

## NEW PREDICTION OF CRACKS PROPAGATION IN REPAIRED STEEL PLATE WITH BONDED COMPOSITE PATCH AT CYCLIC LOADING

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*received 18 October 2023, revised 27 February 2024, accepted 4 March 2024*

**Abstract:** This study presents a numerical prediction of the fatigue life of steel panels repaired by a composite patch. The effect of length cracks, the stress ratio  $R$  and properties of the patch is presented. The obtained results show that the bonded composite repair significantly reduces the stress intensity factors at the tip of repaired cracks. The results are in a good agreement with those in the literature. The Monte Carlo method is used to predict the distribution function governing crack propagation in fatigue analysis. In computing the failure probability of the structure, we consider the statistical uncertainty associated with key variables, along with the previously discussed model uncertainty. The results obtained highlight the considerable impact of variations in crack length and stress ratio on the distribution function. Notably, uncertainty in these parameters significantly amplifies the probability of structural failure in plates, thereby diminishing overall structural durability.

**Keywords:** Repair composite plate, fracture mechanics, fatigue, Monte Carlo method, stress ratio

### 1. INTRODUCTION

Fatigue is a phenomenon where, due to the influence of cyclic, repetitive, or fluctuating stresses or strains, there is a transformation in the material's local characteristics, potentially resulting in the development of cracks and, eventually, structural failure. Accurately predicting fractures and ensuring the reliability of structures across diverse real-world applications are of utmost importance, considering their significant implications for both economic considerations and safety [1].

The repairing damaged structures by composite patch are applied in marine structures, in oil and gas industries for the repair of corroded pipes and pressure vessels as well as in civil engineering. Lifetime prediction is necessary for the design of structures subjected to fatigue loads, [2, 3].

A.A. Baker [4] is credited as the initial trailblazer in these explorations, which took place at the aeronautical and maritime research laboratory of the Royal Australian Air Force. The challenge of implementing composite patches arises from a complex interplay of numerous factors. These factors encompass the mechanical characteristics of diverse materials (such as aluminum panels, composites, and adhesives), the geometric attributes of the composite patch, and the specific loading conditions.

Several authors have computed the stress intensity factor at the crack tip of repairing cracks among them [5 – 14]. These researches have shown that, after repair, the stress intensity factor exhibits an asymptotic behaviour as the crack length increases. The durability and reliability of structures repaired with composite patches depend mainly on the mechanical and thermal behavior of the adhesive layer. These are the most essential points for studying the causes of failure and the degradation of the entire repair [15 – 23].

The primary objectives of this investigation are to estimate the fatigue life of steel panels that have undergone repair using a composite patch and to examine the influence of crack length, the stress ratio  $R$ , and patch properties. The study employs the Monte Carlo method to forecast the distribution function governing crack propagation in fatigue. Additionally, the calculation of the structural failure probability takes into consideration both the statistical uncertainty associated with fundamental variables and the model uncertainty, as previously outlined.

### 2. PRESENTATION OF THE MODEL

#### 2.1. Geometrical of the model

The basic geometry of the cracked structure considered in this study is shown in Fig. 1. Consider a plate with the following dimensions: width  $W_p=100\text{mm}$  and thickness  $T_p=2\text{mm}$ . The plate is subjected to uniaxial tensile following a stress value equal to 100 MPa for elastic analysis. A central crack of length  $2a=6\text{mm}$  perpendicular to the loading axis was supposed to exist in the plate.

This crack is repaired with boron/epoxy and graphite/epoxy composites patches. The laminate consists of 11 symmetrical plies with a thickness of 0.132mm per ply ( $T_r=1.452\text{mm}$ ), and the stacking sequence is as follows  $[-45^\circ, 0^\circ, 0^\circ, +90^\circ, 0^\circ, 0^\circ, 0^\circ, 0^\circ, 0^\circ, +45^\circ]$ s. With width  $W_r=99\text{mm}$ .

The adhesive is used to bond the patch on cracked plate: FM73 (tab. 2), Epoxy adhesive. The adhesive thickness  $T_a$  is taken equal to 0.132mm.

The boundary conditions are:  $U_x = U_z = U_{R_x} = U_{R_y} = U_{R_z} = 0$  and  $U_y \neq 0$ .

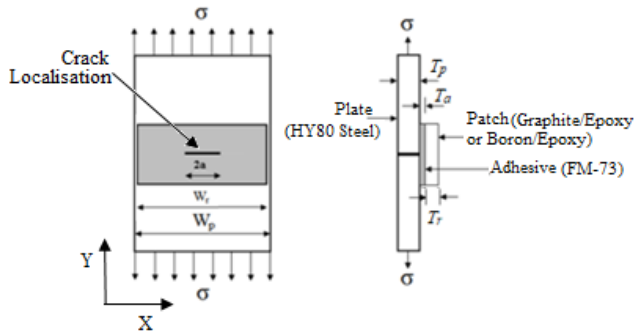


Fig. 1. Geometrical model

## 2.2. Elastic properties of the model

The elastic properties of the HY80 steel plate used for the manufacture of gas pipelines and the composite patch are given in Table 1, and the adhesive is given in Table 2.

Tab. 1. Mechanical properties of model studied [24]

|                               | HY80 steel            | Boron/epoxy           | Graphite/epoxy        |
|-------------------------------|-----------------------|-----------------------|-----------------------|
| $E_L$ [MPa]                   | $20.6843 \times 10^4$ | $20.6843 \times 10^4$ | $17.2369 \times 10^4$ |
| $E_T$ [MPa]                   | -                     | $19.3053 \times 10^3$ | $10.3421 \times 10^3$ |
| $G_{LT}$ [MPa]                | -                     | $5.17107 \times 10^3$ | $4.82633 \times 10^3$ |
| $\nu_{LT}$                    | 0.33                  | 0.3                   | 0.31                  |
| $\nu_{TL}$                    | -                     | 0.14                  | 0.13                  |
| $\alpha_L$ [K <sup>-1</sup> ] | -                     | $4.3 \times 10^{-6}$  | $-7 \times 10^{-7}$   |
| $\alpha_T$ [K <sup>-1</sup> ] | -                     | $1.87 \times 10^{-5}$ | $3.6 \times 10^{-5}$  |

Tab. 2. Mechanical properties of Adhesive FM-73 [24]

|                |         |
|----------------|---------|
| $G_{LT}$ [MPa] | 413.368 |
| $\nu_{LT}$     | 0.33    |

## 2.3. Theoretical model

We used a calculation code for studying fatigue crack propagation to predict the fatigue life of metal structures called AF-GROW.

The NASGRO model was developed and modified by Maierhofer et al. [25] is applied to predict crack propagation. The crack propagation equation used by NASGRO is:

$$\frac{da}{dN} = C \left[ \left( \frac{1-f}{1-R} \right) \Delta K \right]^n \frac{\left( 1 - \frac{\Delta K_{th}}{\Delta K} \right)^p}{\left( 1 - \frac{K_{max}}{K_{cr}} \right)^q} \quad (1)$$

where:

$$\Delta K = K_{max} - K_{op} \quad (2)$$

$K_{max}$  : maximum stress intensity factor;

$$K_{max} = Y \sigma_{max} \sqrt{\pi a} \quad (3)$$

$K_{op}$  : crack opening stress intensity factor corresponds to the minimum value;

$$K_{op} = Y \sigma_{min} \sqrt{\pi a} \quad (4)$$

$K_{cr}$  : critical stress intensity factor

$$K_{cr} = Y \sigma_{cr} \sqrt{\pi a_{cr}} \quad (5)$$

The function  $f$  is:

$$f = \frac{K_{op}}{K_{max}} \quad (6)$$

Tab. 3. NASGRO equation parameters for HY80 steel [24]

| C                        | n   | p    | q    |
|--------------------------|-----|------|------|
| $3.0101 \times 10^{-11}$ | 2.5 | 0.25 | 0.25 |

## 3. RESULTS AND DISCUSSION

### 3.1. Comparison of FEM and analytical results

Fig. 2 shows a comparative analysis of the stress intensity factor SIF variation obtained from the FEM results, developed in this work, and that resulting from an analytical solution (7). The stress intensity factor SIF is the only significant parameter, the relation between the far applied stress on the plate  $\sigma$  and the stress intensity factor  $K_I$  is as follows:

$$K_I = Y \sigma \sqrt{\pi a} \quad (7)$$

with:  $Y = 1.12$

It can be seen, according to Fig. 2 that the presence of the patch has a considerable effect on the stress intensity factor SIF variation at the crack tip. It shows that the patch repair highly decreases the stress intensity factor, the maximum reduction of stress intensity factor  $K_I$  is about 70% of the crack length  $a=40\text{mm}$ .

The analytical solution gives a good agreement of the stress intensity factor compared with the finite element method.

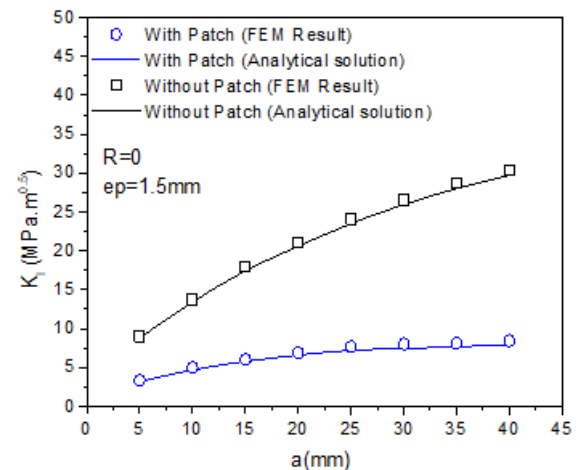


Fig. 2. Variation of the SIF according to the crack length for the cases with and without patch for analytical and FEM solution

### 3.2. Effect of the stress ratio R

In cyclic loading, the stress ratio plays an important role in the development of fatigue cracking in healthy and repaired structures.

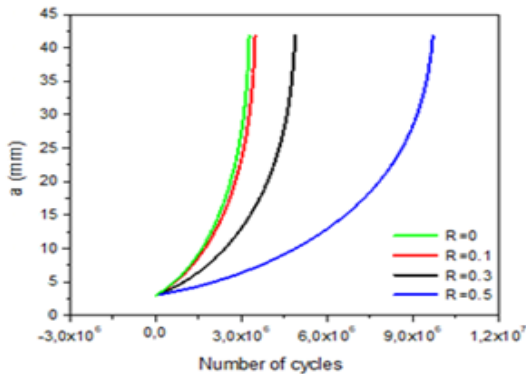


Fig. 3. Illustrates the effect of the variation in crack length as a function of the number of cycles for different value of R

The study showed that the fatigue life increases for the repaired plate compared with the unrepaired one, the cycle of failure

is double for all stress ratios studied. When the stress ratio R increases from 0 to 0.5 the cycle of failure increases because of higher stress values (See Fig. 3).

### 3.3. Effect of patch properties

To evaluate the influence of the patch material on crack propagation (initial crack  $a=3\text{mm}$ ) and to highlight the repair process of a central crack, we chose two patches, boron/epoxy and graphite/epoxy, with identical geometric shapes and different mechanical properties, to repair a cracked HY80 steel plate.

Fig. 4 illustrates the effect of the mechanical properties of the patch on the variation in crack length as a function of the number of cycles without repair and with repair (boron/epoxy and graphite/epoxy).

For the variation in crack propagation life, for the unrepaired and repaired plate with boron/epoxy and graphite/epoxy, for the different stress ratios is very distinct, seeing the better properties of the composite patches for repair and absorption of the stresses that stress the crack head, since these patches improve and increase the life of this plate.

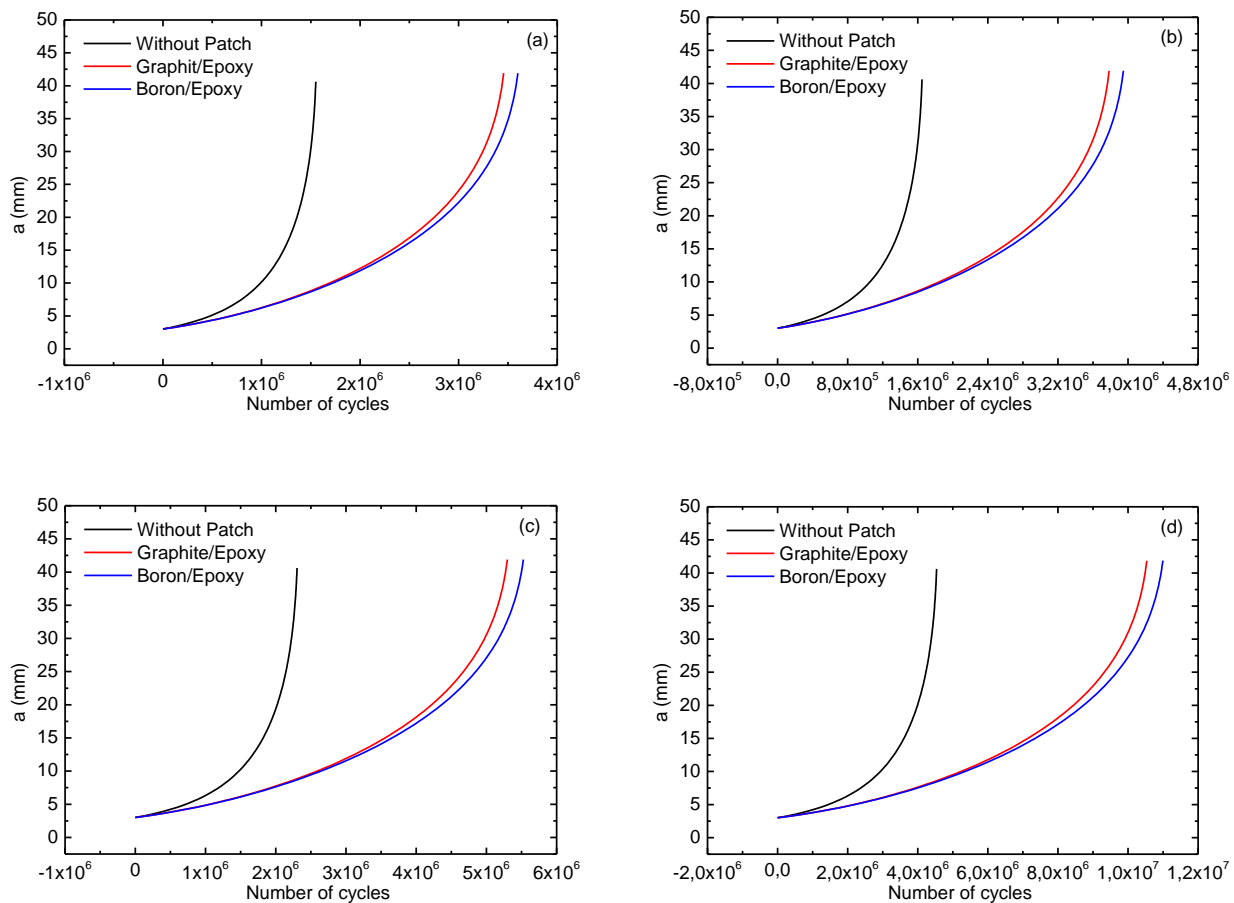


Fig. 4. Illustrates the effect of the specimen structure on the variation in crack length as a function of the number of cycles without repair and with repair process (boron/epoxy and graphite/epoxy). a) R=0, b) R=0.1, c) R=0.3, d) R=0.5

4. PROBABILISTIC ANALYSIS

4.1. Random parameters and fracture response

The density function is evaluated by using Monte Carlo method. The basic idea is to draw random samples for the input parameters, then to compute the mechanical response for each sample. we have realised this work by the FORTRAN program, response by using Monte Carlo method. To achieve a high accuracy of the results, we have carried out 105 simulations.

Consider a cracked structure with uncertain mechanical and geometric characteristics that is subject to random loads. Denote by X an N-dimensional random vector with components characterizing all uncertainties in the system and loading parameters. For example, the possible random components are: geometric parameters, H, W, and L, mechanical parameters, E,  $\nu$ , ratio R varies from 0 to 0.5 and length of cracks a varies from 5 to 30mm: All or some of these variables can be modeled as random variables. Hence, any relevant fracture response, such as the KI and (a), may be evaluated by the probability.

The Monte Carlo method is used to predict the distribution function of the mechanical response, the sensitivities of the mechanical response are evaluated regarding the uncertainties in the design variables. The fundamental concept involves generating random samples for the input parameters and subsequently calculating the mechanical response for each one. As a substantial number of Monte Carlo samples are collected, it enables us to perform statistical analysis on the response datasets, ultimately yielding the probability density function.

The basic idea is to draw random samples for the input parameters, then to compute the mechanical response. In order to capture the role of the random variables, the sensitivity of the R(x) regarding the uncertainties of the input parameters was analysed, Table 4.

Tab. 4. Random variables and corresponding parameters

| Variable | Mean        | Coefficient of variation (COV) |
|----------|-------------|--------------------------------|
| E(MPa)   | 20.6843.104 | 1%                             |
| $\nu$    | 0.3         | 1%                             |
| HP(mm)   | 100mm       | 2%                             |
| WP(mm)   | 100mm       | 2%                             |
| TP (mm)  | 100mm       | 2%                             |
| a(mm)    | 20mm        | 3%                             |

Tab. 5. Mean R(x) and fitting error for probabilistic distributions

| Fitting probability density functions | Average R(x) | Least square fitting error |
|---------------------------------------|--------------|----------------------------|
| Gaussian                              | 2,72474.106  | 0%                         |

Tab. 6. Random variables and corresponding parameters of stress intensity factor

| Length of crack (a) | Mean    | Standard deviation |
|---------------------|---------|--------------------|
| 20                  | 6,88762 | 3,44381            |

4.2. Probabilistic results

Fig. 5 plots the histograms of the R obtained by Monte Carlo simulations. The probability density function PDF is obtained by fitting the histogram with theoretical models three distribution laws are investigated: Gaussian it can be clearly observed that the three distributions give more or less good approximation of the R probability density function, with good estimation of the average.

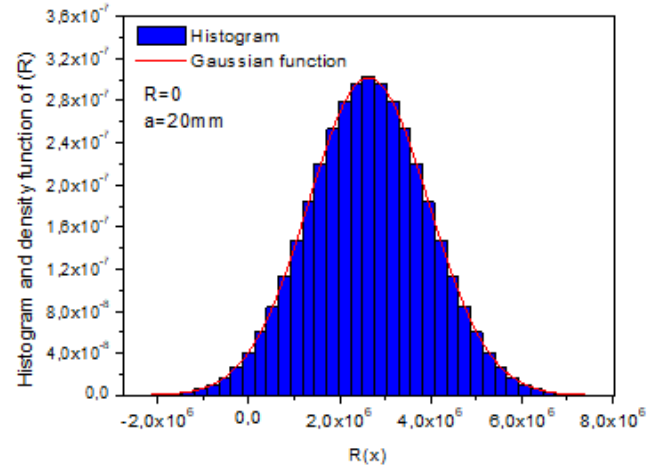


Fig. 5. Histogram and probability density function of R

Figs. 6 - 7 show the probability density of the ratio R and the crack length a. We have noted that when the value of the ratio R and the crack length a is large, the value of the probability density is low and the failure rate is high.

It can be seen that the margin increases significantly with the uncertainties associated with the ratio R is the crack length a, leading to a higher probability of failure.

Finally, we note that the ratio R and the crack length a, are the important factor influencing on the durability of structure.

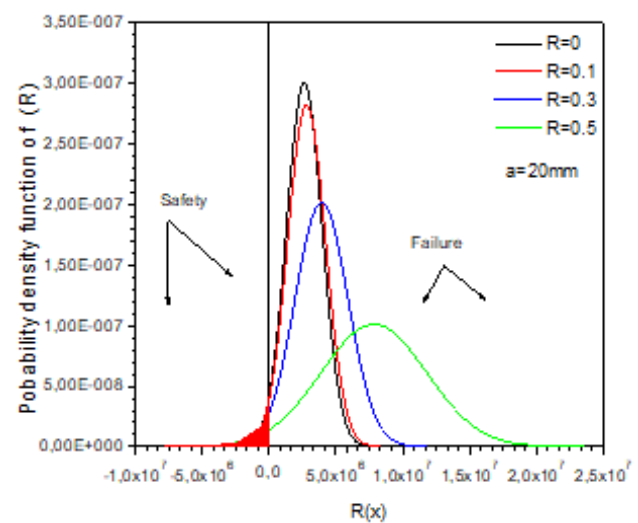


Fig. 6. Probability density function of R

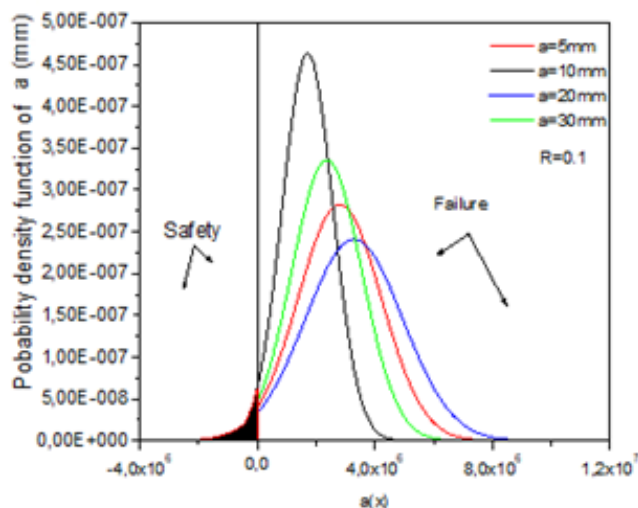


Fig. 7. Probability density function of a

### 5. CONCLUSIONS

The work we have carried out has enabled us to examine the influence of patch material properties on the fracture parameter values calculated for a plate subjected to tensile stress containing a central crack. The qualities of the patch material, the crack length and the stress ratio have a direct influence on the fatigue life of the structures.

Fatigue results have shown that increasing the stress ratio increases crack propagation velocity, leading to an increase in the values of cycles to fracture. For composite patch-repaired plates, an increase in stress ratio leads to a reduction in repair efficiency, resulting in an increase in numbers of cycles to fracture.

The presence of a patch considerably reduces crack propagation, which can delay the speed of cracking and subsequently increase the fatigue life of the structure. The results are in a good agreement with those in the literature [26]. The Monte Carlo method is used to predict the distribution function of propagation of crack in the fatigue.

The failure probability of the structure was calculated by taking into account both the statistical uncertainty on the basic variables and the model uncertainty as previously discussed.

According to the obtained results, we note that the crack length variations and the stress ratio are important factors influencing the distribution function.

Finally, we note that the ratio  $R$  and the crack length  $a$ , are the important factor influencing on the durability of structure.

#### List of Symbols

|            |   |
|------------|---|
| $W_P$      | Width of plate [mm]                                       |
| $T_P$      | Thickness of plate [mm]                                   |
| $\sigma$   | Stress [MPa]  |
| $a$        | Length of crack [mm]                                      |
| $T_r$      | Thickness of patch [mm]                                   |
| $W_r$      | Width of patch [mm]                                       |
| $T_a$      | Thickness of adhesive [mm]                                |
| $E_L$      | Longitudinal Young's modulus [N/mm <sup>2</sup> ]         |
| $E_T$      | Transverse Young's modulus [N/mm <sup>2</sup> ]           |
| $G_{LT}$   | Shear modulus in XY- plane direction [N/mm <sup>2</sup> ] |
| $\nu_{LT}$ | Poisson's ratio modulus in XY- plane direction            |

|                   |   |
|-------------------|---|
| $\nu_{TL}$        | Poisson's ratio modulus in YX- plane direction                    |
| $\alpha_L$        | Longitudinal coefficients of thermal expansion [K <sup>-1</sup> ] |
| $\alpha_T$        | Transverse coefficients of thermal expansion [K <sup>-1</sup> ]   |
| $C, n, p$ and $q$ | Material parameters (see Tab. 3);                                 |
| $R$               | Stress ratio;   |
| $K_{max}$         | Maximum stress intensity factor [MPa.m <sup>1/2</sup> ]           |
| $K_{op}$          | Crack opening stress intensity factor [MPa.m <sup>1/2</sup> ]     |
| $K_{cr}$          | Critical stress intensity factor [MPa.m <sup>1/2</sup> ]          |
| $K_I$             | Stress intensity factor [MPa.m <sup>1/2</sup> ]                   |

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