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Piles system securing road against landslide. 2D/3D method of numerical modeling and design problems

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Abstract. The main objective of this work is to present an innovative method of numerical modeling of anchored piles system acting as a road protection against landslide, called the "2D/3D method". Firstly, short description of the problem and "state of the art" review are included. An effective methodology of the design supported by the numerical analysis, solving the problem of interaction of a periodic system of piles and the unstable soil mass is presented, for which some detailed information about proposed numerical approach is given. The key idea of 2D/3D method is to join the pile with the 2D plane strain continuum by fictitious connectors of Winkler type with P-Y properties identified during the analysis of a subsidiary 3D problem. Practical example of usage of proposed approach to a real case of a road endangered by a landslide then protected by the piles system is presented. On the base of this example, a discussion about important design issues like internal forces in piles (mainly bending moments) and anchors (tensile forces) or overall stability of the soil-structure system is done.

Key words: landslide, numerical analysis, piles, soil-structure interaction, multi-scale analysis.

1. Introduction

A typical situation at the roads crossing landslide-endangered slopes, which is commonly encountered in sub-mountains area of Carpathian region in southern Poland is shown schematically in Fig. 1.

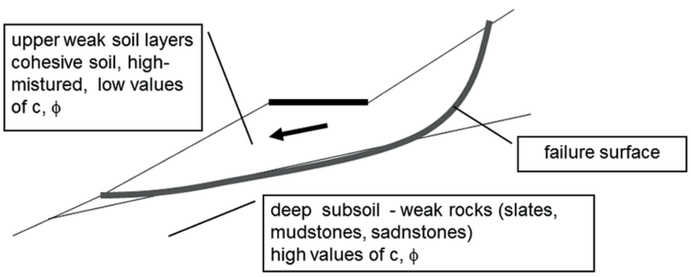

Fig. 1. Schema of geotechnical situation

In these situations, soil mass movements are often initiated by the increase of an amount of water contained in subsoil, leading to decrease the soil strength parameters. This may be caused by an inflow from the above or by heavy rainfalls. When soil parameters reach some critical values, a sliding surface, typically located at the bottom of a weak layer, appears. This landslide causes deformation and substantial deterioration of the road surface, and some action preventing it must be undertaken. In case when the depth at which stronger subsoil may be found reaches several meters, practical needs of securing the road would require introducing the piles system and anchoring it in stronger rocky layers, stabilizing the road subsoil. A schematic view of such a structural solution is given in Fig. 2. It is worth to note, that in described situation any action stabilizing only a slope surface beneath or above the road would not bring the desired effect, because it does not delete the source of the problem – sliding on the deep failure surface. On the other hand, piles system will stabilize the road and its subsoil only, but will not prevent the appearance of some "shallow" landslides, which, if needed, might be stabilized by other means, for example by soil nailing.

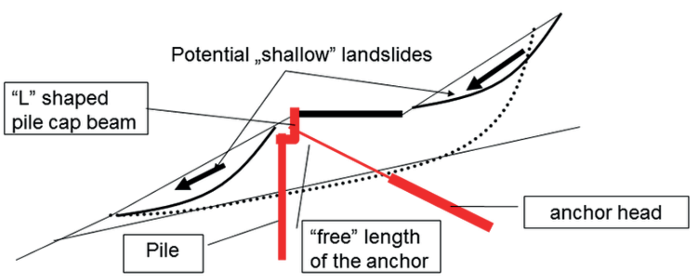

Fig. 2. Schema of structural solution applied to secure the road

The other issue is dewatering of the whole landslide area which is always beneficial. In practice, however, it is difficult to assure its full effectiveness, particularly when impermeable layers of weak cohesive soils are mixed with more permeable soils and morphology of the slope is complex and hardly recognizable. Obviously, the whole idea of securing a road is not a new one and was successfully applied in many similar situations.

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The designer task is setting parameters of the system, which of course must be preceded by estimation of internal forces in structural elements. These are as follows:

- piles: length L and diameter D, distance between them a, reinforcement,
- soil anchor: span, cross-section and required bearing capacity of its head, pre-stress σ_o,
- cap beam: dimensions and reinforcement.

In the paper, an effective methodology of the design supported by numerical analysis, solving the problem of interaction of a periodic system of piles and the soil mass, called 2D/3D is presented. First, however, the analysis of stability of an existing state of the slope is performed, which basing on back-analysis, helps to establish critical parameters of the soil. This will be presented in the next point on one example coming from the engineering practice of the Authors. All computations will be performed using ZSoil.PC v.16 code.

2. A brief "state of the art" review

Problem of the laterally loaded piles was and still is a subject of investigation undertaken by many researchers. Two main sources of the horizontal loading are analyzed: external horizontal force and relative soil-pile movement. Our research is oriented around creating methodology of 2D numerical analysis of real landslide cases, often with complicated geotechnical conditions but considering some essential 3D effects. The 2D/3D method, although approximate, would be more effective and less time consuming than full 3D analysis of a whole soil-piles system. In Urbański [1] the correctness of the 2D/3D method was investigated by comparison with full 3D analysis, giving acceptable results. Similar approach used to analyze stabilization of the building on active landslide is used by Urbański and Grodecki [2].

Winkler-type model for soil (with horizontal stiffness) is often used to obtain relationship between horizontal displacement of the pile and soil resistance (so-called P-Y Curve). Examples are given by Pando [3], but without considering vertical stress. P-Y curves could be obtained from field measurement (detailed procedure is described by Pando [3]), but in recent work Authors prefer to identify the P-Y Curve with the use of numerical modeling, which would be found as more convenient in practical use, by a majority of designers. Identification of the Winkler-type model is also a subject of work by Taheri et al. [4], where influence of vertical stress on soil horizontal reaction is investigated. Obtained relationship is nonlinear.

Interaction between laterally loaded pile and soil in Berlin-type wall (soldier pile wall) is a subject of investigation of Santos [5]. 3D numerical modeling is used in analysis of a real case. In the work of Ruigrok [6] different calculation methods are compared. Blum, Brinch Hansen, Broms, Characteristic Load Method (CLM), Non-dimensional Method (NDM), MSheet, P-Y Curves and numerical analysis are compared. Strong and weak sides of the presented methods are discussed.

Quin, in his PhD Thesis [7], describes the laboratory tests used to model relative soil-pile movement. Special attention is paid to develop simple formulas for maximal bending moment and shear force in the pile. Quin et al. [8] shows analytical formula for ultimate horizontal load of the pile.

Georgiadis et al. [9] shows results of the numerical analysis of the pile-soil interaction, together with analytical upper-bound solution. Special attention is paid to the influence of the pile spacing on the pile-soil interaction.

Won *et al.* [10] shows a solution for a pile in the slope (similar situation to the one presented in this work). Combination of the simplified Bishops method for slope stability and plastic state theory by Ito and Matsui [11] for the estimation of the soil pressure on the pile gives possibility to estimation of the Stability Factor of the slope reinforced with piles. 3D numerical model of the slope reinforced with piles is also presented and effects of piles stiffness, spacing and position are investigated.

Geological approach with simplified mechanical model of the horizontally loaded pile is presented by Kozubal, Bhat and Pradhan [12].

3. Numerical simulations of an existing state

The example of an analysis of stability of the road crossing landslide area, seen in Fig. 3, is briefly summarized. Further on, in this situation structural system presented in Introduction will be described in detail.

The c-ϕ reduction method (described in details by Zimmermann *et al.* [13], Griffiths and Lane [14], Matsui and San [15]) was used to estimate the Stability Factor SF. All simulations were performed in the plane strain conditions. Mohr-Coulomb elastic-plastic model was used for the soil. Soil-water composite was treated as a single-phase media, mainly because geotechnical documentation, which was prepared prior to the authors' involvement in the case, contains data allowing only for this kind of approach, i.e. water content as well as total values of c and ϕ. Moreover, 2-phase treatment of soil-water composite requires exact recognition of pore pressure field, which in the described practical situation, was hard to obtain. The reason is that all weak soil layers are in partial saturation area, with degree of saturation $S_r < 1$. For soils under the water table buoyant unit weight was used.

Table 1
Parameters of the soil used in the stability analysis

ID	Material	E [MPa]	γ [kN/m^3]	ϕ [°]	c [kPa]
1	Ib (uncontrolled embankment)	16	20.6	10.00 (5.5)	9.00 (5.5)
2	2 schist, mudstone, clay	20	19.5	20.77	3.13
3	3 breccia (schist + clay)	20	20.6	11.83 (4.50)	25.04 (21.00)
4	4 mudstone, schist	500	24.5	50.00	50.00

Parameters in brackets are the critical parameters obtained from back analysis of the stability problem, which is described later in this work

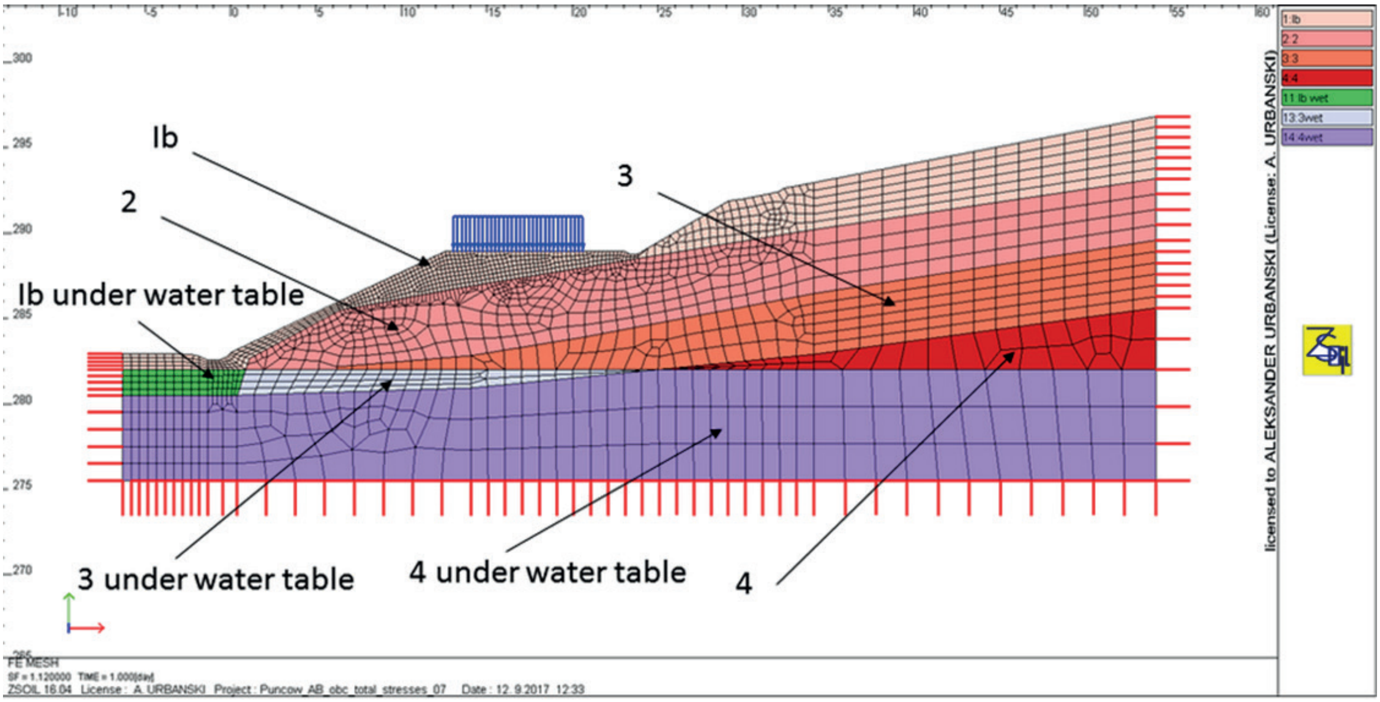

Fig. 3. Numerical model of the landslide area

First of all, the stability analysis of the existing "in situ" state of the slope was performed (numerical model – see Fig. 3).

Obtained value of stability factor SF = 1.20 without road load and 1.12 with road load 25 kPa (according to Polish Regulation of the Minister of Transport and Maritime Economy [16]) shows that failure of the slope is very probable (see Fig. 4). The point is, however, that geotechnical *in situ* examinations were performed in different conditions (mainly state of soil watering), than these encountered in times when soil mass movements have occurred. In existing geotechnical evidence 2 historical failure modes were described. First one, recorded in 2012, is shallow, located mostly in Ib layer. Sec-

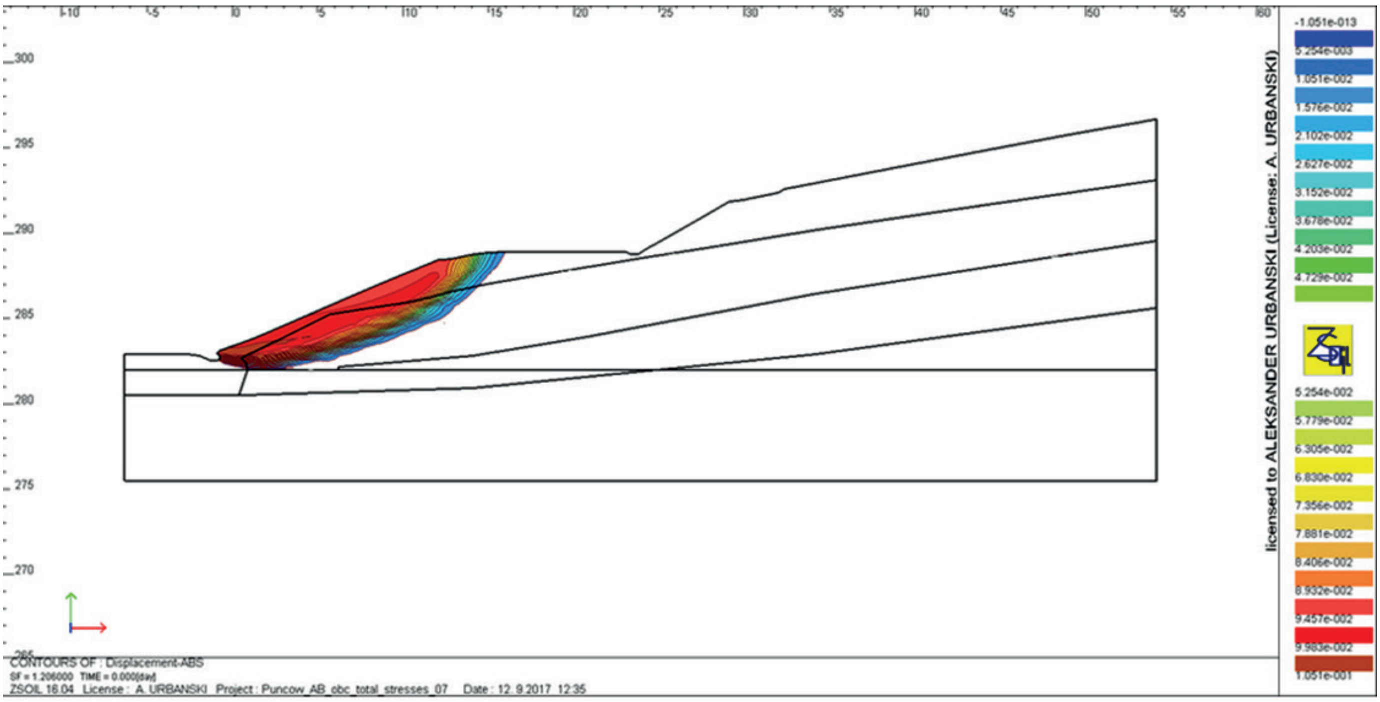

Fig. 4. Failure mode (sliding surface) of the existing landslide, SF = 1.20 without road load

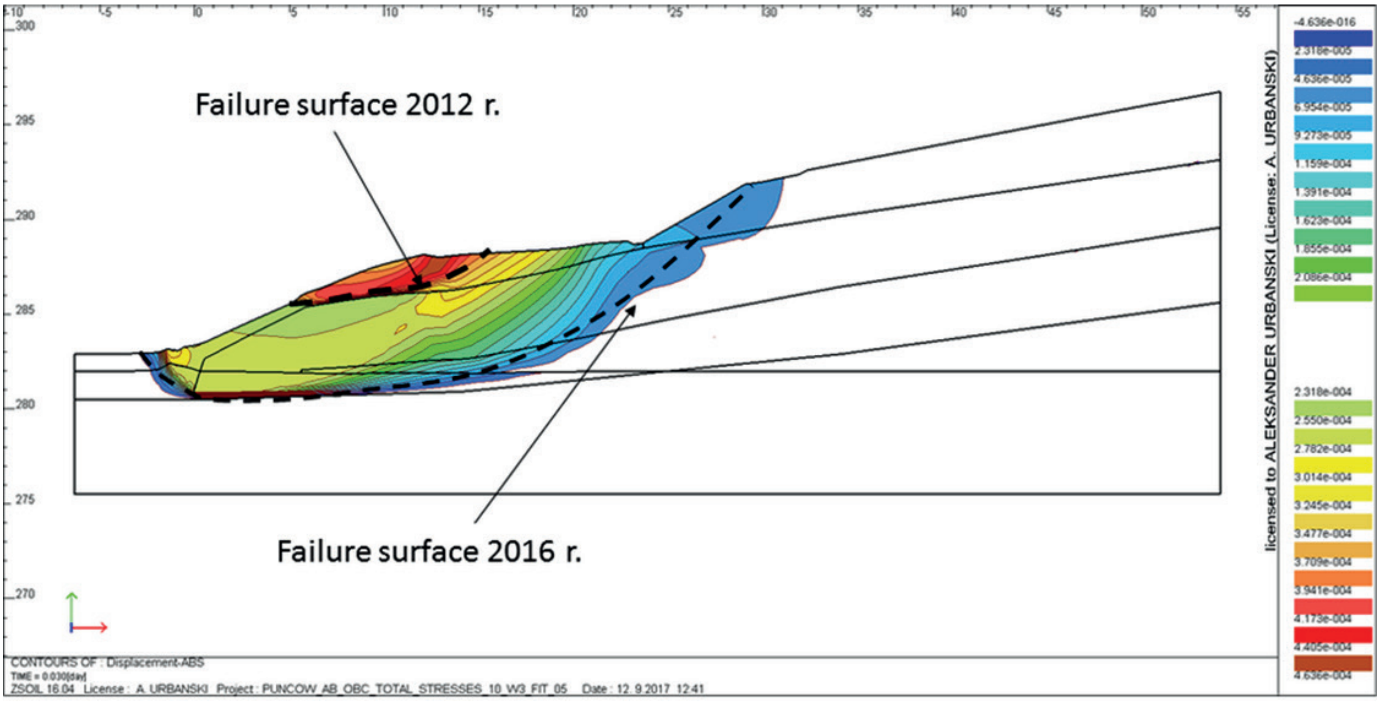

Fig. 5. Failure mode (sliding surface) for critical parameters of the Ib and 3 layers, SF = 1.04 without road load, SF = 1.00 for road load 6.75 kPa

ond one, observed in 2016 is much deeper, located mostly in breccia layer 3. So, back analysis was performed, to obtain SF very close to 1, with failure modes similar to those observed in reality. For critical parameters of uncontrolled embankment Ib ($c_{cr} = 5.5$ kPa $\phi_{cr} = 5.5°$) and breccia 3 ($c_{cr} = 21$ kPa $\phi_{cr} = 4.5°$) SF = 1.04 was obtained (without road load) and stability loss induced by road load of 6.75 kPa was recorded. Obtained critical parameters of layers Ib and 3 were used in further calculations. Obtained failure mode is very similar to described in geotechnical evidence (see Fig. 5).

4. Numerical analysis of the design state

4.1. Assumption and formulation of the problem. The analysis is aimed at establishing two most important results for the design of the described structural system, that is:

- internal forces in piles (bending moments, shear forces),
- tensile forces in anchors.

Global analysis concerning selected cross-section will be performed with the assumption of the plane strains state (PS), mainly because of the effectiveness of the design process and lack of satisfactory information about spatial geometry of the slope morphology. On the other hand, a visible contradiction appears, as all fields at the surroundings of the pile (displacements $\mathbf{u}$, strains $\boldsymbol{\varepsilon}$ and stresses $\boldsymbol{\sigma}$) are 3D in their nature and drastically violate assumptions of the 2D PS model. The point is, that these local 3D periodic fields decide the forces acting on the pile from soil mass, particularly in the case of moving soil mass when the landslide is active, and in consequence, internal forces decomposition in piles as well as forces in anchors. To reliably approximate a full 3D solution of the problem by 2D PS model a method named 2D/3D is proposed. It was originally developed by the Urbański [1], Urbański and Łabuda [17], Urbański and Grodecki [18], for the analysis of a periodic pile system loaded by external horizontal forces, which is crucial in the design of the Berlin-type retaining wall (soldier pile wall).

Here, the situation is reversed, see Fig. 6, because a pile position may be treated as fixed, due to the pile anchoring and its stiffness, but the soil tends to move around it. The 2D/3D method is fully applicable also in that case because the key factor is a relative motion measured as displacement between the piles and surrounding soil.

The method is open for the application of any constitutive model of the soil media, here standard elastic-plastic Mohr-Cou-

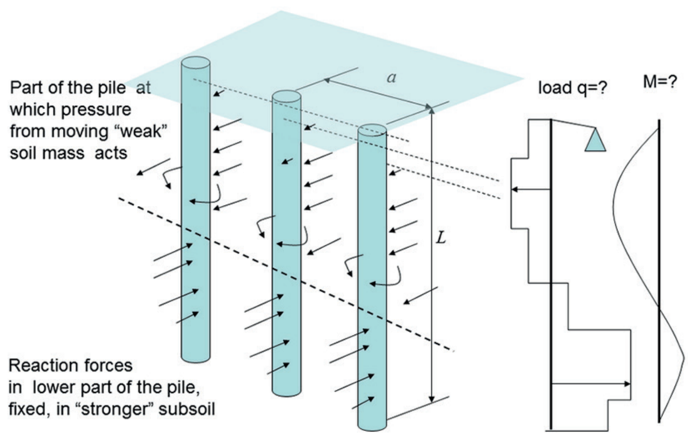

Fig. 6. Schematic spatial view of statics in a pile system

lomb is used. The usage of Hardening Soil –small strain (HSs) model, which is now recognized by many researchers as the most exact one in soil-structure interaction problems, was also considered by the Authors. Finally, this idea was abandoned due to lack of necessary data in geotechnical documentation for the HSs identification procedure. Basing on common knowledge on HSs model, see Truty and Obrzud [19], if it would be applied, some differences might be expected in the description of an initial phase of deformation process, but stability assessment as well as forces in structure, corresponding to the state close to bearing capacity, should not exhibit significant differences. It however may be a subject of future research.

In the 2D/3D method, the plane strain (PS) model is completed by a set of horizontal elastic-plastic connectors between beam elements modelling the pile and 2D continuum elements. The whole methodology of the problem solution may be seen as a kind of multi-scale approach.

4.2. Identification of elastic-plastic connectors in 2D/3D method of analysis periodic system of piles submitted to horizontal load. The crucial point of 2D/3D method is identification of connector properties. The idea of it is shown in Fig. 7 and briefly explained below. As it was mentioned in the previous point, the whole procedure of identification, will be run like for the problem of piles horizontally pressing on the ground, it is with a reference frame attached to the continuum. While analyzing periodic pile system securing landslide theoretically it should be changed, as position of the pile is fixed, but the soil continuum moves. For both cases however, thanks to the notion of relative displacements between the soil and the pile, results i.e. forces interacting between them will be quantitatively identical.

The identification of stiffness and strength properties of the connectors will be performed based on a function relating the reaction force and the imposed displacement, obtained from the subsidiary 3D problem for the layer of arbitrary small thickness t. Only 1/2 of the length a of one segment of periodic structure is taken due to symmetry. The vertical loads acting on the layer is a soil dead weight above the layer p_Y.

In this FE model of a soil layer, considering vertical load and resulting compressive stresses $\sigma_{YY} = x p_Y$ is crucial, because these stresses, when only introduced to the yield condition (in full 3D form), have a great influence on the plastic state and response of the media, particularly when soil possesses low cohesion $c \approx 0$. In ZSoil it can be achieved by building a 3D FE model with one layer of brick elements B8. Between the pile and the soil interface elements are introduced assuming frictionless unilateral contact (with constant topology of elements).

The kinematic boundary conditions are as follows (Fig. 7b):

– imposed horizontal displacements $U_X(0, t, 0) = U_X(0, 0, 0) = U_{XB}$, at the central nodes of the pile;
– YZ planes, no horizontal displacements (at the sufficient distance $l/2$ from the centre): $U_X(x = \pm l/2, y, z) = 0$;
– XY planes (from symmetry and periodicity of the system): $U_Z(x, y, z = \frac{1}{2}a) = U_Z(x, y, z = 0) = 0$;
– XZ plane (at the bottom of the model): $U_Y(x, y = 0, z) = 0$;

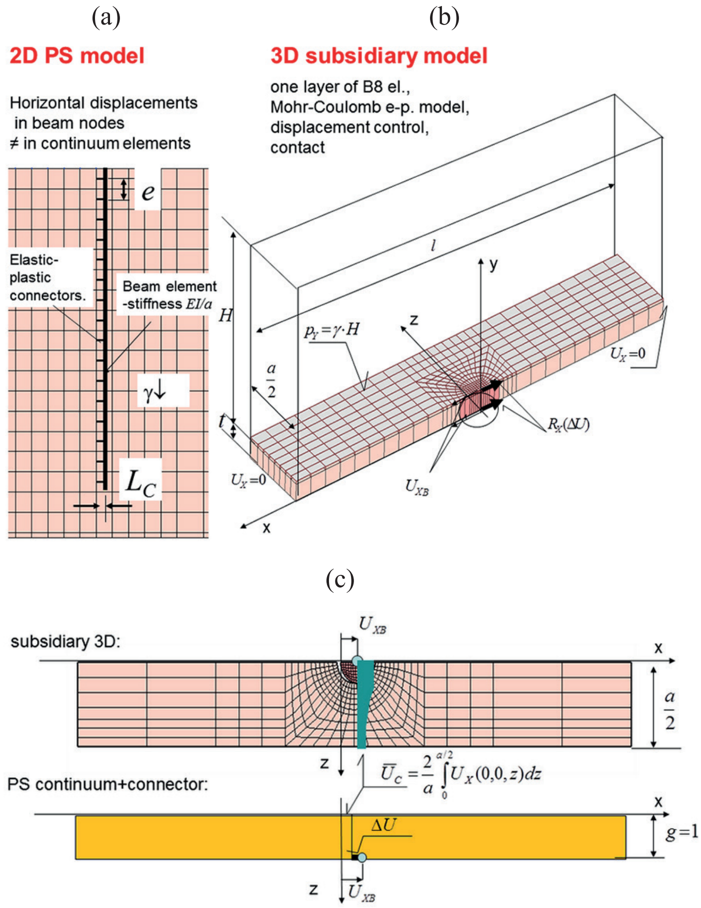

Fig. 7. Idea of FE modeling in 2D/3D method a) plane strain model with a pile and connectors; view in XY plane b) subsidiary 3D FE model of a layer with a pile, c) equivalency of both models: subsidiary 3D – plane strain with a connector in XZ projection

The output is a sum of 2 reaction forces (at the bottom and top node) as a function of the difference ΔU between displacement of the pile U_B and averaged mean displacement of soil $\overline{U}_C$, for $x = 0$:

$$\overline{U}_C = \frac{2}{a} \int_0^{a/2} U_X(0, 0, z)\,dz. \tag{1}$$

It is important to note, that taking the difference ΔU but not the displacement U_{XB} alone as an output argument, is a must, because in this kinematic condition (i.e. periodic in z dir.), the force response on the applied displacement is strongly sensitive to the length of the model l in the x-direction (is linearly dependent) while for the difference ΔU, it is not. This is shown in Fig. 8, for exemplary subsidiary model of the layer. In the subsidiary 3D model U_{XB} is displacement of the pile, which in 2D equivalent model is applied to the outer node of the connector. $\overline{U}_C$ is a value of averaged displacements over the YZ section of 3D model at the location of the pile, as shown in Fig. 7 c, while in 2D model it represents horizontal displacement of continuum in the pile location.

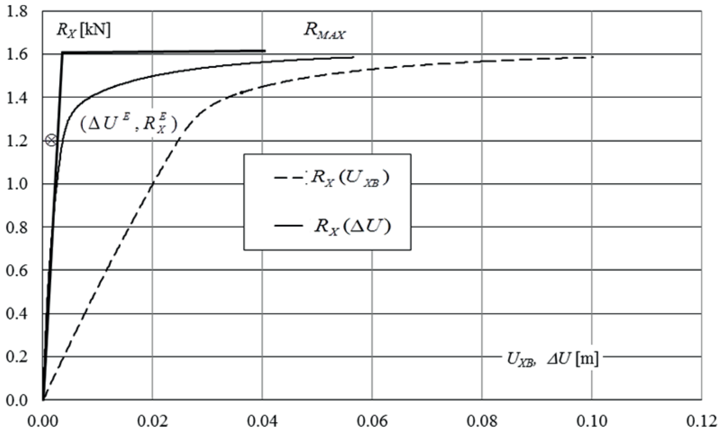

Fig. 8. Reaction force as a function of imposed displacement U_{XB}, and displacement increment ΔU

The difference ΔU represents elongation of the connector:

$$\Delta U = U_{XB} - \overline{U_C} \Rightarrow \varepsilon_C = \frac{\Delta U}{L_C}. \tag{2}$$

This is basic interpretation of ΔU, being a measure of fluctuations of the displacement field caused by the presence of the pile, to be used in plane strain model of whole slope-piles system. Figure 8 shows reaction R_X as a function of ΔU, and its approximation by a bilinear function. Having given reaction forces $R_X(\Delta U)$, lateral resisting stresses q, averaged on surface $t \cdot {}^1\!/_2 a$, can easily be obtained as:

$$q(\Delta U) = \frac{2 \cdot 2 \cdot R_X(\Delta U)}{t \cdot a}. \tag{3}$$

Assuming that in the 2D model, connectors having an area A_C are equally distributed in the distance e, taking into account the static equivalency between the force in the connector $F_C = \sigma_C \cdot A_C$ and force resulting from lateral resisting stresses q at $e \cdot g$ area in the form:

$$\sigma_C \cdot A_C = q \cdot e \cdot g, \tag{4}$$

with $g = 1$ [Lengths Unit] – unit width of a slice (in the direction z) in PS model.

The function between the stress and strain in the connector is:

$$\sigma_C(\varepsilon_C) = \frac{q \cdot e \cdot g}{A_C} = \frac{4e \cdot g}{t \cdot a \cdot A_C} R_X(\varepsilon_C \cdot L_C). \tag{5}$$

Elastic-plastic properties of the connector (or rather its bilinear approximate model) are Young modulus of a fictitious 1D truss type element

$$E_C = \frac{4 \cdot}{t \cdot a} \frac{R_X^E \cdot e \cdot g \cdot L_C}{A_C \cdot \Delta U^E}, \tag{6}$$

where R_X^E, ΔU^E are values used to identify elasticity of the connector and the tensile/compressive strength of its material f_C:

$$f_C = \frac{4 \cdot}{t \cdot a} \frac{R_{MAX} \cdot e \cdot g}{A_C}. \tag{7}$$

These characteristics are dependent on the geometry of the periodic pile system, constitutive properties of the soil and the depth on which considered layer is located.

4.3. Results of the identification problem for 2D/3D method. At first, selected results of the subsidiary 3D problems, concerning identification of connectors are presented. These are related to assumed geometrical set up (pile horizontal distance a, pile diameter D, layer thickness $t = 0.1$ m), constitutive model data of the soil in each layer. For the Mohr-Coulomb model they are: Young modulus E, cohesion c, friction angle ϕ, dilatancy angle ψ assumed $(\psi = 0)$ and, what is not obvious, to the vertical compressive stress p_Y. The latest dependency can be traced on exemplary $R_X(\Delta U)$ graphs, shown in Fig. 9.

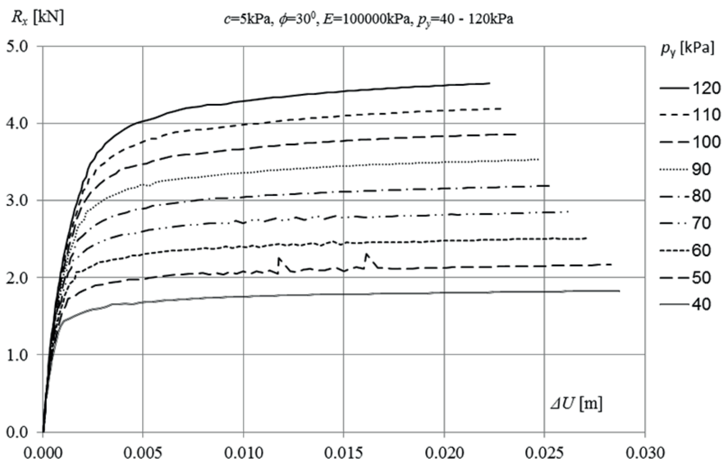

Fig. 9. Relation $R_X(\Delta U)$ for different level of vertical load p_Y. Geometrical set up: $a = 2.0$ m, $D = 0.40$ m, $t = 0.1$ m

In the slope analyzed in p.2, the following pile system was introduced: distance $a = 2.0$ m, pile diameter $D = 0.4$ m, pile length $L = 12.0$ m. The latest was chosen to fix the lower part of the pile in rocky layers at about 4m length. Along the whole pile, connectors are set in a distance $e = 0.25$ m, their length $L_C = 0.30$ m. Note, that modeling this situation in ZSoil is highly simplified and fast, as nodes of background 2D FE mesh may not necessarily coincide with nodes of introduced truss elements representing connectors. Otherwise, if any software does not have a possibility to accept truss node located in arbitrary point within continuum elements, 2D FE mesh must be prepared with care, considering the presence of connectors.

Having established geometry, several fast computations have to be done on subsidiary 3D model, with varied constitutive data of soil layers and vertical load p_Y in order to set connectors properties (E_C, f_C), according to Eq. (6) and (7). Assum-

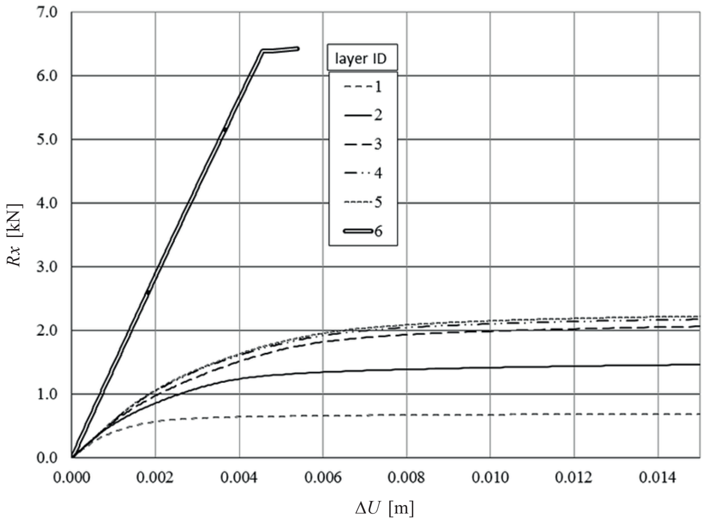

Fig. 10. Reaction $R_X(\Delta U)$ for the connectors. Layers are specified in Table 2

Table 2
Properties of the connectors

Connector layer Id	Soil type, ID in Table 1	Load condition: Depth H [m]	Load condition: Mean v. load p_Y[kPa]	Connector props: E_C [kPa]	Connector props: f_C [kPa]
1	uncontrolled embankment, Ib	2	40	862.3	6.9
2	schist, mudstone, clay, 2	4	80	1280.1	14.7
3	schist, mudstone, clay, 2	6	120	1461.6	20.7
4	breccia (schist + clay), 3	7	140	1582.1	21.8
5	breccia (schist + clay), 3	8	160	1597.0	22.2
6	**mudstone, schist, 4**	**10**	**200**	**4203.9**	**63.9**

ing connectors area per $g = 1$ m (unit thickness of PS model) as $A_C = 1$ m^2, and averaged soil volume weight $\gamma = 20.0$ kN/m^3, their approximate values are given in Table 2, which also gives reference numbers to its position along the pile, used in Fig. 11. Parameters of layer in "stronger" subsoil are typed in bold. In Fig. 10 all characteristics used in the considered example are shown.

4.4. Discussion of the results for stabilized system. First issue concerning the whole system of securing the road against landslide is the result of its stability analysis. As can be seen on the maps $\|\Delta\mathbf{u}\|$, Fig. 12, thanks to the introduced pile system and anchors, the stability factor evaluated for the critical soil parameters, recognized in p.2, given in Table 1 increases to $\mathrm{SF}_1 = 1.47$, from $\mathrm{SF}_0 = 1.00$ obtained for the slope without strengthening, loaded with 20% of required live load at the road. Related sliding surface is moved towards a slope surface beneath the road and does not propagate inside the slope. Thus, a primary goal of securing the road is achieved, although a land-

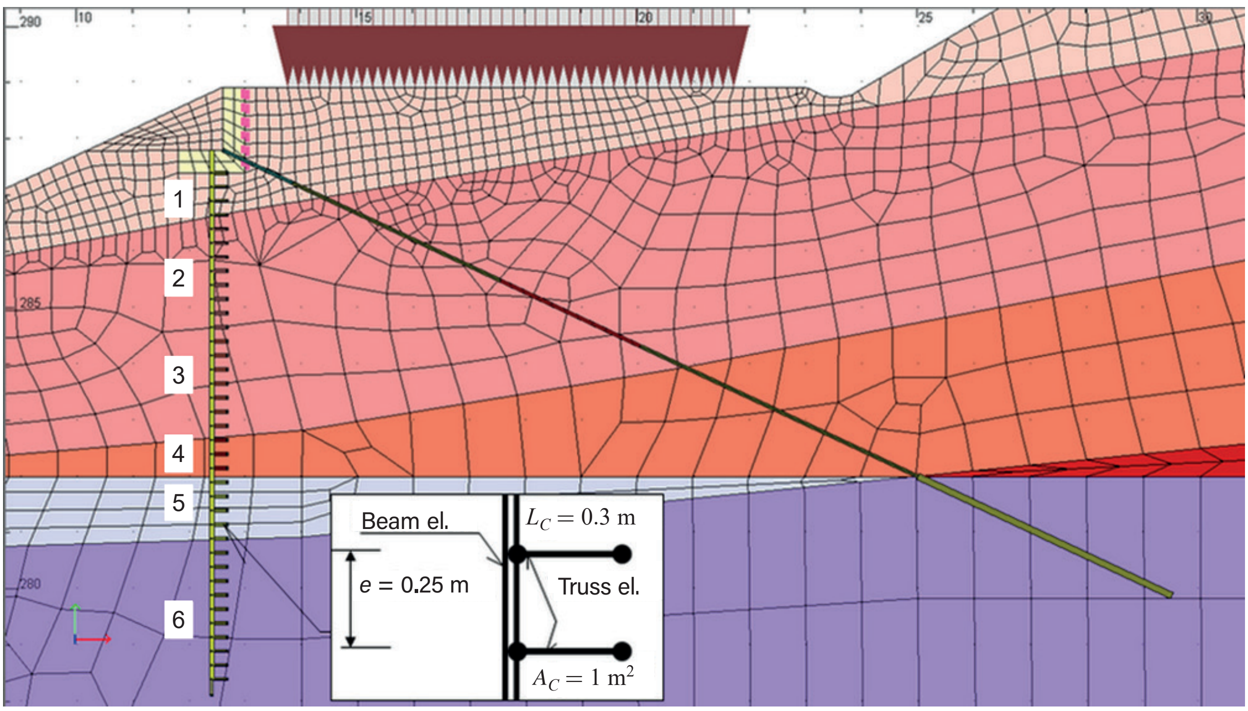

Fig. 11. Pile-connectors system introduced to the plane strain model

a) b)

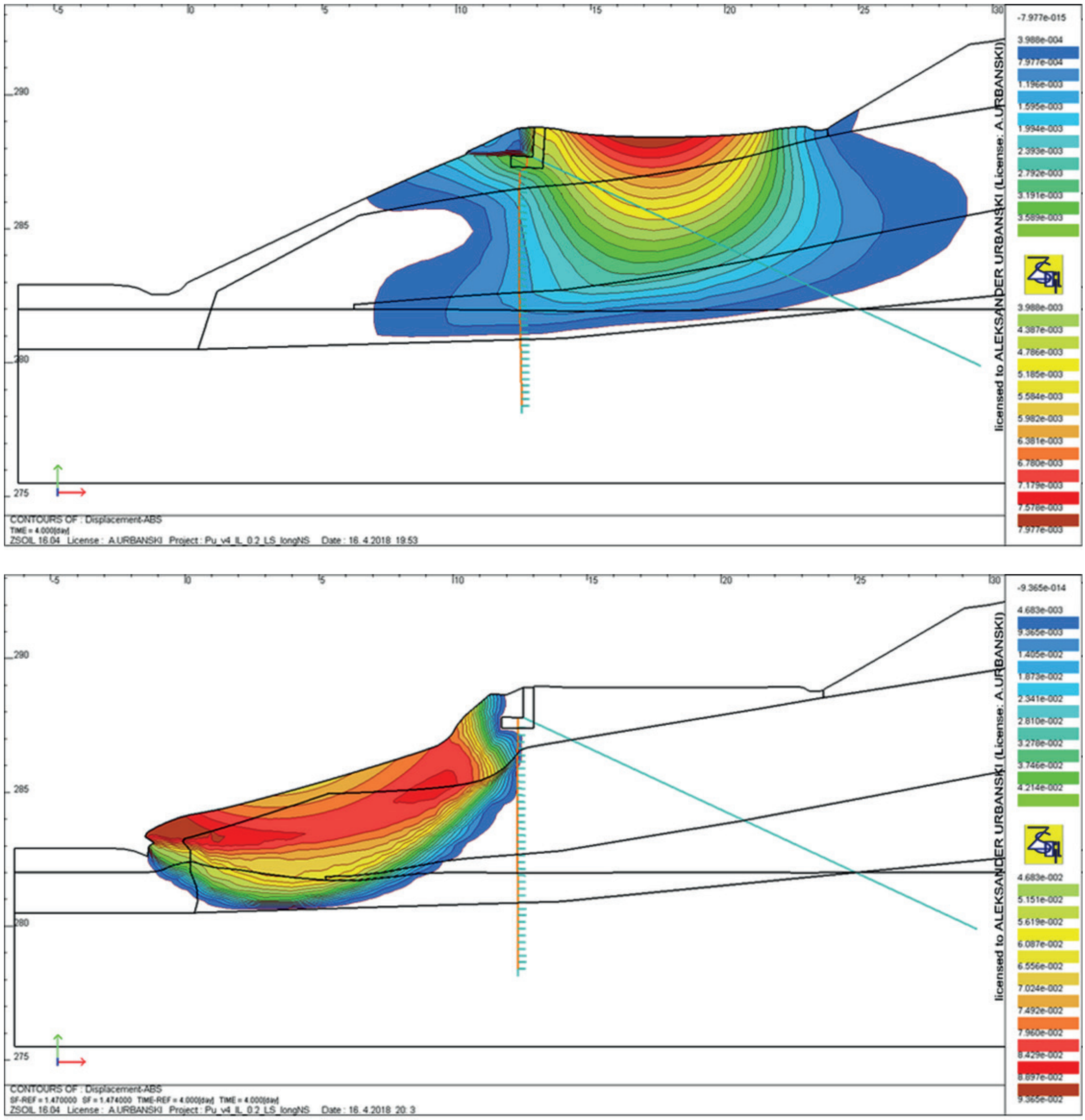

Fig. 12. Strengthened system after introducing pile and anchors. a) deformation accompanying live road load b) failure mode for $SF_1 = 1.47$

slide problem still exists, which now concerns only a slope beneath the road, and, if needed, may be dealt by other means. Moreover, for the strengthened system submitted to live road load q = 25 kPa, prior to c-ϕ reduction algorithm, no signs of stability loss (i.e. localized strain field) are visible.

Another important result concerns the pile itself. In Fig. 12 graphs of bending moments in the pile are given, obtained in different states of c-ϕ reduction algorithm applied to investigate stability of the system. Note, that maximal moments in the pile depend in nonlinear manner on the applied parameters reduction factor SF. This dependency, as well as tensile force in anchors is shown in Fig. 14. The value of SF = 1.25 in c-ϕ reduction algorithm is chosen for presentation of internal forces in structural elements as it exactly corresponds to the material partial safety factor $\gamma_M = 1.25$, which is a value recommended in European design standard (Eurocode 7 [20]), while used to estimate "effects of action". Internal forces for $SF = \gamma_M = 1.25$ are taken to detailed design of piles, anchors, and cap-beam.

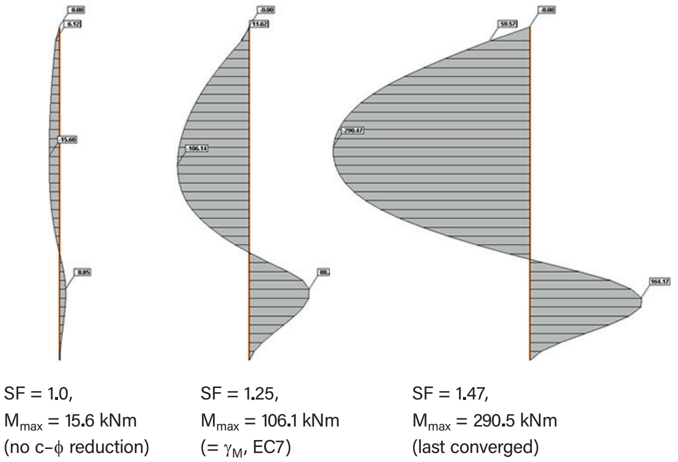

Fig. 13. Bending moments in the pile. Case of road live load 25 kN/m^2: a) without any strength parameter reduction b) with SF = 1.25, c) at last converged state, with SF = 1.47

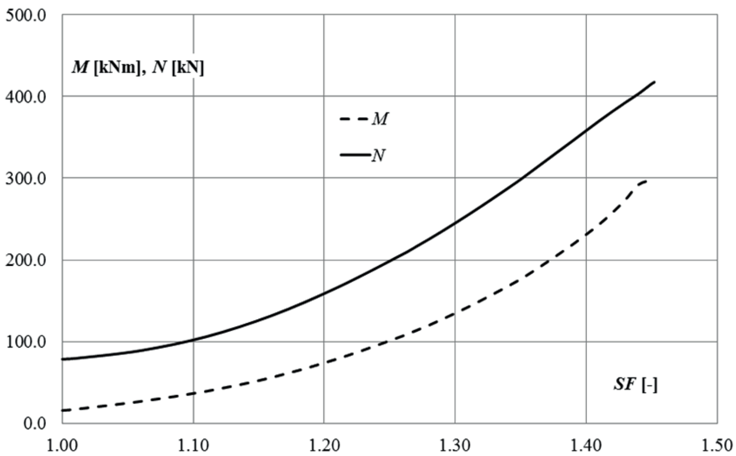

Fig. 14. Maximum of bending moment M in the pile and normal force N in the anchor as functions of SF

Wider discussion on the problem of partial material and load factors is given by Bogusz and Godlewski [21].

Interesting results are the stresses in connectors, shown in Fig. 15. It is a disputable and open question whether during stability analysis by c-ϕ reduction algorithm strength of the connector f_C should be reduced, as it is for the continuum surrounding the pile. If so, interacting forces will reach an ultimate state shown in Fig. 15d), otherwise the majority of connectors will remain in an elastic state.

5. Final remarks

The 2D/3D method, presented in the paper, is a simple computational tool for analysing pile-soil interaction for the case of horizontal loading which is an important factor in the design of different geo-structural systems. It would be particularly useful in practical cases when a periodic system of piles is part

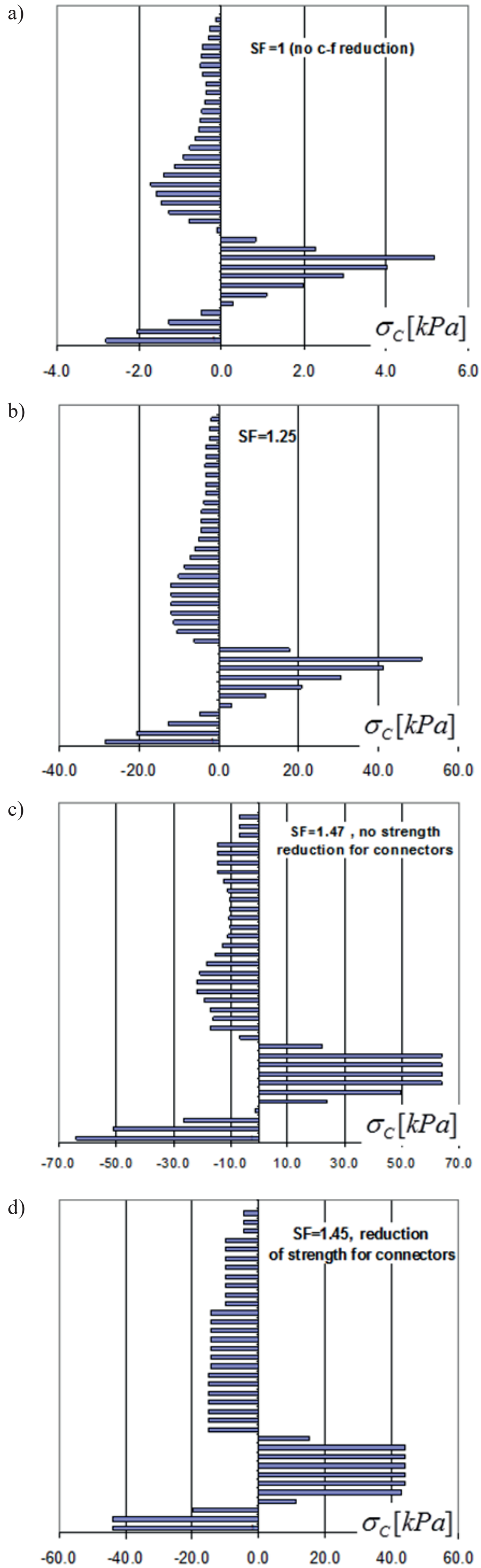

Fig. 15. Stresses in connectors for different strength reduction factors SF

of a larger system (for example a landslide endangered slope) which is analysed by 2D FEM. It allows to consider the local 3D phenomena in the soil surrounding pile, and its consequences on deformation and stability of the whole system.

The key idea of 2D/3D method is to join the pile with the 2D plane strain continuum by fictitious connectors of Winkler type with P-Y properties identified during analysis of subsidiary 3D problem.

During the works described in the paper, ZSoil.PC software system was effectively used, without any customizations. If so, any other FE system specialized towards geomechanics (eg. PLAXIS, DIANA) should be capable to perform described 2D/3D method.

The Authors applied the 2D/3D method in a few designs of securing roads in southern Poland in the last five years. Unfortunately, no monitoring was set on any, mainly due to the lack of investors' interest in it. But all designed structures have withstood heavy rainfalls in the area, in May 2019. Similar episodes, encountered earlier, caused landslides phenomena and gradual deterioration of the roads. Also, economic aspects of construction process were evaluated as satisfactory.

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