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Sensitivity of Transport Object Control Quality for Measuring the Inaccuracy of State Variables

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Abstract – This work analyzes the sensitivity functions and optimum control of a transport and logistics process model. It explains the fundamental model of controlling safe ship movement as a differential game, and optimizing control algorithms through multi-matrix game and multi-stage positioning game. The sensitivity features for controlling safe ship in actual collision scenario are described in relation to inaccurate information of process position and variations in its varables, based on the determination of computer simulation algorithms in Matlab/Simulink software.

Key Words – marine navigation, safety at sea, safe ship control, game control, computer simulation

1 Introduction

Sensitivity theory, described by Wierzbicki [1] and Eslami [2], has become an independent field in cybernetic sciences and automatic control theory recently. This is largely related to the rapid development of adaptive automation systems, designed to operate effectively in situations of significant disturbances, according to Rosenwasser and Yusupov [3]. Cruz [4] distinguishes between the response of the process model to parameters changes and the response of the optimal control to inaccurate state information and disturbances in real life applications.

In the transport and logistics processes of marine, air and land, multiple entities interact, which entails human operators making maneuvering decisions that are influenced by different factors. To safely control these processes, game theory, according to Lisowski [5], can play a significant role here. This allows for the creation of anti-collision control algorithms that consider the uncontrollable path of the transport process resulting from the operator's incorrect assessment of the situation.[1]

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2 DEFINITION OF SENSITIVITY FUNCTION

Kinematics and dynamics of the transport and logistics process, presented in Figure 1, in general terms, describe that the: equations of state (1), equations of output (2), state and control constraints (3) and control quality index are determined by the control objective (4).

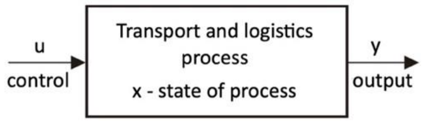


Fig. 1. Flowchart of the transport and logistics process

$$\dot{x}(t) = f[x(t), u(t), t] \tag{1}$$

$$y(t) = g[x(t), y(t), t] \tag{2}$$

$$h[x(t), u(t), t] \leq 0 \tag{3}$$

$$F[x(t), u(t), t] = \int_{t_o}^{t_f} f_o[x(t), u(t), t] \tag{4}$$

where: x is state parameter, u is control parameter, y is output, t is time, F is control quality index, f is state function, f_o is function of functional, t_o and t_f are start and end time of the control process [6].

2.1 SENSITIVITY FUNCTION OF THE TRANSPORT PROCESS MODEL

The method employed for analyzing the model sensitivity entails examining the mathematical representation of the parametric model of the process via state equations. The model parameters **a** ($a_1, a_2, \ldots, a_m$) were analyzed within a certain range of changes. The sensitivity function takes the following form:

$$s_a = \frac{\partial x(t,a)}{\partial a} \tag{5}$$

being a function of the first-order sensitivity of the process model to changes in the values of its parameters [7-10].

Theoretically, one can also consider a kth - order sensitivity function:

$$s_{k,a} = \frac{\partial^k f(t,a)}{\partial a_1^{k_1} \ldots \partial a_m^{km}} \qquad k_1 + \cdots + k_m = k \tag{6}$$

2.2 SENSITIVITY FUNCTION OF TRANSPORT PROCESS OPTIMAL CONTROL

The first-order sensitivity function of optimal control to the error in the state measurement **x** ($x_1, x_2, \ldots, x_n$) can be expressed as follows:

$$s_x^{oc} = \frac{\partial F[x(u)]}{\partial x} \tag{7}$$

and the k^{th} order sensitivity of optimal control:

$$s_{k,x}^{oc} = \frac{\partial^k F[x(u)]}{\partial x_1^{k_1} \ldots \partial x_n^{km}} \qquad k_1 + \cdots + k_m = k \tag{8}$$

3 PROCESS OF SAFELY MANAGING SHIP TRAFFIC

Many mathematical models have been formulated for controlling safe ship voyage process, including the basic and approximate models.

3.1 BASIC MODEL OF THE DIFFERENTIAL GAME

The best model that involves passing one's own ship with J encountered ships is the model of the J players' differential game [11-14].
The state equation describes the properties of the process :

$$\dot{x}_i = f_i\left(x_0^{\zeta_0}, \dots, x_j^{\zeta_j}, \dots, x_J^{\zeta_J}; u_0^{\gamma_0}, \dots\ u_j^{\gamma_j}, \dots, u_J^{\gamma_J}, t\right)$$
$$i = 1, 2, \dots, I$$
$$I = j \cdot \zeta_j + \zeta_0 \quad (9)$$
$$j = 1, 2, \dots, J$$

where: $x_0^{\zeta_0}(t)$ is the ζ_0 ship's own dimensional state vector, $x_j^{\zeta_j}(t)$ is the ζ_j ship j dimensional state vector, $u_0^{\gamma_0}(t)$ is the γ_0 ship's own dimensional control vector, $u_j^{\gamma_j}(t)$ is the γ_j ship j dimensional control vector.

State and control limitations result from maintaining the safe passing distance D_s while observing the legal COLREG maneuvering rules:

$$h_j\left(x_j^{\zeta_j}, u_j^{\gamma_j}\right) \leq 0 \quad (10)$$

The synthesis of safe ship control involves minimizing the objective function given in integral and final form:

$$F_{0,j} = \int_{t_0}^{t_k}\left[x_0^{\zeta_0}(t)\right]^2 dt + r_j(t_f) + d_f(t_f) \rightarrow \min \quad (11)$$

The basic payoff is the ship's path that is lost as it passes the encountered ships, and the final payoff determines the last collision risk $r_j(t_f)$ for the j-th encountered ship and the last aberration of the own ship's trajectory $d_f(t_f)$ from the given one.

3.2 APPROXIMATE MODELS OF THE SAFE SHIP CONTROL PROCESS

The following approximate models of the process of safe ship traffic management are distinguished models of multi-stage positional game and multi-step matrix game [15-20].

- Multi-stage positional game control

The task of determining the optimal ship path can be shortened to a linear programming problem. The least value of the control objective function given in a linear form should be determined, representing the shortest time of arrival of own ship to the closest turning point, corresponding to the maximum longitudinal velocity component for the given direction of motion. The control process takes the form of a multi-stage, multi-object positional game.

- Multi-step matrix game control

The own ship has control in the form of changing course or speed to perform an anti-collision maneuver at a given safe distance. Similarly, every other ship we encounter uses control in the form of its strategies to conduct an anti-collision maneuver. The optimal ship path is determined using the dual linear programming technique according to the collision risk matrix between ships. The control process represents a multi-step positional game of many objects.

4 ALGORITHMS FOR DEFINING A SAFE SHIP COURSE

4.1 ALGORITHM OF MULTI-STAGE POSITIONAL GAME

To synthesize safe ship control, a basic differential game model is used; it is simplified into a multi-stage j positional game of non-cooperative players.

Optimum management of own ship $u_0^*(t)$, equivalent to optimum positional control $u_0^*(p)$ for the current position p(t), is defined below:

- pairs of permissible tactics $U_j^0[p(t)]$ of encountered objects relative to one's own ship and original sets $U_0^j[p(t)]$ of permissible tactics of one's own ship relative to each j of the objects encountered are determined,
- a set of vectors u_j and u_0^j are determined for each j object, and the optimum positioning tactic $u_0^*(p)$ of own ship is determined based on the situation:
-

$$F_{0,j}^* = \min_{u_0 \in \cap_{j=1}^{j} U_0^j} \left\{ \max_{u_j \in U_j} \min_{u_0^j \in U_0^j} D_0[x_0(t_f)] \right\} = D_0^* \tag{12}$$

Variable D_0 means the continuous control objective function of the own ship, describing the ship's distance at time t_0 to the closest turning point P_0 on the given cruise path. The criteria for selecting the optimum course of one's own ship entail determining its trajectory and speed. This would make the least path loss to safely pass the encountered objects, at a distance above the expected D_s value, considering the ship's dynamics based on the operation lead time (Fig. 2).

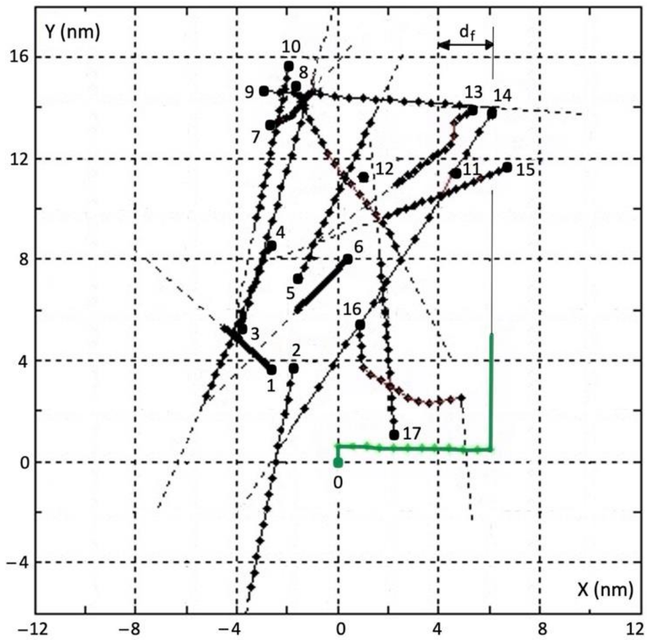


Fig. 2. Computer simulation of the ship's safe trajectory according to the MPG multi-stage position game algorithm in the situation of passing j = 17 ships encountered in the Skagerrak Strait at D_s = 1.0 nm, d_f = 6.06 nm, r_j (t_f) = 0

4.2 ALGORITHM OF MULTI-STEP MATRIX GAME

In digressing from the ship dynamics equations, the basic model of the differential game of the collision prevention process involves a j matrix game of non-cooperative participants. Particpant 0 in a matrix game has the opportunity to utilize u_0 clean tactic , and participant j can utilize u_j clean tactics. Restrictions on the choice of strategy are the resultants of the recommendations of the COLREG sea route law. Since the game usualy lacks a saddle point, there is no guaranteed state of equilibrium. This issue can be solved using dual linear programming. In the dual problem, player 0 strives to reduce collision risk, while participant j strives to increase collision risk r_j. The mechanisms of a combined tsctic describe the probability share of players utilizing their clean methods. Thus,, for the optimal control criterion:

$$F_{0,j}^{*} = \min_{\propto_0} \max_{\propto_j} r_j \tag{13}$$

the probability matrix P of utilizing individual clean tactics is achieved. The answer to the control task is the strategy with the maximum probability (Fig. 3):

$$u_{0,\propto_0}^{*} = u_{0,\propto_0}\left[p_j\left(\propto_0, \propto_j\right)\right]_{\max} \tag{14}$$

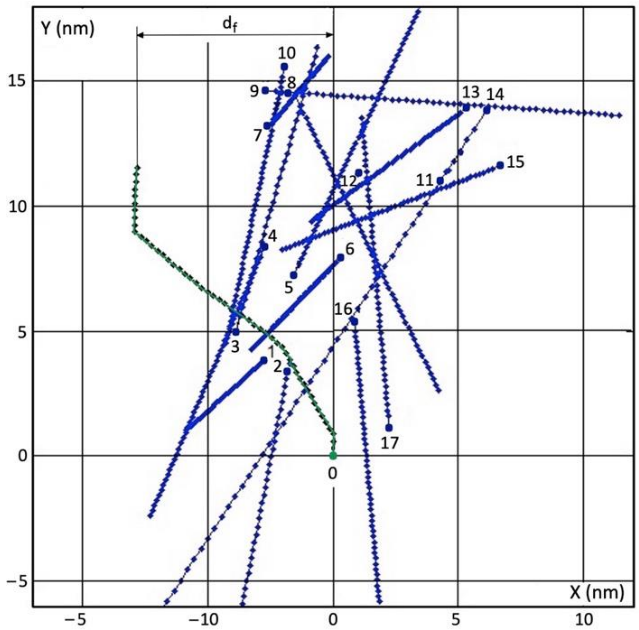


Fig. 3. Computer simulation of the ship's safe trajectory based on the MMG multi-step matrix game algorithm in the state of passing j = 17 ships encountered in the Skagerrak Strait at D_s = 1.0 nm, d_f = 7.59 nm, $r_j(t_f)$ = 0

5 SENSITIVITY OF SAFE SHIP CONTROL

The research on the sensitivity of the game control was reduced to the analysis of the sensitivity of the final payout of the game; it was measured by the comparative final aberration of the game safe course from the set course, as a first-order quality sensitivity.

The response of the operator's control model to the inaccurate information about the approaching situation of ships and to parameters changes in the process model was assessed. The allowable average mistakes that can be introduced by the anti-collision system sensors have the values below for:

- radar:
  - bearing error when rocking the ship ± 0.22 [°],
  - beam shape ± 0.05 [°],
  - pulse shape ± 20 [m],
  - antenna drive play ± 0.5 [°],
  - quantization error for bearing ± 0.01 [°] and distance ± 0.01 [nm],
- gyrocompass: ± 0.5 [°],
- log: ± 0.5 [kn],
- GPS: ± 15 [m].

It was assumed that the total of all mistakes affecting the depiction of a given navigation condition cannot exceed 5%, i.e. the minimum accuracy value with which a given navigation situation will be presented cannot be less than 95%.

5.1 CHARACTERISTICS OF THE SENSITIVITY OF SAFE SHIP CONTROL TO INACCURACY OF INFORMATION FROM ARPA ANTI-COLLISION SYSTEM

Let **x** define the set of information about the process state from the ARPA anti-collision system such that:

$$\mathbf{x} = \{V_0, \psi_0, V_j, \psi_j, D_j, N_j\} \tag{15}$$

and $\mathbf{x_m}$ defines a set of information subject to calculation and processing mistakes:

$$\mathbf{x_m} = \{V_0 \pm \delta V_0, \psi_0 \pm \delta\psi_0, V_j \pm \delta V_j, \psi_j \pm \delta\psi_j, D_j \pm \delta D_j, N_j \pm \delta N_j\} \tag{16}$$

where: V_0 is speed of own ship, V_j is speed of j ship, D_j is distance of own ship to j ship.
The relative sensitivity measure of the final aberration of the ship's safe course d_f from the given course is:

$$s_x^{oc}(x, x_m) = \frac{d_f(x_m)}{d_f(x)} 100\% = \{s_{V_0}, s_{\psi_0}, s_{V_j}, s_{\psi_j}, s_{D_j}, s_{N_j}\} \tag{17}$$

5.2 CHARACTERISTICS OF THE SENSITIVITY OF SAFE SHIP CONTROL TO CHANGES IN PROCESS PARAMETERS

Let **a** be the set of parameters of the transport model of the process:

$$\mathbf{a} = \{D_s, \Delta V_0, t_a, t_s\} \tag{18}$$

and $\mathbf{a_m}$ be the set of information subject to measurement and processing mistakes:

$$\mathbf{a_m} = [D_s \pm \delta D_s, \Delta V_0 \pm \delta\Delta V_0, t_a \pm \delta t_a, t_s \pm \delta t_s] \tag{19}$$

The relative sensitivity measure of the final aberration of the ship's safe course d_f from the given course will be:

$$s_a^m(a, a_m) = \frac{d_f(a_m)}{d_f(a)} = \{s_{D_s}, s_{\Delta V_0}, s_{t_a}, s_{t_s}\} \tag{20}$$

where: D_s is safe distance, ΔV_0 is value of the speed reduction maneuver, t_a is advanced time of the trajectory or speed alteration maneuver, t_s is trajectory stage duration.

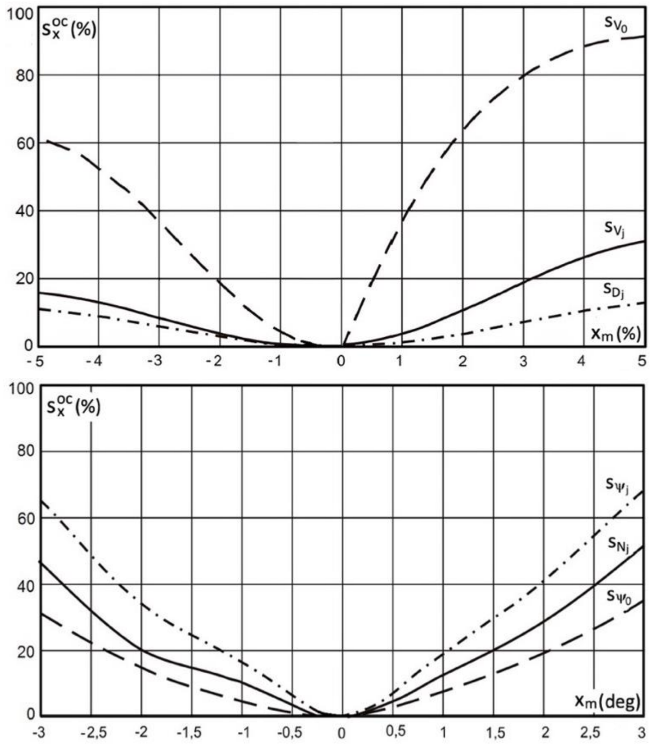
6 ANALYSIS OF SENSITIVITY CHARACTERISTICS

Sensitivity characteristics were obtained by computer simulation using Matlab/Simulink software of individual control algorithms. The sensitivity of the process in the Skagerrak Strait, where j = 17 ships were encountered, to changes in the range of information accuracy and process parameters was analyzed (Table 1).

Table 1. Range of changes in the values of sensitivity characteristics parameters

	Parameter	Unit		Scope of measurement changes
x	V_0	[kn]	$\mathbf{x_m}$	$\pm$ 5 % V_0
	ψ_0	[°]		$\pm$ 3 [°] ψ_0
	V_j	[kn]		$\pm$ 5 % V_j
	ψ_j	[°]		$\pm$ 3 [°] ψ_j
	D_j	[nm]		$\pm$ 5 % D_j
	N_j	[°]		$\pm$ 3 [°] N_j
a	ΔV_0	30 [%]	$\mathbf{a_m}$	(18 $\div$ 100) [%]
	D_s	1.5 [nm]		(0,6 $\div$ 1) D_s = (0,9 $\div$ 1,5) [nm]
	t_a	3 [min]		(0,6 $\div$ 1) t_a = (1,8 $\div$ 3) [min]
	t_s	3 [min]		(0,6 $\div$ 1) t_s = (1,8 $\div$ 3) [min]

Figure 4 shows the sensitivity characteristics of the MPG multi-stage positional game algorithm for safe ship control, based on the navigation condition in the Skagerrak Strait.



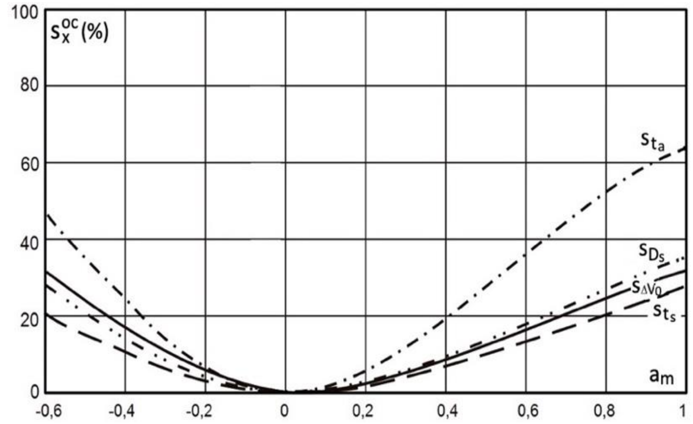
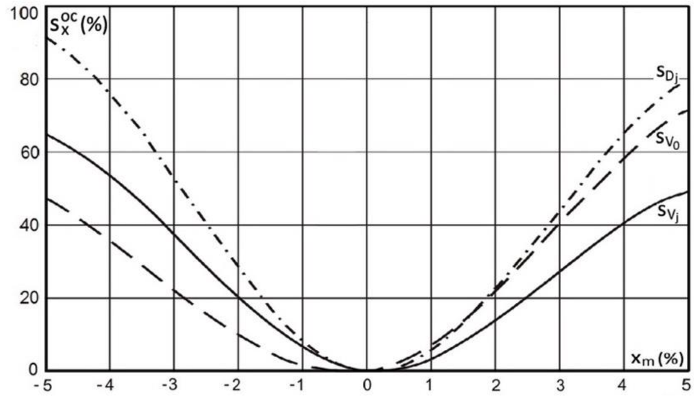


Fig. 4. Sensitivity characteristics of the MPG multi-stage positional game algorithm for safe ship control on the example of the navigation situation in the Skagerrak Strait

Figure 5 shows the sensitivity characteristics of the MMG multi-step matrix game algorithm for safe ship control, based on the navigation condition in the Skagerrak Strait.



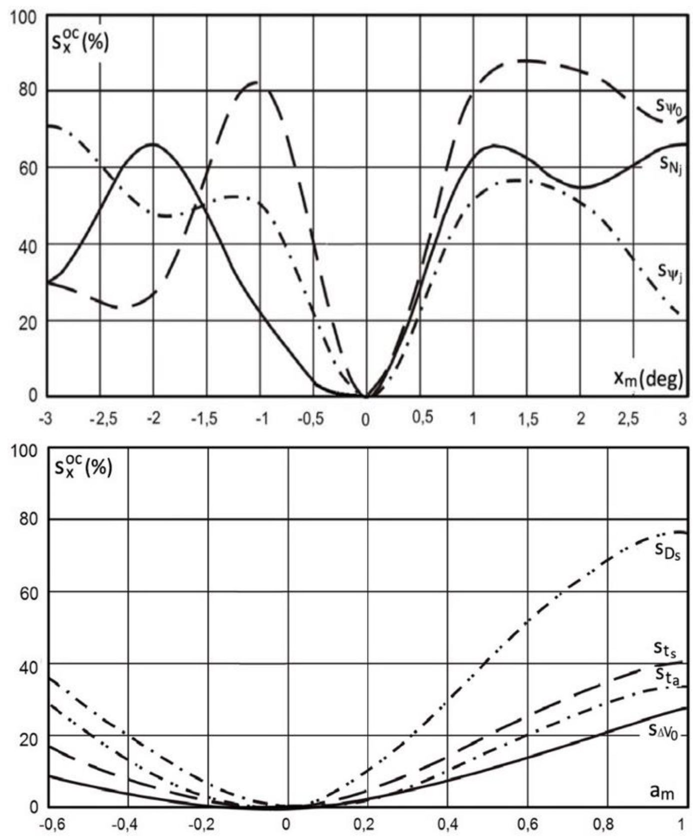


Fig. 5. Sensitivity characteristics of the MMG multi-step matrix game algorithm for safe ship control on the example of the navigation situation in the Skagerrak Strait

7 CONCLUSIONS

Using simplified differential game models for synthesizing safe and best steering allows one to determine the safe course of the ship when passing a larger number of encountered ships at some order of trajectory and speed maneuvers.

The computer programs developed consider the COLREG laws of the sea route and the maneuver advance time similar to the dynamic features of the ship. They also assess the last abberation of the actual course from the target one.

Sensitivity of the safe control of the ship's movement measured by the relative final deviation of the striker's trajectory from the final payoff given as sensitivity:

- depends on the least of the trajectory discretization period and the maneuver advance time,
- depends mostly on changes in the speed and course of your own ship and the ship you encounter,
- increases with the degree of the role-playing nature of the control process and the number of permissible strategies for one's own ship and the ships encountered.

The considered control algorithms are, in a sense, formal models of the decision-making processes of the navigating officer operating the ship and can be used to construct both appropriate training simulators at maritime universities and different alternatives of the on-board ARPA anti-collision system.

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