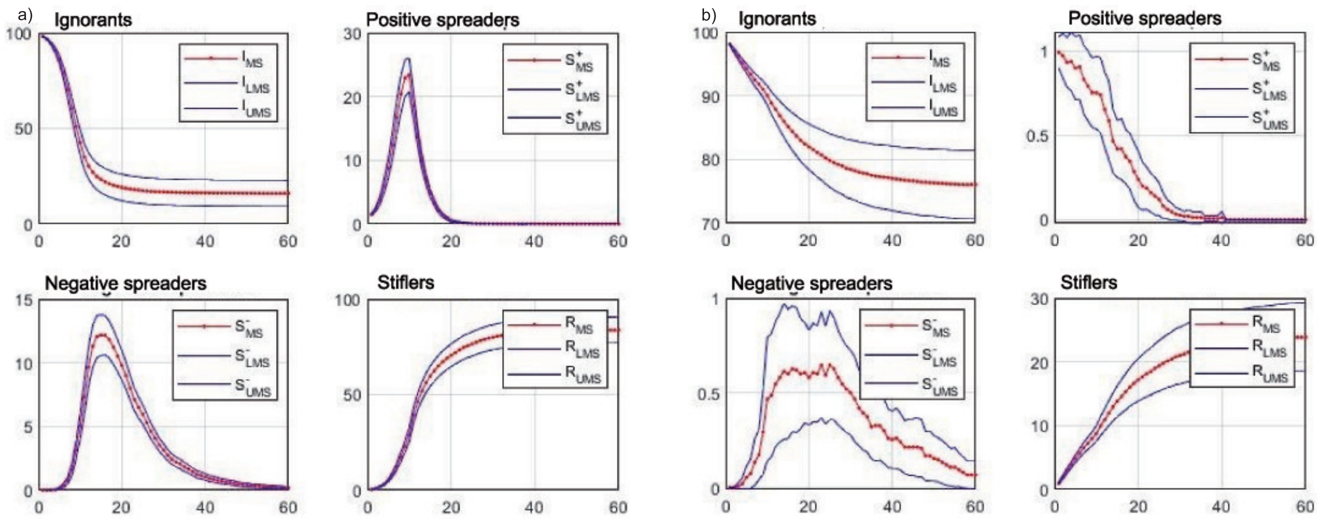

Figure 5. Solution of the simulation. Experiment 2:

$$\text{a) } \alpha = \begin{cases} 0.7, & t \in [0,\ 10] \\ 0.1, & t \in [10,\ T] \end{cases} \quad \beta = 0.1,\ \forall t,\ \text{ b) } \beta = \begin{cases} 0.7, & t \in [0,\ 10] \\ 0.1, & t \in [10,\ T] \end{cases} \quad \alpha = 0.1,\ \forall$$

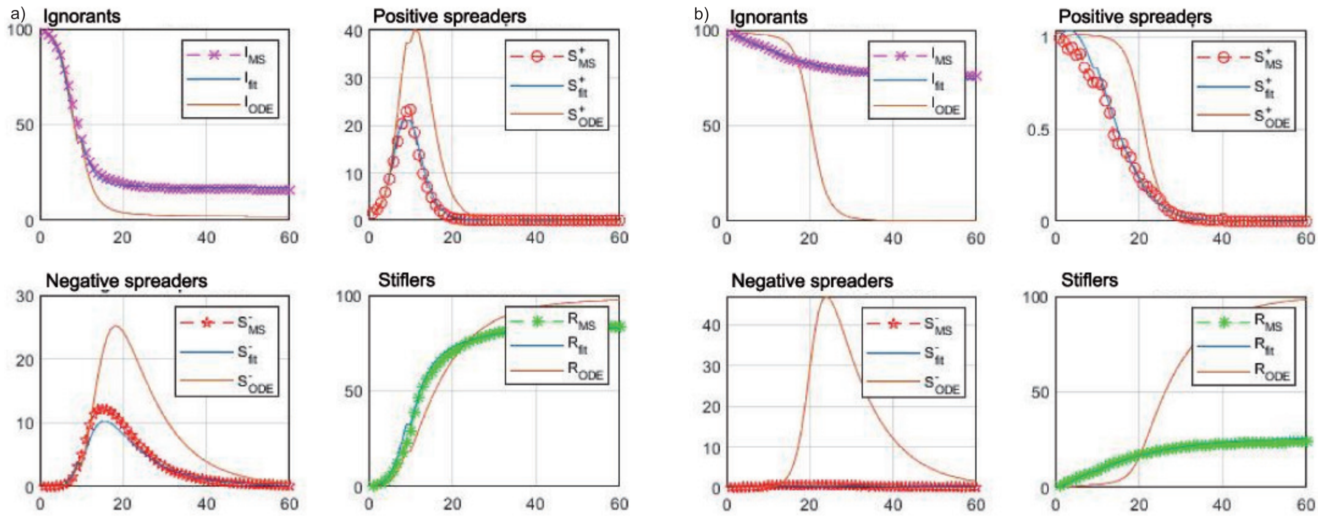

Figure 6. Simulation, ODE and fitted solution. Experiment 2:

$$\text{a) } \alpha = \begin{cases} 0.7, & t \in [0,\ 10] \\ 0.1, & t \in [10,\ T] \end{cases} \quad \beta = 0.1,\ \forall t,\ \text{ b) } \beta = \begin{cases} 0.7, & t \in [0,\ 10] \\ 0.1, & t \in [10,\ T] \end{cases} \quad \alpha = 0.1,\ \forall$$

Experiment 3

In this case, we consider that both rates are piecewise constant functions. They are

$$\alpha = \begin{cases} 0.5 & 0 \leq t \leq 10 \\ 0.1 & 10 < t \leq 60 \end{cases} \qquad \beta = \begin{cases} 0.5 & 0 \leq t \leq 10 \\ 0.7 & 10 < t \leq 60 \end{cases} \qquad \text{(a)}$$

$$\alpha = \begin{cases} 0.5 & 0 \leq t \leq 10 \\ 0.7 & 10 < t \leq 60 \end{cases} \qquad \beta = \begin{cases} 0.5 & 0 \leq t \leq 10 \\ 0.1 & 10 < t \leq 60 \end{cases} \qquad \text{(b)}$$

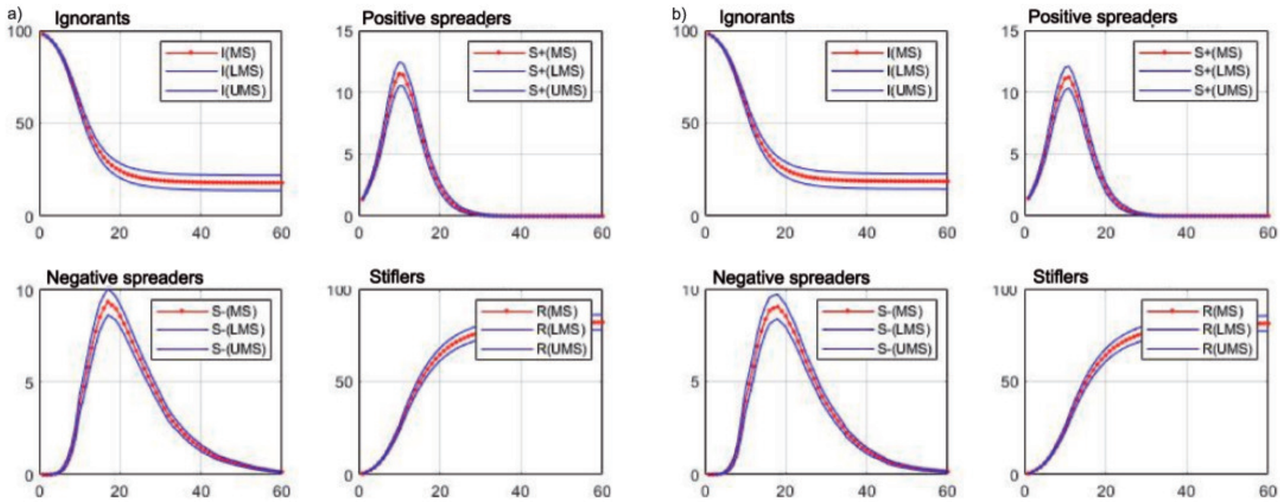

Figure 7. Solution of the simulation. Experiment 3:
a) $\alpha = \begin{cases} 0.5 & 0 \leq t \leq 10 \\ 0.1 & 10 < t \leq 60 \end{cases}$, $\beta = \begin{cases} 0.5 & 0 \leq t \leq 10 \\ 0.7 & 10 < t \leq 60 \end{cases}$, b) $\alpha = \begin{cases} 0.5 & 0 \leq t \leq 10 \\ 0.7 & 10 < t \leq 60 \end{cases}$, $\beta = \begin{cases} 0.5 & 0 \leq t \leq 10 \\ 0.1 & 10 < t \leq 60 \end{cases}$

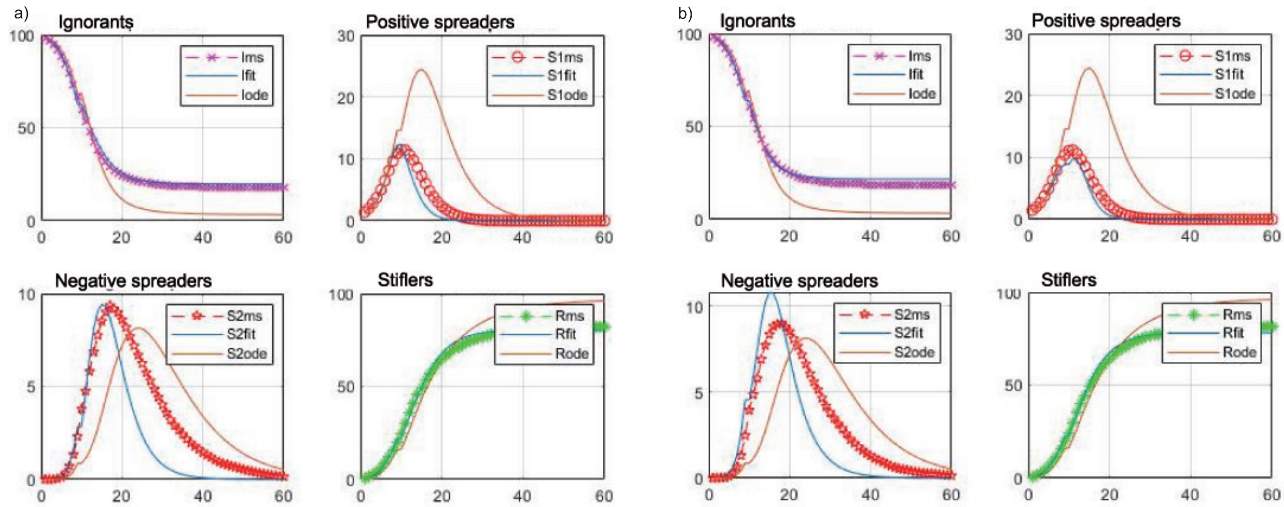

Figure 8. Simulation, ODE and fitted solution. Experiment 3:
a) $\alpha = \begin{cases} 0.5 & 0 \leq t \leq 10 \\ 0.1 & 10 < t \leq 60 \end{cases}$, $\beta = \begin{cases} 0.5 & 0 \leq t \leq 10 \\ 0.7 & 10 < t \leq 60 \end{cases}$, b) $\alpha = \begin{cases} 0.5 & 0 \leq t \leq 10 \\ 0.7 & 10 < t \leq 60 \end{cases}$, $\beta = \begin{cases} 0.5 & 0 \leq t \leq 10 \\ 0.1 & 10 < t \leq 60 \end{cases}$

The example describes a situation that can appear in practice. The rumour starts at $t = 0$. During the first period, $[0, T_0]$, we take $\alpha = \beta$, because new spreaders can indistinctly think that the information is true or false. Suppose that at $t = T_0$, the evidence of the veracity of the information appears. If the rumour is false, then for $t > T_0$, $\beta > \alpha$, as reflected in Case a. Otherwise, see Case b, $\alpha > \beta$. Case a is an example of what can happen with fake news.

Figure 7 shows the results of the simulation. As in the other experiments, the mean of the simulation is a good approximation given the relatively small length of the confidence interval at 99%

Figure 8 shows the mean of the simulation, the numerical solution of the ODE system and the fitted model. We obtain results which are similar to those of Experiments 1 and 2: the fitted model provides very good approximations to the simulation of the spreading of the rumour. The error is always smaller than 4.95. The error using the original parameters in the ODE system is considerably larger. Values larger than 20 appear.

5.2. Different networks

For testing the performance of the approach, the experiments are carried out for $\alpha = 0.7$ $\beta = 0.1$. So, the quality of the solution can be compared with the results obtained in Experiment 1a.

Experiment 4

For the network structure, we generate a random graph with 100 nodes and 500 arcs. Figure 9a shows that the variance of the simulated values is larger than in the other cases. The lengths of the confidence intervals are twice larger than those obtained for the networks considered in Experiment 1. For instance, for ignorants and stiflers at day 60, the length is around 11 for full networks and 25 for the random graph considered in this part.

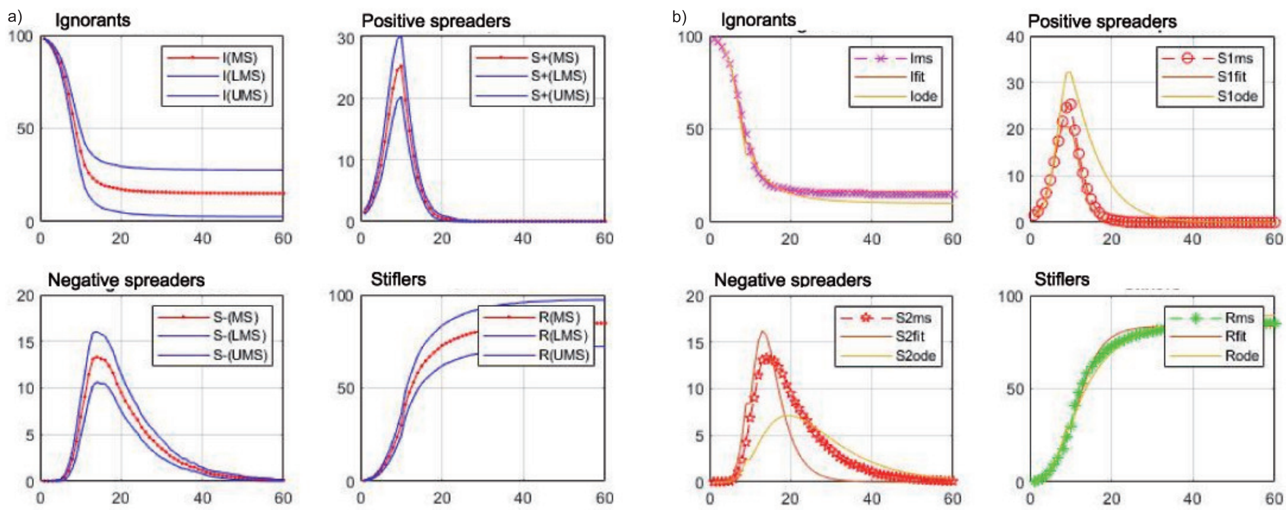

Figure 9. Random graph, $\alpha = 0.7, \beta = 0.1$. Experiment 4:
a) solution of the simulation, b) simulation, ODE and fitted solution

According to the results depicted in Figure 9b, the fitted model is closer to the simulated values than the ODE model, especially for spreaders, where $\max_t \|(\mathcal{P}_{\rm MS}(t) - \mathcal{P}_{\rm FIT}(t)\| \leq \dfrac{\max_t \|(\mathcal{P}_{\rm MS}(t) - \mathcal{P}_{\rm ODE}(t)\|}{2}$.

Negative spreader shows the worst behavior and this error is not larger than 5. We want to point out that for Experiment 1, this value was 7. So, good results have been obtained.

Experiment 5

Here we generate a small world network structure with 100 nodes, an average degree of 6 and a rewiring probability equal to 0.002. The results are depicted in Figure 10. As can be seen, the results are very similar to those obtained in Experiment 4: the length of confidence intervals are larger than the case depicted in Experiment 1, but the solutions of the fitted ODE systems provide good approximations. Indeed, the approximation provided by solving model (3) is better for Experiment 4. In fact, $\max_t \|(\mathcal{P}_{\mathrm{MS}}(t)-\mathcal{P}_{\mathrm{ODE}}(t)\|$ is larger in this case and the $\max_t \|(\mathcal{P}_{\mathrm{MS}}(t) - \mathcal{P}_{\mathrm{FIT}}(t)\|$ is similar to that in Experiment 3.

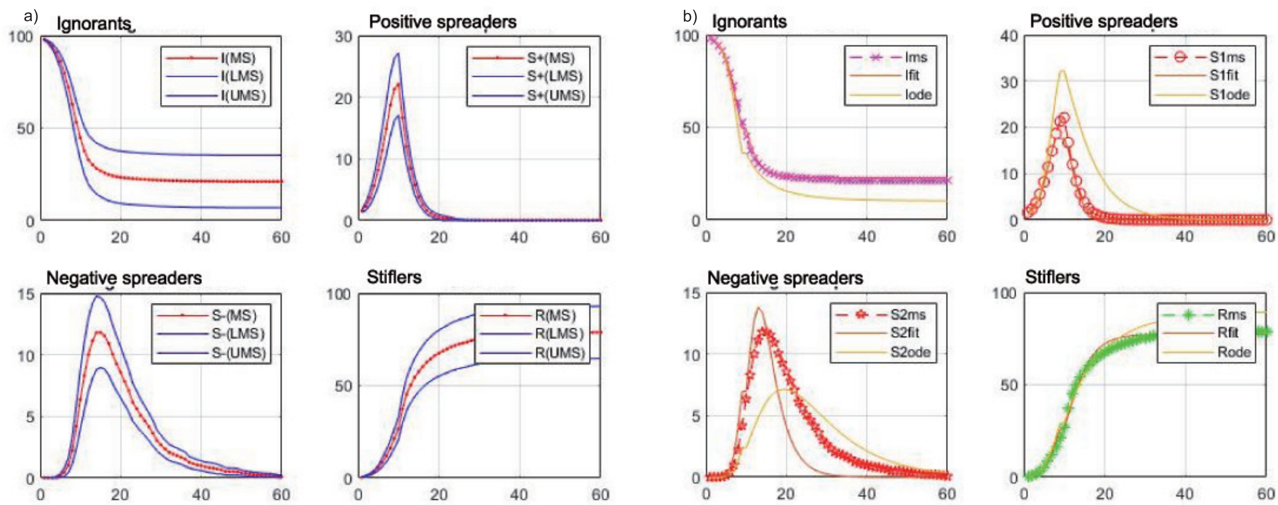

Figure 10. Small world, $\alpha = 0.7$, $\beta = 0.1$. Experiment 5:
a) solution of the simulation, b) simulation, ODE and fitted solution

6. Conclusions

In this paper, a simulation approach was used to study the behaviour of rumour propagation. We considered the rates under which an ignorant becomes a spreader of positive or negative character are constant or piecewise constant functions. Although in the paper we assumed that the rates have a certain value for $t < t_0$ and another for $t > t_0$, this can be easily extended to general piecewise constant functions. With this simulation tool, the dissemination of the rumour can be studied without large costs. In particular, it is easy to find patterns in the process analysing variations in the rates.

For the examples that we have considered, the results are similar to those expected in real-life situations. We also obtained a tool that, given a rumour, fits the parameters of the corresponding dynamical system that describes the phenomenon. Note that we only adjusted the rates α and β. The other rates were taken as equal in the simulation and in the solution of the dynamical system. Even with this simplification, the results were very good. As α and β determine the appearance of new spreaders, they play the main role in the rumour dynamics. This approach can also be applied to their graph structures. Assuming that interactions can only be done among linked individuals, we considered random graphs and small-world networks. Although the variance of the simulated values is larger, the results are still promising because it is expected that more simulations will decrease it. These networks show the need of fitting the ODE system. Indeed, the error between the simulation and the original ODE system is very large, but it decreases a lot if the parameters of the ODE model are fitted.

In future work, more examples will be considered. Random graphs and small worlds networks generated by different parameters and real-life applications, such as the information spread in social networks, will be studied. Furthermore, the positive or negative character of the comments on news platforms can be included in the model proposed in [12] as well as the influence of other rumours and media coverage discussed in [5, 11].

We will also consider the problem of controlling a rumour corresponding to maximize or minimize the spreading of a certain rumour, given the possibility of controlling the rates α and β. The simulation tool will provide more complete and cheap information about the behaviour of the rumour under uncertainty due to its probabilistic character.

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