## 17<sup>th</sup> SYMPOSIUM ON HYDROACOUSTICS

SHA 2000

Jurata May 23-26, 2000

# NEW SIGNAL PROCESSING IN A PASSIVE SONAR WITH GRADIENT ANTENNA

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The well-known set up of an acoustic source bearing consists of four ultrasonic transducers located in the corners of a square and an additional fifth transducer placed in the centre of the square. The subtractions of the signals from the opposite transducers are proportional to sine and cosine of the angle of the arrival wave. The subtracted signals are modulated by the signal from the central transducer and treated as co-ordinates (X, Y) of the points exposed on the display with the Cartesian co-ordinate system. However such a simple solution has one serious limitation, viz., it gives satisfactory results only when a signal to noise ratio is high enough, because the noise frequency band is large compare with the bandwidth of measured signal. The paper presents an improved version of the system. Received signals are sampled and their discrete Fourier transforms are calculated. Every spectral line in DFT is presented as an individual point on the monitor. The angular position of the point shows the direction of the wave arrival and the distance from the co-ordinate origin is proportional to the power of the spectral line.

#### 1. PRINCIPLE OF OPERATION OF A SONAR

The principle of operation of passive sonar with gradient hydrophones is based on the well-known principle of operation of radio bearing devices with a crossed frame antenna. Classic devices of this type receive narrow band signals. When the signals received are broad band and the signal to noise ratio is low, there is not much sense in using a system like that. To avoid this flaw, applying discrete Fourier transformation to the signals received modified the basic principle of operation of the system. The idea of the modification, generally speaking, is to apply the classic method to each spectral line separately.

In the system in question, the gradient antenna is built of four hydrophones placed in the corners of a square with diagonal d. The dimension of d is set to make it much less than the length of wave  $\lambda$  for the highest spectrum component of signals received. Angle  $\alpha$  marks the direction of wave arrival. If the source of sound is far from the antenna, then angle  $\alpha$  is constant – and does not depend on the distance. Let us mark the signal received by the hydrophone as s(t) and assign it the number of the particular hydrophone.

Simple relations show that:

$$s_{1}(t) = s(t + \frac{d}{2c}\cos\alpha) = s(t + \tau_{c})$$

$$s_{2}(t) = s(t + \frac{d}{2c}\sin\alpha) = s(t + \tau_{s})$$

$$s_{3}(t) = s(t - \frac{d}{2c}\cos\alpha) = s(t - \tau_{c})$$

$$s_{4}(t) = s(t - \frac{d}{2c}\sin\alpha) = s(t - \tau_{s})$$

$$(1)$$

where:

$$\tau_c = \frac{d}{2c}\cos\alpha \qquad \tau_s = \frac{d}{2c}\sin\alpha$$
 (2)

In these formulas time t = 0 matches the moment of signal arrival to the central point of the antenna. After we calculate Fourier transform from formulas (1) and subtract them in pairs, we get:

$$S_{c}(j\omega) = S(j\omega) \left( e^{j\omega\tau_{c}} - e^{-j\omega\tau_{c}} \right) = 2jS(j\omega)\sin(\omega\tau_{c})$$

$$S_{s}(j\omega) = S(j\omega) \left( e^{j\omega\tau_{s}} - e^{-j\omega\tau_{s}} \right) = 2jS(j\omega)\sin(\omega\tau_{s})$$
(3)

For  $d \ll \lambda$  we can assume the following approximations:

$$\begin{split} S_{c}(j\omega) &\cong 2j\omega\tau_{c}S(j\omega) = j\omega\frac{d}{c}\cos\alpha \cdot S(j\omega) \\ S_{s}(j\omega) &\cong 2j\omega\tau_{s}S(j\omega) = j\omega\frac{d}{c}\sin\alpha \cdot S(j\omega) \end{split} \tag{4}$$

In the above formulas quotient d/c can be assumed to be constant. The amplitude of both components is a function  $j\omega$ , which makes it increase proportionally to the frequency. This unfavourable effect can be easily fixed by dividing both spectra by  $j\omega$ .

The above functions should best be treated as components of a specific vector. The value of the vector should depend on the spectrum of a signal received, with the direction depending on the angle of wave arrival. Unfortunately, formulas (4) are not suited for this interpretation because  $S(j\omega)$  is a complex number. These formulas should be transformed to make the length of the vector proportionate to the module of complex function  $S(j\omega)$ . To make the measurement of the angle explicit, what we need to know is spectrum  $S(j\omega)$ . To determine the spectrum let us add up all signal spectra. We then get:

$$S_{\Sigma}(j\omega) = S(j\omega)(e^{j\omega\tau_{c}} + e^{-j\omega\tau_{c}} + e^{j\omega\tau_{s}} + e^{-j\omega\tau_{s}}) = 2S(j\omega)(\cos\omega\tau_{c} + \cos\omega\tau_{s})$$
 (5)

For d  $<< \lambda$  the following approximation is true:

$$S_{\Sigma}(j\omega) \cong 4S(j\omega)$$
 (6)

Using this relation we can easily have formulas (4) assume the desired form. By multiplying the formulas on both sides by conjugate functions to functions written with formula (6) we get:

$$Y(j\omega) = S_c(j\omega) \cdot S_{\Sigma}^*(j\omega) = j\omega \frac{4d}{c} |S(j\omega)|^2 \cos \alpha$$

$$X(j\omega) = S_s(j\omega) \cdot S_{\Sigma}^*(j\omega) = j\omega \frac{4d}{c} |S(j\omega)|^2 \sin \alpha$$
(7)

The two above formulas can be treated as the algorithm of a passive sonar operation. The algorithm is based on Fourier transformation of signals received by the gradient antenna. It seems very useful because as we transform signals at the same time we get a measurement of the "amplitude" of the signal's spectrum and its direction.

#### 2. RECEIVING SIGNALS FROM MANY SOURCES IN NOISE BACKGROUND

To receive signals with no noise the result of mathematical operations described with formulas (7) should be imaginary functions. In practice, however, what we are going to get are complex functions, which is because noise is received together with useful signals. Next to the useful imaginary component is the real component. Useful information is contained in the imaginary component. This means that it is the imaginary component that should be used to determine the spectrum module of the signal received and the angle of wave incidence. In theory, this component should not carry any interference. In real signals the imaginary component also carries noise. This causes errors both in evaluating the value of the useful signal spectrum module and in evaluating the direction of acoustic wave arrival. The worse the input signal to noise ratio is, the bigger the errors will be.

When uncorrelated signals are received simultaneously from several directions, what we get are several vectors that are deflected by the desired angles. The length of the particular vectors depends not only on the spectrum, which is associated with the vector. It also depends on the power of the spectrum of all received signals. The situation gets more complicated when the spectra of received signals are correlated with one another. In this case a single spectral line is assigned to various sources of signals, as a result the vector of presentation is not directed towards any of the source angles. The vector direction is dependent on both on the proportions between spectrum modules and their directions. This unfavourable situation hardly ever occurs in practice because the spectra of two different sources will not overlap. This situation, however, occurs when useful signals are received in the background of a broad band noise whose spectral density can be compared to the amplitude of signal spectral lines. In this case, the noise spectrum components occurring in the same frequencies together with lines of useful signals are treated by the system as signals coming from indefinite directions.

To end the general description of the system it needs to be said that it does radically improve the signal to noise ratio compared to the broad-band system in which detection is made based on signal power measured in the entire band of the receiver. The reason for the improvement is that when using Fourier transformation we detect and estimate the direction in the narrow frequency band  $\Delta f$  whose width is equal to the width (on the frequency scale) of a single spectral line. If the width of the observed spectrum is B, the power of noise in band  $\Delta f$  is  $\Delta f/B$  less than in the entire band B. The power of the useful signal is in both cases the same. As a result, the signal to noise ratio improves when Fourier transformation is used  $\Delta f/B$  times.

## 3. EXAMPLES OF BEARINGS

The operation of the passive sonar is illustrated in the drawings given below. They are the result of computer simulation as well as real signals signal processing.

Figures 1 shows the effect of the signal to noise ratio on the accuracy of direction determination. The calculation parameters were the following: harmonic signal whose wave length is equal to  $\lambda=25d,$  wave incidence angle 30°, number of signal samples – 4096, number of sample series – 10, signal to noise ratio 20 dB and 0 dB. As you can see, even a low signal to noise ratio gives a rough idea of the wave arrival direction. Fig. 2 shows the signals spectra used in the system for determination of source direction. The errors of bearings are caused by low signal to noise ratio of differential signals  $S_c(j\omega)$  and  $S_s(j\omega)$ .

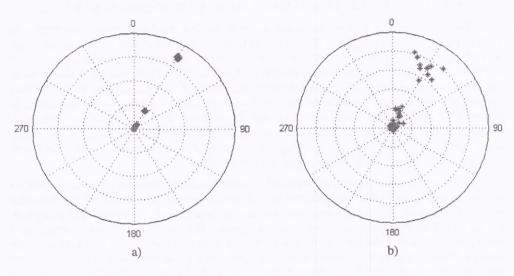


Fig. 1. The result of system operation with signal to noise ratio at a) 20 dB and b) 0 dB  $(\lambda = 25d)$ .

Fig. 3 shows the result of simulation for two signal sources of various frequencies and directions. Fig. 3a shows two sources of harmonic signal whose wavelength is equal to  $\lambda = 50 \, \mathrm{d}$  and  $\lambda = 25 \, \mathrm{d}$ , wave incidence angle - 330° and 30°, respectively. Signal to noise ratio is 20 dB for both sources. On the fig. 3b both sources have the same frequency (wavelength is equal to  $\lambda = 25 \, \mathrm{d}$ ), wave incidence angle of firth source is 30° and signal to noise ratio is 20 dB. The angle of second source is 330° and signal to noise ratio is equal to 14 dB. Number of signal samples—4096, number of sample series—10. This rare in practice case manifest itself as single source of angle, this is not equal to angles of both signals.

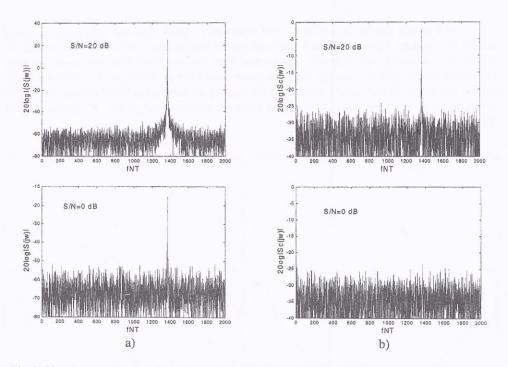


Fig. 2. Signal spectra for signal to noise ratio at 20 dB and 0 dB, ( $\lambda$  = 25 d): a) spectra of  $S_{\Sigma}$ , b) spectra of  $S_{c}$  or  $S_{s}$ .

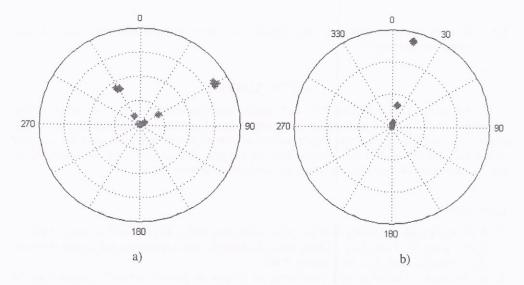


Fig. 3. The result of system operation when two harmonic signals are received: a) signals of different frequencies, b) signals of equal frequencies.

Fig. 4 shows system operation for real broadband signals. The test was made in inland waters and studied signals emitted by an underwater loudspeaker and a motor boat. Fig. 4a shows two concentrations of points. The central ones are connected with the noise, the ones on the left comes from the loudspeaker. The points generated by the boat, presented in fig. 4b, are located along the line, which shows that the bearing on the boat is fixed and that the signal isn't sinusoidal but is periodic. Every point sings the amplitude of particular spectral line of the received signal.

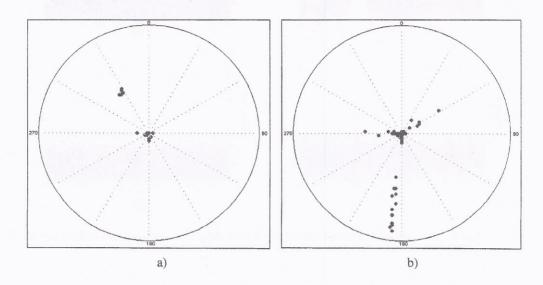


Fig. 4. Result of system operation when real signals are received: a) from a loudspeaker and b) from a motor boat in motion.

### 4. CONCLUSION

The above results of computer simulations and measurements provide proof of the usefulness of the passive method for determining sound source bearing as presented in the paper. The method enables the determination of direction and helps to keep the antenna relatively small ( $10 \div 100$  times smaller than wavelength). The desired signal to noise ratio in this method should be positive, to ensure decent bearing determination.

#### REFERENCE

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