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Weak disposability in the non-parametric analysis of efficiency. An interval abatement factor

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Abstract

The prevalent economic principle of weak disposability has been the foundation for studies in environmental assessment using data envelopment analysis (DEA). Recently, a shift from classic free disposability to weak disposability has been observed as an emerging trend for treating undesirable factors in research. Weak disposability is perceived to have significant analytical power in measuring the efficiency of decision-making units (DMUs). With the aim of decreasing of undesirable outputs, a non-radial model grounded on a non-uniform augment factor is presented. The application of this proposed model anticipates a suitable quantity for the decreasing of undesirable outputs. Concurrently, the model ensures a corresponding and satiable amount for reduction in undesirable outputs. Numerical instances illuminate the practicality and robustness of the proposed model and demonstrate its superior performance over its original counterpart.

Keywords: *undesirable outputs, data envelopment analysis (DEA), decision-making unit (DMU), weak disposability, environmental assessment, interval factor*

1. Introduction

Since the last decade, there has been a growing interest in use of efficiency and productivity management taking undesirable outputs/inputs into account. In production theory, both parametric and non-parametric techniques offer the advantage of imposing the weak-disposability assumption on the functional form of the underlying technology. Data envelopment analysis (DEA), first introduced by Charnes et.al [3] and later extended by Banker et.al [2], has recently made significant contributions to the analysis of undesirable variables. The modeling of undesirable factors has gained considerable attention not only for measuring efficiency and productivity but also for estimating pollution factors. This issue has been investigated in various research studies, with early contributions from Lovell et.al [21], Seiford and Zhu [29],

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Hailu and Veeman [11], Färe and Grosskopf [4–7], Hailu [10], and Kuosmanen [16]. The concept of reducing undesirable outputs by decreasing the level of production activity (output weak disposability axiom) was first proposed by Shepard [30]. The author applied a uniform abatement factor to all observed activities in the sample. Subsequently, Kuosmanen [16] argued that using a uniform abatement factor is inconsistent with the conventional wisdom of focusing abatement factors on firms with lower abatement costs. Kuosmanen and Podinovski [18] demonstrated that a single abatement factor does not suffice to capture all feasible production plans, and leads to the violation of convexity, one of the maintained assumptions of the model. They also proved that the Kuosmanen [16] technology is the correct minimum extrapolation technology under the stated axioms. Podinovski and Kuosmanen [27] developed two additional technologies for modeling weak-disposability under relaxed convexity assumptions.

A methodological contribution of such DEA-based studies, in alignment with the output weak disposability definition posits that a proportional reduction in the level of undesirable outputs can be achieved if accompanied by a reduction in desirable outputs in the same proportion. Kuosmanen and Kazemi--Matin [17] shed new light on the economic interpretation of weak disposability by developing dual formulations of the weakly disposable DEA technology. Referring to the scarcity of natural resources and the need to preserve them by using renewable resources, the concept of undesirable outputs has gained prominence in recent studies. Considering the importance of reduction the hazards of pollutants, both desirable and undesirable outputs become crucial in real-world situations. The researchers have some contributions to deal with undesirable outputs. Jahanshahloo et.al [12] presented a non-radial DEA-based model for managing both undesirable inputs and outputs, aiming to decrease undesirable outputs and increase undesirable inputs simultaneously. Liu et al. [20] treated undesirable inputs and outputs as desirable outputs and inputs, respectively, while assuming the standard strong disposability assumption using a non-radial model applying Russell measure or slack based DEA models. As an example of DEA application in public health in presence of undesirable outputs, Fare et.al [8] reformulated the diet problem with a linear optimization problem with both desirable and undesirable outputs. They also showed that the equivalency of the dual modified model to their proposed Benefit-of-the-Doubt model with forward and reverse indicators. As an application, 180 country was selected to apply the model and a composite indicator of public health was constructed.

A review of the DEA literature reveals numerous DEA models for modeling undesirable outputs using the concept of weak disposability. Roshdi et al. [28] introduced a new concept of exponential weak disposability assumption for undesired outputs, allowing for different types of trade-offs between desirable and undesirable outputs. By satisfying three axioms (concavity, linearity, and convexity), a piecewise Cobb–Douglas environmental technology was derived. Based on this technology, radial and nonradial functions were extracted to measure environmental performance. Mehdiloozad and Podinovski [23] noted that Shepard's technology modification for increasing undesirable input with a single scaling factor can cause problematic side effects, such as congestion measurement issues. To address this deficiency, the authors developed an appropriate technology that incorporates weak input disposability. Then, based on progressively relaxed convexity assumptions, various ranges of technologies were also investigated. Mehdiloo and Podinovski [22] argued that the disposability assumption may not be suitable and could lead to meaningless proportions when inputs or outputs are overlapping or strongly correlated. To address this issue, they developed a production technology in which groups of closely related inputs and outputs are only jointly weakly disposable. Pham and Zelenyuk [25] discussed the use of single or multiple scaling factors in different scenarios and revealed the link between various returns to scale and weak disposability of desirable and undesirable outputs. Another contribution of their study was the construction of a comprehensive taxonomy of reference technology sets for activity analysis models with various return to scale assumptions. Fouladvand et al. [9] developed a linear model to investigate congestion in the presence of undesirable outputs, employing the concept of output weak disposability. Li et al. [19] proposed a model based on a circular economy structure to analyze waste treatment efficacy for solid waste during the 11th and 12th five-year plans from 2011 to 2015. The research showed that efficiency in pollution and disposal of solid waste improved during these periods. Monzeli et al. [24] determined efficiency measurements in the presence of undesirable inputs and outputs using a three-step approach: first, an appropriate production possibility set was defined based on problem assumptions; second, the undesirable effects in DMUs were modeled by considering the weak disposability assumption; and third, the efficiency of DMUs was calculated using a radial DEA model.

Kordrostami et al. [15] expanded the classical definition of weak disposability to accommodate undesirable inputs and introduced a linear formulation. By implementing the concept of simultaneous proportional reduction in desirable and undesirable outputs, along with proportional expansion in favorable and unfavorable inputs, a linear model was applied to efficiency analysis. Further information can be found in different studies, for example Piao et.al [26], Jo and Chang [13] and Yu and Rakshit [31]. Although each approach in the literature has its merits, the application of the weak disposability axiom in activity analysis continues to elicit questions. In the current context, the reduction of hazards of bad outputs is becoming increasingly crucial as it addresses environmental concerns and promotes the incorporation of waste and pollutants into production systems. According to the World Commission on Environment and Development (WCED), pollution levels continue to rise while resource scarcity persists. As a result, there is growing interest in the reducing the effects of undesired outputs and efficiency management, which takes undesirable outputs into account. From a computational standpoint, it is rational to consider a suitable amount for the reduction of undesirable outputs, necessitating the development of an optimization-based axiom approach to address this issue. With respect to weak disposable technology, the most attention has been given to the potential decrease in undesirable outputs. To gain a deeper understanding of the concept of weak output disposability, this study examines to select a satiable amount for reduction of undesirable outputs as the unit capacity allows. That is to say, instead of conflict to achieve the unity as the upper bound which does complies with reality, the units themselves can select the practical and achievable upper bound for reduction of undesirable output based on underlying technology. The focus of this research is on the assumption of weak input disposability, which may yield more realistic results in terms of economic development. The structure of this study is unfolded as follows: Section 2 provides a brief overview of weak output disposability axioms, followed by a modification of weak output disposability in Section 3. Finally, conclusion will end the paper.

2. Weak disposable technology

Modeling undesirable outputs (such as emission of harmful substances in air, energy wasted in power plant) of production activities has attracted considerable attention among researchers. Hailu and Veeman [10] extended non-parametric productivity analysis models to include undesirable outputs. They introduced a non-orthodox monotonicity condition on their technology and claimed it is preferable to weak disposability concept in DEA. Fare and Grosskopf [4] showed that employing monotonicity condition in steal of weak disposability is inconsistent with physical law. Suppose that there are K DMUs and for DMU$_k$ data on the vectors of inputs, desirable and undesirable outputs are presented $x_k = (x_{1_k}, \ldots, x_{N_k}) \geq 0,\ v_k = (v_{1_k}, \ldots, v_{M_k}) \geq 0$ and $w_k = (w_{1_k}, \ldots, w_{J_k}) \geq 0$. Further assume that, $x_k \neq 0, v_k \neq 0$, and $w_k \neq 0$. The production technology can be represented by:

$$P(x) = \{(x, v, w) \mid x\text{can produce}\,(v, w), x \in R_N^+\}$$

Definition 1. Outputs (desirable and undesirable) are weakly disposable if and only if $(v, w) \in P(x)$ and $\theta \leq 1$ imply that $(\theta_v, \theta_w) \in P(x), x \in R_N^+$ [30]. Fare and Grosskopf [4] proposed the following technology under variable return to scale satisfying weak-disposability assumption:

$$\begin{aligned} \hat{Y}_s = \Big\{(x, v, w) \mid & \sum_{k=1}^{K} \theta z^k x_n^k \leq x_n, \quad n = 1, \ldots, N \\ & \sum_{k=1}^{K} \theta z^k v_m^k \geq v_m \geq 0, \quad m = 1, \ldots, M \\ & \sum_{k=1}^{K} \theta w_j^k = w_j, \quad j = 1, \ldots, J \\ & \sum_{k=1}^{K} z^k = 1, \quad z^k \geq 0, \quad \theta \leq 1\Big\} \end{aligned} \tag{1}$$

The abatement parameter θ in (1) corresponds to Shephard's definition of weak disposability. This parameter allows simultaneous contraction of desirable and undesirable outputs. The variable $z = (z^1, z^2, \ldots, z^k)$ is referred to as intensity variables. The non-negativity of undesirable outputs in the third constraint and inputs in the first constraint are automatically stated. Kuosmanen [16] pointed out, this model uses a uniform abatement factor to all firms. To allow non-uniform abatement factor of the individual firms, he proposed the following production technology:

$$\begin{aligned} \hat{Y}_k = \Big\{(x, v, w) \mid & \sum_{k=1}^{K} \theta^k z^k x_n^k \leq x_n, \quad n = 1, \ldots, N \\ & \sum_{k=1}^{K} \theta^k z^k v_m^k \geq v_m \geq 0, \quad m = 1, \ldots, M \\ & \sum_{k=1}^{K} \theta^k w_j^k = w_j, \quad j = 1, \ldots, J \\ & \sum_{k=1}^{K} z^k = 1, \quad z^k \geq 0, \quad \theta^k \leq 1\Big\} \end{aligned} \tag{2}$$

Set of constraints (1) is a special case of (2) with $\theta^1 = \theta^2 = \cdots = \theta^k$. As Kuosmanen [16] argued the above non-linear technology can be restated in an equivalent linear set of constraints by portioning the intensity weight z^k into two components $z^k = \lambda^k + \mu^k$. Using this notation, the linear model of evaluating the efficiency of DMU_0 is stated as follows:

$$\begin{aligned}
&\theta^* = \min \theta \\
&\text{s.t.} \\
&\sum_{k=1}^{K} (\lambda^k + \mu^k) x_n^k \leq x_n^0, \quad n = 1, \ldots, N \\
&\sum_{k=1}^{K} \lambda^k v_m^k \geq v_m^0 \geq 0, \quad m = 1, \ldots, M \\
&\sum_{k=1}^{K} \lambda^k w_j^k = \theta w_j^0, \quad j = 1, \ldots, J \\
&\sum_{k=1}^{K} \lambda^k + \mu^k = 1, \quad \lambda^k, \mu^k \geq 0
\end{aligned} \tag{3}$$

In this linear set, $\lambda^k = z^k \theta^k$ points out the part of outputs that remains active and the other part $\mu^k = z^k(1 - \theta^k)$ presents the reduced part of outputs. As the model (3) stated the right hand sides of the envelopment constraints are faced up with scaling variables.

3. Determining an interval for the abatement factor

The abatement factor θ as discussed in the previously mentioned technologies, belongs to the closed interval $[0, 1]$. In real world scenarios, cases may arise where it is impossible to completely disregard undesirable factors, such as the increasing need for reduction of $CO2$ emission to protect the environment. Furthermore, the abatement factor θ may not attain the upper bound which is equal to unity and its usage has inherent limitations. As a matter of fact, in some cases, the unit's capacity does not allow to attain the upper bound which is equal to unity, whilst the underlying technology addresses to deal with the maximum reduction with the aim of performance measurement. To obtain reliable result and improve applicability, a modification appears warranted. Furthermore, determining this quantity is in line with the underlying technology with reference to unit capacity for reduction. Model (3), discussed in the previous section, and solely focuses on decreasing the undesirable outputs. This perspective may lead to different efficiency measures and, in some cases, deviate from reality. In fact, in all DEA applications, undesirable output reduction is desired and expected. Consequently, it is logical to modify the model to not only support the reduction of undesirable output but also encourage the increment of desirable outputs. In other words, the satiable amount for reduction of undesirable output, $l \leq 1$ replaced with unity. This modification may develop approaches aimed at addressing the problem in the presence of undesirable outputs. Considering the concept of output weak disposability, dual points are replaced in model (3). To achieve this, a model is applied to expect the simultaneous reduction of undesirable outputs. The concept of slack variables is modified to be used in the third constraint related to undesirable output, ensuring the reduction of undesirable outputs. Applying the modified constraint, based on the idea of weak disposability of undesirable outputs, may lead to the reduction of bad outputs, as expected in the production process and underlying technology.

Again suppose that there are k DMUs and for DMU$_k$ data on the vectors of inputs, desirable and undesirable outputs are presented as $x_k = (x_{1k}, \ldots, x_{Nk}) \geq 0$, $v_k = (v_{1k}, \ldots, v_{Mk}) \geq 0$ and $w_k = (w_{1k}, \ldots, w_{Jk}) \geq 0$. Further assume that $x_k \neq 0, v_k \neq 0$, and $w_k \neq 0$. The production technology can be represented by:

$$P(x) = \{(x, v, w) \mid x \, \text{can produce} \, (v, w), x \in R_N^+\}$$

To evaluate the efficiency of DMUs with the above proposition, the production technology of Kuosmanen [16] is considered, and model (3) can be modified accordingly:

$$\begin{aligned}
&\rho = \min \frac{1}{J} \sum_{j=1}^{J} \theta_j \\
&\text{s.t.} \\
&\sum_{k=1}^{K} (\lambda^k + \mu^k) x_n^k + S_n^+ = x_n^0, \quad n = 1, \ldots, N \\
&\sum_{k=1}^{K} \lambda^k v_m^k - S_m^- = v_m^0, \quad m = 1, \ldots, M \\
&\sum_{k=1}^{K} \lambda^k w_j^k = \theta_j w_j^0 - S_j^-, \quad j = 1, \ldots, J \\
&\sum_{k=1}^{K} (\lambda^k + \mu^k) = 1, \quad \lambda^k, \mu^k, S_j^- \geq 0, \quad 0 \leq \theta_j \leq l \leq 1
\end{aligned} \tag{4}$$

Upon close examination, all constraints within the modified model support the idea of DEA weak output disposability. It can be easily seen that the model (4) is bounded and feasible and the objective function is invariant with respect to the units of data and it always holds $\rho^* \geq 0$. The first and second constraints are the usual weak disposable constraints. The requirement for dominance constraint, $0 \leq \theta_j \leq l$ ensures the reduction of undesirable outputs up to a satiable and reliable amount. The third constraint is modified as $\sum_{k=1}^{K} \lambda^k w_j^k = \theta_j w_j^0 - S_j^-$, $j = 1, \ldots, J$ and refers to the proportional undesirable output reduction, admitting that the remaining decrease can be traced back to slack variable $S_j^- \geq 0$. The objective function represent the Russell undesirable measure of efficiency represented as $\frac{1}{J} \sum_{j=1}^{J} \theta_j$. In terms of efficiency measurement, we scope on minimizing the potential change of each unit in the observed data set. The main characteristic of the modified Model (4) is supporting the weak disposability axiom by imposing the constraint $\sum_{k=1}^{K} (\lambda^k + \mu^k) = 1$ that stems from the transformation of non-uniform abatement factor for all units. When evaluating using model (4), the unit being assessed, DMU_0 is considered efficient if the efficiency measure equals one.

To address the potential of a unit in determining the abatement factor in the interval format, suppose that the optimal solution of model (4) is $(S_n^{*+}, S_m^{-*}, S_j^{-*}, \theta_j^*)$. Employing the optimal solution, the satiable and achievable upper bound $0 \leq \theta_j \leq l \leq 1$ for undesirable output reduction can be defined as

$$\frac{1-\left(\dfrac{1}{N}\displaystyle\sum_{n=1}^{N}\frac{S_n^{*+}}{x_n^0}\right)}{1+\left(\dfrac{1}{M+J}\left(\displaystyle\sum_{n=1}^{N}\frac{S_m^{*-}}{v_m^0}+\sum_{j=1}^{J}\frac{S_j^{*-}}{w_j^0}\right)\right)}$$

which does not exceed the optimal value of objective function. Hence, the interval $[0, l]$ can determine a reliable abatement factor with reference to unit's potential. Although, employing this optimal interval can make relatively better discrimination on DMUs. It can be easily seen that the optimal solution of model (4) is lower than or equal to model (3).

4. Numerical examples

4.1. Example 1

The applicability of the approach is illustrated by a real data set consisting of sixteen units. The data set was taken from the paper by Amirteimoori et al. [1]. The data set consists of three inputs (x_1, x_2, x_3), two desirable outputs (D. output) (v_1, v_2) and one undesirable output (Und. output) w_1. Table 1 provides the summary of the data set.

Table 1. Data for sixteen DMUs

DMU	Input 1 x_1	Input 2 x_2	Input 3 x_3	D. output 1 v_1	D. output 2 v_2	Und. output w_1
1	403	1120	5331608	18363085	31227605	154
2	411	1125	5883209	19665025	30840680	127
3	420	1112	5940485	19028804	30586165	152
4	452	1121	6626290	20103803	33175128	140
5	495	1126	8154603	21189140	36092750	194
6	473	1146	8502509	21189140	37271623	175
7	472	1087	9031125	21136856	38828087	161
8	454	1132	8788094	20801147	309107870	142
9	432	1108	9157097	21698043	38143325	180
10	427	1045	8868206	21051481	33723130	171
11	438	979	8398829	20114530	32698925	159
12	450	923	9083108	22023502	32641359	232
13	435	851	7470562	17400575	29972388	150
14	424	869	7188360	17101044	28763740	147
15	420	829	6665653	16937531	25005947	118
16	474	879	6489033	17681062	22596922	123

Running model (3) and the proposed model (4), the results are reported in Table 2. As the Table 2 shows the proposed model (4) presents ten units out of sixteen as efficient unit. In model (3) there are only two inefficient units and the rest of DMUs are reported as efficient. The proposed model (4) considerably decreases the number of DMUs. The defined upper bound l is represented in the last column of Table 2. Additionally, the efficiency score of model (4) is lower than of model (3).

Table 2. Efficiency score and proposed interval

DMU	Model (3) θ^*	Model (4) ρ^*	Propose interval	DMU	Model (3) θ^*	Model (4) ρ^*	Proposed interval
1	1	1	[0, 1]	10	1	1	[0, 1]
2	1	1	[0, 1]	11	1	1	[0, 1]
3	0.87	0.81	[0, 0.99]	12	1	1	[0, 1]
4	1	0.97	[0, 0.98]	13	1	0.83	[0, 0.96]
5	1	0.83	[0, 0.96]	14	0.86	0.80	[0, 0.96]
6	1	0.90	[0, 0.98]	15	1	1	[0, 1]
7	1	1	[0, 1]	16	1	1	[0, 1]
8	1	1	[0, 1]	Average	0.98	0.94	–
9	1	1	[0, 1]	Variance	0.002	0.006	–

As the last column of Table 2 reports the upper bound for reduction of undesirable outputs does not exceed one. The proposed interval admits that the undesirable outputs reduction can achieve an upper bound which is close to the abatement factor ρ^* and emphasis the real upper limit for reduction which can coincide with the unit's potential and obtain more realistic results. Moreover, the last row of Table 2 shows that the average of efficiencies obtained from model (4) is close to that of model (3) with values of 0.98 and 0.94, respectively.

4.2. Example 2

The applicability of the proposed approach is demonstrated using a real data set consisting of thirty units. The data set origins from Kao and Hwang [14]. The data set consist of thirty paper mills along the HUAI River in Anhui Province, China. Each unit employs two sets of inputs to produce two categories of outputs: two desirable and one undesirable output. Table 3 depicts the data set.

Evaluations of these units with models (3) and the proposed model (4) are recorded under the heading of model (3) and model (4) in Table 4. As the Table 4 shows there are five efficient DMUs out of thirty units in evaluating with model (4). On the other hand, the original model (3) evaluates eighteen efficient units. The average of efficiencies are recorded in the last row of Table 4. From the statistical point of view, the average efficiency of proposed model (4) is significantly lower than of model (3) with values of 0.27 and 0.66, respectively.

Interestingly enough, the efficiency scores obtained by model (4), are not greater than the efficiency scores of model (3) formulated by the same technology. In a nutshell, model (4) has been making an effort to make a satiable amount of undesirable outputs reduction utilizing the same technology. The results of the last column in Table 4 advocate that the proposed model (4) is more effective than of the counterpart model (3) and provides more realistic results. The upper bound in the proposed interval reports the satiable amount for reduction regarding the unit's potential and optimal reduction of bad outputs. Clearly, this upper bound can reflect the achievable amount of reduction any unit can catch. In essence, the model (4) outperforms its counterpart model (3).

Table 3. Data Set for thirty Data Set

DMU	Input 1 x_1	Input 2 x_2	D. output 2 v_1	D. output 1 v_2	Und. output w_1
1	437	1438	2015	14667	665
2	884	1061	3452	2822	491
3	1160	9171	2276	2484	417
4	626	10151	953	16434	302
5	374	8416	2578	19715	229
6	597	3038	3003	20743	1083
7	870	3342	1860	20494	1053
8	685	9984	3338	17126	740
9	582	8877	2859	9548	845
10	763	2829	1889	18683	517
11	689	6057	2583	15732	664
12	355	1609	1096	13104	313
13	851	2352	3924	3723	1206
14	926	1222	1107	13095	377
15	203	9698	2440	15588	792
16	1109	7141	4366	10550	524
17	861	4391	2601	5258	307
18	249	7856	1788	15869	1449
19	652	3173	793	12383	1131
20	364	3314	3456	18010	826
21	670	5422	3336	17568	1357
22	1023	4338	3791	20560	1089
23	1049	3665	4797	16524	652
24	1164	8549	2161	3907	999
25	1012	5162	812	10985	526
26	464	10504	4403	21532	218
27	406	9365	1825	21378	1339
28	1132	9958	2990	14905	231
29	593	3552	4019	3854	1431
30	262	6211	815	17440	965

Table 4. Efficiency Score and Proposed Interval

DMU	Model (3) θ^*	Model (4) ρ^*	Proposed interval	DMU	Model (3) θ^*	Model (4) ρ^*	Proposed interval
1	1	1	[0, 1]	17	0.08	0.08	[0,0.90]
2	1	1	[0, 1]	18	0.02	0.01	[0, 0.99]
3	1	0.14	[0, 0.53]	19	1	0.12	[0, 0.94]
4	0.27	0.08	[0, 0.84]	20	1	0.30	[0, 0.93]
5	1	0.64	[0, 0.85]	21	0.06	0.02	[0, 0.99]
6	0.15	0.08	[0, 0.98]	22	1	0.04	[0, 0.91]
7	0.02	0.02	[0, 0.99]	23	1	0.10	[0, 0.90]
8	1	0.10	[0, 0.81]	24	1	0.04	[0, 0.71]
9	0.07	0.07	[0, 0.87]	25	0.86	0.15	[0, 0.82]
10	0.09	0.08	[0, 0.98]	26	1	0.22	[0, 0.91]
11	0.35	0.14	[0, 0.84]	27	0.03	0.02	[0, 0.95]
12	1	1	[0, 1]	28	1	0.09	[0, 0.89]
13	1	0.16	[0, 0.67]	29	0.07	0.06	[0, 0.98]
14	1	0.13	[0, 0.92]	30	1	1	[0, 1]
15	1	1	[0, 1]	Average	0.66	0.27	–
16	1	0.29	[0, 0.52]				

5. Conclusion

Treating undesirable outputs attracts the interest of DEA researchers since the last decade. In the realm of efficiency and productivity analysis various approaches have been developed within DEA research to address the concept of undesirable outputs. One favored approach involves substituting the axiom of weak input disposability with the free disposability assumption. Despite numerous advancements in this area, the debate on weak disposability persists. The present study introduces a non-radial alternative model based on a non-uniform abatement factor for undesirable outputs, addressing two critical issues. Firstly, the proposed model offers a reduction amount for the decreasing of undesirable outputs, yielding different results from existing models in the literature. Secondly, the model achieves an adequate interval supporting the unit's potential for reduction in undesirable outputs, a feature that can be justified in real-world applications. The applicability and strength of the proposed model are demonstrated through two examples. By developing this non-radial alternative model, the study contributes to the ongoing discourse on weak disposability and expands the range of available techniques for handling undesirable outputs in efficiency and productivity analysis. Future research could explore other non-uniform abatement factors, further refining the proposed model and its applicability to various industrial and environmental contexts. Additionally, comparisons with other existing models could provide valuable insights into the strengths and limitations of the proposed approach, facilitating the development of even more robust methods for addressing the challenges posed by undesirable outputs in efficiency and productivity analysis.

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