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ON THE CHARACTERISATION OF *X*-LINDLEY DISTRIBUTION BY TRUNCATED MOMENTS. PROPERTIES AND APPLICATION

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This paper presents the characterisation of *X*-Lindley distribution using the relation between truncated moment and failure rate/reverse failure rate function. An application of this distribution to real data of survival times (in days) of 92 Algerian individuals infected with coronavirus is given.

Keywords: *Lindley distribution, X-Lindley distribution, truncated moments, failure rate function, reversed failure rate function, characterisation of distributions*

1. Introduction

Several papers introduce the new distributions and their applications, including, among others, those of Ducey and Gove [7], Grine and Zeghdoudi [8], Chouia et al. [5], Seghier et al. [11], Beghriche and Zeghdoudi [4], where characterisation of a probability distribution plays an important role in statistical science. Several researchers studied the characterisations of probability distributions. For example, Su and Huang [12] study the characterisations of distributions based on expectations. In addition, Nanda [10] studies the characterisations by average residual life and the failure rates of functions of absolutely continuous random variables. Ahmadi et al. [1] consider the estimation based on the left-truncated and right randomly censored data arising from a general family of distributions. On the other hand, Ahsanullah et al. [2, 3] present two characterisations of Lindley distribution, standard normal distribution, *t*-Student's, exponentiated exponential, power function, Pareto, and Weibull distributions based on the relation of failure rate, reverse failure rate functions with left and right truncated moments. Recently,

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Haseeb and Yahia [9] studied truncated moments for two general classes of continuous distributions.

In this paper, two characterisations of the X-Lindley distribution, introduced by Chouia and Zeghdoudi [5] have been studied. They are based on the failure, relation of the inverse failure rate functions with the left and right truncated moments, respectively. Section 2 gives some properties of X-Lindley distribution. Section 3 discusses the characterisation of general distribution by left truncated and failure rate function and then right truncated and reverse failure rate function. Section 4 studies the characterisation of X-Lindley distribution by using the relation between left/right truncated moment and failure/reverse failure rate function. Finally, an illustrative example of X-Lindley distribution with other one-parameter distributions is given to show the superiority and flexibility of this model.

2. Properties of *X*-Lindley distribution

The X-Lindley distribution with parameters $\alpha = 0$ and $\theta > 0$ has probability density function (p.d.f.)

$$f(x,\theta) = \frac{\theta^2}{(\theta+1)^2}(x+\theta+2)\exp(-\theta x), \quad x>0, \theta>0 \tag{1}$$

The mode of X-Lindley distribution is given by

$$\text{mode}(X) = \begin{cases} -\dfrac{\theta^2+2\theta-1}{\theta} & \text{for } 0<\theta<\sqrt{2}-1 \\ 0 \text{ otherwise} \end{cases} \tag{2}$$

We can find easily the cumulative distribution function (c.d.f) of the X-Lindley distribution

$$F_{XL}(x,\theta) = 1-\left(1+\frac{\theta x}{(1+\theta)^2}\right)\mathrm{e}^{-\theta x}, \quad x>0, \theta>0 \tag{3}$$

The X-Lindley distribution with parameter θ has failure rate function (f.r.f) and reversed failure rate function (r.f.r.f.)

$$r(x)=\frac{f(x)}{1-F(x)}=\frac{\theta^2(x+\theta+2)}{(1+\theta)^2+x\theta},\ \ x>0,\theta>0 \tag{4}$$

$$\eta(x)=\frac{f(x)}{F(x)}=\frac{\theta^2(x+\theta+2)}{(1+\theta)^2\exp(\theta x)-(1+\theta+2\theta)} \tag{5}$$

The rth moment (about the origin) of the X-Lindley distribution with parameter θ is

$$E(X^r)=\frac{(\theta+2\theta+r+1)r!}{(1+\theta)^2\theta^r},\ \ r=1,2,... \tag{6}$$

which is finite for all orders r and $\theta > 0$. For more details, see [5].

3. Characterisation of general distribution

First, we shall introduce the characterisation of general distribution by left truncated and failure rate function and then right truncated and reverse failure rate function. We use the following lemmas to give the characterisation of the general distribution based on left truncated moments.

Lemma 1. Suppose that the random variable X has a continuous c.d.f. $F(x)$ with $F(0)=0,\ F(x)>0$ for all $x>0$, p.d.f. $f(x)=F'(x)$, and f.r.f. $r(x)=\frac{f(x)}{1-F(x)}$. Let $g(x)$ be a continuous function in $x>0$ and $0<E\left[g(x)\right]<\infty$. If

$$E\left[g(x)\middle|X\geq x\right]=h(x)r(x)+k(x),\ \ x>0$$

where $h(x)$ and $k(x)$ are double differentiable functions in $x>0$, then $f(C_1,C_2,x)$ is density function, $x>0$, where $C_1,C_2>0$ are normalizing constants.

Proof. Since

$$E\left[g(x)\middle|\ X\geq x\right]=\frac{1}{1-F(x)}\int_x^\infty g(y)f(y)dy$$

it follows that

$$\int_x^\infty g(y)f(y)dy = h(x)f(x) + k(x)(1 - F(x))$$

Differentiating both sides of the above equation concerning *x*, we have

$$-g(x)f(x) = h(x)f'(x) + h'(x)f(x) + k'(x)(1 - F(x)) - f(x)k(x)$$

Now dividing both sides by $k'(x)$

$$\frac{-g(x)f(x)}{k'(x)} = \frac{h(x)f'(x) + h'(x)f(x) - f(x)k(x)}{k'(x)} + (1 - F(x))$$

differentiating again concerning *x*, and multiplying with $(k'(x))^2$, we get

$$\begin{aligned} &k''(x)g(x)f(x) - f(x)g'(x) - f'(x)g(x) \\ &= k'(x)\left(f(x)h''(x) + 2f'(x)h'(x) + f''(x)h(x) - f'(x)k(x) - f(x)k'(x)\right) \\ &\quad - \left(k''(x)\left(f(x)h'(x)\right) + f(x)h(x) - f(x)k(x)\right) - \left(k'(x)\right)^2 f(x) \end{aligned}$$

After simplifying the above equation, we get

$$\begin{aligned} &h(x)k'(x)f''(x) + (k'(x)g(x) + 2h'(x)k'(x) - k'(x)k(x) - h(x)k''(x))f'(x) \\ &+ (g'(x)k'(x) + h''(x)k'(x) - k''(x)g(x) - k''(x)h'(x) - 2\left(k'(x)\right)^2)f(x) = 0 \end{aligned}$$

Thus, the general solution of this second-order linear differential equation for the unknown function $f(x)$ is $f(C_1, C_2, x)$ which is a density function with $x > 0$ where the initial condition

$$\int_0^\infty f(C_1, C_2, x) = 1, \;\; f(C_1, C_2, x) \geq 0 \text{ and } f(C_1, C_2, 0) = f_0$$

for $x > 0$ and $C_1, C_2 > 0$ is a normalizing constant, which is the proof of the lemma.

Lemma 2. Suppose that the random variable X has a continuous c.d.f. $F(x)$ with $F(0)=0,\ F(x)>0$ for all $x>0,$ p.d.f. $f(x)=F'(x),$ and f.r.f. $r(x)=\dfrac{f(x)}{1-F(x)}$ and $0<E\left[g(x)\right]<\infty.$ If

$$E\left[X \mid X \geq x\right]=r(x)+k(x),\ x>0$$

where $k(x)$ is a double differentiable function in $x>0,$ then $f(C_1, C_2, x)$ is a density function, $x>0,$ where $C_1, C_2>0$ are normalizing constants.

Proof. The lemma can be proved on the lines of Lemma 1 after noting that $g(X)$ $=X, h(x)=1.$

Lemma 3. Suppose that the random variable X has an absolutely continuous c.d.f. $F(0)=0$, $F(x)>0$ for all $x>0$, p.d.f. $f(x)=F'(x)$, and f.r.f. $r(x)=\dfrac{f(x)}{1-F(x)}$ and $0<E\left[g(x)\right]<\infty$. If

$$E\left[X \mid X \geq x\right]=h(x)r(x), x>0$$

where $h(x)$ is a differentiable function in $x>0$, then

$$f(x)=C\exp\left(-\int_0^x \frac{y+h'(x)}{h(y)}dy\right),\ x>0$$

where $C>0$ is a normalizing constant.

Proof. The lemma can be proved on the lines of Lemma 1 with $g(X)=X$ and $k(x)=0$ or one may refer to [3].

The characterisation of general distribution by right truncated and reverse failure rate function is omitted because it is similar to the characterisation of general distribution by left truncated and failure rate function.

4. Characterisation of *X*-Lindley distribution

In this section, the characterisation of *X*-Lindley distribution is presented by using the relation between left/right truncated moment and failure/reverse failure rate function.

Theorem 1. Suppose that the random variable X has a continuous c.d.f. $F(x)$ with $F(0)=0,\ \ F(x)>0$ for all $x>0,$ p.d.f. $f(x)=F'(x),$ and f.r.f. $r(x)=\dfrac{f(x)}{1-F(x)}$. Assume $0<E\left[X^n\right]<\infty$ for a given positive integer n. Then X has the X-Lindley distribution with parameters $\theta>0$ if and only if

$$E\left[X^n \mid X\geq x\right]=\frac{\sum_{k=0}^{n+1}c_k x^k+(1+\theta)\sum_{k=0}^{n}d_k x^k}{x+\theta+2}r(x)$$

where

$$c_0=\frac{n!(n+1+\theta)}{\theta^{n+2}},\ \ c_{k+1}=\frac{\theta}{k+1}c_k,\ \ k=0,1,...,n-1,\ \ c_{n+1}=\frac{1}{\theta}$$

and

$$d_0=\frac{n!}{\theta^{n+1}},\ \ d_{k+1}=\frac{\theta}{n-1}c_k d_k,\ \ k=0,1,...,n-1,\ \ d_{n+1}=\frac{1}{\theta}$$

Proof. First, we shall prove the necessary part:

$$E\left[X^n \mid X\geq x\right]=\frac{1}{1-F(x)}\int_x^\infty y^n f(y)dy=\frac{r(x)}{\left(2+\theta+x\right)e^{-\theta x}}\int_x^\infty y^n\left(2+\theta+y\right)e^{-\theta y}dy$$

$$=\frac{\sum_{k=0}^{n+1}c_k x^k+(1+\theta)\sum_{k=0}^{n}d_k x^k}{x+\theta+2}r(x)$$

Now consider,

$$\int_x^\infty y^n\left(2+\theta+y\right)\mathrm{e}^{-\theta y}dy$$

$$=\int_x^\infty y^n\left(1+y\right)\mathrm{e}^{-\theta y}dy+\int_x^\infty y^n\left(\theta+1\right)\mathrm{e}^{-\theta y}dy$$

$$=\mathrm{e}^{-\theta x}\left(\sum_{k=0}^{n+1}c_k x^k+(1+\theta)\sum_{k=0}^{n}d_k x^k\right) \tag{7}$$

and proceed to get the value of the above integral. To prove sufficiency part, we assume that $g(x)=x^n, n\geq 0$ and

$$h(x)=\frac{\sum_{k=0}^{n+1}c_k x^k+(1+\theta)\sum_{k=0}^{n}d_k x^k}{x+\theta+2}$$

Using the recurrence relations of the c_k and d_k, we have

$$\theta\sum_{k=0}^{n+1}c_k x^k-\sum_{k=1}^{n+1}kc_k x^{k-1}=\sum_{k=0}^{n+1}\left[\theta c_k-(k+1)c_{k+1}\right]x^k$$
$$+\left[\theta c_n-(n+1)c_{n+1}\right]x^n+\theta c_{n+1}x^{n+1}=x^n(1+x)$$

Hence,

$$(1+\theta)\left(\theta\sum_{k=0}^{n}d_k x^k-\sum_{k=1}^{n}kd_k x^{k-1}\right)=(1+\theta)x^n$$

Then, we can write

$$\theta\left(\sum_{k=0}^{n+1}c_k x^k+(1+\theta)\sum_{k=0}^{n}d_k x^k\right)-\left(\sum_{k=1}^{n+1}kc_k x^{k-1}+(1+\theta)\sum_{k=1}^{n}kd_k x^{k-1}\right)=x^n\left(2+\theta+x\right)$$

It follows that

$$\frac{g(x)}{h(x)}=\frac{x^n\left(2+\theta+x\right)}{\sum_{k=0}^{n+1}c_k x^k+(1+\theta)\sum_{k=0}^{n}d_k x^k}$$
$$=\frac{\theta\left[\sum_{k=0}^{n+1}c_k x^k+(1+\theta)\sum_{k=0}^{n}d_k x^k\right]-\left[\sum_{k=1}^{n+1}kc_k x^{k-1}+(1+\theta)\sum_{k=1}^{n}kd_k x^{k-1}\right]}{\sum_{k=0}^{n+1}c_k x^k+(1+\theta)\sum_{k=0}^{n}d_k x^k}$$

$$= \frac{\theta\left[\sum_{k=0}^{n+1} c_k x^k + (1+\theta)\sum_{k=0}^{n} d_k x^k\right]}{\sum_{k=0}^{n+1} c_k x^k + (1+\theta)\sum_{k=0}^{n} d_k x^k} - \frac{\left[\sum_{k=1}^{n+1} kc_k x^{k-1} + (1+\theta)\sum_{k=1}^{n} kd_k x^{k-1}\right]}{\sum_{k=0}^{n+1} c_k x^k + (1+\theta)\sum_{k=0}^{n} d_k x^k}$$

$$= \theta - \frac{\left[\sum_{k=1}^{n+1} kc_k x^{k-1} + (1+\theta)\sum_{k=1}^{n} kd_k x^{k-1}\right]}{\sum_{k=0}^{n+1} c_k x^k + (1+\theta)\sum_{k=0}^{n} d_k x^k}$$

Now, to obtain $\frac{h'(x)}{h(x)}$ we have firstly

$$\log\left(h(x)\right) = -\log\left(2+\theta+x\right) + \log\left(\sum_{k=0}^{n+1} c_k x^k + (1+\theta)\sum_{k=0}^{n} d_k x^k\right)$$

then

$$\frac{h'(x)}{h(x)} = \frac{\delta \log\left(h(x)\right)}{\delta x} = -\frac{1}{2+\theta+x} + \frac{\sum_{k=1}^{n+1} kc_k x^{k-1} + (1+\theta)\sum_{k=1}^{n} kd_k x^{k-1}}{\sum_{k=0}^{n+1} c_k x^k + (1+\theta)\sum_{k=0}^{n} d_k x^k}$$

$$= -\frac{1}{2+\theta+x} + \theta - \frac{g(x)}{h(x)}$$

which implies that

$$\frac{h'(x)+g(x)}{h(x)} = \frac{-1}{2+\theta+x} + \theta$$

Now, for all $x > 0$

$$\int_0^x \frac{g(y)+h'(y)}{h(y)} dy = \int_0^x \left(\frac{-1}{2+\theta+y} + \theta\right) dy = \left(-\log\left(2+\theta+y\right) + \theta y\right)_0^x$$

$$= -\log\left(2+\theta+x\right) + \theta x + \log(2+\theta)$$

Finally, using Lemma 3, for all $x > 0,$ we have

$$f(x) = K\exp\left(-\int_0^x \frac{g(y)+h'(y)}{h(y)}dy\right) = K\left(2+\theta+x\right)\mathrm{e}^{-\theta x}(2+\theta)$$

where $K > 0$ is the normalizing constant, i.e., $k = \frac{(2+\theta)\theta^2}{(1+\theta)^2}$.

This completes the proof of the theorem.

Remark 1. To obtain characterisations of *X*-Lindley distribution by left truncated moments in terms of the failure rate function, we can take integer values of *n* in Theorem 4.

Theorem 2. Suppose that the random variable *X* has a continuous c.d.f. $F(x)$ with $F(0)=0,\ F(x)>0$ for all $x>0,$ p.d.f. $f(x)=F'(x),$ and $\eta(x)=\frac{f(x)}{F(x)}.$ Assume $0<E\left[X^n\right]<\infty$ for a given positive integer n. Then *X* has the *X*-Lindley distribution with parameters $\theta>0$ if and only if

$$E\left[X^n \middle| X \le x\right] = \frac{\left(c_0+d_0+\frac{n!}{\theta^n}\right)\mathrm{e}^{\theta x}\left[\sum_{k=0}^{n+1} c_k x^k + (1+\theta)\sum_{k=0}^{n} d_k x^k\right]}{x+\theta+2}\eta(x)$$

where

$$c_0 = \frac{n!\left(n+1+\theta\right)}{\theta^{n+2}},\ c_{k+1} = \frac{\theta}{k+1}c_k,\ k=0,1,...,n-1,\ c_{n+1} = \frac{1}{\theta}$$

and

$$d_0 = \frac{n!}{\theta^{n+1}},\ d_{k+1} = \frac{\theta}{n-1}c_k d_k,\ k=0,1,...,n-1,\ d_{n+1} = \frac{1}{\theta}$$

Proof. Necessary part

$$E\left[X^n \middle| X \le x\right] = \frac{1}{F(x)}\int_0^x y^n f(y)dy = \frac{1}{F(x)}\left(E\left[X^n\right] - \int_x^\infty y^n f(y)dy\right)$$

$$= \frac{\eta(x)}{\left(2+\theta+x\right)\mathrm{e}^{-\theta x}}\left(\frac{(1+\theta)^2}{\theta^2}E\left[X^n\right]-\mathrm{e}^{-\theta x}\left(\sum_{k=0}^{n+1}c_k x^k+(1+\theta)\sum_{k=0}^{n}d_k x^k\right)\right)$$

$$= \frac{\left(c_0+d_0+\frac{n!}{\theta^n}\right)\mathrm{e}^{\theta x}-\left(\sum_{k=0}^{n+1}c_k x^k+(1+\theta)\sum_{k=0}^{n}d_k x^k\right)}{\left(2+\theta+x\right)e^{-\theta x}}\eta(x)$$

Proof. Sufficiency part. Let $g(x)=x^n$ where n is a positive integer and

$$\omega(x)=\frac{\left(c_0+d_0+\frac{n!}{\theta^n}\right)\mathrm{e}^{\theta x}-\left(\sum_{k=0}^{n+1}c_k x^k+(1+\theta)\sum_{k=0}^{n}d_k x^k\right)}{\left(2+\theta+x\right)}$$

Using the relation mentioned in the above section,

$$\frac{g(x)}{\omega(x)}=\frac{x^n\left(2+\theta+x\right)}{\left(c_0+d_0+\frac{n!}{\theta^n}\right)\mathrm{e}^{\theta x}-\left(\sum_{k=0}^{n+1}c_k x^k+(1+\theta)\sum_{k=0}^{n}d_k x^k\right)}$$

$$=\frac{\theta\left(c_0+d_0+\frac{n!}{\theta^n}\right)\mathrm{e}^{\theta x}-\theta\left[\sum_{k=0}^{n+1}c_k x^k+(1+\theta)\sum_{k=0}^{n}d_k x^k\right]}{\left(c_0+d_0+\frac{n!}{\theta^n}\right)\mathrm{e}^{\theta x}-\left(\sum_{k=0}^{n+1}c_k x^k+(1+\theta)\sum_{k=0}^{n}d_k x^k\right)}$$

$$-\frac{\left[\sum_{k=1}^{n+1}kc_k x^{k-1}+(1+\theta)\sum_{k=1}^{n}kd_k x^{k-1}\right]+\theta\left(c_0+d_0+\frac{n!}{\theta^n}\right)\mathrm{e}^{\theta x}}{\left(c_0+d_0+\frac{n!}{\theta^n}\right)\mathrm{e}^{\theta x}-\left(\sum_{k=0}^{n+1}c_k x^k+(1+\theta)\sum_{k=0}^{n}d_k x^k\right)}$$

$$=\frac{\theta\left(c_0+d_0+\frac{n!}{\theta^n}\right)\mathrm{e}^{\theta x}-\left[\sum_{k=1}^{n+1}kc_k x^{k-1}+(1+\theta)\sum_{k=1}^{n}kd_k x^{k-1}\right]}{\left(c_0+d_0+\frac{n!}{\theta^n}\right)\mathrm{e}^{\theta x}-\left(\sum_{k=0}^{n+1}c_k x^k+(1+\theta)\sum_{k=0}^{n}d_k x^k\right)}-\theta$$

Also, since

$$\log(\omega(x)) = -\log\left(2+\theta+x\right) + \log\left[\left(c_0 + d_0 + \frac{n!}{\theta^n}\right)\mathrm{e}^{\theta x} - \left(\sum_{k=0}^{n+1} c_k x^k + (1+\theta)\sum_{k=0}^{n} d_k x^k\right)\right]$$

then

$$\frac{\omega'(x)}{\omega(x)} = \frac{\delta \log(\omega(x))}{\delta x} = \frac{-1}{2+\theta+x}$$
$$+\frac{\theta\left(c_0 + d_0 + \frac{n!}{\theta^n}\right)\mathrm{e}^{\theta x} - \left[\sum_{k=1}^{n+1} kc_k x^{k-1} + (1+\theta)\sum_{k=1}^{n} kd_k x^{k-1}\right]}{\left(c_0 + d_0 + \frac{n!}{\theta^n}\right)\mathrm{e}^{\theta x} - \left(\sum_{k=0}^{n+1} c_k x^k + (1+\theta)\sum_{k=0}^{n} d_k x^k\right)}$$
$$= \frac{-1}{2+\theta+x} + \theta + \frac{g(x)}{\omega(x)}$$

which implies that

$$\frac{\omega'(x) - g(x)}{\omega(x)} = -\frac{1}{2+\theta+x} + \theta$$

Now, for all $x > 0$,

$$\int_0^x \frac{\omega'(y) - g(y)}{\omega(y)} dy = \int_0^x \left(-\frac{1}{2+\theta+y} + \theta\right) dy$$
$$= \left[-\log\left(2+\theta+y\right) + \theta y\right]_0^x$$
$$= -\log\left(2+\theta+x\right) + \theta x + \log(2+\theta)$$

Finally, using the above Lemma, for all $x > 0$, we have

$$f(x) = C\exp\left(-\int_0^x \frac{\omega'(y) - g(y)}{\omega(y)} dy\right) = K\left(2+\theta+x\right)\mathrm{e}^{-\theta x}(2+\theta)$$

where $C > 0$ is the normalizing constant, i.e., $C = \frac{(2+\theta)\theta^2}{(1+\theta)^2}$.

Remark 7. By taking integer values n in the above theorem, we obtain characterisations of X-Lindley distribution by right truncated moments in terms of the reversed failure rate function.

5. Application

This section presents an illustrative example of X-Lindley distribution with other one-parameter distributions to show the superiority and flexibility of this model. To achieve this goal, we use data of survival times (in days) of 92 Algerian individuals infected with coronavirus, as shown in Table 1 (recorded from 6 to 16 August 2021; available at https://covid19.who.int/), to make a comparison between Lindley, exponential and X-Lindley distributions.

Table 1. Comparison between Lindley, Exponential, and X-Lindley distribution

Survival time $m = 3.2$	Observed frequency	$\text{Lindley}_{\hat{\theta}=0.50}$	$\text{Exponential}_{\hat{\theta}=0.30}$	$X\text{-Lindley}_{\hat{\theta}=0.45}$
[0, 2]	44	35.2	43.94	40.02
[2, 4]	22	27.15	25.46	24.84
[4, 6]	16	17.19	12.46	15.01
[6, 8]	7	7.26	6.62	7.02
[8, 10]	3	3.15	3.52	3.11
Total	92	92	92	92
χ^2	–	3.275	1.572	0.789

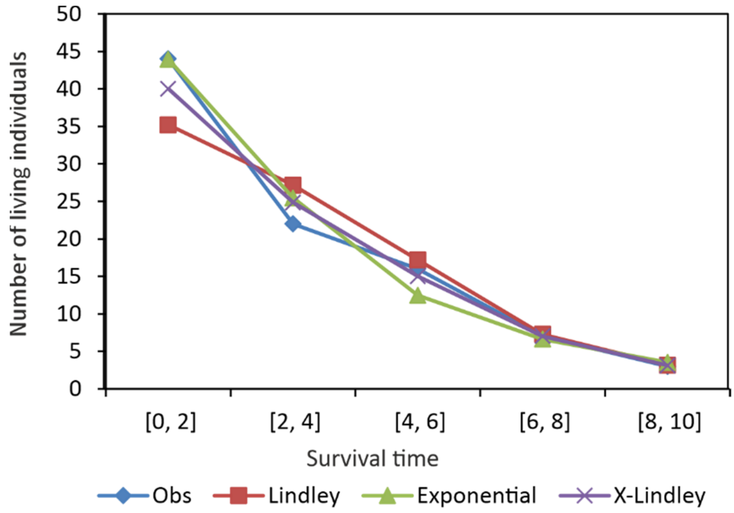

Fig. 1. Goodness of fit of X-Lindley distribution

According to Table 1 and Fig. 1, we can observe that the *X*-Lindley distribution provides smallest χ^2 value compared to Lindley or, exponential distributions, and hence best fits the data among all the models considered.

6. Conclusion

A new continuous probability distribution, namely, *X*-Lindley distribution is considered. Their corresponding characterisation is provided by truncated moments, which may be useful in applied science, actuarial science and physics. The *X*-Lindley distribution is flexible and probable model in describing real life time-to-event data. Illustrative example is given to confirm the goodness of fitting to this model.

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