Vol. 34, No. 1 (2024) | DOI: 10.37190/ord240105

OPEN ACCESS

Operations Research and Decisions

www.ord.pwr.edu.pl

OPERATIONS RESEARCH AND DECISIONS QUARTERLY

A sustainable bi-objective inventory model with source-based emissions and plan-based green investments under inflation and the present value of money

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Abstract

The paper develops a finite-horizon inventory model with source-based emissions, plan-based green investments under inflation, and the present value of money. The cap-and-trade policy is used as the carbon policy. The model is solved in a bi-objective scenario where the two objectives are maximization of the present value of net profit and minimization of the total emission. We find the Pareto optimal solutions represented by a Pareto front using the ϵ-constraint method. A flowchart is provided to find the non-dominated solutions. Pareto solutions for three special cases (no inflation, carbon tax, and no green investments) are also derived. In our sensitivity analyses, we observe that the carbon quota does not affect the optimal policy. It only affects the optimum profit. Our model shows that green investment is beneficial for the polluting firm and also for the environment.

Keywords: *bi-objective, carbon policy, emission, green investment, inflation, Pareto front*

1. Introduction

Global warming is a growing concern worldwide because of its catastrophic effects on the Earth's climate, which is badly damaging our ecosystem. Extreme heatwaves, extreme drought, rise in sea level due to melting of glaciers, frequent wildfires, and extinction of some species are some effects of global warming that we have been experiencing in recent times. The major contributor to global warming is greenhouse gases (GHGs) present in the Earth's atmosphere. Among the GHGs, carbon dioxide plays a crucial role due to its long-lasting heat-trapping property [29]. The concentration of GHGs in the atmosphere

Received 10 November 2022, accepted 12 January 2024, published online 28 March 2024
ISSN 2391-6060 (Online)/

The costs of publishing this issue have been co-financed by the program *Development of Academic Journals* of the Polish Ministry of Education and Science under agreement RCN/SP/0241/2021/1

is rapidly increasing due to various human activities like industrialization, deforestation, urbanization, etc. During the past few decades, the manufacturing industries have flourished at a rapid pace in many countries. These manufacturing industries release a huge amount of GHGs into the atmosphere during their industrial processes. A country's economy largely depends upon industrial growth, so emissions cannot be completely stopped. However, we can slow down the emission rate by taking wise initiatives. The growing environmental awareness of customers and the government's environmental policies are forcing manufacturers to shift towards greener technologies.

Many developing countries are taking the initiative to reduce GHG emissions by implementing stringent environmental policies, also known as carbon policies. Carbon tax and cap-and-trade are two popular carbon policies adopted by many countries. In the carbon tax policy, the government imposes a surcharge on carbon-based fuels, primarily fossil fuels, a tax that polluters pay for using these fuels. In a cap-and-trade policy, the regulatory body places a cap (quota) on the total emissions from industry and reduces this cap every year to achieve the time-based emissions targets. Based on the set cap, the government distributes or auctions carbon quotas fairly to the polluting firms as permits or allowances. Unused permits can be traded. If a regulated firm exceeds its quota, it must buy the excess permit from the market. The European Union's Emissions Trading System (EU ETS) is a major carbon market that works on the cap-and-trade principle and operates in almost all European countries (https://ec.europa.eu/clima/policies/et_en).

Both carbon policies have advantages and disadvantages. Some researchers compare these policies [20, 21]. Traditionally, production and inventory control is an essential part of any manufacturing system. In the presence of a government's carbon policy, a manufacturing firm needs to reset its optimal decisions to reduce the burden of emission-related costs. The carbon policy forces firms to invest in green technologies. There is a growing interest among many researchers to investigate production-inventory systems under carbon policies. A large number of published articles are available in this field. Some researchers analyzed the effects of carbon policies on the optimum policies without green investments. Some recently published articles incorporated green investment as a model parameter. They consider the amount of money invested in green technologies as continuous. This assumption is just an approximation of dollars invested in green technology. The dollar investment in green technology is discrete in a practical situation, depending on the investment plan. For example, if the firm manager wants to replace one of the old machines with a new energy-efficient one, he has to bear the cost of the new machine and its installation cost. So, a more appropriate model should consider plan-based green investments.

As far as the authors' knowledge is concerned, no research has been carried out so far on plan-based green investments. The green installations incur a recurring cost towards their maintenance which might have significant effects on the optimal policy of a firm manager. This concept is also missing in sustainable inventory literature. This research gap needs to be addressed. Another important economic factor that has a significant impact on a firm's optimum policies is inflation. In traditional models, we consider all inventory-related costs as constant. However, these costs are subject to change due to inflation, particularly when we plan to control inventory for more than a year. Furthermore, the purchasing power of money also decreases over time. Some important system characteristics and policies adopted by inventory researchers are price-sensitive demand, inflation, the present value of money, carbon emissions, green investments, multiple objectives, and finite control horizon. However, none of the carbon policy-based models integrate these characteristics in their models. The motivation behind the present work is to address this research gap by integrating price-sensitive demand, plan-based green investments, green maintenance costs, inflation, and the present value of money in a production inventory system under a bi-objective scenario. An environment-conscious manufacturer will always pay some attention to emission reduction to protect our earth and make a trade-off between profit maximization and emission reduction. Hence, finding a solution in a bi-objective scenario will add some value to this field of research.

We integrate the effects of cap-and-trade policy, source-based emissions, and plan-based green investments in a finite horizon multiple-period production-inventory model under inflation and the present value of money. We include maintenance costs for green installations in our model, which is quite realistic. We develop the model in a bi-objective scenario where the maximization of the present value of net profit and minimization of the total emissions are the two objectives. We also perform a sensitivity analysis of some key model parameters. We obtain Pareto solutions by using the ϵ-constraint method (Deb [18]; Mavrotas [38]). A Pareto front represents the set of compromised solutions. To justify the contribution of our paper, we compare our paper with some related published papers and present them in Table 1.

Table 1. Comparison of the present paper with other published papers

Model	Questions[a] 1	2	3	4	5	6	Ref.
Production model	no	no	stochastic	yes	yes	no	[26]
Inventory model	no	no	constant	yes	yes	no	[53]
Production-inventory	no	no	price-dependent	yes	yes	no	[14]
Inventory control (multi-echelon)	no	no	stochastic	yes	no	–	[29]
Inventory	no	no	credit period, price, emission dependent	yes	no	–	[3]
EOQ	no	no	constant	yes	yes	no	[32]
Production-inventory	no	no	price-dependent	yes	yes	no	[16]
Vendor-buyer supply chain	no	no	constant	yes	yes	no	[44]
Sustainable product inventory	no	no	constant (online and offline)	yes	yes	no	[49]
Multi-product inventory	yes (4 objectives)	no	type 2 fuzzy	no	no	–	[40]
Production-inventory	no	no	price-dependent	yes	yes	no	[15]
Production-inventory	no	no	constant	yes	yes	no	[39]
Supply Chain	yes (4 objectives)	no	constant	yes	yes	no	[1]
EPQ	yes	yes	price-dependent	yes	yes	yes	present study

[a] Questions: 1 – Bi-objective? 2 – Inflation and the present value of money? 3 – Demand rate? 4 – Emissions considered in the model? 5 – Green investment? 6 – Plan-based green investment?

The rest of the paper is organized as follows. Section 2 presents a literature review. Section 3 describes the problem descriptions, assumptions, and notations used in the model. The model construction and the solution method are presented in Section 4. Section 5 contains a numerical example and sensitivity analysis of some key model parameters. Solutions to some particular cases are presented in Section 6. Section 7 discusses the analyses of the numerical results, sensitivity analyses, and some managerial insights. Finally, the authors present concluding remarks and future scopes in Section 8.

2. Literature review

The literature review is divided into three categories such as emission-, inflation-, and multi-objective--based studies in the field of production/inventory management. The review is performed only on the existing studies concerning the proposed model at least to some extent.

2.1. Emission- and carbon regulations-based studies

Growing green consciousness of customers and strict carbon regulations of the government force polluting firms to tend towards greener production. Under carbon regulations, firms need to adjust their optimum policies. Many researchers realize the importance of investigating the impact of carbon regulations on the optimum policies of firms. Many articles are available in the field of research where sustainability plays a key role. Bouchery et al. [8] investigated a system in a two-echelon inventory setup. They incorporated emissions but neglected carbon costs. They also presented a solution procedure in a bi-objective scenario. Arsalan and Turkay [4] revised the conventional inventory models by adding environmental and social dimensions of sustainability. Chen et al. [12] derived a condition under which emissions can be reduced just by suitably adjusting the order quantities. They described the applicability of the results to the systems under different carbon policies. Battini et al. [5] investigated an economic order quantity (EOQ) model with sustainability considerations. They used a life cycle assessment (LCA) approach to perform transportation costs analysis and quantification, and external costs integration. Zanoni et al. [56] included carbon emissions in their vendor-buyer supply chain model. They assumed price and environmentally sensitive demand. Liao and Deng [33] analyzed a carbon-constrained EOQ model for a remanufacturing system with demand uncertainty. Other than from the perspective of firms, they also demonstrated the effects of different carbon policies on optimum strategies from the perspective of the administration.

Some sustainable models consider shortages in the systems (cf. [39, 52]). Some authors realized that the growing green consciousness of the customers may affect the demand, and so they used emission-sensitive demand in their sustainable models [3, 24, 56]. Manna and Bhunia [37] developed a sustainable production model with time, price, and electricity consumption reduction-dependent demand. Another important concept in the inventory and supply chain model is the trade credit (also known as delay payment). A customer is allowed for a certain period to make payment without any additional charges. Qin et al. [45] investigated a production-inventory model with trade credit under a carbon tax and cap-and-trade policies. Recently, Chaudhari et al. [11] analyzed an inventory system with deteriorating items under generalized payments and carbon tax. They investigated the impact of down-cash-credit payments on the optimal policy.

Many other authors contributed to this field of research (e.g. [7, 13, 23, 48, 53, 58]). Some researchers proposed a hybrid carbon policy [17, 57]. Ideally, every inventory manager would tend toward green technologies to reduce emissions which, in turn, would reduce the burden of extra costs arising from the carbon policy. Some researchers realized this fact and incorporated green investment in their models as a decision variable [6, 14–16, 26, 28, 32, 44, 49, 53]. The existing models considered the amount of money invested in green technologies as continuous. However, in a practical situation, this is not true. It is, in fact, discrete depending upon the green investment plan. Thus, a plan-based green investment is a more appropriate assumption. To our knowledge, no published article considers a plan-based green investment. Another significant cost that is missing in the existing literature is the green installation maintenance cost, which may be significant and vary from year to year.

2.2. Inflation-based studies

An important economic factor that has a significant impact on a firm's optimum policies is inflation. In traditional models, we consider all inventory-related costs as constant. However, these costs are subject to change due to inflation, particularly when the inventory manager wishes to control inventory for more than a year. Furthermore, the purchasing power of money also decreases over time. Numerous studies are available in inventory research where inflation and the present value of money are considered as model parameters [9, 10, 17, 19, 25, 30, 31, 43, 46, 47, 50, 51]. However, none of these studies focus on sustainability by including emission as a model parameter. A few studies are available where carbon emissions and inflation are integrated into the same model. Alamri et al. [2] analyzed an inventory system with imperfect products that consider emissions, inflation, and learning effects as model parameters. They also included waste management costs in their model. A sustainable two-warehouse inventory model with inflation is developed by Kansal et al. [27]. Their model considers fuzziness. Recently, Vandana et al. [54] analyzed an inventory system with agile manufacturing incorporating learning and forgetting effects, and carbon emissions.

2.3. Multiobjective-based studies

Traditional inventory models focus on a single objective only, either cost minimization or profit maximization. In a complex and competitive business scenario, there is a need to focus on various other objectives. In this situation, the decision-maker needs to find a compromised solution in a multi-objective setup. The Pareto front represents a series of non-dominated compromised solutions. Some researchers investigated multiobjective inventory models without emissions and carbon policies [35, 36, 43]. Recently,Ahmadini et al. [1] analyzed a sustainable multiobjective supply chain model. They considered four objectives in a multi-item system such as maximizing the profit ratio to total back-ordered quantity, minimizing the holding cost in the system, minimizing total waste produced by the inventory system per cycle, and minimizing the total penalty cost due to green investment. Mogale et al. [41] modeled a sustainable bi-objective food grain supply chain system where cost minimization and emission minimization are two objectives. The inflation parameter is not incorporated in this model. Moon et al. [42] investigated an emission-constrained fuzzy bi-objective production planning system without inflation. The objectives considered in their model are maximizing profit and minimizing cumulative shortages.

3. Problem definition and model description

This section provides a brief description of the proposed model and also describes the basic assumptions and notations used in this model.

3.1. Problem description

We consider a polluting firm which produces a particular item and supplies it to the customers. The firm manager wishes to control the production-inventory process over a finite period. He decides to adopt a multiple-period policy which means there will be multiple replenishment cycles. Various costs involved are subject to change due to inflation. During production run-time, the supply takes place directly from the production area, and the excess produced items are stored in a nearby storage facility. When the production stops, supply takes place from the storage facility. The manager is environment-sensitive. He wants a set of compromised non-dominated solutions in the form of a Pareto front in a bi-objective setup where his two objectives are: maximizing the present value of the total profit and minimizing total emissions. We graphically present the problem structure in Figure 1.

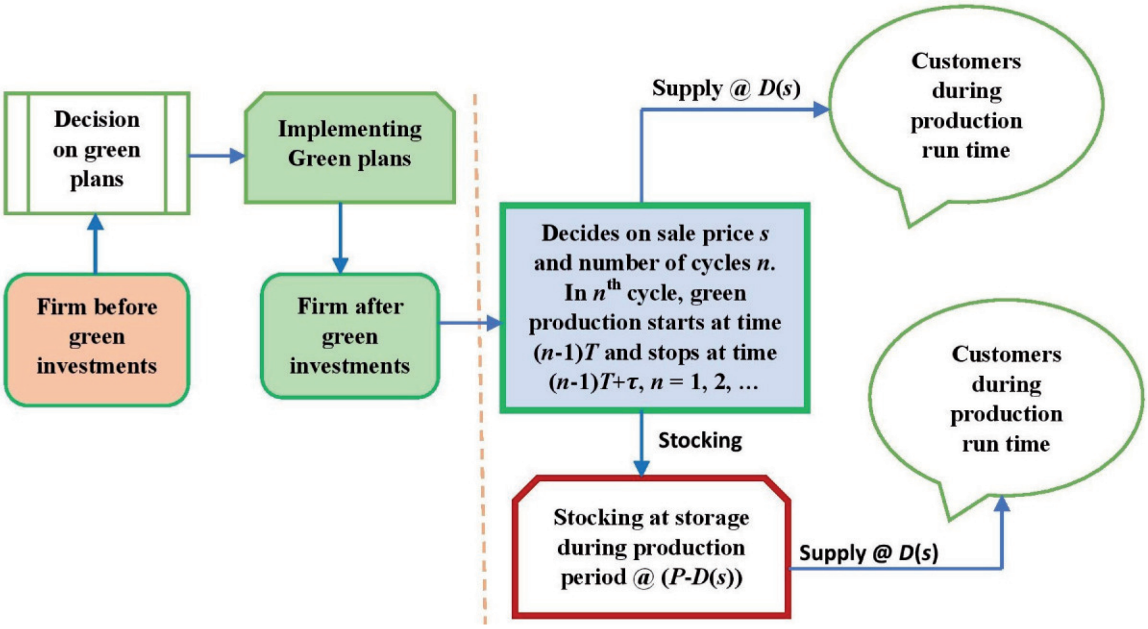

Figure 1. Graphical presentation of the problem

3.2. Assumptions

- The time horizon is finite.
- Shortages are not permitted.
- The production rate is constant.
- The demand rate is a decreasing function of the selling price.
- Inflation is continuous in time, and its rate is constant over the assumed time horizon [17, 50].
- The present value of money is continuous in time, and the discount rate representing the present value of money is constant over the assumed time horizon.
- Carbon price remains constant over the assumed time horizon.
- The carbon quota is not essentially the same for all the years.
- There are multiple and identical production cycles over the assumed time horizon. All cycles are complete.

- Three major sources of emissions are production setup, production process, and stock holding. Emission rates are different in these three sources and are independent.
- The emissions generated from each source can be reduced by investing in appropriate green technologies. Each emission source has a finite number of green investment plans, and the plans are not necessarily mutually exclusive. The set of plans available for each source is the power set of all basic plans in that particular emission source.
- The system starts at time $t = 0$ after upgrading it with green investments.
- Green installations incur recurring maintenance costs for annual maintenance contracts (AMCs), insurance, etc.

3.3. Notations

H – length of the planning horizon, years; this is not necessarily an integer ($H > 0$)

P – production rate, units/year

s – selling price per unit - a decision variable, \$/unit

$D(s)$ – demand rate, units/year; this is a decreasing function of the selling price s, $D(s) = \alpha - \beta s$ where $\alpha, \beta > 0$, $0 \leq s < \frac{\alpha}{\beta}$

Q_i – carbon quota for year $i, i = 1, 2, \ldots, \overline{H}$, where $\overline{H}$ is the ceiling of H, units

$x \times 100\%$ – annual inflation rate

$y \times 100\%$ – annual discount rate representing the present value of money

R – equal to $x - y$, relative inflation rate over discount rate per \$

C_1 – setup cost at time t = 0, \$/setup

C_2 – production cost per unit at time $t = 0$, \$/unit

C_3 – holding cost at time $t = 0$, \$/unit/year

C_4 – carbon price, constant over [0, H], \$/t of emission

e_1 – emission for setup, t

e_2 – average emission per year for machining operations during production run-time, t

e_3 – average emission per unit production, directly linked to production quantity, t

e_4 – average emission for holding/storing activities per unit per year, t

T – length of each cycle, years

τ – length of production run-time in each cycle, year; obviously, $\tau < T$

$\underline{H}$ – the floor value of H (i.e., the integer part of H)

$I_{\max}$ – budget cap on green investment, \$

n – number of complete cycles

p – number of available green investment plans for setup numbered as $1, 2, \ldots, p$

q – number of available green investment plans for the production process numbered as $1, 2, \ldots, q$

r – number of available green investment plans for holding/storage numbered as $1, 2, \ldots, r$

u – decision variable indicating the green plan for the source of emission "setup", $u \in \{0, 1, \ldots, p\}$, $u = 0$ implies no green investment in this source of emission

v – decision variable indicating a green plan for the source of emission "production", $v \in \{0, 1, \ldots, q\}$, $v = 0$ implies no green investment in this source of emission

w – decision variable indicating the green plan for the source of emission "storage",
$w \in \{0, 1, \ldots, r\}$, $w = 0$ implies no green investment in this source of emission

U_u, f_u – green plan u for setup requires an investment $\$U_u$, and this investment reduces e_1 by a proportion $f_u(u = 0,\ 1,\ \ldots,\ p)$, $u = 0$ implies no green investment in setup, obviously, $U_i > U_j \Rightarrow f_i > f_j;\ i, j \in \{0, 1,\ \ldots,\ p\}$ and $U_0 = f_0 = 0$

$V_v,\ g_v$, and h_v – green plan v for the production process requires an investment $\$V_v$, and this investment reduces e_2 by a proportion g_v and e_3 by a proportion h_v $(v = 0,\ 1,\ \ldots,\ q)$, $v = 0$ implies no green investment in the production process, obviously, $V_i > V_j \Rightarrow g_i > g_j$ and $h_i > h_j;\ i, j \in \{0, 1,\ \ldots,\ q\}$ and $V_0 = g_0 = h_0 = 0$

$W_w,\ l_w$ – green plan w for holding/storage requires an investment $\$W_w$, and this investment reduces e_4 by a proportion $l_w(w = 0, 1,\ \ldots,\ r)$, $w = 0$ implies no green investment in storage, obviously, $W_i > W_j \Rightarrow l_i > l_j,\ i, j \in \{0,\ 1,\ \ldots,\ r\}$ and $W_0 = l_0 = 0$

C^i_{mu} – annual maintenance cost in the year i for green installations in setup green plan u, \$, $u = 0, 1, \ldots, p$

C^i_{mv} – annual maintenance cost in the year i for green installations in production green plan v,\$, $v = 0, 1, \ldots, q$

C^i_{mw} – annual maintenance cost in the year i for green installations in storage green plan w,\$, $w = 0, 1, \ldots, r$

$$i = \begin{cases} 1, 2,\ \ldots, H & \text{if } H \text{ is an integer} \\ 1, 2,\ \ldots, \underline{H} + 1 & \text{if } H \text{ is not an integer} \end{cases}$$

The green equipment installation and service providers cover the first year's maintenance, and hence we can set $C^1_{mu} = C^1_{mv} = C^1_{mw} = 0$. Furthermore, the maintenance costs do not decrease in subsequent years

PR – present value of net profit over the period $[0, H]$

n, u, v, w, s – decision variables

4. Model building and solution method

In this section, we construct the mathematical model of the system and provide the solution method of the developed model.

4.1. Model construction

The system starts at time $t = 0$ after upgrading the system with green investments if any. Suppose that the green investment plans u, v, and w are implemented in setup, production process, and storage, respectively. Only one plan will be accepted for each source of emission. There are n identical production cycles over the planning time horizon H. A pictorial representation of the system is shown in Figure 2.

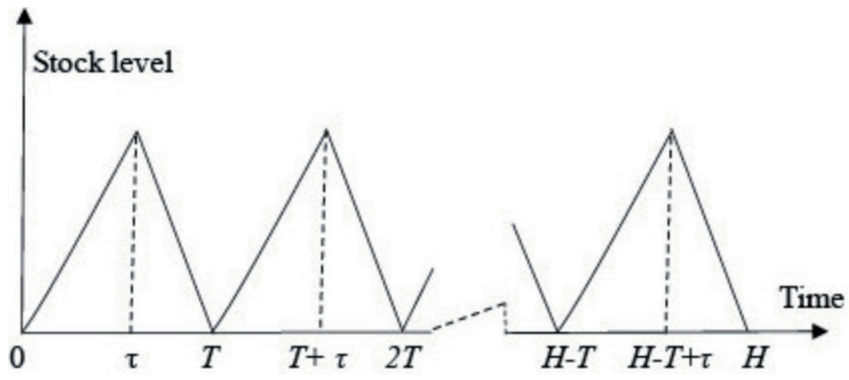

Figure 2. Pictorial representation of the system

The length of the production run-time of each cycle is τ, and the length of each cycle is T. Based on the characteristics of the system, the following relations are obtained:

$$T = \frac{H}{n}, \quad \tau = \frac{HD(s)}{nP}$$

Calculation of average annual emissions quantity

In each cycle

Emission from setup = $(1 - f_u)e_1$.

Emission during production = emission for machining operations + emission for production = $(1 - g_v)e_2\tau + (1 - h_v)e_3\tau P$.

Emission for holding inventory = $0.5(1 - l_w)\{P - D(s)\}e_4\tau T$.

Total emission during the entire period [0, H] is

$$\begin{aligned} E_H &= n \times \text{total emission in a cycle} \\ &= n \times [(1 - f_u)e_1 + (1 - g_v)e_2\tau + (1 - h_v)e_3\tau P \\ &\quad + 0.5(1 - l_w)\{P - D(s)\}e_4\tau T] \\ &= (1 - f_u)ne_1 + \frac{(1 - g_v)e_2D(s)H}{P} + (1 - h_v)e_3D(s)H \\ &\quad + \frac{(1 - l_w)\{P - D(s)\}e_4D(s)H^2}{2nP} \end{aligned} \tag{1}$$

where $u \in \{0, 1, \dots, p\}$, $v \in \{0, 1, \dots, q\}$ and $w \in \{0, 1, \dots, r\}$.

The average annual emissions quantity is

$$\begin{aligned} E_A = \frac{E_H}{H} &= \frac{(1 - f_u)ne_1}{H} + \frac{(1 - g_v)e_2D(s)}{P} + (1 - h_v)e_3D(s) \\ &\quad + \frac{(1 - l_w)\{P - D(s)\}e_4D(s)H}{2nP} \end{aligned} \tag{2}$$

Calculations of present values (PVs) of all costs and profit

PV of green investments is:

$$C_G = U_u + V_v + W_w \tag{3}$$

PV of the total setup cost is

$$C_S = C_1(1 + \mathrm{e}^{RT} + \mathrm{e}^{2RT} + ... + \mathrm{e}^{(n-1)RT}) = C_1\frac{\mathrm{e}^{nRT}-1}{\mathrm{e}^{RT}-1} = C_1\frac{\mathrm{e}^{RH}-1}{\mathrm{e}^{\frac{RH}{n}}-1} \tag{4}$$

PV of total production cost is

$$C_P = C_2P\sum_{i=1}^{n}\int\limits_{(i-1)T}^{(i-1)T+\tau}\mathrm{e}^{Rt}dt = \frac{C_2P(\mathrm{e}^{\frac{RHD(s)}{nP}}-1)(\mathrm{e}^{RH}-1)}{R(\mathrm{e}^{\frac{RH}{n}}-1)} \tag{5}$$

PV of total holding cost is

$$C_H = C_3\sum_{i=1}^{n}\left(\Big(P - D(s)\Big)\int\limits_{(i-1)T}^{(i-1)T+\tau}\Big(t-(i-1)T\Big)\mathrm{e}^{Rt}dt + D(s)\int\limits_{(i-1)T+\tau}^{iT}(iT-t)\mathrm{e}^{Rt}dt\right)$$

After simplifying the above expression, we obtain

$$C_H = \frac{C_3\Big(\mathrm{e}^{RH}-1\Big)\Big(P - D(s) - P\mathrm{e}^{\frac{RHD(s)}{nP}} + D(s)\mathrm{e}^{\frac{RH}{n}}\Big)}{R^2\Big(\mathrm{e}^{\frac{RH}{n}}-1\Big)} \tag{6}$$

Assuming that the maintenance cost is payable at the beginning of each year towards the annual maintenance contract (AMC), the PV of total green installations' maintenance cost is

$$C_M = \begin{cases} \sum_{i=1}^{H}(C_{mu}^i + C_{mv}^i + C_{mw}^i)\mathrm{e}^{-(i-1)R} \quad \text{for } H = \underline{H} \\ \sum_{i=1}^{\underline{H}}(C_{mu}^i + C_{mv}^i + C_{mw}^i)\mathrm{e}^{-(i-1)R} \\ +(C_{mu}^{\underline{H}+1} + C_{mv}^{\underline{H}+1} + C_{mw}^{\underline{H}+1})(H-\underline{H})\mathrm{e}^{-HR} \quad \text{for } H \neq \underline{H} \text{and } H > 1 \\ (C_{mu}^1 + C_{mv}^1 + C_{mw}^1)H \quad \text{for } H \neq \underline{H}\text{and } H < 1 \end{cases} \tag{7}$$

In the above, we assume that the maintenance cost incurs proportionately for the incomplete last year if any. The carbon price (C_4) is assumed to be constant during the entire period [0, H]. So, it has no inflationary effect (i.e., x= 0). We assume that the carbon trading is done at the end of each year. If H is not an integer, the carbon trading for the last period will be done at time t = H. Based on these assumptions, the PV of the emissions cost is

$$C_E = \begin{cases} C_4\sum_{i=1}^{\underline{H}}(E_A - Q_i)\mathrm{e}^{-iy} \quad \text{for } H = \underline{H} \\ C_4\sum_{i=1}^{\underline{H}}\Big((E_A - Q_i)\mathrm{e}^{-iy}\Big) + C_4\Big((H-\underline{H})E_A - Q_{\underline{H}+1}\Big)\mathrm{e}^{-yH} \quad \text{for } H \neq \underline{H}\text{and } H > 1 \\ C_4(E_H - Q_1)\mathrm{e}^{-yH} \quad \text{for } H \neq \underline{H}\text{and } H < 1 \end{cases} \tag{8}$$

Calculation of PV of gross revenue and net profit

PV of gross revenue is

$$R_G = s\int_0^H D(s)e^{-yt}dt = \frac{sD(s)(1-e^{-yH})}{y} \tag{9}$$

The present value of net profit = PV of gross revenue – (PV of green investments + PV of setup cost + PV of production cost + PV of holding cost + PV of green maintenance cost + PV of emission cost). Therefore, the present value of net profit during the planning period H is:

$$PR(n,u,v,w,s) = R_G - (C_G + C_S + C_P + C_H + C_M + C_E) \tag{10}$$

where $C_G, C_S, C_P, C_H, C_M, C_E$ and R_G are given by equations (3)–(9).

The main objective of any firm is to maximize its profit. However, an environment-sensitive firm will always try to find a compromised solution between profit maximization and emission minimization. To address this issue, we model the system in a bi-objective scenario, where the decision maker's two objectives are:

1) maximizing the PV of net profit (PR),

2) minimizing the total emissions (E_H).

The optimizing problem representing this bi-objective scenario is:

$$\begin{aligned}
&\text{Maximize } PR\\
&\text{Minimize } E_H\\
&\text{subject to}\\
&n \in I^+\\
&u \in \{0,\ 1,\ \ldots,\ p\}\\
&v \in \{0,\ 1,\ \ldots,\ q\}\\
&w \in \{0,\ 1,\ \ldots,\ r\}\\
&0 \le U_u + V_v + W_w \le I_{\max}\\
&0 \le s < \frac{\alpha}{\beta}\\
&p,\ q,\ r \text{ are non-negative integers}
\end{aligned} \tag{11}$$

4.2. Solution method

The optimization problem (11) is a bi-objective constrained MINLP problem consisting of discrete variables n, u, v, w, and continuous variable s. We wish to obtain the Pareto optimal solutions represented by a Pareto front [34, 55] with the set of non-dominated solutions. There are several methods for finding the Pareto front, namely, the weighted sum (WS), ϵ-constraint (EC), adaptive weighted sum (AWS), normal boundary intersection (NBI), normalized normal constraint (NNC), NSGA II, etc. [22, 38]. The WS method is simple to use and works well for the convex solution space. However, the method provides poor results if the solution space is non-convex. To find the Pareto front of the numerical problem presented in the next section, we first use the WS method and then refine it using the EC method. A brief description of these two methods is given in the next paragraph.

In the WS method, we assign some non-negative weights to each objective function and then combine the objective functions into a single scalar and composite objective function by taking their weighted sum. We then solve this single objective problem. At optimal, the set of values of the original objective functions represents a point in the solution space, and this solution is non-dominated. Every set of values of weights will give some non-dominated solution point. The set of these non-dominated solutions represents the Pareto front.

In the EC method, we transform the multiobjective problem into a single objective subproblem by converting all but one objective to constraints. The upper bounds of the minimizing constrained objectives and the lower bounds for maximizing constrained are given by an ϵ-vector. We solve this single objective problem. Then we change the ϵ-vector suitably to generate a series of solutions, giving us a more precise Pareto front. Here, because of the presence of discrete decision variables, the solution space is not continuous. We provide the flowchart of an algorithm in the Appendix to find the non-dominated solutions based on the WS method. The proposed algorithm uses a simple method that sets some upper bound of n (number of cycles) discretizes s (selling price) and then searches the entire solution space thus defined for an optimum solution. At optimum, if the number of cycles equals the assumed upper bound of n, we need to increase the upper bound. This algorithm provides us with the exact solution for each weight value. A minor modification of this algorithm will help us to generate the solutions of the EC method. Alternatively, any MINLP-based optimizing software can be used to get the solutions. Microsoft Excel's Evolutionary Algorithm is a powerful algorithm to find the solution to an NLP problem with a single objective. To validate the results obtained using our algorithm, we performed a sensitivity analysis on some key model parameters.

5. Numerical example

To illustrate the model, we present a numerical example. The model parameter values are given in Table 2. We first use the WS method to get a rough Pareto front to solve this problem. Then it is fine-tuned using the EC method. The WS method helps us to get the bounds of the constrained objective necessary for the EC method.

5.1. Pareto solution by the WS method

In this method, we convert the two objectives PR and E_H defined in optimizing problem (11) to a single objective that maximizes $PR_C = \psi PR$ - $(1 - \psi)E_H$ by assigning weights ψ to PR and (1 – ψ) to E_H with $\psi_0 \leq \psi \leq 1, \psi_0 > 0$. We have not taken the lower bound of the weight parameter ψ as zero because, in a real-life situation, no decision-maker assigns zero weight to the profit. In our numerical problem, we have taken $\psi_0 = 0.1$, which we assume as the minimum weight for PR. The minimum value will be used in our sensitivity analysis. We now generate C-codes using the proposed flowchart and find a set of non-dominated solutions for $0.1 \leq \psi \leq 1$ with step size 0.05. We found 19 non-dominated solutions, eight solutions of which are distinct (non-repeated). The solutions are presented in Table 3.

Table 2. Parameter values

H	P	α	β	x	y	C_1	C_2	C_3	C_4	e_1
5.5	7800	7500	80	0.06	0.04	4,000	40	5	20	3
e_2	e_3	e_4	$I_{\max}$	$p=q=r$	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
1.5	0.15	0.5	8,000	3	450	450	400	400	380	380
U_1	U_2	U_3	V_1	V_2	V_3	W_1	W_2	W_3	f_1	f_2
80	90	95	1,000	2,000	2,500	1,000	1,500	2,000	0.2	0.3
f_3	g_1	g_2	g_3	h_1	h_2	h_3	l_1	l_2	l_3	
0.35	0.2	0.25	0.26	0.1	0.15	0.2	0.1	0.2	0.25	
Year↓		C^i_{mu}			C^i_{mv}			C^i_{mw}		
i	$u=1$	$u=2$	$u=3$	$v=1$	$v=2$	$v=3$	$w=1$	$w=2$	$w=3$	
1	0	0	0	0	0	0	0	0	0	
2	10	12	15	100	200	220	120	150	250	
3	10	12	15	100	250	220	130	150	250	
4	12	12	18	150	250	280	150	180	280	
5	12	15	18	150	300	280	150	180	280	
6	15	15	20	180	300	320	180	200	300	

Table 3. Results of WS method (19 points)

Wt. (ϵ)	n	u	v	w	s (\$)	E_H (t)	PR (\$)	Wt. (ϵ)	n	u	v	w	s (\$)	E_H (t)	PR (\$)
0.10	8	3	3	3	73.81	1969.58	167,733.84	0.60	7	0	3	3	73.06	2171.54	168,493.28
0.15	8	3	3	3	73.51	1995.45	167,926.38	0.65	7	0	3	3	73.06	2171.54	168,493.28
0.20	8	3	3	3	73.21	2021.21	168,049.66	0.70	7	0	3	3	72.99	2177.85	168,496.56
0.25	7	0	3	3	73.36	2144.42	168,448.09	0.75	7	0	3	3	72.99	2177.85	168,496.56
0.30	7	0	3	3	73.29	2150.76	168,466.69	0.80	7	0	3	3	72.99	2177.85	168,496.56
0.35	7	0	3	3	73.29	2150.76	168,466.69	0.85	7	0	3	3	72.99	2177.85	168,496.56
0.40	7	0	3	3	73.29	2150.76	168,466.69	0.90	7	0	3	3	72.99	2177.85	168,496.56
0.45	7	0	3	3	73.13	2165.22	168,486.41	0.95	7	0	3	3	72.99	2177.85	168,496.56
0.50	7	0	3	3	73.06	2171.54	168,493.28	1.00	7	0	3	3	72.99	2177.85	168,496.56
0.55	7	0	3	3	73.06	2171.54	168,493.28								

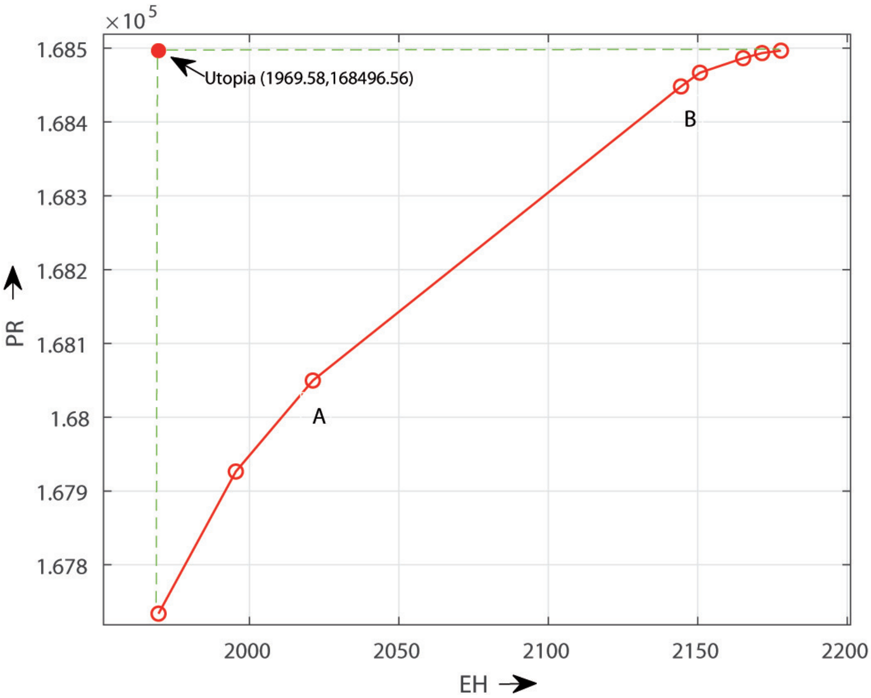

Figure 3. Pareto front - WS method

The Pareto front generated by the WS method is presented in Figure 3. We can observe a long gap between the marked points A and B. The WS method cannot find the actual front pattern between these two points even if we select denser values of ψ. This pattern indicates the presence of an inward turn of the actual Pareto front between these two points. To resolve this problem, we use the EC method described in the following subsection.

5.2. Pareto solution by the EC method

The method considers the primary objective function PR as the objective and the secondary objective function E_H as $\leq$ type constraint. The revised optimization problem is

$$\begin{aligned}
&\text{Maximize } PR\\
&\text{subject to}\\
&n \in I^+\\
&E_H \leq \epsilon\\
&(E_H)_{\min} \leq \epsilon \leq (E_H)_{\max}\\
&u \in \{0,\,1,\,\ldots,\,p\}\\
&v \in \{0,\,1,\,\ldots,\,q\}\\
&w \in \{0,\,1,\,\ldots,\,r\}\\
&0 \leq U_u + V_v + W_w \leq I_{\max}\\
&0 \leq s < \frac{\alpha}{\beta}\\
&p, q, r \text{ are non-negative integers}
\end{aligned} \tag{12}$$

Table 4. Results of EC method (22 points)

ϵ	n	u	v	w	s (\$)	E_H (t)	PR (\$)	ϵ	n	u	v	w	s (\$)	E_H (t)	PR (\$)
1970	8	3	3	3	73.81	1969.58	167733.84	2080	8	0	3	3	72.76	2068.06	168124.25
1980	8	3	3	3	73.69	1979.94	167786.06	2090	7	0	3	3	73.96	2089.83	168149.50
1990	8	2	3	3	73.59	1989.76	167876.59	2100	7	2	3	3	73.78	2099.96	168219.19
2000	8	2	3	3	73.51	1996.65	167926.78	2110	7	3	3	3	73.66	2109.83	168295.81
2010	8	2	3	3	73.36	2009.54	167997.47	2120	7	2	3	3	73.59	2117.25	168334.28
2020	8	3	3	3	73.23	2019.50	168022.13	2130	7	0	3	3	73.59	2123.55	168368.56
2030	8	0	3	3	73.21	2029.61	168068.47	2140	7	0	3	3	73.43	2138.07	168424.91
2040	8	2	3	3	73.06	2035.25	168086.06	2150	7	0	3	3	73.36	2144.42	168448.09
2050	8	0	3	3	72.99	2048.44	168109.75	2160	7	0	3	3	73.20	2158.90	168476.25
2060	8	0	3	3	72.91	2055.27	168123.06	2170	7	0	3	3	73.13	2165.22	168486.41
2070	8	0	3	3	72.76	2068.06	168124.25	2180	7	0	3	3	72.99	2177.85	168496.56

We observe that the worst and the best values of E_H are 2177.85 and 1969.58, respectively, i.e., $(E_H)_{\min} = 1969.58$ and $(E_H)_{\max} = 2177.85$ (Table 3). We set $\epsilon = 1970 + 10\Omega$, where $\Omega = 0, 1, \ldots, 21$. Here Ω denotes the step number while increasing ϵ. We have taken ϵ values from 1970 to 2180 by increasing with step size 10. A different starting point and ϵ value might give us a different set of non-dominated solutions. Table 4 displays the non-dominated solutions generated by the EC method.

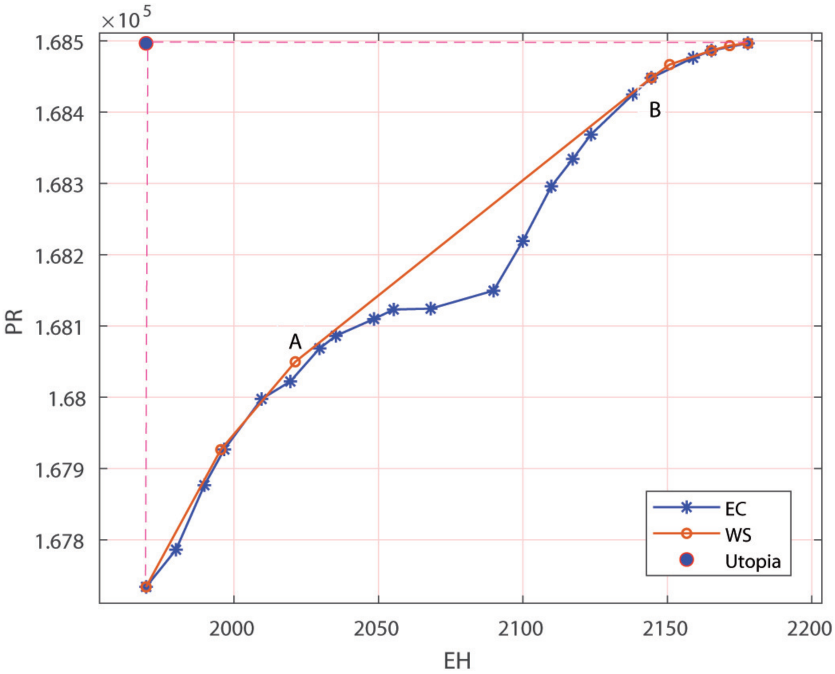

Figure 4. Pareto fronts - both methods

Figure 4 shows the Pareto fronts generated by WS and EC methods. The EC method gives a finer Pareto front, and it captures the front pattern between A and B. The EC method gives us more distinct non-dominated solutions. It is evident from Figure 4 that the EC method gives a more accurate Pareto front and captures the pattern between points A and B. To fine-tune the Pareto front, we should take denser ϵ values.

5.3. Sensitivity analysis

For the sensitivity analysis, we choose the environment-related parameters carbon price (C_4), emission rates (e_1, e_2, e_3, e_4), carbon permits (quotas) ($Q_1, Q_2, Q_3, Q_4, Q_5, Q_6$), and the inflation-related parameters inflation rate (x), discount rate (y).

Table 5. Sensitivity analysis results (single objective)

Changing parameter	% change	% change in E_H	% change in P_R	Changing parameter	% change	% change in E_H	% change in P_R
	–50	27.43	–0.15		–50	1.70	27.04
	–25	8.24	–0.55		–25	3.76	13.03
	–10	0.55	–0.28	Inflation rate x	–10	1.38	5.13
Carbon price C_4	0	0.00	0.00	discount rate y	0	0.00	0.00
	10	–1.24	0.30	(simultaneous)	10	–1.74	–5.02
	25	–7.81	0.86		25	–3.99	–12.31
	50	–9.55	2.07		50	–8.68	–23.90
	–50	0.00	–12.82		–50	–36.27	12.66
	–25	0.00	–6.41		–25	–18.82	5.86
Carbon quotas	–10	0.00	–2.56	Emission rates	–10	–9.50	2.29
$Q_i, i = 1, 2, \ldots, 6$	0	0.00	0.00	e_1, e_2, e_3, e_4	0	0.00	0.00
(simultaneous)	10	0.00	2.56	(simultaneous)	10	8.63	–2.26
	25	0.00	6.41		25	15.24	–5.54
	50	0.00	12.82		50	35.63	–10.74

At first, we study the effects of changes in these parameters on the optimum PV of net profit (PR) and total emission (HS) in a single objective scenario (profit maximization). Then, we perform the sensitivity analysis to see the effects of the changes in these parameters on the Pareto front in the bi-objective scenario. We consider the above numerical example and perform our analysis based on the results obtained by the EC method. In a single objective case, we change the parameters by –50%, –25%, –10%, 10%, 25%, and 50% and observe the percentage changes in PR and the total emissions (HS). The results obtained are shown in Table 5. All remaining parameters are left at their original values, except for those listed in the "Changing parameter" column. The 0% change indicates the results of the original problem. To get a better idea, we graphically presented the results in Figure 5.

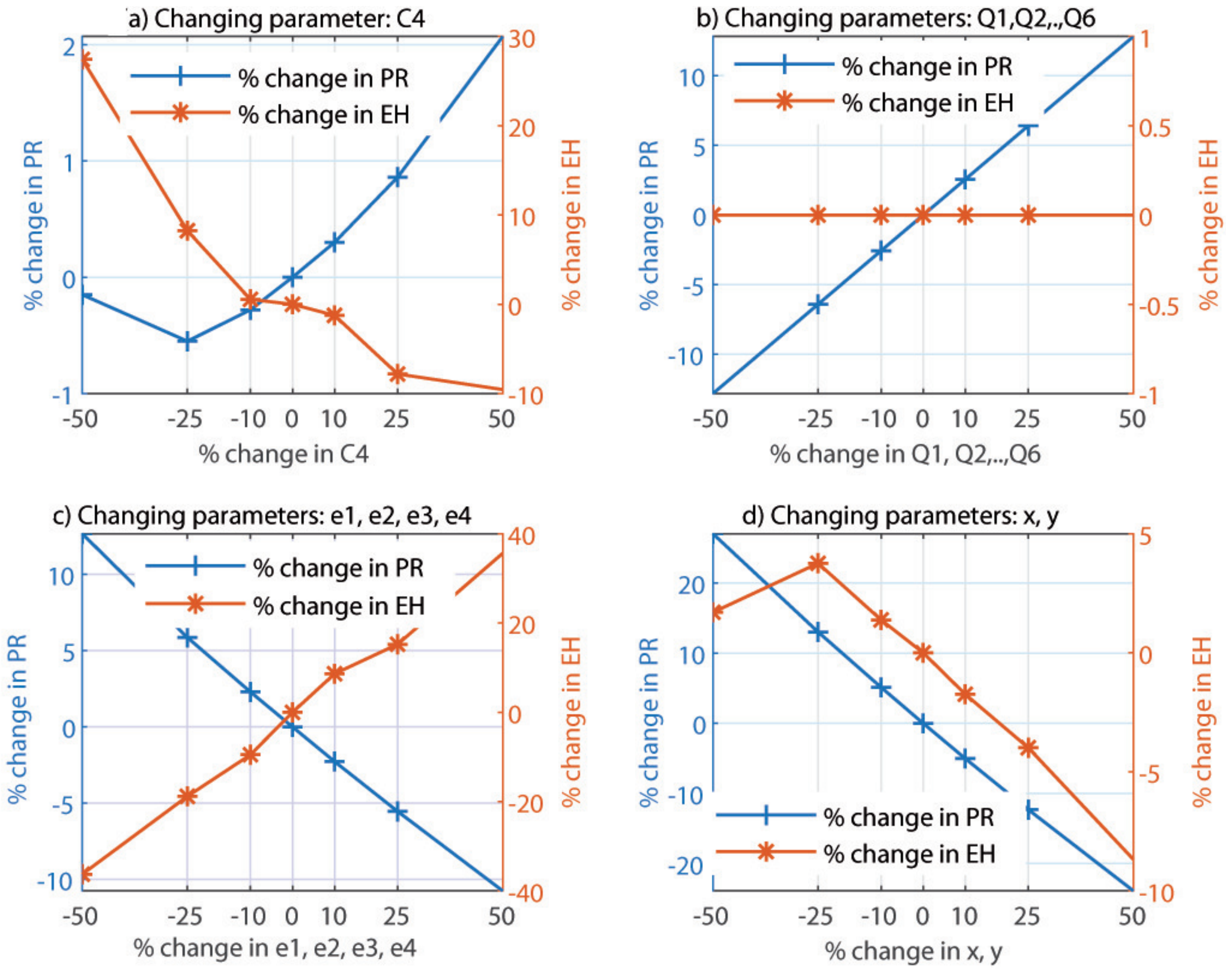

Figure 5. Graphical display of sensitivity results

Following, are some important observations from our sensitivity analysis (single objective). Section 7 provides some comprehensive managerial insights.

- In Figure 5a, we observe that the carbon price (C_4) and total emission (E_H) are negatively correlated. The E_H value is more sensitive to C_4 for lower values of C_4 and is comparatively less sensitive for higher values. The PV of net profit PR is positively correlated with C_4. But it is weakly sensitive to C_4. In this figure, we observe a decrease in the value of PR at a certain point (-25%). Then it increases monotonically. Occasionally, this type of behavior can be observed due to the presence of discrete decision variables.

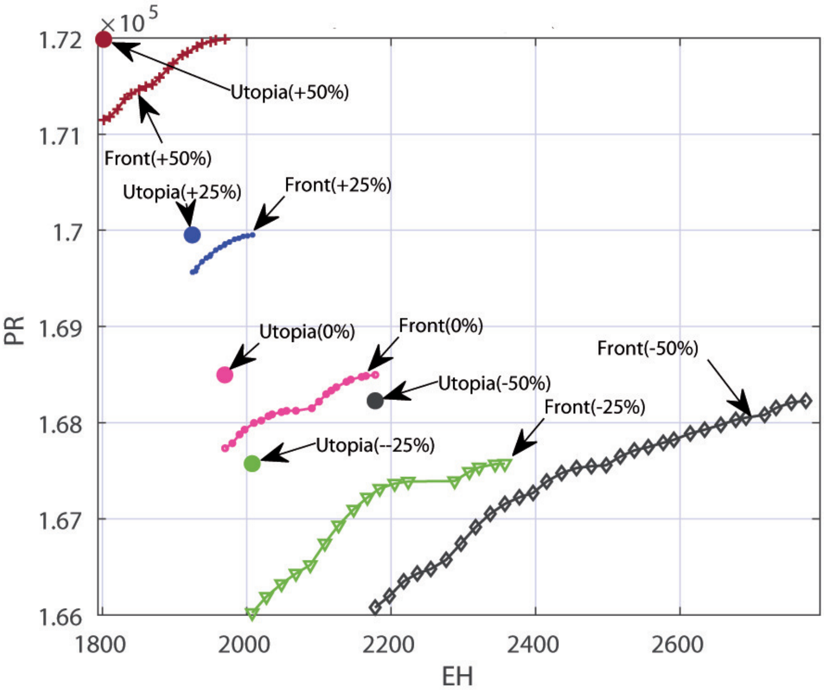

Figure 6. Pareto fronts for changes in C_4

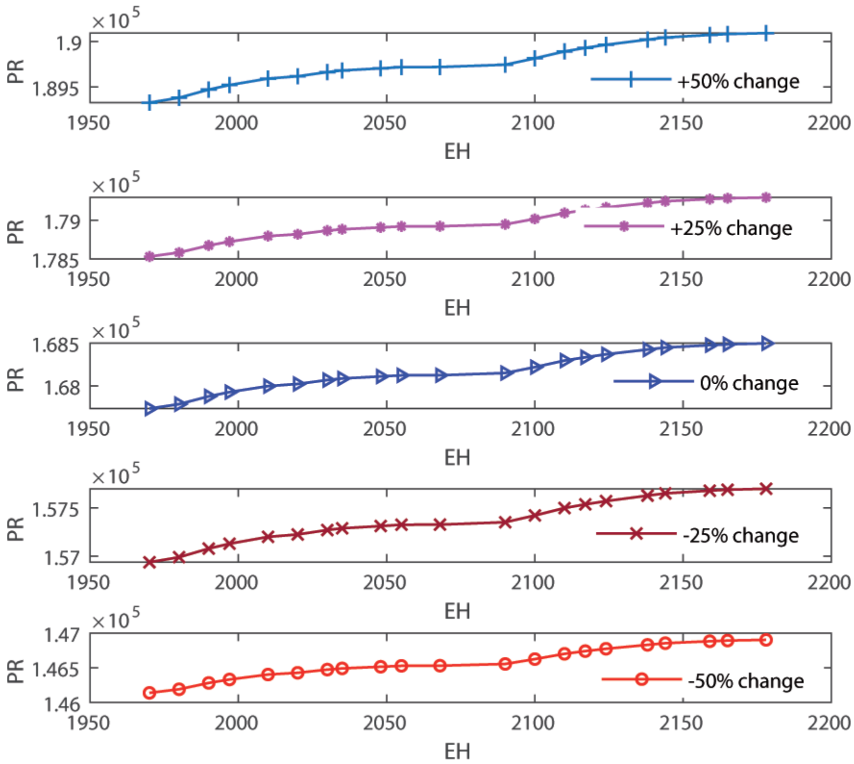

Figure 7. Pareto fronts for changes in $Q_1, Q_2, ...Q_6$

- Figure 5b shows that the E_H is purely insensitive to the changes in the freely distributed carbon permits (quotas) Q_i (i = 1, 2,...,6) from this firm's point of view. PR has a moderately positive correlation with Q_i. We observe a linear relationship between PR and Q_i.
- In Figure 5c, we observe that emission rates $e_i, i = 1, 2, 3, 4,$ and E_H have a positive and moderately strong correlation. This sensitiveness of E_H is almost similar to positive or negative changes

in e_i values from their original values. The PR and e_i have a negative relation. This means that an increase in the emission rate results in a decrease in net profit.

- In Figure 5d, we observe that E_H is weakly sensitive to the changes in inflation parameters x and y. It has a negative relation with x and y. At one value (–25%), E_H increases. We have already explained the reason for such behavior when we discussed the sensitivity of C_4. PR has a negative correlation with x and y. The sensitivity is average.

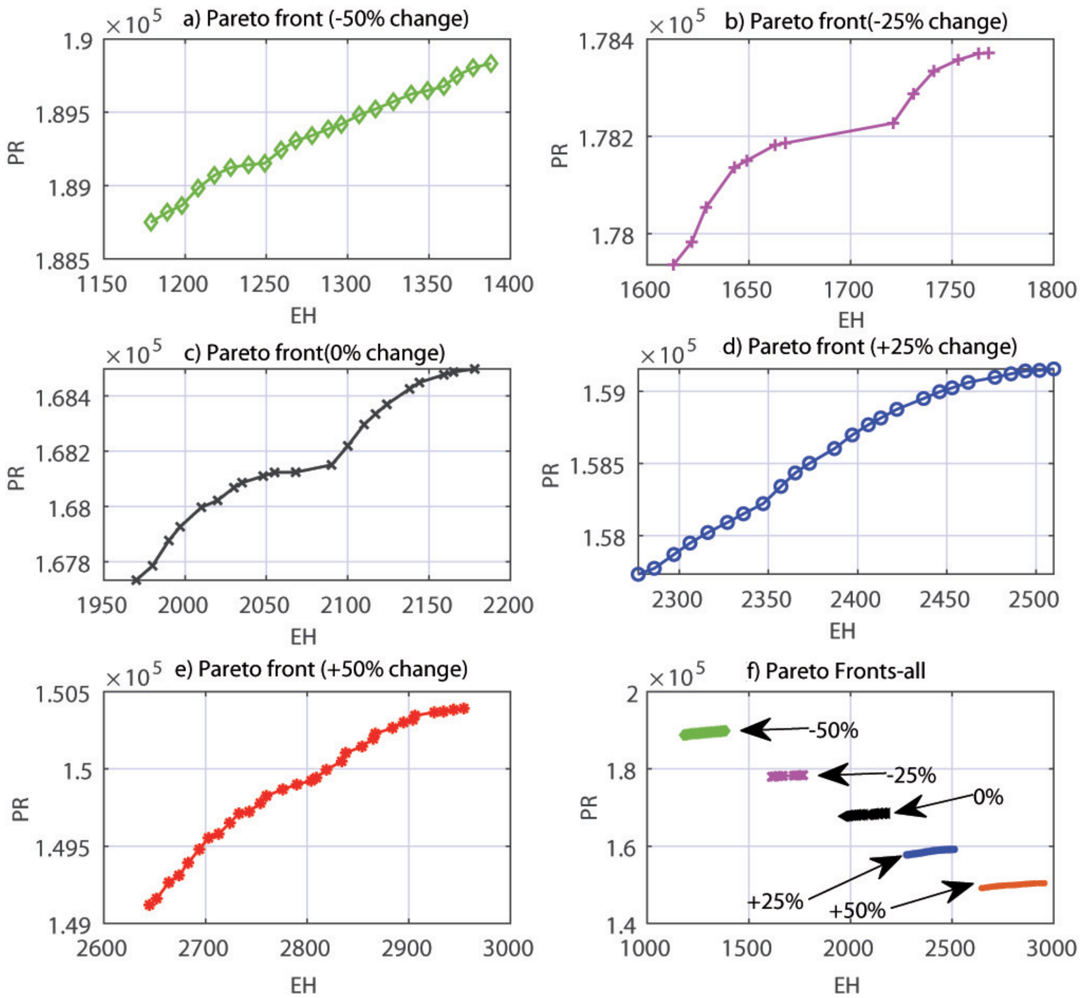

Figure 8. Pareto fronts for changes in e_1, e_2, e_3, e_4

In our analysis of the bi-objective scenario, we change the parameters by –50%, –25%, 25%, and 50%. The Pareto fronts obtained are displayed in Figures 6–9.

- Figure 6 shows that the Pareto front shifts towards the northwest corner as C_4 increases except in the case of –25%. Some portion of the front at –25% is below some portion of the front at –50%. In general, higher values of C_4 ensure better non-dominated solutions. The fronts have different structures but almost similar trends. All five fronts are well-structured.
- An increase in the carbon quotas in Figure 7 will shift the Pareto front vertically upward direction. So, changes in carbon quota will change PR, but not E_H. All five fronts have similar structures.
- Figure 8f shows that the Pareto front shifts towards the southeast corner as the emission rates (e_i) increase. Thus, an increase in emission rate provides us with an inferior compromised solution. The

remaining five subplots show that the front structure changes with the change in emission rates, but the trend remains almost the same.

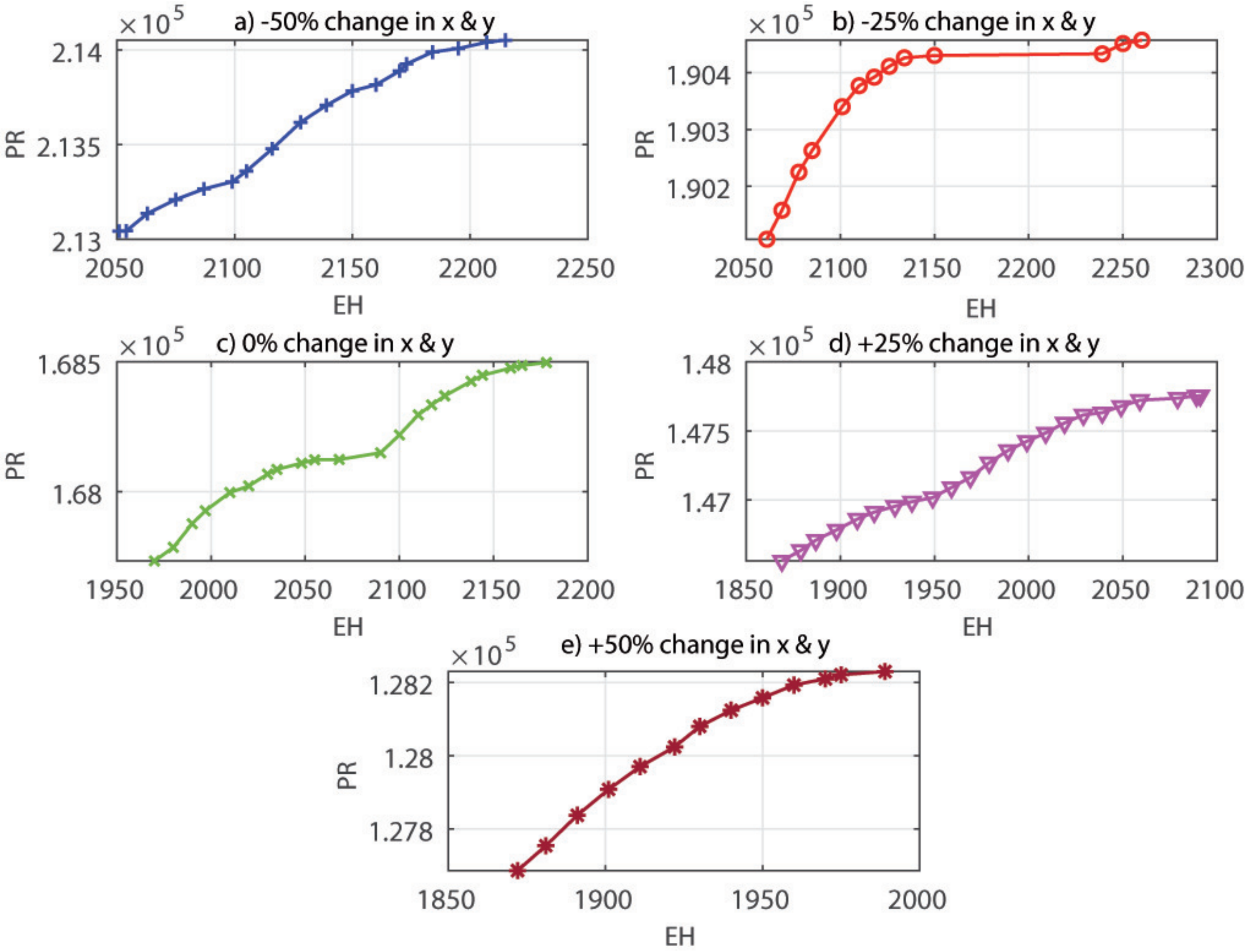

Figure 9. Pareto fronts for changes in x and y

- Figure 9 shows that if inflation parameters increase, the Pareto front moves towards the southeast corner except at –25%. Thus, in general, PR decreases, and E_H increases when inflation parameters increase.

6. Some special cases

This section finds and describes the Pareto fronts of the above numerical problem for some particular cases. The non-dominated solutions are provided in the Appendix (see Table A1 through Table A3). We use the EC method in all three cases. In each case, we first find the maximum and minimum values of E_H using the WS method and then set the values of ϵ.

Case 1: Without inflation and the present value of money

Substitute $x = y = 0$. This gives, $R = 0$. Replace PR and HS with their limiting values when $R \to 0$. The Pareto front, in this case, is shown in Figure 10. A comparison between Figure 4 and Figure 10 indicates that more profit can be generated in the absence of inflation and the present value of money. However, total emissions will increase in his case.

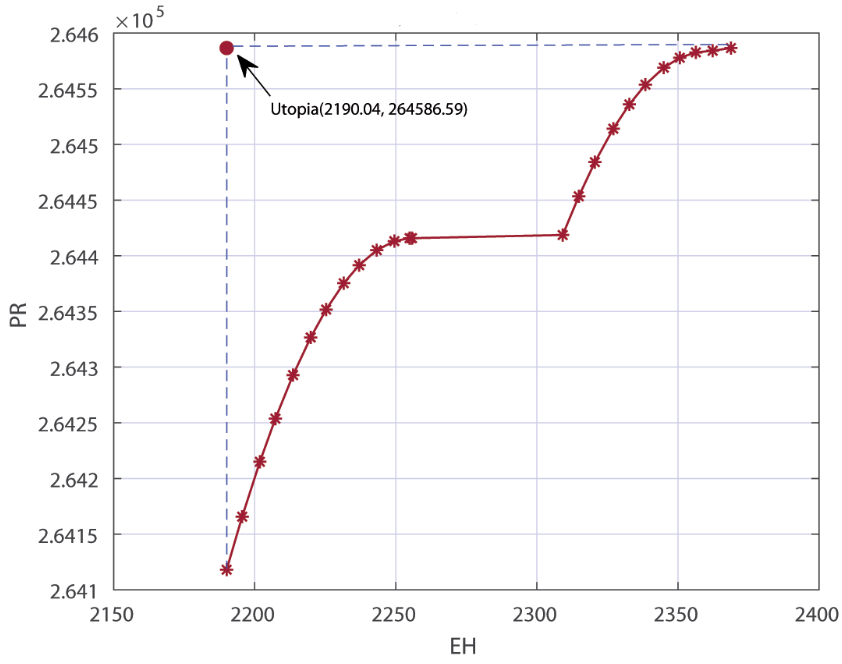

Figure 10. Pareto fronts (no inflation)

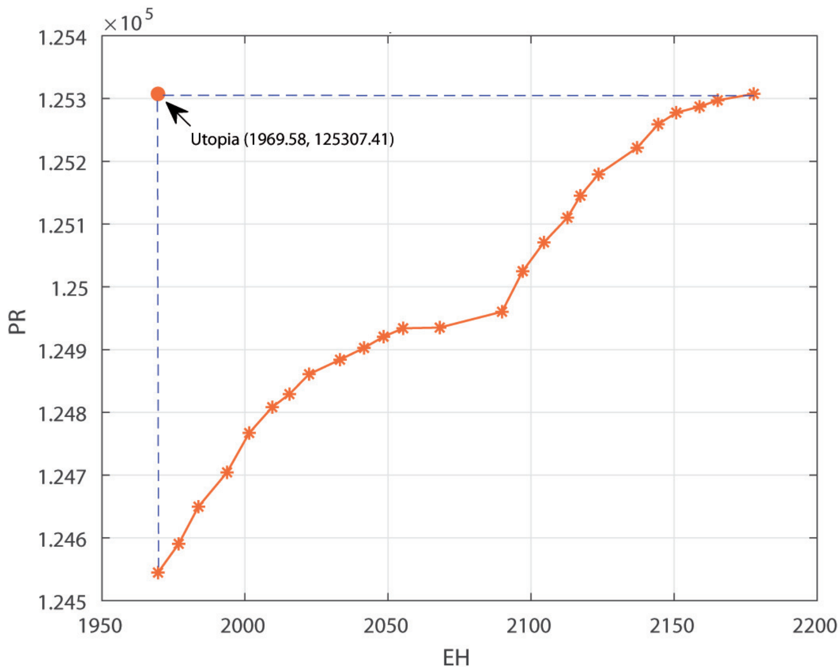

Figure 11. Pareto front (carbon tax)

Case 2: The model with carbon tax

We develop the model based on the cap-and-trade policy. The same model can be applied to the carbon tax policy model by making a minor change. To get the results under a carbon tax policy, substitute Q_i=0 (i =1, 2, ...), and replace the carbon price(C_4) by the carbon tax per ton. Figure 11 shows the Pareto front in this case. If we compare Figure 11 with Figure 4, we can observe that there is no significant change in emissions, but the profit declines in the case of a carbon tax. In carbon tax policy, the carbon tax is levied on every ton of emission. On the contrary, in the cap-and-trade policy, the polluting firm has to buy a carbon permit only if it exceeds the permitted cap. It can sell the excess unused permits in the carbon market. This might be the reason for the declining profit in carbon tax policy.

Case 3: The model without green investment

To get the results without green investment, substitute $I_{\max} = 0$. Figure 12 shows the Pareto front. A comparison between Figure 12 and Figure 4 shows that without green investment the profit declines and total emission significantly increases. So, green investment is beneficial for polluting firms.

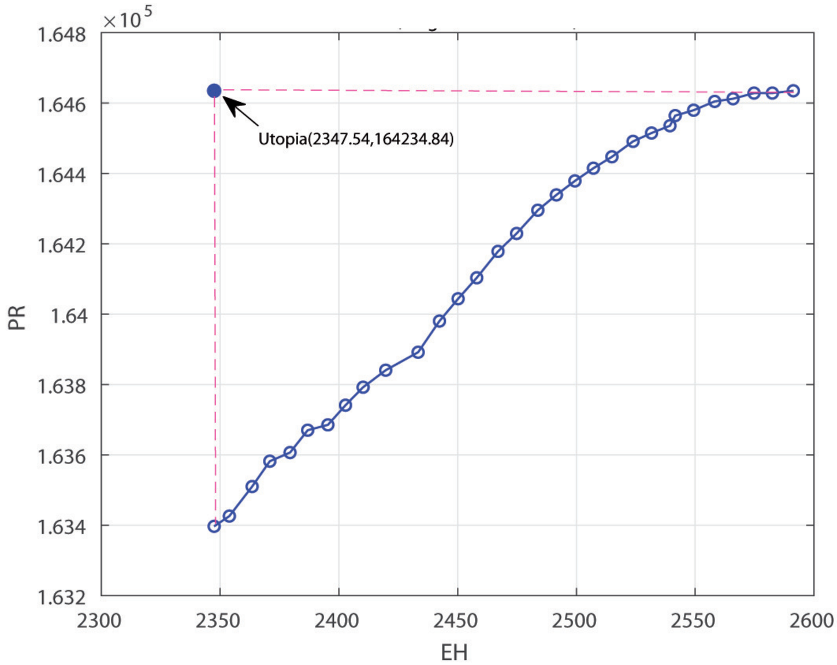

Figure 12. Pareto front (without green investments)

7. Analysis of results and managerial insights

Many research articles available in the existing literature consider inflation, carbon emissions, and green investments as important model parameters. However, none of them integrates all these parameters in a single model under a bi-objective scenario. The uniqueness of the present study is the inclusion of plan-based green investments, source-based emissions, and the maintenance cost of green installations in a single model under a bi-objective scenario. This is quite realistic and is missing in the existing literature. Our bi-objective model integrates inflation, plan-based green investment, the green installation maintenance cost, and cap-and-trade policy in a single finite-horizon model. Two objectives are - profit maximization and emission minimization. In this section, we wish to interpret the results obtained in the numerical example and the special cases, presented in Sections 5 and 6. In Tables 4, A1, A2, and A3, the last row indicate the values of the decision variables and output variables at optimum corresponding to a single objective problem with "profit maximization" as the only objective. The first row indicates the results corresponding to the best environmental performance (least emissions) as set by the decision-maker in a compromised solution. We observe that the firm must sacrifice a part of its optimum profit to reduce emissions to a certain level in a compromised non-dominated solution. Table 4 shows that the total decrease in profit to reduce emissions from 2177.85 t to 1969.58 t is \$762.72(=168496.56 – 167733.84). So, the average rate of decrease in profit from its optimum value for reducing emissions is \$3.662 per ton.

The same rate for three special cases - “without inflation”, “carbon tax”, and “without green investments” are \$2.621, \$3.662, and \$5.079, respectively. Let us use the term “rate of profit loss” for this average loss of profit from its optimum value for reducing emissions by 1 t in a compromised solution. The numerical example, sensitivity analysis, and special cases provide the following managerial insights:

- The rate of profit loss increases in the absence of green investment and decreases in the absence of inflation.
- Green investment increases profit and decreases emissions.
- It is evident from the last rows of Table 4 and Table A2 that the cap-and-trade and the carbon tax policies have a similar impact on emissions. The carbon tax policy has more negative effects on the optimum profit than the cap-and-trade policy.
- Inflation reduces the optimum profit.
- The carbon quota has no impact on the total emission at optimum. It has no role in the firm’s optimal policy. It only changes the profit linearly. However, if the size of the carbon market is limited and a polluter has to pay a hefty fine for every unit of emission exceeding the quota, then the carbon quota might have an impact on the total emissions. In this situation, the polluting firm might reduce its production size to get rid of the hefty fine, which in turn affects the GDP.
- The carbon price plays an important role in reducing emissions. An increase in the carbon price results in a decrease in emissions. The total emission is more sensitive to the carbon price for lower carbon prices. So, if a state’s carbon price is already very high, then a small change in the carbon price will not make any major change in the emissions.
- The emission rates from different sources of a manufacturing firm have a negative covariance with the optimum net profit. So, a firm must upgrade its production system using green technologies to improve its net profit. This will benefit the firm in the long run.

8. Conclusions and future research directions

Conventional emission-based inventory models with green investment assume green investment amount as a continuous decision variable. However, this assumption is not valid in a real situation. Installing energy-efficient machines, hiring experts for manufacturing, etc., are some areas where the manufacturer can invest in reducing emissions. The manufacturer must bear the cost of new machines and hire the experts, not part of the total cost. Hence, we must consider the dollar investment in green technology as discrete, depending upon the green investment plan. The present article presented a finite horizon production inventory model with plan-based green investment, inflation, and the present value of money under a cap-and-trade policy. We also considered source-based emissions. We solved the model in a bi-objective scenario – maximizing the present value of net profit and minimizing the total emissions. We found the Pareto front based on the EC method. We also found the Pareto fronts for some particular cases. This is the first paper that integrates inflation, the present value of money, and plan-based green investment in a bi-objective scenario. We solved the numerical problem using the flowchart provided in Appendix. To validate our model, we perform a sensitivity analysis of the key model parameters. We observe in the EC method that the decision variables n, u, v, w, s are the non-increasing functions of the parameter ϵ. Maintaining sustainability in a production-inventory process is a big challenge to decisionmakers. The growing environmental consciousness of the customers forces the firms to tend towards sustainability of the processes involved in a production-inventory system. An environment-conscious firm might be interested in finding an optimal policy in a bi-objective scenario where maximization of profit and minimization of emissions are the two objectives. A firm manager can apply this model with minor modifications as per his need. The model can guide him in making an appropriate decision about the compromised solution. If the manager wants to maintain the inventory for a very short period, then inflation might not be significant. Our special case 1 mentioned how the present model can be used without any inflationary effect. This model is also valid in a situation where the firm does not want to invest in green technology (special case 3). Our price-sensitive demand structure can help the manager to decide on the optimal selling price.

For future research, our model can be extended by considering deteriorating items with or without shortages. This model can be re-analyzed for different types of demand. The trade-credit option can be incorporated into this model. This idea can be used in a supply chain model. This model can be restructured by incorporating price breaks. The model can be revisited considering imprecise emission rates.

Conflict of interest: None.

Data availability: No data used.

Acknowledgement

The authors are indebted to the referees for their valuable comments to improve the quality of this paper.

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A. Appendix

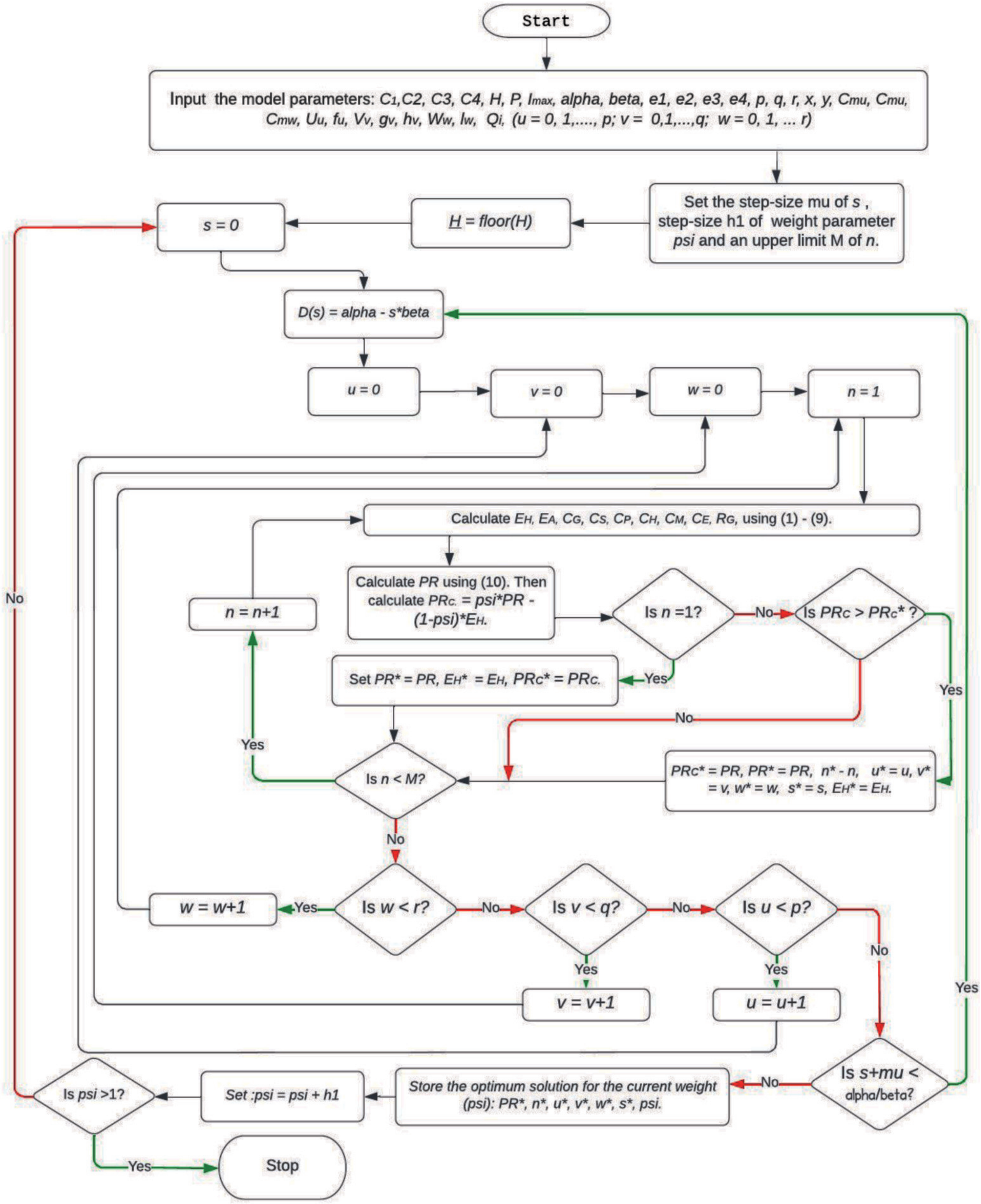

Figure A1. Flowchart of the proposed algorithm

Table A1. Non-dominated solutions (no inflation) – EC method

ϵ	n	u	v	w	s ($)	HS (t)	PR ($)	ϵ	n	u	v	w	s ($)	HS (t)	PR ($)
2190.10	9	3	3	3	69.86	2190.04	264118.06	2285.46	9	3	3	3	69.02	2255.69	264415.81
2196.06	9	3	3	3	69.79	2195.54	264165.69	2291.42	9	3	3	3	69.02	2255.69	264415.81
2202.02	9	3	3	3	69.71	2201.82	264215.00	2297.38	9	3	3	3	69.02	2255.69	264415.81
2207.98	9	3	3	3	69.64	2207.31	264253.69	2303.34	9	3	3	3	69.02	2255.69	264415.81
2213.94	9	3	3	3	69.56	2213.57	264292.84	2309.30	8	3	3	3	69.77	2309.13	264418.66
2219.90	9	3	3	3	69.48	2219.83	264326.66	2315.26	8	3	3	3	69.70	2314.84	264453.41
2225.86	9	3	3	3	69.41	2225.30	264351.69	2321.22	8	3	3	3	69.63	2320.55	264484.13
2231.82	9	3	3	3	69.33	2231.55	264375.38	2327.18	8	3	3	3	69.55	2327.07	264514.13
2237.78	9	3	3	3	69.26	2237.01	264391.59	2333.14	8	3	3	3	69.48	2332.77	264536.00
2243.74	9	3	3	3	69.18	2243.24	264405.06	2339.10	8	3	3	3	69.41	2338.46	264553.69
2249.70	9	3	3	3	69.10	2249.47	264413.16	2345.06	8	3	3	3	69.33	2344.96	264568.94
2255.66	9	3	3	3	69.03	2254.91	264415.81	2351.02	8	3	3	3	69.26	2350.64	264577.84
2261.62	9	3	3	3	69.02	2255.69	264415.81	2356.98	8	3	3	3	69.19	2356.32	264582.66
2267.58	9	3	3	3	69.02	2255.69	264415.81	2362.94	8	0	3	3	69.22	2362.28	264584.16
2273.54	9	3	3	3	69.02	2255.69	264415.81	2368.77	8	0	3	3	69.14	2368.77	264586.59
2279.50	9	3	3	3	69.02	2255.69	264415.81								

Table A2. Nondominated solutions (carbon tax) – EC method

ϵ	n	u	v	w	s ($)	HS (t)	PR ($)	ϵ	n	u	v	w	s ($)	HS (t)	PR ($)
1970.00	8	3	3	3	73.81	1969.58	124544.69	2082.00	8	0	3	3	72.76	2068.06	124935.09
1978.00	8	2	3	3	73.74	1976.83	124590.50	2090.00	7	0	3	3	73.96	2089.83	124960.34
1986.00	8	2	3	3	73.66	1983.73	124649.81	2098.00	7	3	3	3	73.80	2097.08	125024.97
1994.00	8	3	3	3	73.53	1993.73	124704.41	2106.00	7	2	3	3	73.73	2104.51	125070.88
2002.00	8	3	3	3	73.44	2001.47	124767.19	2114.00	7	2	3	3	73.64	2112.70	125110.25
2010.00	8	2	3	3	73.36	2009.54	124808.31	2122.00	7	2	3	3	73.59	2117.25	125145.13
2018.00	8	2	3	3	73.29	2015.55	124829.13	2130.00	7	0	3	3	73.59	2123.55	125179.41
2026.00	8	2	3	3	73.21	2022.41	124860.88	2138.00	7	3	3	3	73.36	2137.07	125221.63
2034.00	8	3	3	3	73.07	2033.20	124883.91	2146.00	7	0	3	3	73.36	2144.42	125258.94
2042.00	8	0	3	3	73.07	2041.60	124902.72	2154.00	7	0	3	3	73.29	2150.76	125277.53
2050.00	8	0	3	3	72.99	2048.44	124920.59	2162.00	7	0	3	3	73.20	2158.90	125287.09
2058.00	8	0	3	3	72.91	2055.27	124933.91	2170.00	7	0	3	3	73.13	2165.22	125297.25
2066.00	8	0	3	3	72.91	2055.27	124933.91	2178.00	7	0	3	3	72.99	2177.85	125307.41
2074.00	8	0	3	3	72.76	2068.06	124935.09								

Table A3. Non-dominated solutions (no green investment) – EC method

ϵ	n	u	v	w	s ($)	HS (t)	PR ($)	ϵ	n	u	v	w	s ($)	HS (t)	PR ($)
2348.00	9	0	0	0	74.41	2347.54	163396.94	2476.00	8	0	0	0	74.41	2474.83	164229.28
2356.00	9	0	0	0	74.35	2353.94	163426.06	2484.00	8	0	0	0	74.33	2483.76	164295.31
2364.00	9	0	0	0	74.26	2363.52	163510.03	2492.00	8	0	0	0	74.26	2491.57	164338.94
2372.00	9	0	0	0	74.19	2370.97	163582.00	2500.00	8	0	0	0	74.19	2499.37	164378.66
2380.00	9	0	0	0	74.11	2379.48	163606.66	2508.00	8	0	0	0	74.12	2507.16	164414.78
2388.00	9	0	0	0	74.04	2386.91	163670.09	2516.00	8	0	0	0	74.05	2514.95	164447.22
2396.00	9	0	0	0	73.96	2395.40	163685.28	2524.00	8	0	0	0	73.97	2523.84	164491.06
2404.00	9	0	0	0	73.89	2402.82	163741.16	2532.00	8	0	0	0	73.90	2531.61	164514.81
2412.00	9	0	0	0	73.82	2410.23	163792.25	2540.00	8	0	0	0	73.83	2539.37	164535.53
2420.00	9	0	0	0	73.73	2419.75	163840.69	2548.00	8	0	0	0	73.81	2541.58	164564.28
2428.00	9	0	0	0	73.73	2419.75	163840.69	2556.00	8	0	0	0	73.74	2549.34	164579.63
2436.00	8	0	0	0	74.78	2433.38	163891.72	2564.00	8	0	0	0	73.66	2558.19	164604.63
2444.00	8	0	0	0	74.70	2442.36	163980.31	2572.00	8	0	0	0	73.59	2565.92	164612.28
2452.00	8	0	0	0	74.63	2450.21	164043.72	2580.00	8	0	0	0	73.51	2574.75	164628.22
2460.00	8	0	0	0	74.56	2458.05	164103.31	2588.00	8	0	0	0	73.44	2582.47	164628.28
2468.00	8	0	0	0	74.48	2467.00	164178.34	2596.00	8	0	0	0	73.36	2591.29	164634.84