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Linear programming models for optimal workload and batching in pick-and-pass warehousing systems

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Abstract

Pick-and-pass systems are a part of picker-to-parts order-picking systems and constitute a very common storage solution in cases where customer orders are usually small and need to be completed very quickly. As workers pick items in the zones connected by conveyors, their work needs to be coordinated. The paper presents MILP models that optimize the order-picking process. The first model uses information about the expected demand for items to solve the storage location problem and balance the workload across zones. The task of the next model is order-batching and sequencing – two concepts are presented that meet different assumptions. The results of the exemplary tasks solved with the use of the proposed MILP models show that the total picking time of a set of orders can be reduced by about 35-45% in comparison with random policies. The paper presents an equation for the lower bound of a makespan. Recommendations about the number of zones that guarantee the required system efficiency are also introduced.

Keywords: *linear programming, order-picking, optimal storage, pick-and-pass systems*

1. Introduction

Order-picking is an essential process that plays a critical role in many mail-order stores, where a considerable amount of usually small orders must be picked quickly. Many researchers indicate that the order-picking process can generate about 50–75% of the total operating costs of a warehouse [7, 11, 38]. Order-picking is identified as the highest priority activity for productivity improvements in warehouses [38]. For this reason, the process is constantly being improved. Roodbergen and Vis [35] indicate three groups of operating policies that can improve the operational efficiency of order-picking: routing, batching and storage assignment. Routing includes moving order pickers between storage locations to retrieve needed products. Creating order batches to minimize the distance travelled by the pickers is called the Order Batching Problem [29]. The goal of the storage assignment is to determine where to store items in the warehouse storage area.

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In the pick-and-pass systems (also called progressive zoning systems) analysed in this paper, the storage area is divided into zones (connected by a conveyor) where pickers pick only a part of the items from the customer order. Moreover, in e-commerce, standard orders usually contain a small number of items [27, 31], so for the effective order-picking process, they have to be accumulated into batches, generating pick-lists. As the storage area assigned to one picker is relatively small and all items (stock keeping units, SKU) are easily accessible, the routing does not play as essential a role in pick-and-pass systems as in more common one-zone picker-to-parts systems. The main goals of this paper are proposals for improving the assignment of items to zones, and the optimization of the order batching procedure, together with establishing the proper sequence of picking batches. Both issues should balance the workload in zones and make the performance of the order-picking process smoother. Two other questions will also be discussed: proper storage inside each zone that reduces the distance covered by the pickers, and the risk of emergency replenishment of items that can interrupt the picking process.

The paper is organized as follows. The following chapter presents a literature review concerning pick-and-pass systems, storage location assignment and order-batching problems. Section 3 describes the pick-and-pass systems and outlines the assumptions adopted in this paper. The following two chapters present the mixed integer linear programming (MILP) models for optimizing the order-picking process in the pick-and-pass systems. The storage assignment optimization model is described in section 4, while section 5 is dedicated to the problem of determining batches. The results of the examples solved using the presented MILP models are shown in section 6. The paper is concluded in section 7, where the directions of future work are highlighted.

2. Literature review

The order-picking process can be carried out in many ways. De Koster et al. [11] present the most general classification of order-picking systems, where the main criterion of division is the degree of automation. The number of warehouses with picking robots or automated picking facilities is still increasing, but facilities where humans do most of the picking still constitute the majority. A few companies (e.g., Amazon) have adopted both solutions, but the work of humans is aided by electronic devices and other modern technology [13, 21, 28, 36]. De Koster et al. [11] divide the systems employing humans into picker-to-parts, parts-to-picker, and put systems. Order-picking in picker-to-parts systems can be organized in only one zone or in many picking zones, where the following picking strategies can be used: sort-while-pick, pick-and-pass, pick-and-sort, and wave picking. The pick-and-pass concept usually assumes that the picking process runs without losing the integration of orders: each order is picked separately [23, 34]. However, in a few papers, order-batching (during the order-picking process, orders lose their integrity, after which additional effort is needed for sorting) is considered for pick-and-pass systems [11, 31, 43, 44].

De Koster [9] precisely describes two types of pick-and-pass systems and provides a method based on Jackson's modelling for evaluating the influence of many parameters on the performance of the order-picking process. The presented approach can be used for establishing, e.g., the required number of zones, the conveyor speed, the size of batches, or the number of batches processed per period. Yu and De Koster [43] use the G/G/m queuing network to analyse pick-and-pass systems. The methods for estimating the number of zones and the number of pickers in each zone in pick-and-pass systems are described by Melacini et al. [27]. Pan et al. [31] present an optimization model for storage location assignment. The authors emphasize that the storage location assignment models belong to the class of NP-hard problems and, to solve this, propose a heuristic method based on genetic algorithms. The paper considers the issue of designating the required storage space for each item, as well as problems with the urgent necessity of their replenishment. The model proposed by Pan et al. [31] is described in detail in the next chapter. The problem of space allocation for items is also considered by Anken et al. [1]. The authors propose a linear programming model that minimizes the expected number of replenishments during a period of time. A two-phase heuristic is introduced and recommended, as the problem is solvable in a reasonable time only for small instances. A more general problem (than analysing pick-and-pass systems) is considered by Jewkes et al. [24], where dynamic programming is used for determining the optimal policies for storage location assignment, the picker home base location (equivalent to the dwell point in automated storage and retrieval systems) and the size of zones. Heuristics for space allocation and stock replenishment that help decrease order-picking time are proposed by Gagliardi et al. [20]. Pan and Wu [32] use a Markov chain to estimate the expected distance travelled by the pickers in the zones. Tarczyński and Jakubiak [37] use a simulation tool to compare pick-and-pass and pick-and-sort systems.

The issue of storage location assignment is usually considered from the perspective of the correlation between items (e.g., [6, 19, 25, 41]). Jane and Laih [23] investigate the pick-and-pass system without considering the problem of order-batching. The authors measure the co-appearance of items in one order and propose a heuristic for storage assignment that reduces the workload differences in zones. A similar issue is analysed by Jane [22]. However, when orders are very small, looking for a correlation between SKUs does not make sense. Such a problem is studied by Pan et al. [31], and constitutes the subject of consideration in this paper.

A significant problem when considering very small orders is how to aggregate them so as to improve the efficiency of the order-picking process, i.e., order-batching. In classical picker-to-parts systems, picking orders in batches can reduce the distance covered by the picker and diminish the order-picking time [10]. Won and Olafsson [40] indicate the other goals of order batching: reduction of the number of batches (which implies a lower number of trips) and diminishing the waiting time of orders in the system. A review of batching problems and methods is presented by e.g., [3]. When considering pick-and-pass systems, the other optimization problem occurs with zone workload balance [4]. This issue is not as well explored as classic picker-to-parts systems. Pan et al. [30] propose a batching heuristic based on a genetic algorithm concept for pick-and-pass systems. The method optimizes two criteria: it minimizes the number of batches and reduces the difference in workload across zones, as in subsequent zones, workers pick items from different batches at the same time. In this paper, batching is considered together with the sequencing of batches. The goal is to pick items from all orders (in a specified time period) as soon as possible. Such a problem can be treated as a scheduling problem with a minimal makespan criterion. As the workload in zones should be balanced, the problem seems relatively easy to solve.

Attari et al. [42] discover a MILP model for batching and routing, while Ardjmand et al. [2] present a model for classic manual order-picking systems that minimizes the makespan. Simultaneously, three problems are optimized: order assignment to the pickers, order-batching, and picker routing. In this paper, the first MILP model balances the expected workload in the zones. In contrast, the second model, based on the results from the use of the previous model, simultaneously solves two other problems: order-batching and sequencing of batches. In the literature, zone picking systems are sometimes analysed separately from the batch picking concept. For example, Parikh and Meller [33] treat zone picking and order-batching separately, and evaluate both ideas using the cost model.

As mentioned earlier, one of the issues connected with zone picking is the designation of the number of zones. De Koster et al. [12] propose a MILP model that minimizes the throughput time to finish a batch for synchronized zoning in a classic picker-to-parts system. The model can be used to determine the number of zones. The models presented in this paper can be successfully used for establishing the number of zones in progressive zoning systems in cases when a set of orders (divided into batches) have to be completed in a specified time window. This paper presents the joint solution for storage location assignment, order batching, and sequencing as a new concept that fills the research gap.

3. Problem description

The pick-and-pass system analysed in this paper (Figure 1) consists of a central roller conveyor connecting all zones where pick stations are located ([9], [30]). Only one picker works at each pick station. The pickers move along the shelves with items (storage locations) located parallel to the conveyor. All items are easily accessible to the pickers. Very often, computer-aided picking systems (e.g., pick-to-light systems) are applied in order to make the picker's job faster. Each storage location has light indicator modules that guide the picker to the appropriate SKU and show the amount of the item to be retrieved. After picking the needed item quantity, the picker presses the lighted button to confirm the picking operation. Then the picker puts the items directly on the conveyor or inserts them in the bin together with other items from the same order. After that, the other light indicator switches on, and the procedure repeats. The bar codes attached to the containers or directly to the items (when bins are not in use) are automatically scanned in the sorting area to help ensure the integrity of customer orders.

The major inconvenience of pick-and-pass systems is the possible imbalance in workload in the zones, which may lead to congestion and reduce the performance of the order-picking process. The main optimization goals are: determining the number of zones, assignment of items to the zones, order-batching, and sequencing of batches. Another problem not considered in this paper is establishing the storage space needed for each SKU.

The assumptions taken for the MILP models described in the following chapters are presented below. Some of them are adopted from Pan et al. [31]:

- The speed of the conveyor does not affect the system's capacity. The transport process of items through the zones does not affect the total order-picking time at all.
- The workload for all items is the same and does not depend on the size of an item or the number of items needed to be picked to fulfil the requirement associated with one order.
- The workload connected with picking items from the same zone is additive, even when the picker picks the same SKU for a few orders accumulated into one batch.
- The picking time is a function that increases the workload.
- Customer orders are very small, so there is no correlation between the occurrence of items in orders.
- Each SKU can be allocated to many storage locations but only in one zone. The number of storage locations is designated based on the predicted demand and the size of the SKU (heuristic determin-

ing the required storage space for items is presented by Pan et al. [31].

- In one storage location, only one SKU can be stored.
- As the storage locations are narrow and easily accessible to the picker, the travel time between locations is not considered. For the same reason, the items are assigned to a zone without determining a particular storage location.
- The number of storage locations can differ in each zone.
- Only one picking line is considered (Figure 1). Pan et al. [31] investigate many picking lines working simultaneously but for the adopted problem of balancing the workload in the zones, the number of picking lines does not matter. Only at the stage of order-batching does this issue take on significance.

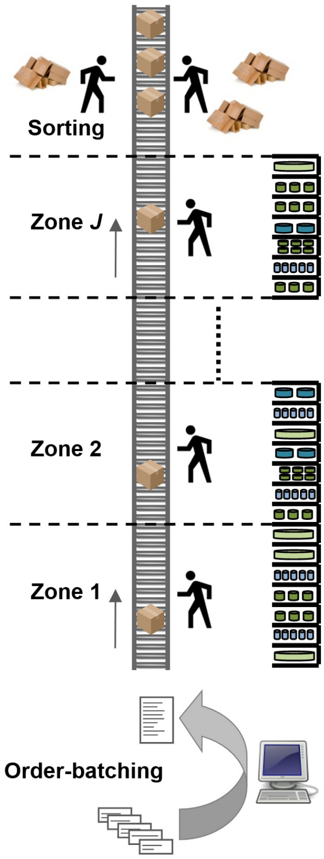

Figure 1. Pick-and-pass system with one picking line

The notation used for the models presented in the next two chapters is as follows.

Indexes

i – item index, $i = 1, 2, \ldots, I$

j – picking zone index, $j = 1, 2, \ldots, J$

m – picking line index, $m = 1, 2, \ldots, M$ (only in [31])

l – storage location index, $l = 1, 2, \ldots, L$ (only in [31])

k – order index, $k = 1, 2, \ldots, K$

b – batch index, $b = 1, 2, \ldots, B$

d – time interval index, $d = 1, 2, \ldots, B + J - 1$

Parameters

I – number of SKUs to be stored in the warehouse
J – number of picking zones in the warehouse
M – number of picking lines (only in [31]; in this paper M=1)
L_j – number of storage locations in zone j
p_i – expected number of orders containing item i to be picked during one time period (one day, one shift)
s_i – maximal number of items with index i that can be stored in one storage location
n_i – the average number of items with index i in one order (picked by the picker at the same time)
N_i – the number of storage locations dedicated for item i (integer);
the N_i values can be established arbitrarily or calculated based on other parameters
K – number of orders
w_{kj} – workload in zone j when picking items from order k
s_{kj}^{bin} – the accumulated size of all items from order k picked in zone j as a percent of the bin's capacity
W – maximal workload in a zone for one batch
B – maximal number of batches

Decision variables

$$x_{ij} = \begin{cases} 1 & \text{when item } i \text{ is assigned to zone } j, \\ 0 & \text{otherwise} \end{cases}$$

y – maximal workload in storage zones (real or integer)

$$v_{kb} = \begin{cases} 1 & \text{when order } k \text{ is assigned to batch } b \\ 0 & \text{otherwise} \end{cases}$$

u_d – maximal workload in zones in time interval d (real or integer)
t_{bj}^{st} – starting time of picking items from batch b in zone j (real or integer)

4. The MILP model for the problem of storage location assignment (P&P assignment model)

Before presenting the MILP models, the proposal of Pan et al. [31] is shown in a more general form. The authors introduce a nonlinear model that optimizes two criteria: the difference in the workload in the zones and the necessity of emergency replenishment of a particular SKU. For the first criterion, the authors propose the objective function:

Minimize:

$$z_1 = \sum_{m=1}^{M} \sum_{j=1}^{J} \sum_{l=1}^{L_j} \sum_{i=1}^{I} n_i p_i x_{mjli} - \frac{\sum_{l=1}^{L_j} n_l p_l}{MJ} \tag{1}$$

The second objective function is as follows:

Minimize:

$$z_2 = \sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{j=1}^{J}\sum_{L=1}^{L_j} x_{mjli} - \frac{n_i p_i}{s_i} \tag{2}$$

Subject to:

$$\sum_{i=1}^{I} x_{mjli} = 1 \quad \text{for } m = 1, 2, \ldots, M;\ j = 1, 2, \ldots, J;\ l = 1, 2, \ldots, L \tag{3}$$

$$\sum_{m=1}^{M}\sum_{j=1}^{J}\sum_{l=1}^{L} x_{mjli} - \sum_{l=1}^{L} x_{mjli} \leq 0 \quad \text{for } m = 1, 2, \ldots, M;\ j = 1, 2, \ldots, J;\ l = 1, 2, \ldots, L \tag{4}$$

$$x_{mjli} \in \{0, 1\} \quad \text{for } m = 1, 2, \ldots, M;\ j = 1, 2, \ldots, J;\ l = 1, 2, \ldots, L;\ i = 1, 2, \ldots, I \tag{5}$$

where x_{mjli} denotes the assignment of SKU i to the particular storage location l in the picking line m and zone j. The objective function (1) minimizes the sum of absolute values of the differences between the workload in each zone and the average workload in one zone. The objective function (2) minimizes the sum of absolute values of the differences between the number of storage locations assigned for item i and the expected number of locations needed for this item to meet demand.

In further analysis, only the first of the presented goals is investigated. As the assignment of items to a particular storage location in the zone does not affect the value of the objective function (1), only storage to a particular zone is considered in this paper. For larger sizes zones and different layouts of racks, the storage location of a particular item may significantly influence order-picking time, and such a simplification is not possible.

The balance in workload in the zones can be achieved in a simpler way than described above: by minimizing the maximal workload in all zones. Such a concept is equivalent to the criterion (1). The model becomes even more straightforward if we omit the picking line indexes and the assignment to particular locations.

The P & P assignment model in a general form is:

Minimize:

$$z = \max_{j} \sum_{i=1}^{I} p_i x_{ij} \tag{6}$$

Subject to:

$$\sum_{j=1}^{J} x_{ij} = 1 \quad \text{for } i = 1, 2, \ldots, I \tag{7}$$

$$\sum_{i=1}^{I} N_i x_{ij} \leq L_j \quad \text{for } j = 1, 2, \ldots, J \tag{8}$$

$$x_{ij} \in \{0, 1\} \quad \text{for } i = 1, 2, \ldots, I;\ j = 1, 2, \ldots, J \tag{9}$$

The minimax problem can be converted to a linear form by defining the additional decision variable y that will measure the highest workload in the zones (see e.g., [39]). Now:

Mminimize:

$$z = y \tag{10}$$

Subject to:

$$\sum_{j=1}^{J} x_{ij} = 1 \quad \text{for } i = 1, 2, \ldots, I \tag{11}$$

$$\sum_{i=1}^{I} N_i x_{ij} \leq L_j \quad \text{for } j = 1, 2, \ldots, J \tag{12}$$

$$\sum_{i=1}^{I} p_i x_{ij} \leq y \quad \text{for } j = 1, 2, \ldots, J \tag{13}$$

$$x_{ij} \in \{0, 1\} \quad \text{for } i = 1, 2, \ldots, I;\ j = 1, 2, \ldots, J \tag{14}$$

$$y \geq 0 \tag{15}$$

The objective function (10) minimizes the maximal expected workload in all zones. Equations (11) are responsible for assigning each item to precisely one zone. The capacity of all zones cannot be exceeded (constraints (12)), and the workload in the zones should be balanced. This goal can be achieved when the maximal workload is minimized (constraints (13) are connected with an objective function (10)). The model in such a simplified form is solvable even for real-life instances (the results of computational examples are presented in Section 6).

5. The MILP model for the problem of batching and sequencing

5.1. P & P batch and sequence model

After the items (based on the size and the predicted picking frequency) are assigned to the zones and the workload is balanced, the current orders need to be batched for the picking process each day. The model presented is as follows:

Minimize:

$$t = \sum_{d=1}^{B+J-1} f(u_d) \tag{16}$$

Subject to:

$$\sum_{b=1}^{B} v_{kb} = 1 \quad \text{for } k = 1, 2, \ldots, K \tag{17}$$

$$\sum_{k=1}^{K} w_{kj} v_{k,d-j+1} \leq u_d \quad \text{for } d = 1, 2, \ldots, B + J - 1;\ j = \max\{d - B + 1; 1\}, \ldots, \min\{d, J\} \tag{18}$$

$$\sum_{k=1}^{K} w_{kj} v_{kb} \leq W \quad \text{for } j = 1, 2, \ldots, J;\ b = 1, 2, \ldots, B \tag{19}$$

$$v_{kb} \in \{0, 1\} \quad \text{for } k = 1, 2, \ldots, K;\ b = 1, 2, \ldots, B \tag{20}$$

$$u_d \geq 0 \quad \text{for } d = 1, 2, \ldots, B + J - 1 \tag{21}$$

The throughput time of batches is divided into time intervals. Time intervals refer to the time needed for simultaneously picking items from batches in all zones (it is assumed that the picking time in a zone is a function that increases the workload in this zone, for simplicity, let $f(u_d) = u_d$. The length of intervals can therefore vary. In detail: during the time interval $d = 1$, only the first batch in the first zone is being processed. Time interval $d = 2$ means that the picker in zone 2 collects items from batch 1, and the picker in zone 1 picks SKUs from batch 2. For $d = 3$ (where the number of zones and the number of batches is at least 3): items from batches 1, 2, and 3 are picked in the zones: respectively, 3, 2, and 1. The number of time intervals is therefore $B + J - 1$. The algorithm can sometimes generate batches with no job in certain zones. To balance the workload, it may happen that in a time interval, the pickers do not work at all. This means they start picking items from subsequent batches. During the last time interval $d = B + J - 1$ only the last batch is being processed in the final zone. As the optimization criterion is the makespan, for the first few batches generated and sequenced by the model, the workload in the first zones should be diminished. In contrast, the workload in the first zones should be greater for the last batches processed than in the last zones. Only then can the workload in the zones be balanced across the time intervals (for example use of the model, see the next chapter).

In the presented model, the objective function (16) minimizes the makespan, i.e., the total time for picking all orders. The set of equations (17) ensures that all orders will be assigned to batches. Constraints (18) designate the maximal workload in picking zones for all time intervals. In the first time interval, only items from zone 1 can be picked ($d = 1 \rightarrow j = 1$ in constraints (18)). In the second time interval, in zone 1 the items from batch 2 are picked, and in zone 2 the items from batch 1 ($d = 2 \rightarrow j \in \{1, 2\}$). And so on. In the last time interval, only items from the last batch can be picked in the last zone ($d = B + J - 1 \rightarrow j = J$). Moreover, in zone j during time interval d the batch $d - j + 1$ is processed. Constraints (19) can be used in order to limit the workload in zones for one batch.

In some approaches, the maximal bin capacity is considered (see, e.g., [30]). In this case, the total size of all items from each order has to be calculated, and additional constraints can be added to the model:

$$\sum_{k=1}^{K} \sum_{j=1}^{J} s_{kj}^{\text{bin}} v_{kb} \leq 1 \quad \text{for } b = 1, 2, \ldots, B \tag{22}$$

The model presented in this subsection has one deficiency. In the case when the number of batches is lower than the number of picking zones, the balance of the workload across the zones can be impossible to achieve, and the congestion effect may occur. The reason for such situations is that the pickers always start their job together at the beginning of the time interval and, after picking all the required items, wait until the picker with the most extensive workload finishes. To avoid this problem, the assumption about time intervals must be skipped in the next model.

5.2. P & P batch & sequence W/O congestion model

This subsection presents an alternative batching and sequencing model that minimizes the makespan and avoids blockage (congestion) situations. In this concept, another set of decision variables is needed that measures starting time for picking particular batches in each zone.

Minimize:

$$t = t_{BJ}^{\mathrm{st}} + \sum_{k=1}^{K} w_{kJ} v_{kB} \tag{23}$$

Subject to:

$$\sum_{b=1}^{B} v_{kb} = 1 \quad \text{for } k = 1, 2, \ldots, K \tag{24}$$

$$\sum_{k=1}^{K} w_{kj} v_{kb} \le W \quad \text{for } j = 1, 2, \ldots, J;\ b = 1, 2, \ldots, B \tag{25}$$

$$t_{11}^{\mathrm{st}} = 0 \tag{26}$$

$$t_{b+1,j}^{\mathrm{st}} \ge t_{bj}^{\mathrm{st}} + \sum_{k=1}^{K} w_{kj} v_{kb} \quad \text{for } j = 1, 2, \ldots, J; b = 1, 2, \ldots, B-1 \tag{27}$$

$$t_{b,j+1}^{\mathrm{st}} \ge t_{bj}^{\mathrm{st}} + \sum_{k=1}^{K} w_{kj} v_{kb} \quad \text{for } j = 1, 2, \ldots, J-1;\ b = 1, 2, \ldots, B \tag{28}$$

$$v_{kb} \in \{0, 1\} \quad \text{for } k = 1, 2, \ldots, K;\ b = 1, 2, \ldots, B \tag{29}$$

$$t_{bj}^{\mathrm{st}} \ge 0 \quad \text{for } j = 1, 2, \ldots, J;\ b = 1, 2, \ldots, B \tag{30}$$

In the P & P batch and sequence W/O congestion model, the objective function (23) minimizes the makespan, i.e., the value of the starting time of picking the items from the last batch in the furthest zone increased by the time (connected with the appropriate workload) needed for collecting items assigned to batch B in zone J. The equations (24) assign each order to precisely one batch. The maximal workload allocated to one zone for a batch can be limited using constraints (25). Equation (26) sets the starting time of picking batches (i.e., the starting time of the first batch in the first zone) to zero. The constraints (27) ensure that each starting time does not have to be lower than the ending time of picking the previous batch in the same zone. Similarly, the constraints (28) guarantee that order picking will not start before the same batch is finished in the previous zone.

6. Computational results

The P & P assignment model, the P & P batch and sequence model, and the P & P batch and sequence W/O congestion model were used to solve a few exemplary problems. The results presented in this chapter were obtained on a ten-year-old PC computer with a 32-bit OS using Open Solver [26] for MS Excel. Tables 1 and 2 show the size of the problems solved and the computation times for the assignment problem. The computation times do not include the work of the Open Solver in entering the models into the Simplex tables. It is purely the solving time. It is assumed that the SKUs can occupy a different number of storage locations which in the warehouse is slightly larger than needed (the items are assigned to about 99% of storage locations). Table 1 shows the results of the calculations when exact solutions of the P & P assignment model were expected. Only problems solved in a reasonable time for quick decision-making (less than 1 h) are presented. In the cases where only two zones are considered, the solution was obtained very fast – in less than 1 s. For larger problems – 10 zones and 10,000 SKUs, the model becomes quite big: 10,0001 decision variables and 10,020 constraints. The exact solution (where the workload is nearly perfectly balanced) was obtained in about 34 min. Approximate solutions (5% branch and bound tolerance level) were gained for even greater examples (Table 2). The obtained values are much better than the assumed tolerance level and do not exceed the value of 2.1%. For the biggest solved example: 10000 SKUs and 15 zones, the model had 150001 decision variables and 10030 constraints, and the nearly-optimal solution was obtained after 5 minutes with a 0.05% real value of the branch and bound coefficient.

Table 1. Exact solutions of example problems
for the P & P assignment model - size of the issues and computation time

1	2	3	4	5	6	7	8	9	10	11
2	1000	680	1352	99.41	2001	1004	13019	13020	0.01	0.07
2	5000	3310	6609	99.83	10001	5004	63555	63556	0.00	0.29
2	10000	6600	13169	99.77	20001	10004	128214	128214	0.00	0.76
6	1000	230	1352	97.97	6001	1012	4339	4340	0.02	5.05
6	5000	1110	6609	99.23	30001	5012	21181	21186	0.02	77.76
6	10000	2200	13169	99.77	60001	10012	42738	42738	0.00	133.95
10	1000	140	1352	96.57	10001	1020	2603	2604	0.04	621.31
10	5000	665	6609	99.38	50001	5020	10703	10707	0.04	442.99
10	10000	1330	13169	99.02	100001	10020	25642	25643	0.00	2029.93
15	1000	92	1352	97.97	15001	1030	1735	1736	0.06	1359.27

1 – No. of zones, 2 – No. of SKUs, 3 – No. of storage locations in each zone, 4 – Total No. of storage locations to be filled, 5 – Storage location fill factor, 6 – No. of decision variables, 7 – No. of constraints, 8 – Minimum workload in the zone, 9 – Objective value (maximum workload in the zone), 10 – difference between maximum and minimum workload,% 11 – time of computation (CPU sec).

Table 3 shows the computational results of the P & P batch and sequence model. Only examples where the number of batches was not smaller than the number of zones were analysed. In such situations,

Table 2. Approximate solutions of example problems for the P & P assignment model (branch and bound tolerance for all issues set to 5%) – size of the problems and computation time

1	2	3	4	5	6	7	8	9	10	11	12	13
2	1000	680	1352	99.41	2001	1004	13017	13022	13019	0.04	0.02	0.03
2	5000	3310	6609	99.83	10001	5004	63554	63557	63555	0.00	0.00	0.27
2	10000	6600	13169	99.77	20001	10004	128207	128221	128214	0.01	0.01	0.75
6	1000	230	1352	97.97	6001	1012	4334	4343	4340	0.21	0.07	2.28
6	5000	1110	6609	99.23	30001	5012	21182	21187	21186	0.02	0.00	25.29
6	10000	2200	13169	99.77	60001	10012	42735	42740	42738	0.01	0.00	86.45
10	1000	140	1352	96.57	10001	1020	2597	2608	2604	0.42	0.16	4.03
10	5000	665	6609	99.38	50001	5020	10693	10731	10706	0.36	0.23	33.47
10	10000	1330	13169	99.02	100001	10020	25635	25645	25643	0.04	0.01	112.88
15	1000	92	1352	97.97	15001	1030	1712	1748	1736	2.10	0.70	7.3
15	5000	445	6609	99.01	75001	5030	8451	8482	8474	0.37	0.09	62.47
15	10000	880	13169	99.77	150001	10030	17076	17103	17095	0.16	0.05	296.97

1 – No. of zones, 2 – No. of SKUs, 3 – No. of storage locations in each zone, 4 – Total No. of storage locations to be filled, 5 – Storage location fill factor, 6 – No. of decision variables, 7 – No. of constraints, 8 – Minimum workload in the zone, 9 – Objective value (maximum workload in the zone), 10 – Lower bound for objective criterion, 11 – Difference between maximum and minimum workload,% 12 – Branch and bound tolerance reached %, 13 – Time of computation (CPU sec).

the results obtained from both models, the P & P batch and sequence model and the P & P batch and sequence W/O CONGESTION model, were the same, and the computation time was similar. For the largest dataset example, 5000 orders containing 1-5 items were divided into 20 batches. The model had 100029 decision variables and 5400 constraints, and was solved with 0.63% possible loss to the real optimal value in about 5 minutes.

Table 3. Approximate solutions of example problems for the P & P batch and sequence model (branch and bound tolerance for all issues set to 5%)

1	2	3	4	5	6	7	8	9	10	11	12
1000	2	10	2423	10011	1040	1265	1265	9.73	104	0.00	1.43
1000	6	10	4008	10015	1120	754	750	9.00	62	0.60	4.75
1000	10	10	5562	10019	1200	893	883	7.84	114	1.13	5.51
5000	2	10	12024	50011	5040	6095	6093	0.36	1	0.03	26.52
5000	6	10	19953	50015	5120	3642	3638	20.60	181	0.12	49.67
5000	10	10	27698	50019	5200	4339	4330	18.79	294	0.20	96.09
1000	2	20	2423	20021	1080	1267	1265	4.29	24	0.16	5.07
1000	6	20	4008	20025	1240	701	690	3.96	27	1.59	9.79
1000	10	20	5562	20029	1400	628	612	11.79	63	2.61	11.45
5000	2	20	12024	100021	5080	6096	6093	0.62	2	0.05	150.69
5000	6	20	19953	100025	5240	3433	3421	7.24	59	0.35	233.31
5000	10	20	27698	100029	5400	3019	3000	23.72	162	0.63	311.03

1 – No. orders, 2 – No. zones, 3 – Maximum No. batches, 4 – Total workload, 5 – No. of decision variables, 6 – No. of constraints, 7 – Objective value, 8 – Lower bound for objective value, 9 – Average difference between workload in zones during one-time intervals, 10 – Maximum difference between workload in zones, 11 — Branch and bound tolerance reached (%), 12 – Computation time (CPU sec).

Additional experiments were conducted to verify how the order-picking process optimized by the presented algorithms works in comparison to random storage and random batching. For the experiments, it was assumed that the picking frequency follows the 80-20 Pareto rule, which can be described by the ABC curve [5]:

$$F(x) = \frac{(1+s)x}{s+x} \tag{31}$$

where x is the number of SKUs (items are sorted based on picking frequency in descending order), s – shape factor ($s = 0.07$ for 80-20 rule; see [5] for s values for different skewness of the ABC curve), $F(x)$ – accumulated picking frequency for x most frequently picked items.

For all experiments, a specified number of orders containing a small number of items was generated using the equation (31). It was assumed that 50% of orders contained only one SKU, while the other 50% could have 1–5 different SKUs. The probability distributions of the number of items in an order are presented in Table 4.

Table 4. Probability distribution for the number of items in an order

Number of items in an order	1	2	3	4	5
Probability for $k < 0.5K$	0.4	0.3	0.15	0.1	0.05
Probability for $k \geq 0.5K$	1	0	0	0	0

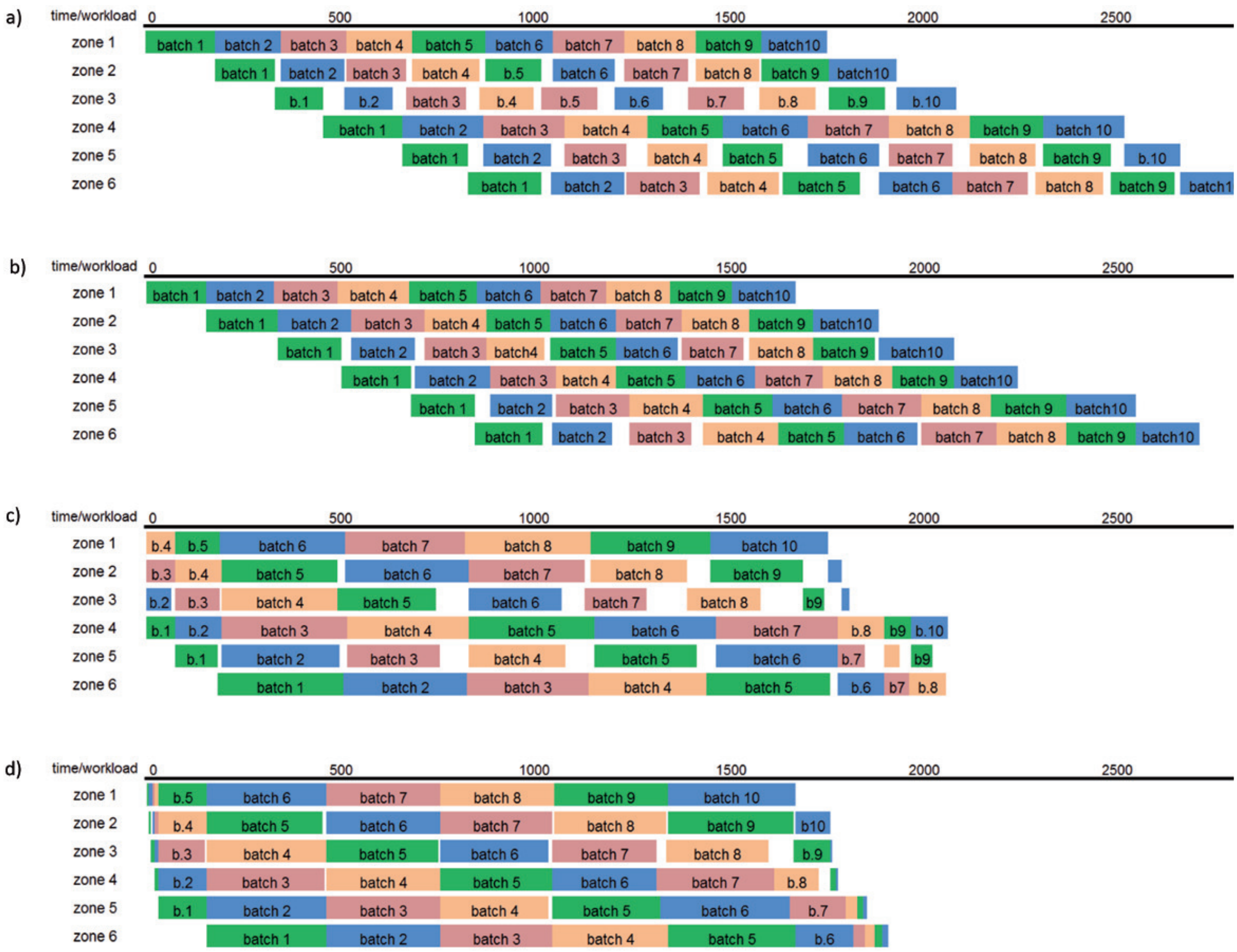

Figure 2. Gantt charts generated for the same data (6 zones, 5000 orders, 10 batches) and different assignment and batching policies: a) not optimized (random storage and random batching), b) workload optimized, random batching, c) batching optimized, random storage, d) storage and batching optimized

For the assignment model, it was assumed that for all SKUs stored in the warehouse, the workload is proportional to the picking frequency calculated (similarly to orders) using equation (31). Figure 2 presents an exemplary comparison of Gantt charts obtained for different combinations of random policies and optimization models. The results from a larger number of experiments are included in Table 5. The values in the brackets in the last three columns show the percentage improvement of makespan compared to random storage and batching. The most significant benefits were achieved by batching optimization. The application of storage and batching optimization algorithms caused a 35–45% reduction in total order-picking time. Ensuring only the expected balance in the zones with a random batching process is not very effective (10–20% improvement). Table 5 also presents the lower bounds for makespan. These values are impossible to achieve in practice. Storage assignment was conducted for experiments assuming the 80-20 Pareto rule. The orders were generated from the same probability distribution, but as the process was random, the workload for a particular set of orders was not perfectly balanced. The lower bound for the makespan can be achieved only when the optimization procedure for storage assignment is launched for a specified set of items to be picked.

Table 5. Makespan values for different combinations of storage and batching policies

No. of zones	No. of batches	Makespan (lower bound)	Storage: Random		Storage: Optimized	
			Batching: Random	Batching: Optimized	Batching: Random	Batching: Optimized
			Makespan[1]			
4	5	625	1213	895 (26.22)	1045 (13.85)	821 (32.32)
4	10	625	1055	874 (17.16)	854 (19.05)	664 (37.06)
4	15	625	1002	874 (12.77)	798 (20.36)	641 (36.03)
4	20	625	967	874 (9.62)	764 (20.99)	641 (33.71)
6	5	500	944	708 (25.00)	895 (5.19)	679 (28.07)
6	10	417	748	536 (28.34)	703 (6.02)	493 (34.09)
6	15	417	677	535 (20.97)	628 (7.24)	459 (32.2)
6	20	417	646	535 (17.18)	595 (7.89)	459 (28.95)
8	5	500	844	640 (24.17)	778 (7.82)	615 (27.13)
8	10	313	650	452 (30.46)	581 (10.62)	409 (37.08)
8	15	313	584	424 (27.4)	509 (12.84)	356 (39.04)
8	20	313	546	424 (22.34)	475 (13.00)	344 (37.00)
10	5	500	856	606 (29.21)	727 (15.07)	590 (31.07)
10	10	250	647	432 (33.23)	526 (18.7)	380 (41.27)
10	15	250	578	412 (28.72)	458 (20.76)	317 (45.16)
10	20	250	546	412 (24.54)	425 (22.16)	292 (46.52)
12	5	500	757	595 (21.4)	747 (1.32)	563 (25.63)
12	10	250	565	392 (30.62)	525 (7.08)	350 (38.05)
12	15	209	486	342 (29.63)	453 (6.79)	285 (41.36)
12	20	209	448	334 (25.45)	407 (9.15)	257 (42.63)

[1] Percentage improvements in comparison to random/random policy values in brackets.

A significant issue when implementing new warehousing concepts is establishing the values of parameters that will provide the proper efficiency of the projected system. One of the crucial problems for pick-and-pass systems is the number of zones (and the number of picking lines for cases where a very high capacity is required). Knowing the total workload for picking all orders, the completion time of a set of orders depends mostly on the number of batches and the number of zones. The lower bound for the makespan can be designated using the equation:

$$t \geq \frac{f(u)}{\min\{J;B\}} \tag{32}$$

where $f(u)$ is the summarized time (calculated based on the workload) for picking all needed items, J – number of zones, B – number of batches. The value of the lowest bound of makespan can be achieved only when the workload for all needed items is equally distributed in all zones or when the number of zones is much greater than the number of batches (but such a picking line is very ineffective).

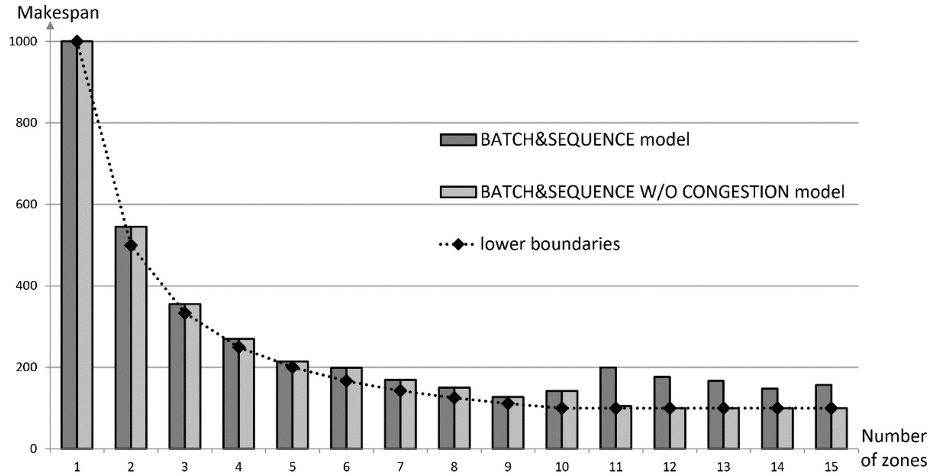

Figure 3. Makespan values for an example with a total workload equal 1000, and 10 batches for different numbers of zones.

Table 6. Recommended number of zones for a specified number of batches completed during a day (or a shift) and the projected makespan expressed as a per cent of the total workload

Makespan	Number of batches														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
100	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
90	–	2	2	2	2	2	2	2	2	2	2	2	2	2	2
80	–	2	2	2	2	2	2	2	2	2	2	2	2	2	2
70	–	2	2	2	2	2	2	2	2	2	2	2	2	2	2
60	–	2	2–3	2–3	2–3	2–3	2–3	2–3	2–3	2–3	2–3	2–3	2–3	2–3	2–3
50	–	2	2–3	2–3	2–3	2–3	2–3	2–3	2–3	2–3	2–3	2–3	2–3	2–3	2–3
40	–	–	3	3–4	3–4	3–4	3–4	3–4	3–4	3–4	3–4	3–4	3–4	3–4	3–4
30	–	–	–	4	4–5	4–5	4–5	4–5	4–5	4–5	4–5	4–5	4–5	4–5	4–5
20	–	–	–	–	5	5–6	5–7	5–7	5–7	5–7	5–7	5–7	5–7	5–7	5–7
10	–	–	–	–	–	–	–	–	–	10	10–11	10–12	10–13	10–13	10–13

Figure 3 shows the example of makespan values obtained for a warehouse with 10 zones, 5000 SKUs and 1000 items to be picked. It is assumed that the demand for SKUs at the stage of storage assignment and the number of items in orders follow the 80-20 Pareto rule (equation (31)). As we see from the figure, increasing the number of zones can reduce system performance when the number of zones becomes lower than the number of batches. In practice, the storage should be optimized based on the ABC curve obtained from historical data (including seasonal changes in demand). The orders from one working day (or shift) do not entirely map the mean demand for SKUs. In the analysed example, for the P & P batch and sequence model, increasing the number of zones from 10 to 11 means that although both the average and maximal workload in the zones are smaller, the number of time intervals rises, and the total efficiency of the system is reduced. For the P & P batch and sequence W/O congestion model with 11 zones, the makespan reaches the lower bound value, and further increasing the number of zones does not lead to a better objective function value.

For a more general analysis, Table 6 shows the recommended number of picking zones in the warehouse in cases where a particular makespan value as a % of total workload has to be achieved. For the calculations, it was assumed that a specific batch could generate an imbalanced workload in the zones, and that the picking time could be 30% longer than the lower bound.

7. Conclusions

Proper storage assignment and order-batching, together with sequencing, are crucial elements in increasing the efficiency of pick-and-pass systems. This paper presents three MILP models: the P & P assignment model, the P & P batch and sequence model, and the P & P batch and sequence W/O congestion model, and can be used for solving quite large dataset problems. In cases where obtaining optimal solutions is too time-consuming, approximate values for a branch and bound tolerance less than 5% can be easily obtained. The decision about storage assignment is usually based on historical data, which means that it includes risk and uncertainty. Due to the fact that even in perfectly balanced zones, a particular set of customer orders may generate different workloads in the zones, the optimization of batching and sequencing plays an important role. Using both concepts can significantly reduce the makespan. However, the wrong number of zones and batches can disturb the order-picking process. Some suggestions for this are included in the paper.

There are still a few research fields connected with pick-and-pass systems that have not been examined thoroughly.

The MILP models presented in the paper consider only one picking line. The number of picking lines for the storage assignment model does not matter. However, the batching and sequencing model assumes only one picking line. For more picking lines, additional constraints are needed. This problem should be analysed thoroughly in the future. The models can be time-consuming for huge tasks, and therefore it is still worth developing fast and simple heuristics.

An issue still not investigated by researchers in the context of pick-and-pass systems is the optimization of storage of the same items (usually with a higher picking frequency ratio) in many locations (in many zones). Such a problem in warehouses without considering zone divisions was analysed by Daniels et al. [8], Dmytrów [14–17], and Dmytrów and Doszyń [18].

Another problem connected with pick-and-pack systems is establishing the necessary storage space for items so as to diminish the number of emergency replenishment. This issue was not analysed in this paper, and although there are some solutions [31], it can still be investigated in the future.

In this paper, it was assumed that the workload for picking items from orders is additive. However, when the picker picks the same SKU only once in more significant amounts for two orders accumulated in one batch, the effort may be lower than picking the same SKU twice in smaller amounts for two batches. This problem of the non-additivity of the workload should also be analysed in detail in the future. The next issue for future research is modelling optimal storage assignment, batching and sequencing in pick-and-pass systems for different warehouse layouts or shelving systems. Taking into consideration gravity-flow racks will make the models much more complicated; the proposition of Anken et al. [1] for determining the storage space for items shows that obtaining precise solutions for real-life problems is very difficult and still unreachable.

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