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ANALYSIS OF THE PRECISION OF DETERMINATION OF AIRCRAFT COORDINATES USING EGNOS+SDCM SOLUTION

Kamil KRASUSKI[1], Marta LALAK[2], Paweł GOŁDA[3], Adam CIEĆKO[4], Grzegorz GRUNWALD[5], Magda MROZIK[6], Jarosław KOZUBA[7]

[1, 2] Institute of Navigation, Polish Air Force University, Dęblin, Poland

[3] Faculty of Aviation, Polish Air Force University, Dęblin, Poland

[4, 5] Faculty of Geoengineering, University of Warmia and Mazury in Olsztyn, Olsztyn, Poland

[6, 7] Silesian University of Technology, Faculty of Transport and Aviation Engineering, Gliwice, Poland

Abstract:

This paper presents an algorithm for determining the precision parameter for aircraft position coordinates based on a combined GPS/EGNOS and GPS/SDCM solution. The proposed algorithm uses a weighted average model that combines a single GPS/EGNOS and GPS/SDCM position navigation solution to determine the resulting aircraft coordinates. The weighted mean model include the linear coefficients as a function of: the inverse of the number of tracked GPS satellites for which EGNOS and SDCM corrections have been generated, and the inverse of the geometric coefficient of the PDOP (Position Dilution of Precision). The corrections between the single GPS/EGNOS and GPS/SDCM solution to the aircraft's resultant coordinates are then calculated on this basis. Finally, the standard deviation for the aircraft resultant BLh (B-Latitude, L-Longitude, h- ellipsoidal height) coordinates is calculated as a measure of precision. The research experiment used recorded on-board GPS+SBAS data from two GNSS receivers mounted on a Diamond DA 20-C1 aircraft. The test flight was carried out on the Olsztyn-Suwałki-Olsztyn route. The calculations of aircraft position based on GPS/EGNOS and GPS/SDCM solution were performed in the RTKLIB v.2.4.3 program in the RTKPOST module. Next, aircraft resultant coordinates and standard deviations were computed in Scilab v.6.0.0 software package. Based on the tests performed, it was found that for the Trimble Alloy receiver, the standard deviation values for the ellipsoidal coordinates BLh of the aircraft do not exceed 1.77 m. However, for the Septentrio AsterRx2i receiver, the values of standard deviations for the aircraft's ellipsoidal BLh coordinates do not exceed 5.04 m. The use of linear coefficients as the inverse of the number of tracked GPS satellites with SBAS corrections in the GPS/EGNOS+GPS/SDCM positioning model resulted in a reduction in standard deviations of approximately 50-51% relative to the solution with linear coefficients calculated as the inverse of the PDOP parameter. In paper, the standard deviation was also obtained using arithmetic mean model. However the values of standard deviation from weighted mean model are lower than arithmetic mean model.

Keywords: *EGNOS, SDCM, precision, standard deviation, aircraft coordinates*

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Contact:
1) k.krasuski@law.mil.pl [https://orcid.org/0000-0001-9821-4450] – corresponding author; 2) m.lalak@law.mil.pl [https://orcid.org/0000-0001-5485-4720]; 3) p.golda@law.mil.pl [https://orcid.org/0000-0003-4066-7814]; 4) adam.ciecko@uwm.edu.pl [https://orcid.org/0000-0002-3984-0846]; 5) grzegorz.grunwald@uwm.edu.pl [https://orcid.org/0000-0001-9252-7624]; 6) magda.mrozik@polsl.pl [https://orcid.org/0000-0003-4496-8331]; 7) jaroslaw.kozuba@polsl.pl [https://orcid.org/0000-0003-3394-4270]

1. Introduction

In GNSS satellite navigation systems, there are four basic parameters for assessing the performance of the system, which ensure that they operate at a high level, resulting in greater opportunities for their use in various scientific applications. These basic parameters are:
- integrity, which defines the level of confidence in the information provided by the system;
- continuity, which is the ability of the system to operate without interruption;
- availability, which determines the time for which the navigation service is available at any given moment;
- accuracy, which is the difference between obtained aircraft coordinates and reference position of flight (Felski et al., 2011; Kaleta, 2014; ICAO, 2006).

In addition to the above-mentioned parameters for evaluating GNSS performance, one can encounter measurement precision, which is often confused with accuracy. Both parameters are very important in GNSS satellite measurements, especially from a statistical point of view. Measurement accuracy is a measure of how close the measurement results are to the true value or assumed to be true, which results in accuracy indicating the quality or correctness of the results (Uznański, 2012; Figurski, 2007). Measurement precision, on the other hand, is a measure of how consistently the measurement is carried out or determines how repeatable the measurement is, so it de facto refers to the quality of the performance of the instrument or method (Figurski, 2007; Oleniacz & Świętoń, 2018). Various positioning methods are used to perform GNSS satellite measurements to improve precision. The methods differ in the time it takes to obtain a position measurement and in the way the GNSS observations are processed. The precise determination of an object's position is achieved by absolute or differential methods. In addition, code or phase positioning methods can be distinguished here (Oleniacz, 2015).

2. State of the art review of the research area

The SBAS augmentation systems allow to determinate the four GNSS positioning quality parameters in air transport. In addition, SBAS systems are used in APV-I and APV-II approach procedures (Kaleta, 2014; ICAO, 2006) and other air operations (Gołda, 2018; Gołda et al., 2021). This chapter presents examples of research work on aeronautical applications of EGNOS and SDCM systems, which will be used for the research in the presented scientific publication. The majority of research work using EGNOS is concerned with determining the positioning accuracy of aircraft in air navigation (Grzegorzewski, 2005; Jafernik, 2016; Felski & Nowak, 2011; Grunwald et al., 2016). In addition, in the context of airborne application, interesting research is being carried out in the SBAS APV approach procedure using the EGNOS or EGNOS+SDCM solution (Felski et al., 2011; Kaleta, 2014; Krasuski et al., 2022; Fellner, 2018; Fellner et al., 2016; Fellner & Jafernik, 2014; Oliveira & Tiberius, 2008; Secretan et al., 2001; Azoulai et al., 2009; Breeuwer et al., 2000; Fonseca et al., 2006; Veerman & Rosenthal, 2006). The integrity of SBAS positioning in air transport is also a particularly relevant research problem, as discussed in publications (Grunwald et al., 2016, Krasuski et al., 2022; Krzykowska-Piotrowska et al., 2021). Furthermore, when determining SBAS satellite positioning in aviation, studies have used the EGNOS or EGNOS+SDCM solution (Kaleta, 2014; Krasuski et al., 2022; Tabti et al., 2021; Beldjilali et al., 2020; Tabti et al., 2020; Fellner et al., 2010; Fellner et al., 2009; Kaleta, 2015; Fellner, 2014; Fellner et al., 2008; Butzmuehlen et al., 2001; Perrin et al., 2006; Muls & Boon, 2001; Hvezda, 2021; Vassilev & Vassileva, 2012). Research on the use of EGNOS has not only been concerned with flight tests, but with the realisation of static measurements at airports, as presented in papers (Felski et al., 2011; Kaleta, 2014; Jafernik, 2016). In the context of the use of EGNOS or SDCM systems in aviation, their compatibility and operability with other GNSS satellite systems is also important, as shown in works (Januszewski, 2010; Januszewski, 2011; Januszewski, 2012a; Januszewski, 2012b).

3. Research problem

In the state-of-the-art analysis presented in Chapter 2, the following conclusions are drawn:
– in the implementation of the flight experiments, the positioning accuracy from the SBAS solution was determined,
– the testing with SBAS systems was carried out within the SBAS APV approach procedure in accordance with ICAO certification,

- an important parameter tested with SBAS was the integrity of positioning,
- tests using SBAS were also carried out in static GNSS measurements,
- a separate element determined during the studies was the quality of SBAS satellite positioning in aviation,
- compatibility and interoperability with other GNSS navigation systems was a topic that emerged in studies of SBAS application in aviation.

In the context of the implementation of flight experiments using SBAS augmentation systems, the problem of the precision of aircraft position determination has so far not been addressed in many scientific works. This is, of course, related to the fact that only a single SBAS is used in the aircraft position navigation solution. Furthermore, the primary measure of precision is the standard deviation, the calculation of which requires at least 2 measurements, i.e. in practice the use of 2 SBAS. This, in turn, translates into the calculation of the degrees of freedom of the position navigation solution and, with the number of measurements equal to $N = 2$, the degree of freedom is $f = N - 1 = 2 - 1 = 1$. This paper will therefore present an aircraft position navigation solution using EGNOS and SDCM systems, in which the precision of the determination of aircraft coordinates will be calculated. The paper proposes a modified algorithm for determining the precision from the SBAS solution based on the combination of position determination from the GPS/EGNOS+GPS/SDCM model. For this purpose, a linear combination with different linear coefficients was used for the development of the aircraft position model from the GPS/EGNOS+GPS/SDCM solution. The linear coefficients were calculated as a function of the inverse of the geometric coefficient PDOP and the inverse of the number of GPS satellites being tracked and corrected with SBAS (EGNOS and SDCM).

The article is divided into 8 parts: 1) Introduction, 2) State of the art review of the research area, 3) Research problem, 4) Research methodology, 5) Research test, 6) Results of the study, 7) Discussion, 8) Conclusions. The whole publication ends with a rather extensive literature list.

4. Research methoology

In the presented computational strategy, a combined GPS/EGNOS+GPS/SDCM navigation solution based on a weighted average model is used. The EGNOS solution is understood to be a solution using GPS observations with EGNOS corrections in SPP positioning mode. An SDCM solution, on the other hand, is a solution using GPS observations with SDCM corrections in SPP positioning mode. On this basis, the aircraft position model will be determined. In a general description, the aircraft position model takes the form:

$$\begin{cases} B_m = \frac{\sum A \cdot B_j}{\sum P_j} \\ L_m = \frac{\sum A_j \cdot L_j}{\sum P_j} \\ h_m = \frac{\sum A_j \cdot h_j}{\sum P_j} \end{cases}, \tag{1}$$

where: j – EGNOS or SDCM solution index; B_j – geodetic latitude from a single SBAS solution; L_j – geodetic longitude from a single SBAS solution; h_j – ellipsoidal height from a single SBAS solution; A_j – linear coefficients for the EGNOS or SDCM solution, allowing a linear combination to be created for the combined GPS/EGNOS+GPS/SDCM solution.

Taking into account the EGNOS and SDCM navigation solution in formula (1), the final result is:

$$\begin{cases} B_m = \frac{A_e \cdot B_{EGNOS} + A_s \cdot B_{SDCM}}{A_e + A_s} \\ L_m = \frac{A_e \cdot L_{EGNOS} + A_s \cdot L_{SDCM}}{A_e + A_s} \\ h_m = \frac{A_e \cdot h_{EGNOS} + A_s \cdot h_{SDCM}}{A_e + A_s} \end{cases}, \tag{2}$$

where: B_{EGNOS} – geodetic latitude from the EGNOS solution; B_{SDCM} – geodetic latitude from the SDCM solution; L_{EGNOS} – geodetic longitude from the EGNOS solution; L_{SDCM} – geodetic longitude from the SDCM solution; h_{EGNOS} – ellipsoidal height from the EGNOS solution; h_{SDCM} – ellipsoidal height from the SDCM solution; A_e – linear coefficient from the EGNOS solution; A_s – linear coefficient from the SDCM solution.

Equation (2) defines a weighted average model that ties a single SBAS solution (EGNOS and SDCM). Equation (2) takes into account the linear coefficients that have been determined as a function of:

– the inverse of the number of tracked GPS satellites for which EGNOS and SDCM corrections have been generated,
– the inverse of the geometric coefficient of the PDOP.

The linear coefficients are shown in equations (3) and (4):

$$A_e = \frac{1}{N_e} \text{ and } A_s = \frac{1}{N_s}, \tag{3}$$

$$A_e = \frac{1}{PDOP_e} \text{ and } A_s = \frac{1}{PDOP_s}, \tag{4}$$

where: N_e - the number of GPS satellites with EGNOS corrections; N_s - the number of GPS satellites with SDCM corrections; $PDOP_e$ – PDOP parameter from EGNOS solution; $PDOP_s$ - PDOP parameter from SDCM solution.

Taking into account the linear coefficients from equations (3) and (4) we finally obtain:

$$\begin{cases} B_m = \dfrac{\frac{1}{N_e} \cdot B_{EGNOS} + \frac{1}{N_s} \cdot B_{SDCM}}{\frac{1}{N_e} + \frac{1}{N_s}} \\ L_m = \dfrac{\frac{1}{N_e} \cdot L_{EGNOS} + \frac{1}{N_s} \cdot L_{SDCM}}{\frac{1}{N_e} + \frac{1}{N_s}} \\ h_m = \dfrac{\frac{1}{N_e} \cdot h_{EGNOS} + \frac{1}{N_s} \cdot h_{SDCM}}{\frac{1}{N_e} + \frac{1}{N_s}} \end{cases}, \tag{5}$$

$$\begin{cases} B_m = \dfrac{\frac{1}{PDOP_e} \cdot B_{EGNOS} + \frac{1}{PDOP_s} \cdot B_{SDCM}}{\frac{1}{PDOP_e} + \frac{1}{PDOP_s}} \\ L_m = \dfrac{\frac{1}{PDOP_e} \cdot L_{EGNOS} + \frac{1}{PDOP_s} \cdot L_{SDCM}}{\frac{1}{PDOP_e} + \frac{1}{PDOP_s}} \\ h_m = \dfrac{\frac{1}{PDOP_e} \cdot h_{EGNOS} + \frac{1}{PDOP_s} \cdot h_{SDCM}}{\frac{1}{PDOP_e} + \frac{1}{PDOP_s}} \end{cases}, \tag{6}$$

The standard deviation, a measure of precision in the presented mathematical algorithm, was also calculated for the determined aircraft position:

$$\begin{cases} \delta B = \sqrt{\dfrac{[v_{B,EGNOS}^T \cdot A_e \cdot v_{B,EGNOS} + v_{B,SDCM}^T \cdot A_s \cdot v_{B,SDCM}]}{N-1}} \\ \delta L = \sqrt{\dfrac{[v_{L,EGNOS}^T \cdot A_e \cdot v_{L,EGNOS} + v_{L,SDCM}^T \cdot A_s \cdot v_{L,SDCM}]}{N-1}} \\ \delta h = \sqrt{\dfrac{[v_{h,EGNOS}^T \cdot A_e \cdot v_{h,EGNOS} + v_{h,SDCM}^T \cdot A_s \cdot v_{h,SDCM}]}{N-1}} \end{cases}, , \tag{7}$$

where: v_B – corrections along the B axi;$v_{B,EGNOS} = B_{EGNOS} - B_m$; $v_{B,SDCM} = B_{SDCM} - B_m$; v_L - corrections along the L axis; $v_{L,EGNOS} = L_{EGNOS} - L_m$; $v_{L,SDCM} = L_{SDCM} - L_m$; v_h - corrections along the h axis; $v_{h,EGNOS} = h_{EGNOS} - h_m$; $v_{h,SDCM} = h_{SDCM} - h_m$; N – number of measurements, represents the number of solutions for a single aircraft position using EGNOS and SDCM systems, $N = 2$.

Taking into account the number of measurements N=2, equation (7) becomes:

$$\begin{cases} \delta B = v_{B,EGNOS}^T \cdot A_e \cdot v_{B,EGNOS} \\ \qquad + v_{B,SDCM}^T \cdot A_s \cdot v_{B,SDCM} \\ \delta L = v_{L,EGNOS}^T \cdot A_e \cdot v_{L,EGNOS} \\ \qquad + v_{L,SDCM}^T \cdot A_s \cdot v_{L,SDCM} \\ \delta h = v_{h,EGNOS}^T \cdot A_e \cdot v_{h,EGNOS} \\ \qquad + v_{h,SDCM}^T \cdot A_s \cdot v_{h,SDCM} \end{cases}, \tag{8}$$

In the example analysed, the number of measurements represents the number of SBAS navigation solutions, hence $N = 2$. Therefore, the number of degrees of freedom of the proposed GPS/EGNOS+GPS/SDCM solution is $f = N - 1 = 1$. It can therefore be concluded that the determined coordinates of the aircraft from the GPS/EGNOS+GPS/SDCM solution are determined at the level of one degree of freedom.

5. Research test

The main objective of the research test was to determine the precision value of the aircraft's coordinates. Therefore, the first step was to register raw GNSS satellite data in the form of GPS observations and navigation data, as well as EGNOS and SDCM corrections. Code observations and GPS navigation data in RINEX format were recorded by two GNSS receivers placed on a Diamond DA 20-C1 aircraft, which was performing a test flight between Olsztyn and Suwałki in north-eastern Poland (Krasuski et al., 2022). Septentrio AsterRx2i and Trimble Alloy geodetic receivers were placed on board. In turn, EGNOS and SDCM corrections in EMS format were also registered in real time. The GPS satellite data in RINEX format and SBAS corrections were used to calculate the aircraft position twice, i.e. separately using the EGNOS corrections and SDCM solution. The final position of the aircraft was obtained in BLh ellipsoidal coordinates (B-geodetic latitude, L-geodetic longitude, h- ellipsoidal altitude). The calculations at this stage were performed in the RTKLIB v.2.4.3 program in the RTKPOST module (RTKPOST Website, 2023). After the implementation of the calculations in the RTKLIB program, the implementation of the algorithm (1-8) was proceeded to determine the precision of the GPS/EGNOS+GPS/SDCM solution. For this purpose, the Scilab v.6.0.0 software (SCILAB Website, 2023) was used, in which an author's script was written to implement the calculations according to the scheme of mathematical equations (1-8). An analysis of the obtained precision results from the proposed GPS/EGNOS+GPS/SDCM solution are presented in Chapter 6.

6. Results of the study

The presentation of the test results began by showing the precision of the determination of aircraft coordinates from the GPS/EGNOS+GPS/SDCM solution for the Trimble Alloy receiver. Figure 1 shows the results of the standard deviations of the aircraft coordinates for the Trimble Alloy receiver when considering the values of the linear coefficients equal to $(A_e = \frac{1}{N_e},\ A_s = \frac{1}{N_s})$ in the GPS/EGNOS+ GPS/SDCM positioning model. The values of the standard deviations along the B-axis are up to 0.53 m, along the L-axis up to 0.71 m, and along the h-axis up to 0.97 m.

Figure 2 shows the results of the standard deviations of the aircraft coordinates for the Trimble Alloy receiver when considering the values of the linear coefficients equal to $(A_e = \frac{1}{PDOP_e},\ A_s = \frac{1}{PDOP_s})$ in the GPS/EGNOS+GPS/SDCM positioning model. The standard deviation values along the B-axis are up to 0.89 m, along the L-axis up to 1.60 m and along the h-axis up to 1.77 m. Comparing the results in Fig. 1 and Fig. 2, it can be seen that the application of linear coefficients $(A_e = \frac{1}{N_e},\ A_s = \frac{1}{N_s})$ in the GPS/EGNOS+GPS/SDCM positioning model resulted in a reduction of standard deviations of about 50-51% compared to the solution with linear coefficients of $(A_e = \frac{1}{PDOP_e},\ A_s = \frac{1}{PDOP_s})$.

Figures 3 and 4 present the results of the standard deviations $(\delta B, \delta L, \delta h)$ for the Septentrio AsterRx2i receiver to show the repeatability of the proposed test method for the mathematical equations (1-8).

Fig. 3 shows the GPS/EGNOS+GPS/SDCM positioning precision results for the Septentrio AsterRx2i receiver considering linear coefficients equal to $(A_e = \frac{1}{N_e}, A_s = \frac{1}{N_s})$. The values of the standard deviations are respectively: for the B coordinate up to 5.01 m, for the L coordinate up to 3.64 m, for the h coordinate up to 5.04 m. It is worth noting that there are high precision values in the initial phase of the flight. This is due to the small number of tracked GPS satellites with SBAS corrections. In this case, the number of tracked GPS satellites by the Septentrio AsterRx2i receiver was equal to 4 or 5 in this phase of the flight. In the later phase of the flight, the number of tracked GPS satellites was considerably higher, which translated into precision results of up to 0.85 m maximum along the BLh coordinate axes.

Fig. 4 shows the GPS/EGNOS+GPS/SDCM positioning precision results for the Septentrio AsterRx2i receiver with linear coefficients equal to $(A_e = \frac{1}{PDOP_e}, A_s = \frac{1}{PDOP_s})$. The values of the standard deviations are respectively: for coordinate B up to 4.23 m, for coordinate L up to 3.08 m, for coordinate h up to 4.26 m. As in Fig. 3, if the first observation epochs were omitted, then the precision values would be up to 1.60 m maximum for the BLh coordinate. Comparing the results in Fig. 3 and Fig. 4 it can be seen that the use of linear coefficients $(A_e = \frac{1}{N_e},\ A_s = \frac{1}{N_s})$ in the GPS/EGNOS+GPS/SDCM positioning model resulted in a reduction of standard deviations of about 50-51% compared to the solution with linear coefficients of $(A_e = \frac{1}{PDOP_e},\ A_s = \frac{1}{PDOP_s})$).

7. Discussion

The discussion of the results obtained is divided into 3 parts. The first part shows the impact of the obtained results from the weighted average model in relation to the arithmetic average model. The second part of the discussion shows the analysis of the obtained results in relation to the guidance in 3D space. Therefore, the resultant standard deviation values in 3D space will be shown. The last strand of the discussion will concern the analysis of the obtained research results in the context of the existing state of knowledge of the research problem undertaken.

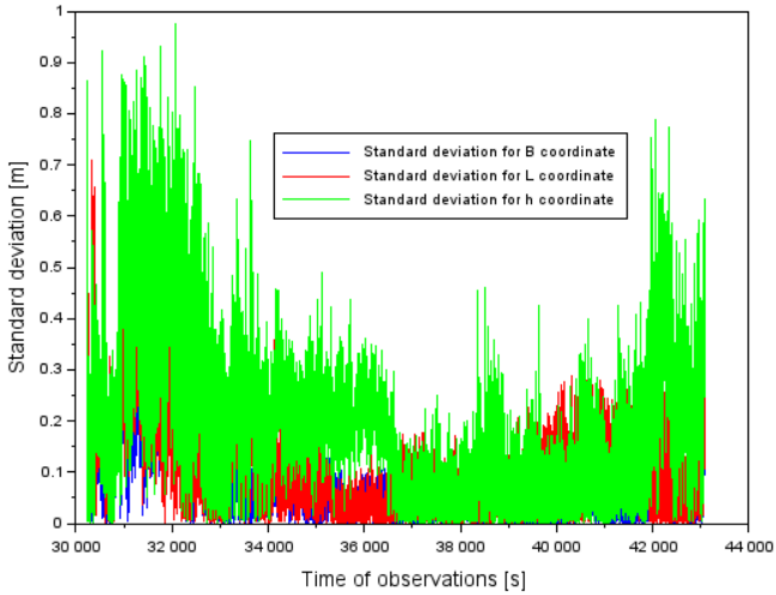

Fig. 1. Standard deviations of aircraft coordinates for the Trimble Alloy receiver in the GPS/EGNOS+ GPS/SDCM positioning model when linear coefficients ($A_e = \frac{1}{N_e}, A_s = \frac{1}{N_s}$) are taken into consideration

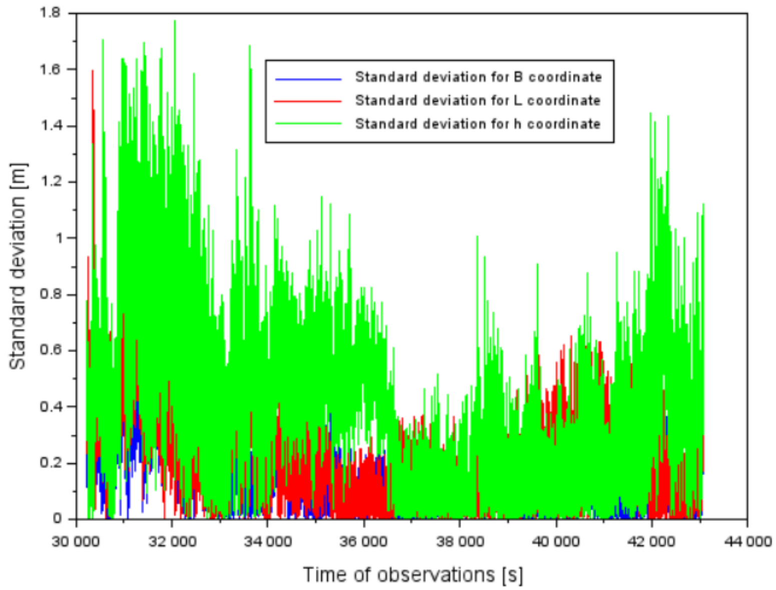

Fig. 2. Standard deviations of aircraft coordinates for the Trimble Alloy receiver in the GPS/EGNOS+ GPS/SDCM positioning model when linear coefficients ($A_e = \frac{1}{PDOP_e}, A_s = \frac{1}{PDOP_s}$) are taken into consideration

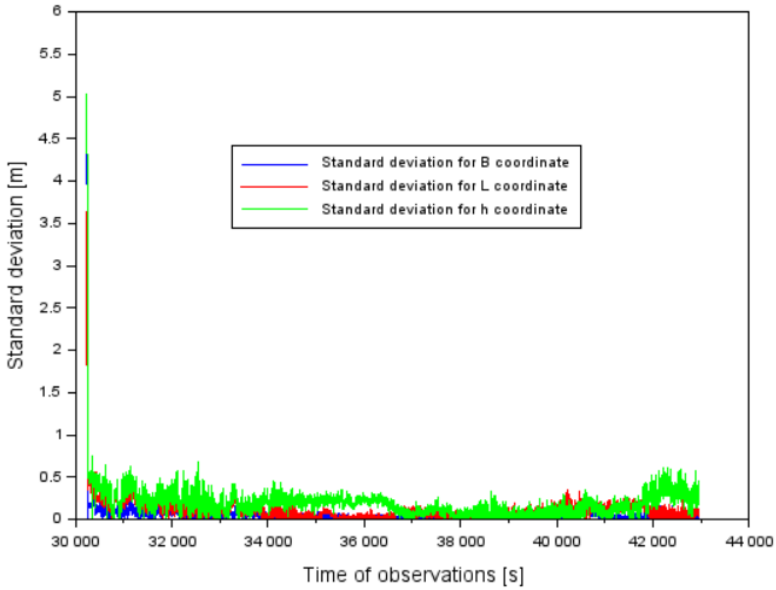

Fig. 3. Standard deviations of aircraft coordinates for the Septentrio AsterRx2i receiver in the GPS/EGNOS+GPS/SDCM positioning model when linear coefficients ($A_e = \frac{1}{N_e}, A_s = \frac{1}{N_s}$) are taken into consideration

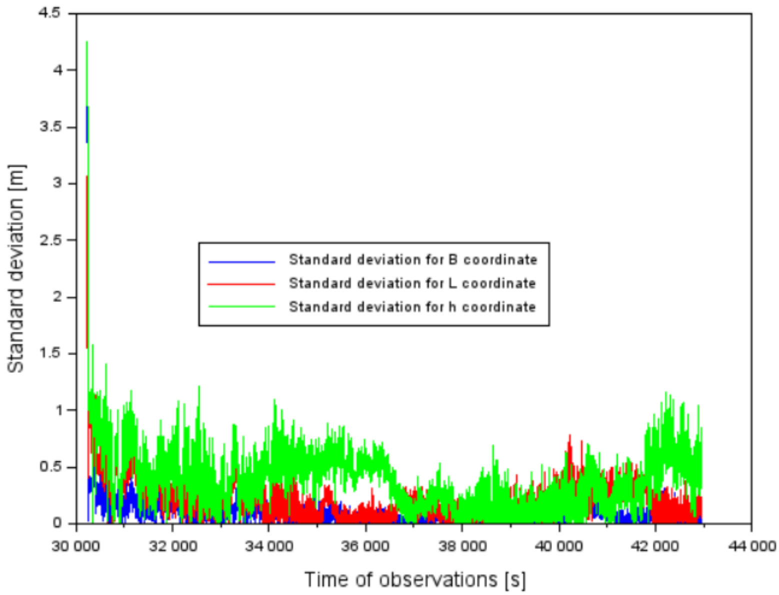

Fig. 4. Standard deviations of aircraft coordinates for the Septentrio AsterRx2i receiver in the GPS/EGNOS+GPS/SDCM positioning model when linear coefficients ($A_e = \frac{1}{PDOP_e}, A_s = \frac{1}{PDOP_s}$)) are taken into consideration

The first part of the discussion concerns the comparison of the obtained research results from the mathematical equations (1-8) with the GPS/EGNOS+GPS/SDCM solution for the arithmetic mean model. The solution of GPS/EGNOS+GPS/SDCM for the arithmetic mean model is described by the mathematical equations (9-11) as written below:
– linear coefficients:

$$A_e = 1 \ \text{ i } \ A_s = 1, \tag{9}$$

– aircraft BLh coordinate values from the GPS/EGNOS+GPS/SDCM solution for the arithmetic mean model:

$$\begin{cases} B_m = \frac{B_{EGNOS}+B_{SDCM}}{2} \\ L_m = \frac{L_{EGNOS}+L_{SDCM}}{2} \\ h_m = \frac{h_{EGNOS}+h_{SDCM}}{2} \end{cases}, \tag{10}$$

– standard deviation values as a measure of the precision of the determination of the aircraft BLh coordinates from the GPS/EGNOS+GPS/SDCM solution for the arithmetic mean model:

$$\begin{cases} \delta B = v_{B,EGNOS}^T \cdot v_{B,EGNOS} + v_{B,SDCM}^T \cdot v_{B,SDCM} \\ \delta L = v_{L,EGNOS}^T \cdot v_{L,EGNOS} + v_{L,SDCM}^T \cdot v_{L,SDCM} \cdot \\ \delta h = v_{h,EGNOS}^T \cdot v_{h,EGNOS} + v_{h,SDCM}^T \cdot v_{h,SDCM} \end{cases}, \tag{11}$$

Table 2 compares the determined precision measures for the equations from the weighted average model (1-8) and the equations from the arithmetic average model (9-11). In the case of the Trimble Alloy receiver, it can be said that:
– the use of linear coefficients ($A_e = \frac{1}{N_e}$, $A_s = \frac{1}{N_s}$) in the GPS/EGNOS+GPS/SDCM positioning model resulted in a reduction of standard deviations of approximately 63-67% compared to the GPS/EGNOS+GPS/SDCM solution for the arithmetic mean model,
– the use of linear coefficients ($A_e = \frac{1}{PDOP_e}, A_s = \frac{1}{PDOP_s}$) in the GPS/EGNOS+GPS/SDCM positioning model resulted in a reduction in standard deviations of approximately 27-30% relative to the GPS/EGNOS+GPS/SDCM solution for the arithmetic mean model.
– In the case of the Septentrio AsterRx2i receiver, it can be observed that:
– the use of linear coefficients ($A_e = \frac{1}{N_e}$, $A_s = \frac{1}{N_s}$) in the GPS/EGNOS+GPS/SDCM positioning model resulted in a reduction of standard deviations of approximately 61-66% compared to the GPS/EGNOS+GPS/SDCM solution for the arithmetic mean model,
– the use of linear coefficients ($A_e = \frac{1}{PDOP_e}, A_s = \frac{1}{PDOP_s}$) in the GPS/EGNOS+GPS/SDCM positioning model resulted in a reduction of standard deviations of approximately 28-31% compared to the GPS/EGNOS+GPS/SDCM solution for the arithmetic mean model.

In summary, the effectiveness of the proposed weighted average model compared to the arithmetic average model for the GPS/EGNOS+GPS/SDCM solution can be seen in Table 1.
The second part of the discussion shows the results of tests relating to the use of the precision values obtained to determine the resultant standard deviations ($\delta B, \delta L, \delta h$) as given below:

$$M = \sqrt{\delta B^2 + \delta L^2 + \delta h^2}, \tag{12}$$

The parameter M defines the resultant standard deviation values of the determined position of the aircraft in 3D space. The M parameter should be regarded as the resultant measure of precision for the determined BLh coordinates for 3D navigation. Fig. 5 shows the results of determining the M parameter for the Trimble Alloy receiver, taking into account all linear coefficients ($A_e = \frac{1}{N_e}, A_s = \frac{1}{N_s}, A_e = \frac{1}{PDOP_e}, A_s = \frac{1}{PDOP_s}$).
The values of the M parameter when using linear coefficients ($A_e = \frac{1}{N_e}, A_s = \frac{1}{N_s}$) in the GPS/EGNOS+GPS/SDCM positioning model are up to 1.14 m maximum. On the other hand, the values of M parameter when using ($A_e = \frac{1}{PDOP_e}, A_s = \frac{1}{PDOP_s}$) linear coefficients in GPS/EGNOS+GPS/SDCM positioning model are up to 2.09 m maximum. Based on

the comparative analysis, it can be said that the application of linear coefficients ($A_e = \frac{1}{N_e},\ A_s = \frac{1}{N_s}$) in the GPS/EGNOS+GPS/SDCM positioning model resulted in a 56% reduction of the resultant standard deviations compared to the solution with linear coefficients ($A_e = \frac{1}{PDOP_e},\ A_s = \frac{1}{PDOP_s}$).

Figure 6 shows the results of the M parameter determination for the Septentrio AsterRx2i receiver considering all applied linear coefficients ($A_e = \frac{1}{N_e}, A_s = \frac{1}{N_s}, A_e = \frac{1}{PDOP_e}, A_s = \frac{1}{PDOP_s}$). The values of the M parameter when using linear coefficients ($A_e = \frac{1}{N_e}, A_s = \frac{1}{N_s}$) in the GPS/EGNOS+ GPS/SDCM positioning model are up to 7.99 m maximum. On the other hand, the values of M parameter when using ($A_e = \frac{1}{PDOP_e}, A_s = \frac{1}{PDOP_s}$) linear coefficients in GPS/EGNOS+GPS/SDCM positioning model are up to 6.75 m maximum. Based on the comparative analysis, it can be said that the application of linear coefficients ($A_e = \frac{1}{N_e},\ A_s = \frac{1}{N_s}$) in the GPS/EGNOS+GPS/SDCM positioning model resulted in a reduction of the resultant standard deviations by 52% relative to the solution with linear coefficients ($A_e = \frac{1}{PDOP_e},\ A_s = \frac{1}{PDOP_s}$).

The final stage of the discussion concerns the comparison of the obtained research results in the context of the existing state of knowledge. By comparing the obtained precision results for the applied test method in relation to the analysis of the state of the art, it can be concluded that:

– in the case of the GPS/EGNOS+GPS/SDCM solution, it is possible to determine the precision of the determination of the aircraft coordinates, similarly to what was done in the works (Krasuski et al., 2022; Krasuski et al., 2021),
– in this paper, two GNSS receivers were used to determine the precision of determining the coordinates of the aircraft, similarly to the work (Grunwald et al., 2016),
– the obtained values of standard deviations from the weighted average model are lower than those from the arithmetic average model similarly to the work (Krasuski et al., 2021),
– in the calculation of the precision, correction data from EGNOS and SDCM satellites were used, similarly as in papers (Krasuski et al., 2022; Krasuski et al., 2021).

8. Conclusions

This paper shows the results of a study on determining the positioning precision of GPS/EGNOS+GPS/SDCM in airborne navigation. The paper proposes a weighted average model to determine the ellipsoidal coordinates of the aircraft BLh from the EGNOS and SDCM solution, and then calculates the positioning precision. The algorithm uses linear coefficients as a function of the inverse of the PDOP parameter and the inverse of the number of GPS satellites tracked with SBAS corrections. The developed algorithm was tested for GPS data recorded by Trimble Alloy and Septentrio AsterRx2i geodetic receivers, during a flight test carried out with a Diamond DA 20-C1 aircraft in north-eastern Poland. Navigation calculations were carried out in RTKLIB and Scilab software. The calculations used GPS observation and navigation data as well as corrections from EGNOS and SDCM augmentation systems. On the basis of the tests performed, it was found that for the Trimble Alloy receiver, the standard deviation values for the ellipsoidal coordinates BLh of the aircraft do not exceed 1.77 m. On the other hand, for the Septentrio AsterRx2i receiver, the standard deviation values for the aircraft's ellipsoidal BLh coordinates do not exceed 5.04 m. The use of linear coefficients as the inverse of the number of tracked GPS satellites with SBAS corrections in the GPS/EGNOS+GPS/SDCM positioning model resulted in a reduction in standard deviations of approximately 50-51% relative to the solution with linear coefficients calculated as the inverse of the PDOP parameter. The mathematical algorithm used in the study can also be applied to other SBAS augmentation systems in air navigation, in particular the Indian GAGAN system, which can be used in Poland.

Table 1. Comparison of computed measures of precision from the weighted average model and the arithmetic average model

GNSS receiver	GPS/EGNOS+GPS/SDCM solution model with linear coefficients ($A_e = \frac{1}{N_e}, A_s = \frac{1}{N_s}$)	GPS/EGNOS+GPS/SDCM solution model with linear coefficients ($A_e = \frac{1}{PDOP_e}, A_s = \frac{1}{PDOP_s}$)	GPS/EGNOS+GPS/SDCM solution model with linear coefficients ($A_e = 1, A_s = 1$)
Trimble Alloy	The mean value of the standard deviations equals respectively: δB=0.08 m, δL=0.11 m, δh=0.20 m.	The mean value of the standard deviations equals respectively: δB=0.16 m, δL=0.23 m, δh=0.41 m.	The mean value of the standard deviations equals respectively: δB=0.22 m, δL=0.33 m, δh=0.57 m.
Septentrio AsterRx2i	The mean value of the standard deviations equals respectively: δB=0.05 m, δL=0.10 m, δh=0.19 m.	The mean value of the standard deviations equals respectively: δB=0.09 m, δL=0.20 m, δh=0.39 m.	The mean value of the standard deviations equals respectively: δB=0.13 m, δL=0.29 m, δh=0.54 m.

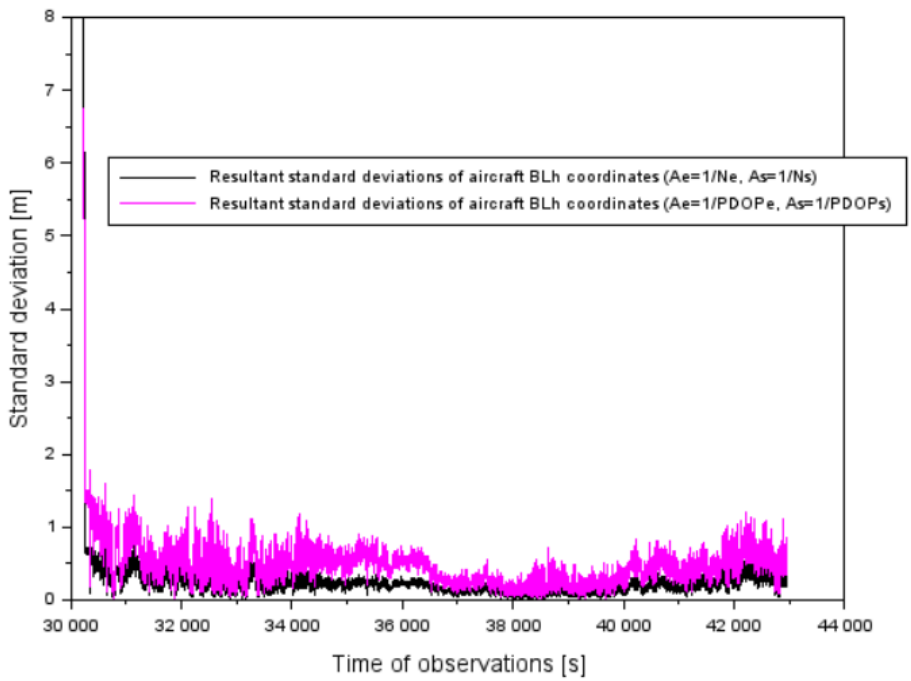

Fig. 6. Resultant standard deviations of aircraft coordinates for the Septentrio AsterRx2i receiver in the GPS/EGNOS+GPS/SDCM positioning model when linear coefficients ($A_e = \frac{1}{N_e}, A_s = \frac{1}{N_s}, A_e = \frac{1}{PDOP_e}, A_s = \frac{1}{PDOP_s}$) are taken into consideararation

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