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A MATHEMATICAL STUDY ON NON-LINEAR TEMPERATURE-DEPENDENT THERMAL CONDUCTIVITY

Vembu Ananthaswamy, Sathyamoorthy Sivasankari

Research Centre and PG Department of Mathematics, The Madura College (Affiliated to Madurai Kamaraj University), Madurai, Tamil Nadu, India
ananthu9777@gmail.com, sivasankari561997@gmail.com

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Abstract. The two main cases of convective fins with temperature-dependent thermal conductivity and heat generation are studied using the Ananthaswamy-Sivasankari method (ASM) approximately. The semi-analytical expressions for the temperature distribution in two cases are given in their clearest and most simple form. The obtained results are then compared with the numerical results, which show significant agreement. The dimensionless fin efficiency is also evaluated. The impacts of the numerous parameters involved in the problem are displayed graphically. With this approach, convergence can be attained more quickly. The application of this method to higher-order problems with boundary values in the applied sciences is possible.

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1. Introduction

Nowadays, many industrial uses, including air conditioning, refrigeration, chemical processing equipment, automobiles, and electrical chips, make extensive use of fins. An extensive analysis of the fins that accelerate heat transmission was provided by Kraus et al. [1]. Despite the fact that there are many different fin designs, the rectangular fin is the most popular one, presumably because of how straightforward its design is and how simple it is to produce. The thermal conductivity was typically assumed to be constant for problems involving ordinary fins, but when there is a significant difference in temperature between the fin's tip and base, the response of temperature upon thermal conductivity should be taken into account. By taking into account the circular fins of the arbitrary thermal source distribution and even a non-linear temperature-dependent heat diffusivity, Razani and Ahmadi [2] were able to determine the ideal fin design. Unal [3] performed a detailed investigation of a rectangular, longitudinal fin having temperature-dependent internal thermal dissipation and a temperature-dependent heat transfer coefficient. Shouman [4] made yet another investigation into this problem (internal heat formation and temperature-dependent heat capacity [5] of a convective fin).

Analytical approaches like HAM [6, 7], HPM [8], VIM, and ADM are crucial for addressing non-linear difficulties, especially in applied sciences. Aziz and Khani [9] used HAM to resolve the heat transfer problem in a rapidly moving fin with varying heat conductivity that is losing heat to its surroundings simultaneously through radiation and convection. In order to assess the effectiveness of the fin efficiency and temperature distribution, Domairry and Fazeli [10] adopted the Homotopy analysis technique to resolve the non-linear differential equation of the straight fin. Moreover, Ganji et al. [11] applied HPM to investigate the temperature gradient of annular fins featuring temperature-dependent thermal conductivity. Analytical techniques were utilized by Hatami and Ganji [12] and Hatami et al. [13] to analyze the heat transfer across porous fins of various geometries. A straight fin featuring temperature-dependent thermal conductivity was evaluated for effectiveness by Inc [14], employing HAM. Ganji and Dogonchi [15] utilized DTM to handle the heat transfer issue in a rectangular fin containing temperature-dependent thermal conductivity, heat transfer coefficient, and heat generation. He reported that the temperature profile is caused by an increase in the thermo-geometric parameter. On the premise of local thermal non-equilibrium, Buonomo et al. [16] tackled a reduced thermal analysis of a porous fin having a finite length and an adiabatic tip. The temperature profile in such a straight, rectangular convective-radiative fin exhibiting temperature-dependent heat capacity was solved simply and precisely using the Double Optimal Linearization Method (DOL) by Bouaziz and Aziz [17].

The efficacy of a triangular fin that is completely wet with dehumidifying conditions suiting a cubic polynomial connection between a particular humidity and the related optimum temperature was analytically examined by Kundu et al. [6] with the aid of the differential transform method. A problem involving the thermal evaluation and improvement of longitudinal as well as pin fins subjected to entirely wet [18], partially wet, and completely dry surface conditions was treated by Kundu [7]. Moradi [19] evaluated the DTM on the straight rectangular fin's thermal properties with all forms of heat transport (radiation and convection) but also compared the outcomes with the fourth-order Runge-Kutta method by employing the shooting method.

The convective-conductive-radiative transfer of heat process via an annular fin is examined by Kumar et al. [20]. In order to assess the heat transport in a porous radial fin under the influence of an inclined magnetic field, Kumar et al. [21] looked into the execution of probabilists' Hermite collocation approach and regression technique. With the help of DTM and the modified residual power series method (MRPSM), Sowmya et al. [22] address the temperature feature of a convective-radiative rectangular patterned annular fin regarding the presence of a magnetic field. Sowmya et al. [23] adopted the Sumudu transform technique with Pade approximant (STM-PA) and the differential transform approach with Pade approximation (DTM-PA)

[24] to explore the thermal behavior of a longitudinal fin exposed to a magnetic field. An analysis of temperature transmission via a longitudinal rectangular fin with linear and exponential temperature-dependent thermal conductivity was described by Kumar et al. [25].

When examining heat transfer properties and fin efficiency, temperature-dependent thermal conductivity is essential. All of the aforementioned studies make the assumption that thermal conductivity is either a constant or an exponential function of temperature, or linearly temperature-dependent [25]. Furthermore, the temperature distribution can be derived by a lengthy computation process using a variety of analytical and numerical techniques that have been used in numerous investigations [20-26]. Thus, the aim of this work is to offer a simple calculation-based solution for the temperature distribution of a longitudinal fin in both linear and non-linear cases and to demonstrate the effectiveness of the technique.

In the present research, we examine two cases of non-linear convective fins with heat generation. The first is thermal conductivity that is constant, while the second is thermal conductivity that varies with temperature. We use ASM to obtain the semi-analytical solution for these two cases. The outcomes are finally compared regarding the results of the numerical solution, and a good agreement is observed. The fin efficiency for both cases is computed and graphically depicted. Furthermore, the outcomes are used to come up with conclusions about the advantages of the suggested approach.

2. Mathematical description of the problem

Now, let us look more closely at a longitudinal fin only with a steady rectangular profile, cross-section area *A*, length *L*, perimeter *P*, thermal conductivity *k* and heat generation q^* specified in [27]. A fin is affixed to a surface that is always the same temperature T_b and loses heat on the environment with temperature T_∞ through a fixed convective heat transfer coefficient *h*. A schematic for the geometry of the described fin and other properties is shown in Figure 1 [27].

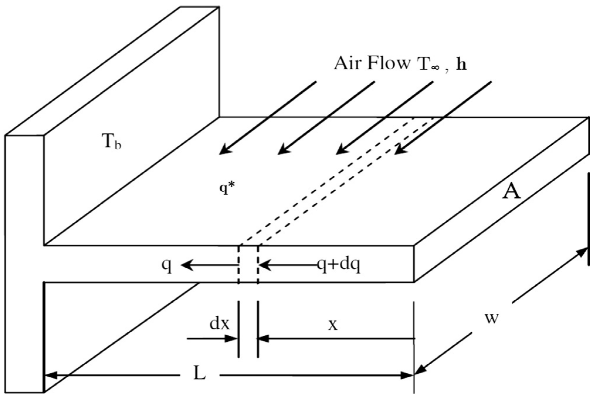

Fig. 1. Schematic illustration of the fin configuration with the heat generation source [27]

The following assumptions have been considered:

- The loss of heat caused by the fin tip is considered as being negligible when compared to the fin's top and bottom surfaces.
- To ensure fin effectiveness, the transverse Biot number needs to remain small [28]. Therefore, variations in temperature in the direction of the transverse tend to be ignored.
- Throughout the problem, we consider that the temperature change in the transfer direction is small, so heat conduction occurs only in the longitudinal direction (x direction).

For this issue, the regulating differential equation with the conditions at the boundary can be expressed as follows [28]:

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}\left(T - T_\infty\right) + \frac{q^*}{k} = 0 \tag{1}$$

$$x = 0,\ \frac{dT}{dx} = 0 \tag{2}$$

$$x = L,\ T = T_b \tag{3}$$

This problem is divided into two main cases. The governing equations for these two cases are introduced below:

Case 1: Fin with constant thermal conductivity and temperature-dependent generation of internal heat

For the first case, we assume that the heat generation in the fin varies with temperature [17, 19, 27, 28], as in Eqn. (4), and that the thermal conductivity is a constant k_0.

$$q^* = q_\infty^*\left(1 + \varepsilon\left(T - T_\infty\right)\right) \tag{4}$$

where q_∞^* is the internal heat generation at temperature T_∞.

We introduce the dimensionless quantities as given below:

$$\theta = \frac{T - T_\infty}{T_b - T_\infty},\ X = \frac{x}{L},\ N^2 = \frac{hPL^2}{k_0 A},\ G = \frac{q_\infty^* A}{hP\left(T_b - T_\infty\right)},\ \varepsilon_G = \varepsilon\left(T_b - T_\infty\right) \tag{5}$$

Using Eqn. (5), the dimensionless form of the Eqns. (1)-(3) can be written as follows:

$$\frac{d^2\theta}{dX^2} - N^2\theta + N^2 G\left(1 + \varepsilon_G\,\theta\right) = 0 \tag{6}$$

$$X = 0,\ \frac{d\theta}{dX} = 0 \tag{7}$$

$$X = 1,\ \theta = 1 \tag{8}$$

Case 2: Fin having temperature-dependent thermal conductivity as well as temperature-dependent generation of internal heat
In the second case, the fin's thermal conductivity varies with temperature and depends on how much heat is produced internally [28]. Given that it fluctuates in response to temperature, we have

$$k = k_0\left[1+\beta\left(T-T_\infty\right)\right] \tag{9}$$

The aforementioned equation's non-dimensional form is represented by

$$\frac{k}{k_0} = \left[1+\varepsilon_C\,\theta\right] \tag{10}$$

where the thermal expansion coefficient is defined as

$$\varepsilon_C = \beta\left(T_b - T_\infty\right) \tag{11}$$

Equation (6) for this condition becomes,

$$\frac{d}{dX}\left[\left(1+\varepsilon_C\,\theta\right)\frac{d\theta}{dX}\right] - N^2\theta + N^2 G\left(1+\varepsilon_G\,\theta\right) = 0 \tag{12}$$

With the boundary conditions specified in Eqns. (7) and (8).

3. Semi-analytical solution of the non-linear boundary value problem using ASM

An effective technique known as the Ananthaswamy-Sivasankari method (ASM) is presented [29-31] for the purpose of employing non-linear ordinary differential equations. Both linear and non-linear differential equations can be solved with this method. This method can also be easily modified to address other non-linear problems, such as boundary value problems in the physical, chemical, and biological sciences, particularly fin issues. The majority of approximation techniques provide only reliable estimates under weak nonlinearities. It is applicable to highly non-linear differential equations [31]. Moreover, the boundary and initial value problems can be solved using the new approach that has been suggested. For the differential equation and its derivatives, additional boundary conditions can be generated. The basic idea of ASM is described in Appendix A.

The advantages of the proposed method are given below:

- The procedure is relatively easy to comprehend.
- Convergence is faster as compared to numerical and analytical techniques.
- The strategy provides simple and precise analytical results which is closer to exact solution.
- The error percentage occurred within an acceptable range (less than 1%).
- Analytical results will aid researchers in understanding how various parameters affect fin temperature.

3.1. Semi-analytical solution for Case 1

The semi-analytical solution for the temperature profile in Eqn. (6) that satisfies the conditions at the boundary is as follows:

$$\theta(X) = l\,e^{aX} + m\,e^{-aX} \tag{13}$$

$$\theta'(X) = l\,a\,e^{aX} - m\,a\,e^{-aX} \tag{14}$$

Utilizing the boundary conditions in Eqns. (7) and (8), we determine the parameter values of *l*, *m* & *a* in the manner described below:

$$l = m \text{ and } l = \frac{1}{e^{a} + e^{-a}} \Rightarrow \theta(X) = \frac{e^{aX} + e^{-aX}}{e^{a} + e^{-a}} \tag{15}$$

Now, by putting the Eqn. (15) into Eqn. (6), and then simplifying, we obtain

$$a^2\left(\frac{e^{aX} + e^{-aX}}{e^{a} + e^{-a}}\right) - N^2\left(\frac{e^{aX} + e^{-aX}}{e^{a} + e^{-a}}\right) + N^2 G\left[1 + \varepsilon_G\left(\frac{e^{aX} + e^{-aX}}{e^{a} + e^{-a}}\right)\right] = 0 \tag{16}$$

Now taking $X = 1$, Eqn. (16) becomes

$$a^2 - N^2 + N^2 G\left(1 + \varepsilon_G\right) = 0 \Rightarrow a = \pm\sqrt{N^2\left[1 - G\left(1 + \varepsilon_G\right)\right]} \tag{17}$$

Hence, the semi-analytical solution of the temperature is obtained by substituting Eqn. (17) into Eqn. (15) as follows:

$$\theta(X) = \frac{\cosh aX}{\cosh a} \tag{18}$$

3.2. Semi-analytical solution for Case 2

The semi-analytical solution for the temperature distribution in Eqn. (12) that satisfies the conditions at the boundary is as follows:

$$\theta(X) = l\,e^{bX} + m\,e^{-bX} \tag{19}$$

$$\theta'(X) = l\,b\,e^{bX} - m\,b\,e^{-bX} \tag{20}$$

Utilizing the boundary conditions in Eqns. (7) and (8), we obtain the parameters *l*, *m* & *b* values as follows:

$$l = m \text{ and } l = \frac{1}{e^{b} + e^{-b}} \Rightarrow \theta(X) = \frac{e^{bX} + e^{-bX}}{e^{b} + e^{-b}} \tag{21}$$

Now, by putting the Eqn. (21) into Eqn. (12) and then taking $X = 0$, we obtain

$$\left(1+\varepsilon_C \sec h(b)\right)b^2 - N^2 + N^2 G\left(\cosh(b)+\varepsilon_G\right) = 0 \tag{22}$$

On solving Eqn. (22), we get the value of the parameter b.
Hence, the semi-analytical solution of the temperature is given by

$$\theta(X) = \frac{\cosh bX}{\cosh b} \tag{23}$$

3.3. Fin efficiency

Newton's simple law of cooling is used to calculate the fin's heat transfer rate:

$$Q = \int_0^b P(T - T_a) \tag{24}$$

The term "fin efficiency" refers to the ratio between the actual transfer of heat from the surface of the fin and the heat transfer that would occur if the complete fin surface existed at exactly the same temperature as the base [32]:

$$\eta = \frac{Q}{Q_{ideal}} = \frac{\int_0^b P(T-T_a)}{Pb(T_b - T_a)} = \int_{X=0}^{1} \theta(X)\,dX \tag{25}$$

where Q is the heat-transfer rate and T is the temperature.
For Case 1: Substituting Eqn. (18) in the above Eqn. (25), we get

$$\eta = \int_{X=0}^{1} \theta(X)\,dX = \int_{X=0}^{1} \frac{\cosh(aX)}{\cosh(a)}\,dX = \frac{\tanh(a)}{a} \tag{26}$$

For Case 2: Substituting Eqn. (23) in Eqn. (25), we get

$$\eta = \int_{X=0}^{1} \theta(X)\,dX = \int_{X=0}^{1} \frac{\cosh(bX)}{\cosh(b)}\,dX = \frac{\tanh(b)}{b} \tag{27}$$

4. Results and discussion

In this portion, we have discussed the dimensionless temperature of the convective fin graphically. The attained semi-analytical results were compared with the numerical solution and the differential transformation technique (DTM) reported in [27]. Variations in several parameters involved in this problem were graphically displayed for both cases. As compared to the numerical method, our analytical method reveals good accuracy for the temperature of the convective fin. Figures 2 to 7 show a comparison of the numerical solution and semi-analytical result of dimensionless temperature $\theta(X)$ with the dimensionless axial coordinate X for each case.

For Case 1: Figure 2 depicts a comparison of present work (ASM), previous work (DTM), and the numerical solution [27]. Figures 3 to 4 represent the dimensionless temperature $\theta(X)$ with the dimensionless axial coordinate X using eqn. (18) for several physical parameters N, G and ε_G. From Figure 3, raising the amounts of G and ε_G causes the temperature profile to increase. Figure 4 illustrates the effects of N for specified amounts of G and ε_G. From this Figure, it is noted that, by increasing the amount of N, the temperature decreases.

For Case 2: Figures 5 and 6 demonstrate the dimensionless temperature $\theta(X)$ with the dimensionless axial coordinate X using Eqn. (23) for several physical parameters N, G, ε_C and ε_G. As seen in Figure 5, the temperature increases as the value of ε_C raises. Figure 6 depicts the influence of G and ε_G for numerous values of N and some particular value of ε_C. From this Figure, it is clear that, by increasing the amount of G and ε_G, the temperature increases.

a)

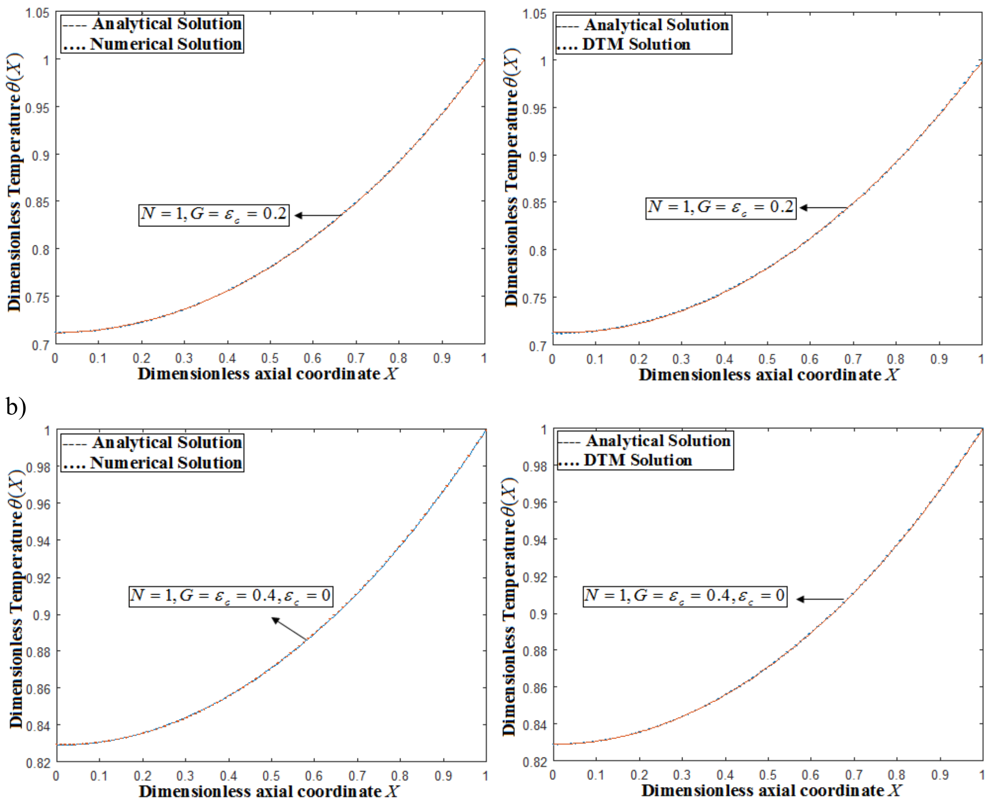

Fig. 2. Comparison between analytical results (ASM and DTM) and numerical solution [27] for: a) Case 1, b) Case 2

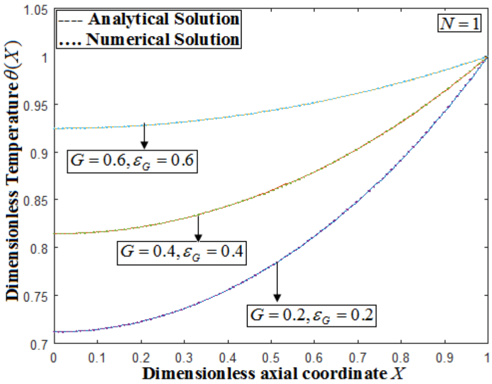

Fig. 3. Dimensionless temperature $\theta(X)$ with dimensionless axial coordinate X for different amounts of ε_G, G when $N = 1$

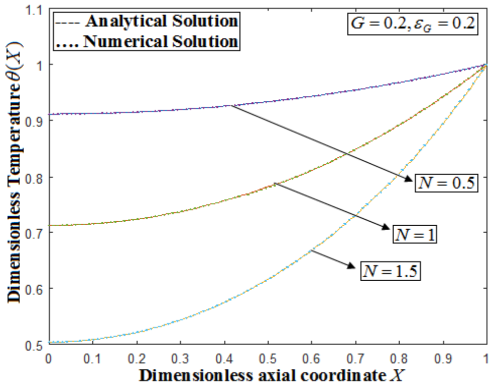

Fig. 4. Dimensionless temperature $\theta(X)$ with dimensionless axial coordinate X for various amounts of N when $\varepsilon_G = G = 0.2$

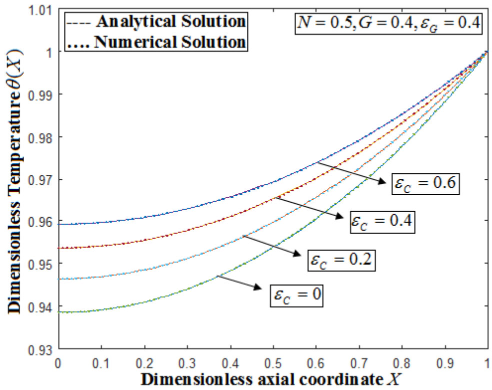

Fig. 5. Dimensionless temperature $\theta(X)$ with dimensionless axial coordinate X for distinct values of ε_C when $N = 0.5$

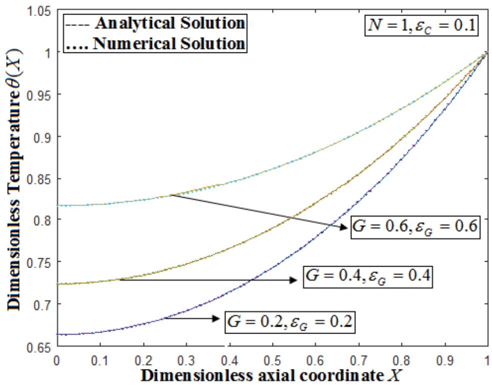

Fig. 6. Dimensionless temperature $\theta(X)$ with dimensionless axial coordinate X for several amounts of ε_G, G and particular values of ε_C when $N = 1$

Fin Efficiency: Figure 7 displays the fin efficiency using Eqn. (26) for Case 1. It portrays the fin efficiency η with the dimensionless parameter N. From this Figure, it can be noted that by increasing the value of G and ε_G, the fin efficiency increased. Figure 8 provides the fin efficiency η using Eqn. (27) for Case 2. It shows the fin efficiency along with the dimensionless thermal expansion coefficient ε_C. In this Figure, the fin efficiency increases as the dimensionless parameters G, ε_G increase. Table 1 compares the outcomes with the numerical solutions to cases 1 and 2, respectively, for particular parameter values. This table shows that convergence is quite accurate and that the average absolute error percentage is very small. The error estimation agrees extremely well with the numerical solution. The graphical representation shows that the approach converges quickly and produces a good fit.

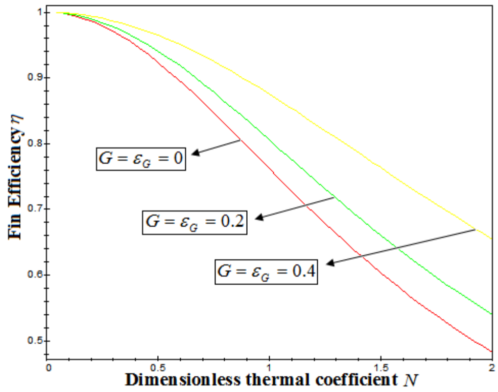

Fig. 7. Dimensionless fin efficiency η with dimensionless thermal coefficient N at various values of G and ε_G

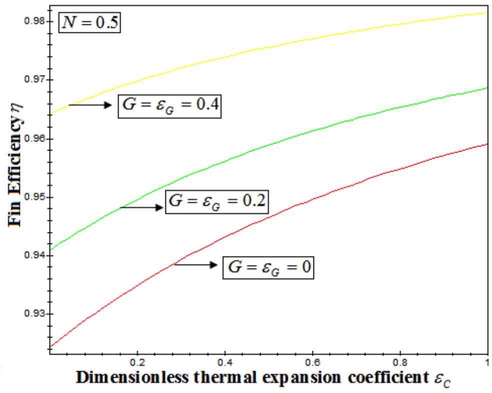

Fig. 8. Dimensionless fin efficiency η with dimensionless thermal expansion coefficient ε_C at various values of G, ε_G and $N = 0.5$

Table 1. Comparison between numerical and analytical solutions of the temperature distribution for Case 1 and Case 2

X	Temperature distribution $\theta(X)$					
	Case 1			Case 2		
	$N = 0.5$, $G = \varepsilon_G = 0.2$			$N = 1$, $\varepsilon_C = 0.2$, $G = \varepsilon_G = 0.4$		
	Analytical Solution	Numerical Solution [27]	Error [%]	Analytical Solution	Numerical Solution [27]	Error [%]
0	0.9119812882	0.9119600000	0.002334	0.8521109814	0.8520500000	0.007157
0.1	0.9128478078	0.9128506448	0.000310	0.8535493737	0.8535606825	0.001320
0.2	0.9154490128	0.9154549664	0.000650	0.8578694059	0.8578902200	0.002430
0.3	0.9197898463	0.9197896016	0.000027	0.8650856633	0.8650880675	0.000280
0.4	0.9258785571	0.9258711872	0.000796	0.8752225080	0.8752036800	0.002151
0.5	0.9337267156	0.9337163600	0.001109	0.8883141635	0.8882865125	0.003113
0.6	0.9433492358	0.9433417568	0.000793	0.9044048274	0.9043860200	0.002080
0.7	0.9547644034	0.9547640144	0.000041	0.9235488230	0.9235516575	0.000310
0.8	0.9679939104	0.9679997696	0.000610	0.9458107814	0.9458328800	0.002340
0.9	0.9830628978	0.9830656592	0.000280	0.9712658612	0.9712791425	0.001370
1	1	1	0	1	1	0
Average error percentage			0.000296			0.000587

5. Conclusion

The convective fin, having heat generation and thermal conductivity that is temperature-dependent was analytically studied with the help of the Ananthaswamy--Sivasankari method (ASM) for two distinct cases. The semi-analytical expressions regarding the temperature distribution in two cases were derived and described in explicit form. The semi-analytical results exhibit good accuracy when compared to the numerical results. The dimensionless fin efficiency was also evaluated. The fin efficiency will be helpful in determining heat transmission from the fin and optimum design analysis. The impacts of numerous parameters involved in the problem were graphically interlined.

The following points have been reached from the results:

- Even with an increase in thermal conductivity and internal heat generation, the outcomes coincide with numerical findings with an error maximum of 0.01%, considering their analytical complexity.
- In comparison with earlier semi-analytical approaches and numerical results, the presented analytical results provide a more effective level of precision.
- Unlike perturbation techniques, this method does not necessitate a convergence control parameter.
- The approach we propose fails to require more iteration in comparison to the numerical solution and DTM. It has no iteration.
- The DTM provides a complicated solution with more than three iterations, but the proposed technique offers a simple solution that contains all the parameters.
- In the applied sciences, this approach can be extended to address boundary value issues of a higher-order.

Appendix A: Basic Concept of the Ananthaswamy-Sivasankari Method (ASM) [29, 30]

Let us consider the non-linear boundary value problem

$$p : g(z, z', z'') = 0 \tag{A.1}$$

where p represents the third-order non-linear differential equation such that $z = z(x, a, b, \ldots)$ in which a, b are given parameters and $x \in [L, U]$ can be finite or infinite with the following boundary conditions:

$$\left.\begin{aligned} &\text{At } \ x = L,\ z(x) = z_{L_0} \ \text{ (or) } \ z'(x) = z_{L_1} \\ &\text{At } \ x = U,\ z(x) = z_{U_0} \ \text{ (or) } \ z'(x) = z_{U_1} \end{aligned}\right\} \tag{A.2}$$

Assume that the semi-analytical solution of the non-linear equations is an exponential function of the form

$$z(x) = k\, e^{\alpha x} + l\, e^{-\alpha x} \tag{A.3}$$

The unknown coefficients k and l are obtained by solving the non-linear differential equations as follows:

$$\left.\begin{aligned} z(L) &= k\,e^{\alpha L} + l\,e^{-\alpha L} = z_{L_0} \\ z'(L) &= \alpha\,k\,e^{\alpha L} - \alpha\,l\,e^{-\alpha L} = z_{L_1} \end{aligned}\right\} \quad \text{(A.4)}$$

$$\left.\begin{aligned} z(U) &= k\,e^{\alpha U} + l\,e^{-\alpha U} = z_{U_0} \\ z'(U) &= \alpha\,k\,e^{\alpha U} - \alpha\,l\,e^{-\varepsilon U} = z_{U_1} \end{aligned}\right\} \quad \text{(A.5)}$$

Equations (A.4) and (A.5) may be used to get the unknown parameters k and l. The following non-linear differential equations are obtained by substituting an Eqn. (A.3) into the Eqn. (A.1)

$$p : g(z(x,k,l,\alpha,a,b), z'(x,k,l,\alpha,a,b), z''(x,k,l,\alpha,a,b)) = 0 \quad \text{(A.6)}$$

This equation is valid at x where $x \in [L,U]$. Solving the Eqn. (A.6), the unknown parameter α can be obtained in terms of the given parameters a and b.

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