

ORIGINAL ARTICLE

Modernization of the basic gravimetric control network Polish Geological Institute – implementing a modern international gravimetric system and relationship with the historical Potsdam system

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Abstract

The project of the gravimetric database reambulation from the collections of the Central Geological Archives set itself the goal of ending the era of gravimetric bases dualism and transferring both databases: gravimetric base and archival gravimetric semi-detailed measurements to the modern frame of gravity reference, consistent with ITGRS/F definitions and realizations. This will strengthen the interpretational potential of gravimetric data and will open the possibility of fully integrating Polish gravity data with international databases of this type, and will also allow for the performance of modern works in relation to the modern gravimetric control network, i.e., in relation to the highest possible precision of the reference definition of gravimetric measurements in the country. Whenever the article refers to the units in which the gravity vector is determined, milliGal [mGal] is used, and the relation to the SI system is as follows: 1mGal = 10^{-5}ms^{-2} or microGal [μGal] where $1\mu\text{Gal} = 10^{-8}\text{ms}^{-2}$.

Key words: gravity, gravimetric network, gravimetric frame, Potsdam gravity system

1 Introduction

The nationwide collection of archival gravimetric measurements (Central Database of Geological Data) currently includes over 1 million points, of which 36 per cent are points of detailed local surveys, the remaining ones being semi-detailed measurements. These measurements have been conducted for several decades, as part of multiple separate studies. To link

them to a consistent database and bring to a common reference level, a special set of points called the gravimetric control network was applied. The Polish Geological Institute began preparation of the point sets in the early 1950s. For some time, these works were coordinated with the geodetic service, until the strategies for creating gravimetric "geophysical" and geodetic networks began to diverge. Geodetic surveyors needed fully accurate rather than dense gravimetric control, defining

the absolute values of gravity acceleration at points. Meanwhile, to facilitate the "geophysical" practice of conducting a gravimetric reconnaissance, the control point set had to be sufficiently dense, and accessible over relatively short distances during the gravimetric survey, which motivated the strategy of creating it as a dense, several-thousand-point structure. The methodology of locating and stabilizing the points of the geodetic and "geophysical" network was also completely different. From the 1970s, two structures of the gravimetric base came into existence in Poland: assumed, stabilized, and developed completely differently. This dualism did not facilitate the data interchange ability, i.e., the geodetic service could not easily use geophysical data, whose level of detail and distribution are incomparably greater and better than those of the geodetic data. There was also no possibility of transferring the technology of precise determination of gravity values, which marked the end of the 20th century in geodetic control networks as a result of the application of the ballistic absolute gravimeters. This meant a significant improvement in the implementation of the gravimetric reference level to an accuracy of 10^{-8} ms^{-2} . Since 2014, the basic geodetic gravimetric control network has been used following the definition of the modern international gravimetric reference system ITGRS/F. At that time, the "geophysical" network (PIG-62 network), established in the 1960s, did not undergo such a transformation and became increasingly detached from the modern standard. Problems appeared in contemporary works related to the geodetic control network, the results of which were difficult to fit into the archival system implemented by the PIG-62 control network.

The restoration of the gravimetric database from the collections of the Central Geological Archives was meant to end the era of a dual gravimetric base by transferring both databases – the gravimetric base and the archival gravimetric semi-detailed measurements – to the modern frame of gravity reference, consistent with ITGRS/F definitions and applications, with a view to strengthen the interpretational potential of gravimetric data and open the possibility of a full integration of the Polish gravity data with international databases of this type. It will also enable modern works within the modern gravimetric control network, i.e., with the highest possible precision of the reference definition of gravimetric measurements in the country. In this article, the gravity vector is determined in milliGal [mGals], and the relation to the SI system is as follows: $1 \text{ mGal} = 10^{-5} \text{ ms}^{-2}$ or microGals [μGal] where $1 \mu\text{Gal} = 10^{-8} \text{ ms}^{-2}$.

2 Historical basic gravimetric control network in the PIG-62 system (PIG62)

The gravimetric base, which was used as a reference for geophysical works after 1945, was created under the aegis of the then Geological Institute (currently: Polish Geological Institute – National Research Institute). Creating the basis for geophysical work using relative gravimeters and the study of the gravity field as a set of uniform, consistent, and accurate gravimetric network points began as early as in the 1950s. After 1953 earlier regional measurements, at that time taken at local reference levels, were amalgamated. The foundations of a uniform three-class gravimetric control network were thus created, under the direction of the Geological Institute, and in cooperation with the geodetic service. At that time, a combination of observations of base or resistance points was applied, as they were then called, as well as their alignment. The preparation of this process was greatly facilitated by the works of Kaczkowska (1953) related to the inventory of existing points in the areas of former partitioners of Poland and lands joined to Poland after World War II, and the network was approved by the geo-

logical and geodetic institutions. The study described the first technical standards of the network of first and second-order base points. Measurement work began in 1956 under the direction of J. Skorupa with the participation of employees of the Geophysical Department of the Geological Institute (Bujnowski, 1975). A purchase of two precise Askania GS relative gravimeters proved to be a breakthrough in the implementation of this task. The contemporary technical standard involved the use of precise relative spring gravity meters and the location of points enabling air transport between first-class points. Each point of this data set existed as a "double point": with a location on the apron and a second point, stabilized with the appropriate sign in the centre of the set of the two points. In this structure, an exceptionally durable stabilization of the point centre with a concrete block of a size of approximately 3m by 3m was used, and many of these structures are still in operation today. In half of cases, the existing network of pendulum gravimetric measurements, created by the Institute of Geodesy and Cartography and the Warsaw University of Technology, was selected as the centre (Ząbek and Dobaczewska, 1959). This provided a link to the gravimetric level and gravity unit defined by the absolute measurements made by the pendulum method in mid-20th century.

Designed in 1964, the first-class network consisted of 18 points, regularly distributed throughout the country and located near airports. In the structure of the network, 37 gravimetric spans (sides connecting two adjacent network points, e.g. A and B) and 20 closed figures were determined to control the measurements by maintaining the condition that the sum of differences in the gravity on the sides constituting a closed figure was equal to 0. Points designated at airports were not permanently stabilized, only marked. The measurement of long spans was conducted using air transport in the ABAB scheme. Measurements linking an airport point with the corresponding centre were made using road transport with a much larger number of measurement repetitions, again in the chain method scheme. Air transport later proved to be a suboptimal solution from the point of view of the drift characteristics of Askania Askania gravimeters, with span measurement errors of several tens of microGal. For comparison, the measurements tied to airport points were made with errors ranging from 9 to 29 μGal . The adjustment of the observations realizing the set of the 1st class control network was made using the LSM algorithm, assuming the observations Δg on spans as unequally accurate observations. The obtained results of determination of gravity at the points of the 1st class network constituted adjustment errors from 0.04 to 0.07 mGal. The reference level for gravimetric measurements was adopted based on the Warszawa Okęcie point, determined through pendulum and relative observations of this point in the years 1934–1949 (Pawłowski, 1939). In the following years, determinations at this point were repeated based on the implementation of a uniform reference level of the gravimetric network of socialist countries and through direct connections to the point in Potsdam, East Germany. Each of these values differed from Pawłowski's results by sometimes even 0.427 mGal. Today, this is seen as a completely unacceptable value, and a systematic error of the difference in levels between gravimetric maps or databases of neighbouring countries.

Based on the experiences and results of setting the network of 1st class, a network of 2nd class was developed, as a dense set of well-accessing points, mainly for road transport. It was carried out with the use of identical instruments and road transport. Unfortunately, in the structure of 144 points connected by 239 gravimetric spans (observations) with a mean length of about 50–60 km, temporarily stabilized points dominate, which currently makes it difficult and sometimes even impossible to find such points. Hence, with using the old observations

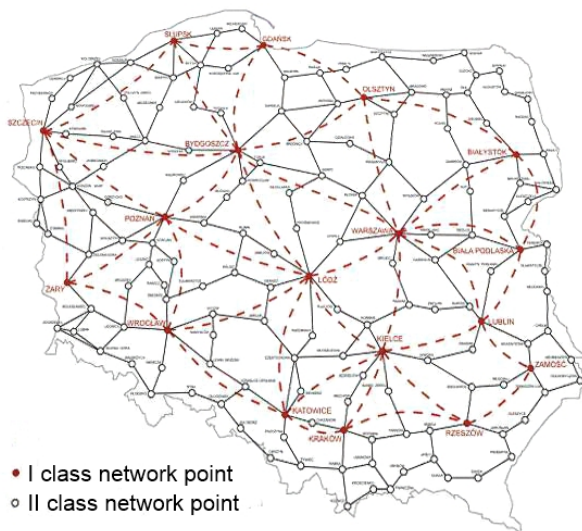


Figure 1. Sketch of spans of the PGI basic gravimetric control network (source: IGIK <http://www.igik.edu.pl/pl/cgig-PIG62>)

and their repeated adjustment we can now assume that each of these points is a mathematical realization of the system, physically inaccessible to establish, but related to the system through the possibility of using historical observations, i. e., gravity differences between two neighbouring points. The adjustment of the 2nd class network was performed using a similar method to the 1st class, i.e., by finding such corrections v to the measured gravity difference values (old measurements results) that the sum of the squares of these corrections satisfies the postulate of the method of least squares, known as Legendre's criterion [$\sum v = \min$], using the weights resulting from loop closures error in triangles (LSM). The LSM method is still the current standard in the algorithm adjustment of geodetic networks. A catalogue of gravity values concerning the 2nd class network was obtained in a system defined by the 1st class points as a base with an average error of 0.036 mGal, and the network itself is characterized by a span error after the alignment value of 0.051 mGal (Bujnowski, 1975).

In conclusion, crucially, the 1st class network was very well stabilized in some centric points, however, unfortunately, this is not true for airport points and the 2nd class points, as the good practice of stabilizing points as separate structures (pillars) was abandoned. Multiple 1st class points based on PIG-62 were used in the implementation of later gravimetric networks by the geodetic service and are still used today. For years, the gravimetric control network was used to conduct prospecting works, for which the structures of the 3rd class control network were established, locally adjusted to the 1st and 2nd class control networks, not always using the LSM method. Importantly, the first- and second-class warp was excellently developed and described, mainly in the study Bujnowski (1975), which contains a detailed description of the network structure, describes the method of span measurements, analyses related to the instruments used for measurements, and detailed sketches and lists. The results of the development of this network in the Potsdam system were recognised by the International Gravimetric Commission. Let us recall that at that time the base implementation standard mentioned 0.05 mGal as the upper limit of accuracy of national networks in the early 1960s. A sketch of point localisations and aviation spans (red) between 1st class stations and car spans (black) between 2nd class is presented in Figure 1.

3 Present-day basic gravimetric control network in Poland

The era of modern gravimetric base in Poland began in the early 1990s, when the measurements of the Basic Gravimetric Network of the Country (POGK), established in the 1970s, were completed. The base consisted of almost 400 points of relative measurements adjusted by linking to 12 absolute points. The network of absolute points was established with a non-homogeneous set of absolute ballistic gravimeters, therefore in this control network there were some errors caused by non-homogenous reference levels. However, adjustment of errors to up to 0.014 mGal was obtained with the accuracy of g determination at absolute points of about 0.005 mGal (Sas-Uhrynowski et al., 2000; Kryński et al., 2013). Each point had a separate pillar, which facilitated the relative measurements.

After 2011, the basic gravimetric control network underwent a thorough modernization and became the first gravimetric control network in Poland established using absolute gravimetric measurement methods, thanks to the dissemination of FG-5 and A10 absolute gravimeters. The team of the Faculty of Geodesy and Cartography of the Warsaw University of Technology together with the Institute of Geodesy and Cartography designed 30 points of the fundamental control network (1st class) and 168 points of the base control network (2nd class). The gravimetric control network was designed to become an integrated control network to the extent that it was possible, i.e., points were located on properly stabilized points of the basic height and horizontal control network.

The reduction in the number of points compared to the previous version of the base gravity network (POGK points) resulted not only from economics, but mainly from the fact that modern gravimeters have a much larger measuring range than in the past and have thermal conditions that allow drift compensation to a much more effective extent than in the past. For the first time in history, the implementation of those base points covered the use of only absolute observations, so there was no need to adjust the network. This method of implementing the gravimetric base is currently common and, within certain technical rigors, meets the conditions for the implementation of the International Terrestrial Gravity Reference Frame ITGRF/S (Wziontek et al., 2021). In the last implementation of the fundamental and base sets, these technical standards were fulfilled. The points of the fundamental control network were located inside buildings, and their function is, apart from the reference, also the definition of spans for the purposes of calibrating relative gravimeters and integration with the points of fundamental geodetic control networks of other types (horizontal EPN control, height UELN control). The points of the base control network are located outside, have the possibility of easy access, and are used exclusively to establish relative measurements. Full information about the location, topographic descriptions, and gravity values are available free of charge on the geoportal.gov.pl website in the "specialist geodetic information" section. Vitaly, the fundamental and basic gravimetric control network are in the base sets basic, ordered to be updated every 10 years. This means that every 10 years there is a review of the condition of points, possible damage of their stabilization of, and a remeasurement of all points of the set. The last construction of absolute fundamental and base nets created two sets of stations: fundamental points with a maximum error of reduced-on pillar gravity values $\pm 4 \mu\text{Gals}$ with using FG-5 absolute gravimeter, and $\pm 10 \mu\text{Gals}$ for base 2nd class set, measured by A10 absolute gravimeters. For all stations, the real non-linear function of dependence of gravity with H was measured to reduce observed gravity value from height of g determination by the ballistic method. A map of the distribution of fundamental and base points after 2014 is presented in Figure 2.

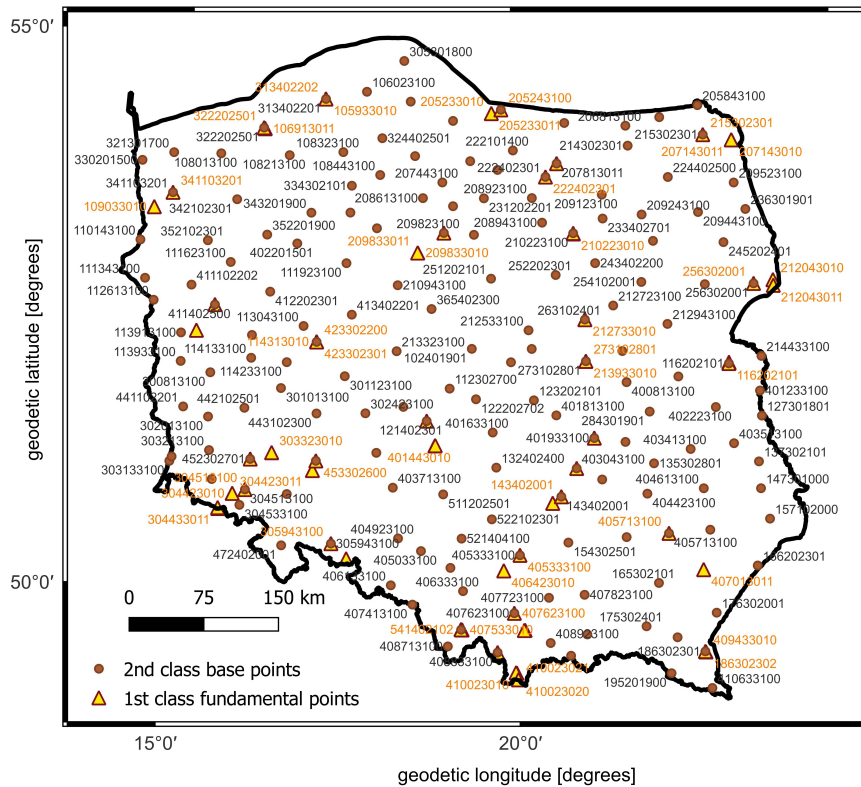


Figure 2. Contemporary Polish Basic Gravimetric Control Network

4 Modernization of the PIG-62 gravimetric control network

4.1 The concept of modernization

PGI-NRI is currently implementing a project named "Verification, consolidation of collections and gravimetric databases in the CBDG resource along with the modernization of the structure in terms of modern technical requirements and increase in functionality – part one: Western Petroleum Province", which is part of the tasks of the state geological service. It was proposed to modernize the PIG62 gravimetric control network in its archival structures of 1st and 2nd class, by applying the approach used in similar works within geodetic control networks, which assumes the use of archival raw observations (the measured gravity differences between the points of nets) together with their errors, used to weigh the observations in the adjustment. These observations are re-adjusted in two systems:

- old, related to one point of the implementation of the Potsdam system in Poland (Warsaw Okęcie), i.e. repetition of the work from the study Bujnowski (1975), but in an integrated structure of the bases: 1st together with the 2nd class, adopted in archival adjustments based on Pawłowski's determinations of gravity of Okęcie station (Pawłowski, 1939),
- new, part of the implementation of the International Gravimetric System, applying the modern basic fundamental and basic gravimetric control network in as many fixed points as possible, after review of old networks and measurements the gravity difference between all existing points of old network to closest points of modern sets.

The concept of modernization of the semi-detailed measurements by a one-rank adjustment of 1st and 2nd class in first step and a similar adjustment of all available observations creating of 3rd class with a base defined in the previous step

as flawless sets includes the correction of several error sources in the existing gravimetric database. One of the basic sources was the problem of the inconsistency of 3rd class gravimetric network, based on which the detailed field measurements were made. The network of 3rd class control points consist of about 7,000 points connected by relative observations and was made in different time periods, as part of taking successive surveys and their documentation. Each of such sub-networks appeared next to the previous, neighbouring one connected with another object. Often, these networks were adjusted with strict or approximate methods, but only within one local object. The created network was therefore a set of unconnected structures indirectly related to each other by referring to 1st and 2nd classes. This resulted in significant errors at the interfaces of studies regarding the existence of step differences in gravity acceleration. Such non-geological artifacts can lead to misinterpretation.

4.2 Fieldwork – measurement of gravity difference between old stations and modern base

Measurement works and adjustments of the networks were conducted by PBG Geophysical Exploration Ltd. in a consortium with Geopartner Geofizyka Ltd. in 2022–23 under the supervision of PGI-NRI. As part of the work, the entire control network of PGI 1st and 2nd classes was covered by a field survey. The interview regarded:

- finding of a review of existing points,
- measurements of geodetic coordinates in the ETRF2000 reference system, valid in Poland, and heights in the modern PL-EVRF2007-NH height system, along with updating the topographic description and photographic documentation.

In the case of finding an existing point in good condition, measurements were made to link this point to the nearest point

of the national geodetic base points in the ABABA scheme, following the standard of gravimetric measurements of the basic control network. The tie-down measurements were made at 79 out of 168 points, enabling the determination of absolute gravity values at the points of the PIG62 network. Not all set of the PIG network was linked to the modern base because majority of stations were impossible to find or were rebuilt or destroyed, mainly in the case of 2nd class points and as a result of road upgrade works, and rebuilding of the closest vicinity of points, yielding significant gravity value changes in comparison to old determinations. In this way, all points of the archival PIG62 network which were still in existence and were confirmed as certain and stable were given values in the modern gravimetric system, in accordance with the reference level and accuracy standard of the ITRGS/F system. As a result, a database of binding points was created with a much more reliable characteristic of eliminating systematic errors related to the scale factor of instruments and recreating the gravity unit countrywide.

4.3 Adjustment of the network of the Polish Geological Institute

Section presents the stage of preparation for adjustment the archival gravimetric observations of the combined 1st and 2nd class PIG62 networks in a uniform structure. The proposed strategy of combining both network classes into a uniform structure originated in the postulate of increasing the number of observations and not introducing network ordering. The row pattern appeared in the 1960s, when the PIG62 control network was being created, and it was caused by the sequence of network formation and the inability to adjust larger control network structures. Nowadays, however, there is no need to maintain such a division, especially that the technical standard for laying the control panel of 1st and 2nd classes was similar. Both structures differed only in the density of points, the length of the spans connecting them, and, only slightly, the accuracy of their measurement.

In the adjustment of the old PIG62 system, the Warsaw Okęcie reference point and the value resulting from the Pawłowski determination of $g_{Wawa} = 981236.582 \text{ mGal}$ were adopted (Pawłowski, 1939). The search for the acceptable level of errors in the observation of spans Δg using the network error coefficient m_0 indicated as a typical observation error of 0.049 mGal for 1st class and 50 per cent higher for spans connecting 2nd class. These results correspond to the previous conclusions (Bujnowski, 1975), estimating the accuracy of the span at 0.051 mGal. Differentiation of errors for PIG62 1st and 2nd class structures results from the difference in the number of measurements, but also from the quality of drift elimination resulting from shorter observations between 1st class points between airports. At present, spans are the "pass" to the basic network, the accuracy of which is 0.020 mGal, so against today's standards, historical observations should be considered as about 2–3 times less accurate.

In the one-point adjustment of the PIG62 control network, 273 observations were adopted at the archival reference level defined by the Warsaw Okęcie point at 161 points. In the alignment, the values of the absolute acceleration of gravity were obtained at 162 points, the average error of determining the absolute g value was 0.053 mGal and the maximum was 0.079 mGal at Augustów, NE part of Poland, point of Polish border opposite to Warsaw–Potsdam direction (see Figure 3). In the analysis of observation corrections, it should be mentioned that the correction to the measured value of the acceleration difference ranged from -0.026 mGal to 0.029 mGal . The dominant error values are clearly visible in the regions of the Polish eastern border and the vicinity of the Carpathians. They should

be associated with the impact of the scale error of historical observations against today's definition of the reference level. To sum up, the results of the re-adjustment of old networks in the PIG62 system as a single-rank network yielded satisfactory results.

The adjustment in the new system defined by the modern basic gravimetric control network was made based on the largest possible number of tie points. The new gravity values were determined using the linking measurements, which covered 78 control points of PIG62 1st and 2nd classes confirmed as existing. In this variant, the same number of historical observations was retained, but the span error of 0.061 mGal and the same proportion of 1st and 2nd class observation errors as in the adjustment in the PIG62 system were used for weighting.

The analysis of alignment variants consisted in supplementing the base of reference points in the form of points with new acceleration values and consistently analysing the network alignment results in terms of alignment errors. This procedure started with a single-point link variant and progressed to a variant with 47 tie-points. Optimum results were obtained for the base of 31 reference points, which was when the best parameters of the alignment statistics and the distribution of corrections to the measured values of acceleration differences were obtained. There are 131 unknowns with an average error of 0.047 mGal and a maximum error of 0.094 mGal. Detailed quantitative characteristics of the adjustment of PIG network in two different frames are presented in Table 1, cartodiagrams visualised distribution of adjustment error of both networks are presented in Figure 3.

Summing up, importantly, the idea of using historical observations to align the PIG62 gravimetric control network and combining the structures of the 1st and 2nd class control networks turned out to be an exceptionally good idea. The digitization of historical observations made them reusable in the process of determining the accuracy characteristics of observations, and the network – readjustable, as a result of which gravity values were obtained in the old PIG62 system and the new one, compatible with the modern gravimetric base, i.e., the ITRGF system. Both alignments of the network ended with satisfactory results, and in addition, many of the points of these sets were found and determined using precise coordinates. A network modernized in this way can be used to establish and readjust a lower-class network, i.e. a 3rd class network that is the base for gravimetric reconnaissance works of a semi-detailed and detailed surveys. For the whole set of points of the 3rd class, after combined adjustment of all linked sub-networks to the base networks the statistics of the obtained results gives 10707 gravimetric spans, 162 reference stations (base networks), the maximum error of the gravity value adjusted points of 3rd class = 0.127 mGal, the mean error of adjusted gravity = 0.020 mGal.

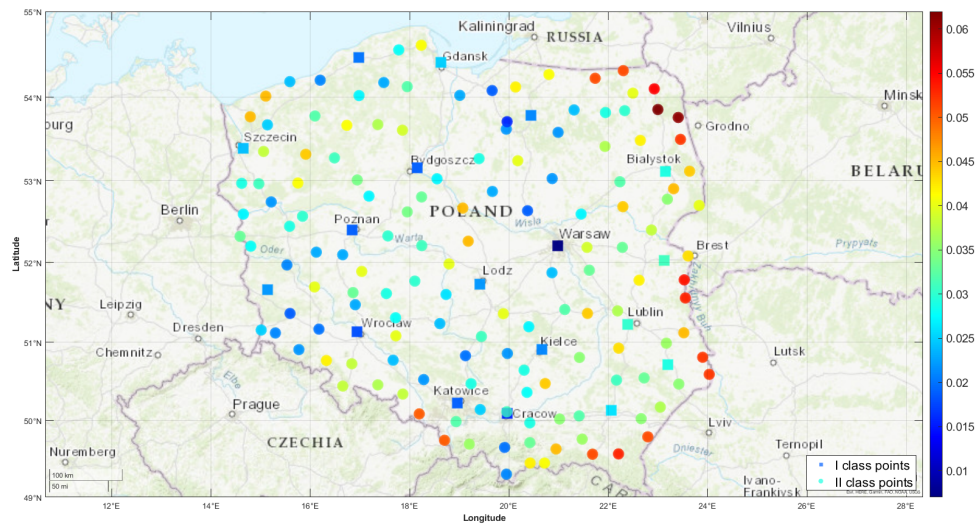
Thanks to the adjustment in both systems, it was possible to uniformly equalize this network in the past and contemporary reference level.

5 Differences of gravity between the past (PIG62) and new (IGSN71 – PIG2023) systems

It is a common mistake to assume the difference in gravity between the Potsdam (Helmert) system implemented through the base in the PIG62 system and the modern one as a constant value (Sas-Uhrynowski, 1999; Sas-Uhrynowski et al., 2000; Barlik, 2009; Kryński et al., 2013; Wziontek et al., 2021). This approach is not logical because it assumes that only the error in determining the value at the reference point is responsible for the gravity differences. This is not the case either in Helmert's

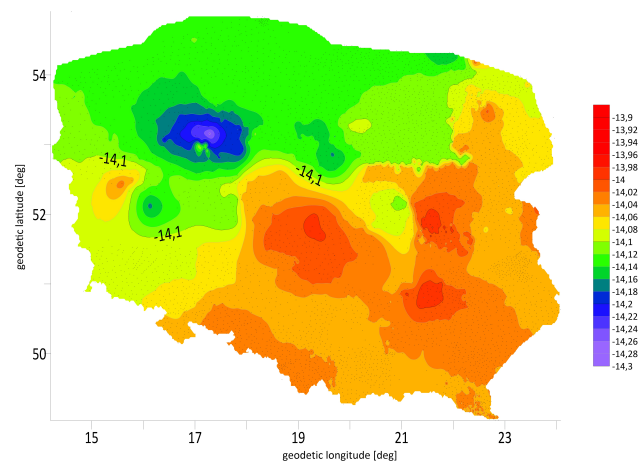
Table 1. Characteristics of results of the adjustment the PIG's network in both frames

Variant	Assumed span error [mGal]	Number of observations	Number of adjusted points	Maximum value adjusted mg [mGal]	Mean value adjusted mg [mGal]	Maximum value adjusted g difference mdg [mGal]	Mean value adjusted g difference mdg [mGal]
1 st + 2 nd networks in PIG62 system (OLD)	0.049	273	161	0.079	0.053	0.118	0.027
1 st + 2 nd networks in PIG62 system (NEW)	0.061	273	131	0.093	0.047	0.166	0.037

**Figure 3.** Distribution of adjustment errors of PIG's first and second class network in IGSN71/ITGRF system [mGals]

time or now, because each of the relative gravimeters, operating on the principle of spring balancing, imposes an error on the scale of the instrument, proportional to the difference of the gravity, on the measurement results. Only the transition to creating control networks based only on absolute observations significantly eliminates this error. Figure 4 presents a graphical illustration of the differences between the two systems together with the points on which these differences were calculated, based on the results of the control network adjustment of 1st, 2nd and 3rd classes in both systems, as described in section 4. A set more than 5000 points of three sets is used to create base of empirical g difference between new (ITGRF) and old (PIG62) frames.

Summarizing determining the differences using the full database of 5037 points in total for 1st, 2nd and 3rd classes, the following results were obtained: maximum value -13.977 mGal, minimum value -14.268 mGal; the average value of the difference is -14.072 mGal, the range of variability is -0.291 mGal and the RMS is 0.043 mGal. Clearly, assuming a constant value of the difference of 14 mGal causes an error of even ca. 0.25 mGal. In Figure 4, additional regional deformations of the difference field caused by errors in the implementation of the 2nd class control network can be noticed. These were transferred to the fragments of the 3rd class control network tied to these points. This should not be interpreted as an effect of geophysical factors, but only as a result of systematic errors occurring in the structure of past control networks against the background of accurate, modern gravimetric observations of a precise modern gravimetric control network.

**Figure 4.** Visualization of differences between the values of gravity in the PIG62 system (Potsdam) and IGSN71 (ITGRF) in Poland [mGals]

5.1 Mathematical model of system conversion

With adjustment results and gravity difference at many points on Poland's territory of the same class network structure in the old (PIG62) and new (PIG2023, realisation ITGRF) systems, an attempt to build a model of differences of gravity in an analytical form is the strategy adopted for converting point sets or map sheets to a new layout, when it is not possible to process raw observation logs and align them or develop a raw version. There can be a process defined as a transformation for which a certain mathematical continuous or discrete formula is created using information about the position of a point or other unique characteristic of its property. In this aspect, three approaches have been explored:

- App 1.** Analytical conversion of formulas for a complete set based on the use of polynomial functions with arguments: geodetic latitude and longitude, as a function of point coordinates (Akima, 1970).
- App 2.** Interpolation grids defining the differences between the levels at points defined by the resolution, as a function of geodetic latitude and longitude (Akima, 1970).
- App 3.** Scale-dependent recalibration functions appropriate for the transformation of gravimetric systems.

Approaches 1 and 2 require the knowledge of the coordinates of the point for which we want to calculate the differences Δg . After adding it to the value of g in the previously-used frame of reference, the value of acceleration in the new frame is obtained. Unfortunately, these approaches suffer from the problem of point localization. In the databases of the networks and semi-detailed measurements, the quality of georeferencing is insufficient. The points were located on maps in the Borowa Góra archival cartographic system, locating them graphically. The precision of the coordinate recording itself is only 20 m, but errors up to ten times greater are still common, resulting from errors in entering the point, its modern digitization, errors in the transformation from the Borowa Góra system to the modern one, e.g. PL-1992. Therefore, method 3 looks safer.

Let's analyse each approach in turn:

Approach 1: The analytical transformation formula is based on the estimation of the parameters (p_{00} , p_{10} , p_{01}) of the polynomial of a certain degree after the latitude (φ) and geodetic longitude (λ) coordinates. For example, for a linear polynomial of degree (1,1) it has the following form:

$$dg(\varphi, \lambda) = p_{00} + p_{10} \cdot \varphi + p_{01} \cdot \lambda \quad (1)$$

The degrees of polynomials from (1,1) to (5,5) were analysed.

For degree (1,1), a function with a simple analytical structure was obtained:

$$\Delta g(\varphi, \lambda) = -13.15 + 0.005904 \cdot \varphi - 0.01997 \cdot \lambda \quad (2)$$

guaranteeing the transformation of systems with a mean square error equal to 0.025 mGal. This is about half of the average error in determining the gravity value in the base structure of 1st and 2nd classes. From this point of view, the result should be considered as a good one in the aspect of gravity adjustment errors. Visualization of the results in the form of a model and residuals, i.e. differences between the observed and model values, is shown in Figure 5.

The presentation of successive results of the estimation of polynomials from (2,2) to (5,5) seems pointless, because the transformation function is extended by non-linear factors. Let us, however, comment on the change in the model error, expressed as the RMS. For model (2,2) it is 0.023 mGal and for model (5,5) it is 0.016 mGal. The result is shown in Figure 6.

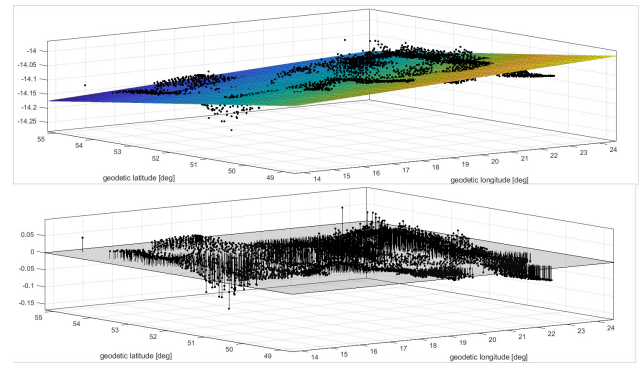


Figure 5. Visualization of the model of recalibration of gravimetric systems as a polynomial model of degree 1 (isoline map, 3D model, residual spatial graph)

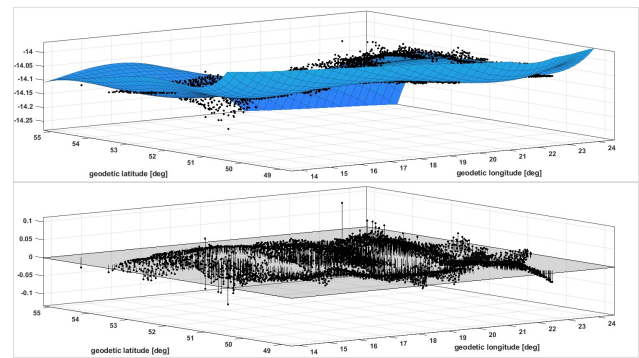


Figure 6. Visualization of the gravimetric system conversion model as a polynomial model of degree 5 (3D model, residual spatial graph)

Note that reducing the error by a third is at the expense of increasing the number of coefficients of the polynomial to 21 in model (5,5) starting from 3 in model (1,1) of first degree.

Approach 2: The interpolation approach seems to be the most difficult to operate later. It consists in creating a set of regularly gridded points and determining differences in them, which in the final grid form a discrete interpolation set, requiring that the user can avail of an appropriate program and use a specific interpolation method. However, this method has the advantage over the analytical method discussed in point 1 that it "notices" local system distortions. The first of these elements, the accuracy of interpolation to a grid with a density of 1 km by 1 km, was analyzed. For linear (1st degree, Figure 7) and non-linear (3rd degree polynomial surface, Figure 8) identical results were obtained, characterized by an error of 0.032 mGal, which was not large, compared to gravity value alignment errors, but still slightly higher than in the case of method 1 at this point. Different interpolation method approaches were not investigated because those groups of methods yielded worse results than the easier method of building mathematical function defined dg with geodetic coordinates relationship (No. 1). This interpolation method has more sensitivity to error of positions, which are the weakest feature of gravimetric PIG's network, cumulating error of positioning on old cartographic map and error in the transformation to the modern frame. In some areas, this error reached more than several hundred meters.

Approach 3: The approach released from the relationship with the coordinates of the point was proposed by Torge (1989), and it is often used to implement the conversion of two gravimetric systems. The idea of the transformation is extremely simple, and it relies on finding coefficients a and b in the fol-

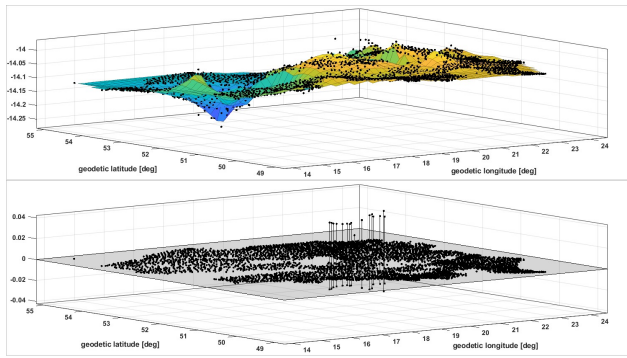


Figure 7. Visualization of the linear model for interpolation of gravimetric systems (map of isolines, 3D model, spatial plot of residues)

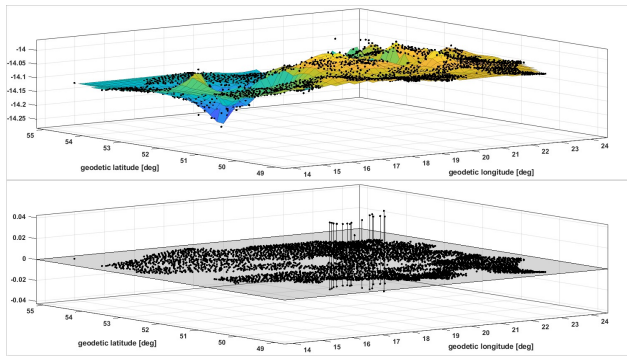


Figure 8. Visualization of the non-linear model for interpolation of gravimetric systems (3D model, spatial plot of residues)

lowing equation:

$$g_{new} = g_{old} + a + b \cdot (g_{old} - g_0) \quad (3)$$

Let us note that coefficient a is an offset, a shift of one system relative to another; b is the coefficient related to the difference in the scale of two systems; as the old system is understood as the past PIG62 system and as the new system – modern ITGRF system convergent with the new IGSN71 implementation. The g_0 value is the reference value in the old system. It can be, for example, the lowest value from the "old" set. An estimation of parameters a , b , and g_0 of transformations gave the result:

$$g_{ITGRF} = g_{PIG62} + (-13, 991091) + (-0, 0002030) \cdot (g_{PIG62} - 980796, 369) \quad (4)$$

by creating a simple function that only uses the gravity value in the old system.

The analysis of the accuracy of such an approach leads to the value of the standard deviation of 0.032 mGal for a set of control points of the 1st and 2nd classes. The value errors here are similar to the previous methods, but this approach is unaffected by the position of the point mistakes, therefore, from this point of view, it is the first choice among those analysed.

Visualizations of differences and residuals (deviations) between the modelled and observed values are shown in Figures 9 and 10.

Approach 3 is therefore less accurate than the analytical approach, but it is simple, and applicable even when we do not have a certain or no point location. Against the backdrop of errors in determining the gravity value based on the 1st and 2nd

classes, it should be reiterated that the model error is about half of the error in determining the g value, implementing in the PIG62 system.

6 Conclusions

The implementation of this work marks the end of the validity of two gravimetric reference systems in Poland. The first of them, the historical PIG62 system, was based on geophysical works conducted since the mid-1950s, and grew much faster than the geodetic system. In the 1990s, "new life" appeared in geodetic base through the possibility of using more precise absolute gravimeters to establish them, thanks to which the gravimetric control network is in operation today. It is included in the State Register of Basic Geodetic Controls of the Central Office of Geodesy and Cartography, as the most accurate structure implementing the modern International gravimetric system (ITGRS/F).

As part of the work, an analysis of raw gravimetric observations in the structure of 1st and 2nd class control networks was conducted, coming from measurements in the 1960s, and new values of gravity in the IGTRF system were determined for all points of the past 1st and 2nd class control network. This enabled the joint adjustment of the entire archival network in the new system without re-measuring the gravity differences on every span. The results of the adjustment should be considered good, considering the average error of determining the g value of only 0.05 mGal. The value is high compared to today's technical standard, but it is the result of using historical observations.

This operation opened a similar possibility for the structure of the third-class control network, and in this way, a set of almost 6,000 points with values of gravity was obtained. Differences in the g values between the past PIG62 system and the present ITGRF system (compatible with IGSN71) were determined and thoroughly analyzed in search of the optimal methodology for mathematical modelling of differences. This effect is as important as the definition of new values itself, as it will enable recalculations of gravimetric data sets for which field measurement logs do not exist. This step was successful, i.e., the indication of the easiest methodology together with the analysis of the conversion error with the average error of 0.03 mGal. These steps give us the possibility to convert every gravimetric data-set collected by geologists and geophysicists to one, modern and common international gravity system and considerably extend the capability of the utilisation of these data in geological and geodetical aspects, i.e. gravimetric prospecting, geoid/quasi geoid modelling, combined global geodetic geopotential models filling, building modern height systems and more.

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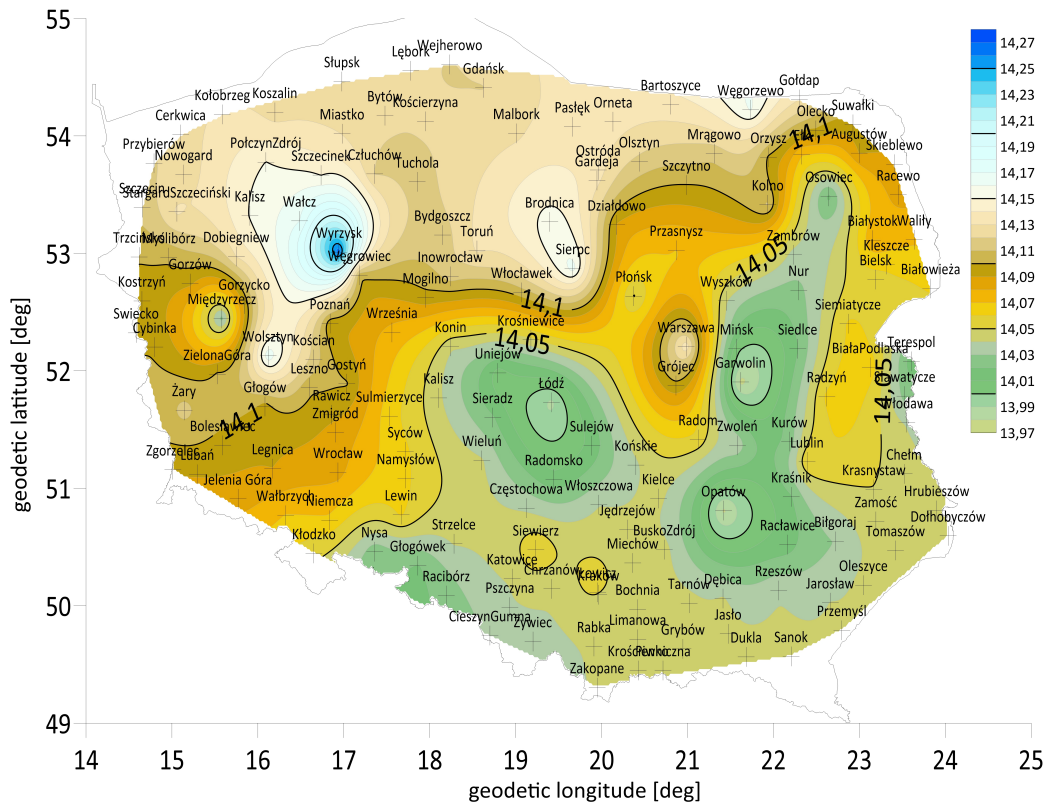


Figure 9. Visualization of differences between PIG62 and PGI2023 within the control network of class I and II in mGals

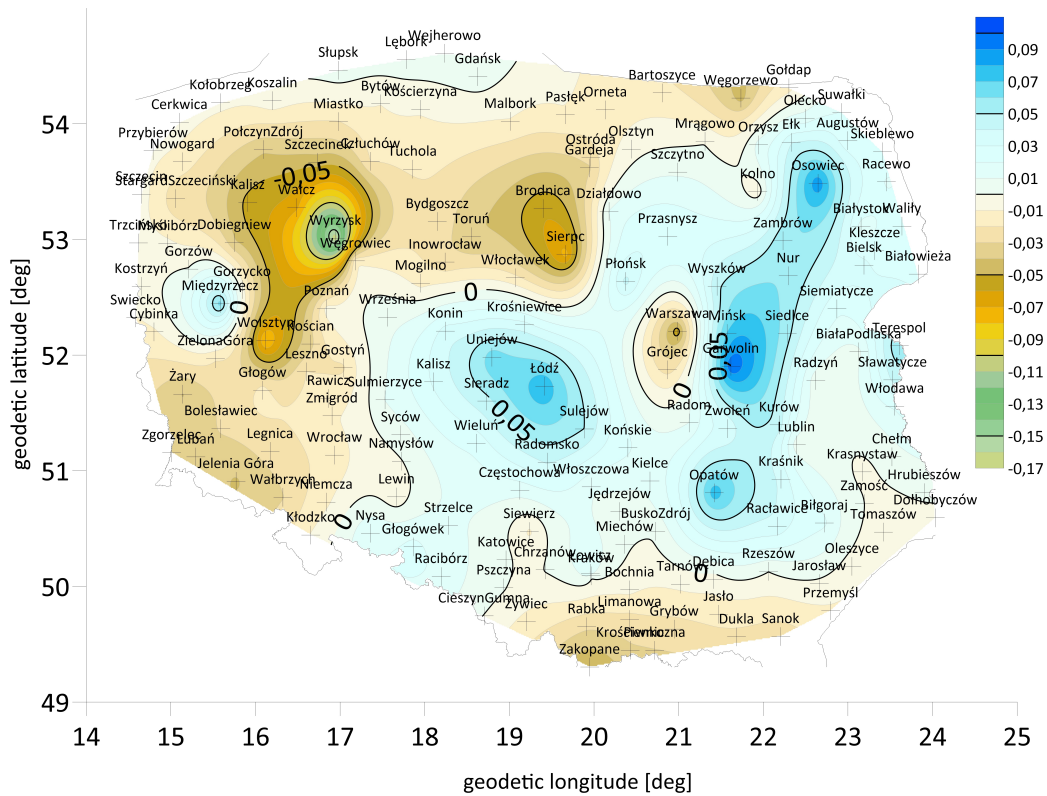


Figure 10. Visualization of residuals (differences in measured values against the model) between the PIG62 and PGI2023 within the control network of class I and II based on Torge approach

along with the modernization of the structure in the aspect of modern technical requirements and increased functionality – part one: Oil Province of Western Poland", financed by National Fund of Environmental Protection and Water Management.

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