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ANALYTICAL-EXPERIMENTAL METHOD OF DETERMINING FATIGUE CHARACTERISTICS FOR DESIGN ELEMENTS

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Summary

Frequently during the initial designing stage, one must use fatigue characteristics of the design element. Producing such a plot by making an experiment is not possible at most times. This paper suggests determining such characteristics based on the fatigue plot for the material. The assumptions of that method have been developed drawing on own experiment and experimental data presented in literature. The name 'analytical and experimental' assigned to the method proposed comes from the fact that it is based on the description of the fatigue properties of the material received experimentally and phenomenological properties received experimentally presented in literature. In order to verify the method proposed, a comparative plot was made for the experimental and analytical fatigue life. Based on the curve plotted, one can demonstrate that the fatigue life predicted according to own proposal range in the assumed scatter band. For the purpose of a comparison, developing the fatigue characteristics according to the FITNET method. As for this algorithm, it was found that the fatigue limit determined according to this method is much greater than the one received experimentally, which means that the method does not work for materials heavily sensitive to the effect of the notch, including e.g. material C45+C.

Key words: *high-cycle fatigue strength, fatigue curves, analytical methods of estimating Wőhler's curve.*

1. Introduction

The designer, for an adequate dimensioning of a design element, must be provided with an adequate fatigue characteristics of this detail. Acquiring such experimental characteristics experimentally is time-consuming and costly. Often at the initial stage of the design, performing experiments used to acquire such characteristics is impossible.

With the above in mind, literature reports on many analytical methods to determine the approximate fatigue curve, e.g. [4] and [6]. Unfortunately there is no data on the possible error the designer can make using one of them. The attempt at verifying the above methods are given in e.g. [7,10,11,12].

This paper presents the proposal of determining the fatigue characteristics for design element based on the material characteristics. Additionally the FITNET method and the characteristics defined according to this algorithm for material C45+C.

The name 'analytical-experimental' used for the proposed method comes from the fact that it is based on the description of the material fatigue properties received experimentally and phenomenological properties received experimentally, reported in literature.

2. Relationship between the fatigue characteristics of the material and a design element

Being exposed to the load variable in time, strength is of high importance, e.g. notch coefficient K_t defined as the ratio of the value of the maximum stress at the bottom of the notch to nominal stress. The value of that coefficient depends on the shape of the design element. As for the element with no change in the cross-section (e.g. smooth normative specimens) the value of the notch coefficient is ~1. As for the monotonic load, the effect of the changes in cross-section of the element on strength does not occur. It is common knowledge that the effect increases with the executed number of cycles until base number of cycles N_0 is reached. In the publication [9] the author presents that it is possible to make an approximation stating that the effect of the notch coefficient and other factors disappears for 10^3 number of cycles, which is seen from the literature review involving the determination of the number of cycles at which the fatigue strength of the notched specimen and the smooth one is the same. The results are broken down in Table 1, showing that the range of variation of the number of cycles of the crossing of the curves for the notched specimens and smooth specimens ranges from $1.15 \cdot 10^2$ to $7.24 \cdot 10^3$. Based on that range, it seems justifiable to assume the value of 10^3 cycles as the strength at which the notch effect disappears. Based on this assumption own method has been proposed to allow for determining fatigue characteristics of the design element based on the material characteristics. This proposal has been presented in the next item.

Table 1. Fatigue strength and fatigue life in the point of curves crossing for the material and the design element

Material	Source	Slope coefficient for material m	Notch coefficient K_t	Slope coefficient for notched specimen m_k	Number of cycles from the curves crossing	Strength value in the point of crossing σ_p [MPa]	Ultimate strength R_m [MPa]	Ratio σ_p/R_m
25CrMo4	[9]	13.1	2.16	5.9	325	663	806	0.823
25CrMo4	[9]	13.1	4.0	4.5	181	693	806	0.860
42CrMoS4	[8]	14.0	1.75	5.55	3329	841	1100	0.765
14CrMoV69	[1]	16.1	2.0	4.5	1374	887	980	0.905
C45+C	Own tests	8.1	2.2	3.9	1068	775	826	0.940
C45+N	[3]	11.0	1.65	4.56	7240	443	730	0.607
39NiCrMo3	[1]	8.2	7.2	4.8	484	884	995	0.888
En3B (C22E)	[13]	19.7	3.8	3.75	4981	453	678	0.668
SUJ2 (100Cr6)	[14]	21.5	2.39	9.0	465	1430	2241	0.638
4140 (42CrMo4)	[5]	13.7	2.11	6.3	115	970	1100	0.882
4140 (42CrMo4)	[5]	13.7	5.03	3.1	1565	802	1100	0.729
Low-carbon steel	[2]	11.5	2.5	6.5	258	547	500	1.094
Low-carbon steel	[2]	11.5	2.9	5.8	309	538	500	1.077
Range of variation					115-7240			0.607-1.094

Additionally the table above presents the strength values in the point of crossing of experimental curves for smooth and notched specimens. Based on those values the ratios of strength in the point of crossing and tensile strength were calculated.

3. Method proposed

The method involves the determination of the points characteristic for the curve. Those are the point of crossing in the region of fatigue limit and the point limiting the range of limited fatigue life (the region of about 10^3 cycles).

The method proposed is based on plotting the fatigue curve for the material by making an experiment. Based on that characteristics, strength σ_3 is determined for fatigue life of 10^3 number of cycles. Yet another step is determining fatigue limit for the notched element - Z_{Gk}. Next we plot the curve with two points showing coordinates (10^3,σ_3) and (10^6,Z_{GK}). The schematic procedure is given in Fig. 1.

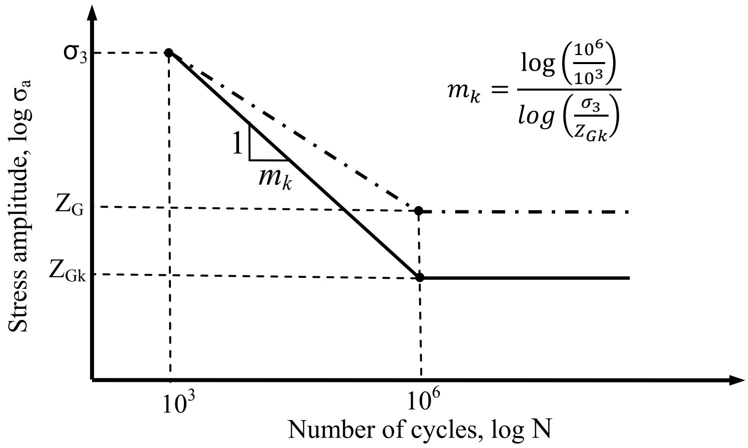

Fig. 1. Proposed method of determining the fatigue characteristics for design element (solid line) based on the fatigue characteristics of the material (dash and point line).

4. Method of determining the fatigue limit

The method of determining the fatigue limit for the material according to the FITNET method [6] involves multiplying tensile strength (R_m) by coefficient $f_{W,\sigma}$ dependent on the type of the material. The values of that coefficient are given in Table 2.

Table 2. Values of coefficient $f_{W,\sigma}$ derived from [6]

Material type	$f_{W,\sigma}$
Carburized steel	0.40
Stainless steel	0.40
Forged steel	0.40
Cast steel	0.34
Steels other than above	0.45
Sferoid cast-iron	0.34
Malleable cast-iron	0.30
Grey cast iron	0.30
Plastic-worked aluminium alloys	0.30
Cast aluminium alloys	0.30

For the notched elements, the fatigue limit for the material must be multiplied by coefficients: size K_d, surface roughness K_S and the operation of notch η. To determine the coefficient determined at the end, there must be calculated stress gradient at the bottom of the notch, according to the formula:

$$\chi = -\frac{d\sigma_a}{dx}\bigg|_{x=0} \frac{1}{\sigma_{a,(x=0)}} \tag{1}$$

Then we calculate coefficient η according to the formula:

$$\eta = \begin{cases} 1 + \chi \cdot \mathrm{mm} \cdot 10^{-\left[a_G - 0.5 + \frac{Rm}{b_G \ \mathrm{a}}\right]} & \text{dla } \chi < 0.1\mathrm{mm}^{-1} \\ 1 + \chi \cdot \mathrm{mm} \cdot 10^{-\left[a_G + \frac{Rm}{b_G \ \mathrm{a}}\right]} & \text{dla } 0\ 1\ \ \chi < 1\mathrm{mm}^{-1} \\ 1 + \sqrt[4]{\chi} \cdot \mathrm{mm} \cdot 10^{-\left[a_G + \frac{Rm}{b_G \ \mathrm{a}}\right]} & \text{dla } \chi > 1\mathrm{mm}^{-1} \end{cases} . \tag{2}$$

Coefficients a_G and b_G are read from the table below.

Table 3. Values of coefficients a_G and b_G

Material	a_G	b_G
Stainless steel	0.4	2400
Other steels	0.5	2700
Cast steel	0.25	2000
Spheroidal cast iron	0.05	3200
Malleable cast-iron	-0,05	3200
Grey cast iron	-0,05	3200
Plastic-worked aluminium alloys	0.05	850
Cast aluminium alloys	-0.05	3200

For material C45+N (the data derived from publication [3]) tested using the specimens showing coefficient K_t equal 1.65 the fatigue limit was defined according to the FITNET method. The material fatigue limit value determined was 292 MPa, calculated by multiplying tensile strength (730 MPa) by coefficient $f_{W,\sigma}$ equal 0.45. Then there was calculated the value of gradient of stress which was 2.4 mm^{-1}. By substituting the value calculated to formula 2 we receive value of coefficient η equal 0.781. The fatigue limit of the notched specimen was received by multiplying the material fatigue limit by coefficient η and the value of 230 MPa was received. The value received differs considerably from the value received experimentally being 165 MPa considerably. With the experimental values reported, it was found that a given material shows a high value of the coefficient of material sensitivity to the effect of notch η_k which assumes values from 0 to 1. This coefficient is used to calculate the coefficient of the operation of notch K_f (defined as the ratio of material fatigue limit Z_G and the fatigue limit of the notched specimen Z_{Gk}) expressed with the formula [3]:

$$K_f = 1 + \eta_k(K_t - 1), \tag{3}$$

For the material analysed, coefficient K_f when substituting η_k with value 1 we receive the value equal K_t, namely 1.65. Calculating the fatigue limit of the notched specimen, dividing the material fatigue limit Z_G by coefficient K_f we receive the value of 169.7 MPa (280 MPa/1.65). To provide the characteristics for the notched specimens from material C45+N according to own proposal, there was assumed the value of the fatigue limit equal 169.7 MPa.

Similar material properties was noted in material C45+C which has the same chemical composition as material C45+N, however, it is in another state of the treatment. The value of the fatigue limit for material C45+C according to the FITNET method was 330.4 MPa. As for the notched specimens with K_t equal 2.2 there was calculated the fatigue limit equal 224.7 MPa (R_m = 826 MPa, χ = 4.61 mm^{-1}, η = 0.77). To define the fatigue limit for notched specimens applying coefficient K_f equal K_t we receive the value of 142.9 MPa (314.4 MPa / 2.2), which is closer to the experimental value of 139 MPa. To provide the characteristics according to the proposed algorithm, the fatigue limit for the notched specimens with material C45+C, the value of 142.9 MPa was assumed.

5. Method of verification

To verify the proposed method, there was made an experiment to determine fatigue properties of smooth and notched specimens of coefficient K_t equal 2.2 made from material C45+C. The curve is presented in Fig. 3 in which the red line stands for the results for notched specimen, while black line – for the results for smooth specimens.

With the points received from the experiment, it was possible to plot a curve to compare the experimental fatigue life with the estimated fatigue life (Fig. 4). The figure provides the black line for the situation when the experimental life equals the evaluated fatigue life (the desired situation) stand for the scatter band which was determined according to the following equations:

$$N_{pg} = s \cdot N_f \tag{4}$$

$$N_{pd} = s \cdot N_f \tag{5}$$

where:
N_f – value of the fatigue life received experimentally,
s – coefficient of the scatter band (there was assumed the value of 3),
N_{pg} – fatigue life value for the upper interval of the scatter band,
N_{pd} – value of fatigue life for the lower interval of the scatter band.

6. Results of the verification

Figs 2 and 3 presents fatigue curves received from the experiment for smooth and notched specimens and estimated curves according to the FITNET method and own proposal. Additionally Fig. 4 presents the curve to compare the fatigue life evaluated to the experimental fatigue life.

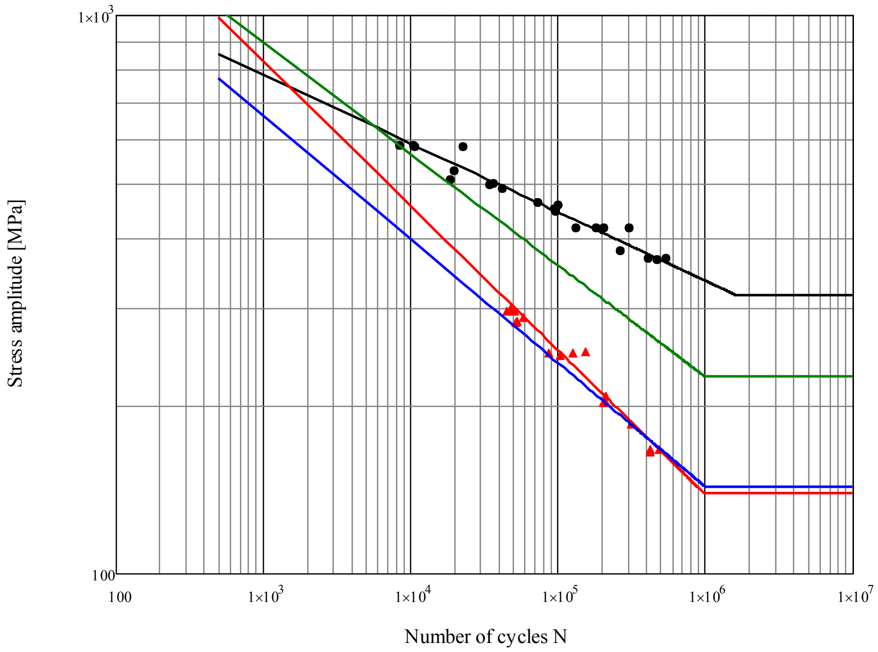

Fig. 2 Fatigue curve for steel C45+C – the black line is smooth, the red line – the circumferential notch K_t=2.2, green line stands for the green curve estimated according the FITNET method and blue line estimated according to own proposal

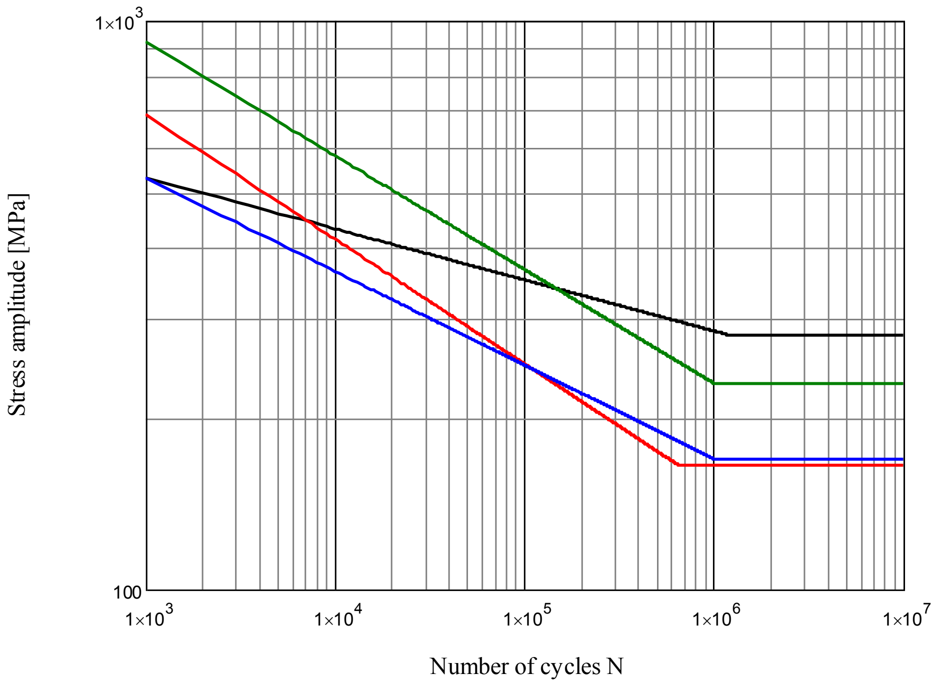

Fig. 3 Fatigue curve for steel C45+N– black line – smooth specimen [3], red line – the specimen with circumferential notch K_t=1.65 [3], green line – the curve estimated according to the FITNET method and blue line estimated according to the own proposal

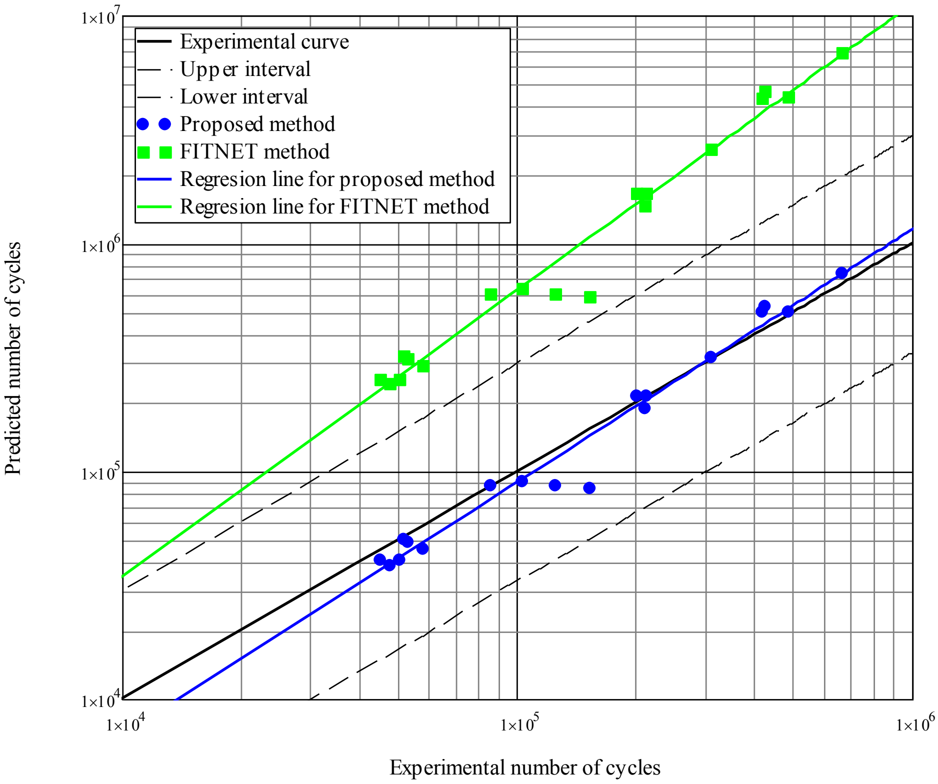

Fig. 4 Comparative curve for fatigue life received experimentally and with the estimated fatigue life according to the method proposed (blue colour) and the FITNET method (green colour)

7. Conclusions

Based on the curves plotted, one can state that the method of determining the fatigue limit according to the FITNET method results in a big error in the case of materials showing high sensitivity to the notch effect. Defining the characteristics for that group of materials can cause a high error in the fatigue life evaluation. Fig. 4 shows that the values estimated with that method are beyond the assumed scatter band, which means that the evaluating the fatigue life is over 3-fold of the experimental value.

As for determining the plot according to the method proposed where determining the fatigue limit the coefficient of the operation of the notch was used; it was assumed at the level equal to the notch coefficient. The characteristics provided are similar to the figure received experimentally. It is an clearly visible in Fig. 4 where the points corresponding to the experimental fatigue life and the evaluated one fall within the assumed scatter band.

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