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RELIABILITY ANALYSIS OF TWO-STATE CONSECUTIVE "*M* OUT OF *L*:F"-SERIES SYSTEMS

ANALIZA NIEZAWODNOŚCI DWUSTANOWEGO SYSTEMU PROGOWO-SZEREGOWEGO TYPU KOLEJNYCH "*M* Z *L*: F"

Sambor Guze

Gdynia Maritime University, Department of Mathematics
83-87 Morska Street, 81-226 Gdynia, Poland
E-mail: (1) sambor@am.gdynia.pl

Abstract: A non-stationary approach to reliability analysis of two-state series and consecutive "*m* out of *k*: F" systems is presented. Further, the consecutive "*m* out of *k*: F"-series system is defined and the recurrent formulae for its reliability function evaluation are proposed.

Keywords: series systems, consecutive "*m* out of *k*: F" systems

Streszczenie. Zaprezentowano niestacjonarne podejście do analizy niezawodności dwustanowych systemów szeregowych oraz progowych typu kolejnych „m z k: U". Następnie zdefiniowano dwustanowe systemy progowo-szeregowe typu kolejnych „m z k: F" oraz zaproponowano formuły rekurencyjne do obliczania niezawodności tych systemów.

Słowa kluczowe: system szeregowy, system progowy typu „m z k: F"

1. Introduction

The basic analysis and diagnosis of systems reliability are often performed under the assumption that they are composed of two-state components. It allows us to consider two states of the system reliability. If the system works its reliability state is equal to 1 and if it is failed its reliability state is equal to 0. Reliability analysis of two-state consecutive "*k* out of *n*: F" systems can be done for stationary and non-stationary case. In the first case the system reliability is the independent of time probability that the system is in the reliability state 1. For this case the main results on the reliability evaluation and the algorithms for numerical approach to consecutive "*k* out of *n*: F" systems are given for instance in (Antonopoulou, Papstavridis, 1987), (Barlow, Proschan, 1975), (Hwang, 1982), (Malinowski, Preuss, 1995), (Malinowski, 2005). Transmitting stationary results to non-stationary time dependent case and the algorithms for numerical approach to evaluation of this reliability are presented in (Guze, 2007a), (Guze, 2007b). Other more complex two-state systems are discussed in (Kołowrocki, 2004). The paper is devoted to the combining the results on reliability of the two-state series and consecutive "*m* out of *n*: F" system into the formulae for the reliability function of the consecutive "*m* out of *l*: F"-series systems with dependent of time reliability functions of system components (Guze, 2007a; Guze, 2007b).

2. Reliability of a series and consecutive "*m* out of *n*: F" systems

In the case of two-state reliability analysis of series systems and consecutive "*m* out of *n*: F" systems we assume that (Kołowrocki, 2004; Guze, 2007b):

- n is the number of system components,
- E_i, $i = 1,2,\dots,n,$ are components of a system,
- T_i are independent random variables representing the lifetimes of a components E_i, $i = 1,2,\dots,n,$
- $R_i(t) = P(T_i > t), t \in < 0,\infty),$ is a reliability function of a component E_i, $i = 1,2,\dots,n,$
- $F_i(t) = 1 - R_i(t) = P(T_i \le t), t \in < 0,\infty),$ is the distribution function of a component E_i lifetime T_i, $i = 1,2,\dots,n,$ also called an unreliability function of a component E_i, $i = 1,2,\dots,n.$

In further analysis we will use one of the simplest system structure, namely a series system.

Definition 1 A two-state system is called series if its lifetime T is given by

$$T = \min_{1 \le i \le n} \{T_i\}.$$

The scheme of a series system is given in *Figure 1.*

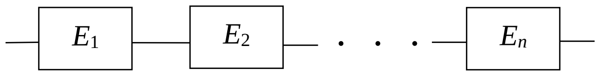

Fig. 1 The scheme of a series system

The above definition means that the series system is not failed if and only if all its components are not failed or equivalently the system is failed if at least one of its components is failed. It is easy to motivate that the series system reliability function is given by

$$\overline{\boldsymbol{R}}_n(t) = \prod_{i=1}^{n} R_i(t), \ t \in < 0, \infty). \tag{1}$$

Definition 2. A two-state series system is called homogeneous if its component lifetimes T_i have an identical distribution function

$$F(t) = P(T_i \le t), \ t \in < 0, \infty), \ i = 1,2,...,n,$$

i.e. if its components E_i have the same reliability function

$$R(t) = 1 - F(t), \ t \in < 0, \infty).$$

The above definition results in the following simplified formula

$$\overline{\mathbf{R}}_n(t) = [R(t)]^n, \ t \in < 0, \infty), \tag{2}$$

for the reliability function of the homogeneous two-state series system.

Definition 3. A two-state system is called a two-state consecutive "*m* out of *n*: F" system if it is failed if and only if at least its *m* neighbouring components out of *n* its components arranged in a sequence of $E_1, E_2, \ldots, E_n$, are failed.

After assumption that:

– T is a random variable representing the lifetime of the consecutive "*m* out of *n*: F" system,
– $\boldsymbol{CR}_n^{(m)}(t) = P(T > t), t \in < 0, \infty),$ is the reliability function of a non-homogeneous consecutive "*m* out of *n*: F" system,
– $\boldsymbol{CF}_n^{(m)}(t) = 1 - \boldsymbol{CR}_n^{(m)}(t) = P(T \le t), t \in < 0, \infty),$ is the distribution function of a consecutive "*m* out of *n*: F" system lifetime T ,

we can formulate the following auxiliary theorem [6].

Lemma 1. The reliability function of the two-state consecutive "*m* out of *n*: F" system is given by the following recurrent formula

$$
\boldsymbol{CR}_n^{(m)}(t) = \begin{cases} 1 & \text{for } n < m, \\ 1 - \prod_{i=1}^{n} F_i(t) & \text{for } n = m, \\ R_n(t)\boldsymbol{CR}_{n-1}^{(m)}(t) & \\ + \sum_{j=1}^{m-1} R_{n-j}(t)\boldsymbol{CR}_{n-j-1}^{(m)}(t) & \\ \cdot \prod_{i=n-j+1}^{n} F_i(t) & \text{for } n > m, \end{cases} \tag{3}
$$

for $t \in < 0, \infty)$.

Definition 4. The consecutive “*m* out of *n*: F“ system is called homogeneous if its components lifetimes T_i have an identical distribution function

$$F(t) = P(T_i \le t),\ i = 1,2,\ldots,n,\ t \in < 0, \infty),$$

i.e. if its components E_i have the same reliability function

$$R(t) = 1 - F(t),\ t \in < 0, \infty).$$

Lemma 1 simplified form for homogeneous systems takes the following form.

Lemma 2. The reliability of the homogeneous two-state consecutive “*m* out of *n*: F” system is given by the following recurrent formula

$$\boldsymbol{CR}_n^{(m)}(t) = \begin{cases} 1 & \text{for } n < m, \\ 1-[F(t)]^n & \text{for } n = m, \\ R(t)\boldsymbol{CR}_{n\text{-}1}^{(m)}(t) + R(t)\sum_{j=1}^{m-1} F^{j-1}(t) \cdot \boldsymbol{CR}_{n\text{-}j\text{-}1}^{(m)}(t) & \text{for } n > m, \end{cases} \tag{4}$$

for $t \in <0, \infty)$.

3. Reliability of two-state consecutive "*m* out of *l*: F"-series system

To define a two-state consecutive "*m* out of *l*: F"-series systems, we assume that

E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i,

are two-state components of the system having reliability functions

$R_{ij}(t) = P(T_{ij} > t), \ t \in <0, \infty)$,

where

T_{ij}, i = 1,2,...,k, j = 1,2,...,l_i,

are independent random variables representing the lifetimes of components E_{ij} with distribution functions

$F_{ij}(t) = P(T_{ij} \leq t), \ t \in <0, \infty)$.

Moreover, we assume that components E_{i1}, E_{i2}, …, E_{il_i}, i=1, 2,… , k, create a consecutive "m_i out of l_i: F" subsystem S_i, i=1, 2,… , k and that these subsystems create a series system.

Definition 5. A two-state system is called a consecutive "m_i out of l_i: F"-series system if it is failed if and only if at least one of its consecutive "m_i out of l_i: F" subsystems S_i, i = 1, 2,… , k, is failed.

According to the above definition and formula (4) the reliability function of the subsystem S_i is given by

$$
\boldsymbol{CR}^{(m_i)}_{i,l_i}(t) = \begin{cases} 1 & \text{for } l_i < m_i, \\ 1 - \prod_{j=1}^{l_i} F_{ij}(t) & \text{for } l_i = m_i, \\ R_{il_i}(t)\boldsymbol{CR}^{(m_i)}_{i,l_i-1}(t) & \\ + \sum_{j=1}^{m_i-1} R_{il_i-j}(t)\boldsymbol{CR}^{(m_i)}_{i,l_i\text{-}j\text{-}1}(t) & \\ \cdot \prod_{\nu=l_i-j+1}^{l_i} F_{i\nu}(t) & \text{for } l_i > m_i, \end{cases} \tag{5}
$$

and its lifetime distribution function is given by

$\boldsymbol{CF}^{(m_i)}_{l_i}(t) = 1 - \boldsymbol{CR}^{(m_i)}_{l_i}(t),$ i = 1,2,... , k.

Hence and by (1), denoting by $\overline{\boldsymbol{CR}}^{(m_1,m_2,...,m_k)}_{k,l_1,l_2,...,l_k}(t) = P(T > t),\ \ t \in < 0, \infty),$ the reliability function of the consecutive "*m* out of *l*: F"-series system, we get the next result.

Lemma 3. The reliability function of the two-state consecutive "m_i out of l_i: F"-series system is given by the following recurrent formula

$$
\overline{\boldsymbol{CR}}^{(m_1,m_2,...,m_k)}_{k,l_1,l_2,...,l_k}(t) = \prod_{i=1}^{k} \boldsymbol{CR}^{(m_i)}_{i,l_i}(t) \tag{6}
$$

$$
= \begin{cases} 1 & \text{for } l_i = m_i, \\ \prod_{i=1}^{k}[1 - \prod_{j=1}^{l_i} F_{ij}(t)] & \text{for } l_i < m_i, \\ \prod_{i=1}^{k}[R_{il_i}(t)\boldsymbol{CR}^{(m_i)}_{i,l_i-1}(t) & \\ + \sum_{j=1}^{m_i-1} R_{il_i-j}(t)\boldsymbol{CR}^{(m_i)}_{i,l_i\text{-}j\text{-}1}(t) & \\ \cdot \prod_{\nu=l_i-j+1}^{l_i} F_{i\nu}(t)] & \text{for } l_i > m_i, \end{cases} \tag{7}
$$

for $t \in <0, \infty)$.

Motivation. Assuming in (1) that $R_i(t) = \boldsymbol{CR}_{il_i}^{(m_i)}(t)$, we get (6) and next considering (5), we get the formula (7).

Definition 6. The consecutive "*m* out of *l*: F"-series systems is called regular if

$$l_1 = l_2 = \ldots = l_k = l \text{ and } m_1 = m_2 = \ldots = m_k = m,$$

where

$l, m \in N, \quad m \le l.$

Definition 7. The consecutive "m_i out of l_i: F"-series system is called homogeneous if its components lifetimes T_{ij} have an identical distribution function

$$F(t) = P(T_{ij} \le t), \; i = 1,2,\ldots, k, \; j = 1,2,\ldots, l_i, \quad t \in <0, \infty),$$

i.e. if its components E_{ij} have the same reliability function

$$R(t) = 1 - F(t), \; i = 1,2,\ldots, k, \; j = 1,2,\ldots, l_i, \; t \in <0, \infty).$$

Under *Definition 6* and *Definition 7* and formula (7), denoting by $\overline{\boldsymbol{CR}}_{k,l}^{(m)}(t) = P(T > t)$, $t \in <0, \infty)$, the reliability function of a homogeneous and regular consecutive "*m* out of *l*: F"-series system, we get following result.

Lemma 4. The reliability function of the homogeneous and regular two-state consecutive "m_i out of l_i: F"-series system is given by

$$\overline{\boldsymbol{CR}}_{k,l}^{(m)}(t) = \begin{cases} 1 & \text{for } l < m, \\ [1 - F^{l}(t)]^k & \text{for } l = m, \\ \prod_{i=1}^{k} [R(t)\boldsymbol{CR}_{i,l-1}^{(m)}(t) & \\ + R(t)\sum_{j=1}^{m-1} \boldsymbol{CR}_{i,l-j-1}^{(m)}(t) & \\ \cdot F^{j-1}(t)] & \text{for } l > m, \end{cases} \tag{8}$$

$t \in <0, \infty)$.

4. Conclusions

The paper is devoted to a non-stationary approach to reliability analysis of two-state systems. Two recurrent formulae for two-state reliability functions, a general one for non-homogeneous and its simplified form for regular and homogeneous two-state consecutive “*m* out of *l*: F”-series system have been proposed.
The proposed methods and solutions may be applied to any two-state consecutive “*m* out of *l*: F”-series systems.

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Sambor Guze is an Assitant in Department of Mathematics at the Faculty of Navigation of Gdynia Maritime University. His field of interest is mathematical modeling reliability of consecutive systems. He has published 10 papers in scientific journals and conference proceedings.