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EVALUATION OF THE SELECTED FACTORS EFFECT ON THE FATIGUE LIFE OF SPECIMENS WITH SIZED AND RIVETED HOLE PART II. STATISTICAL ANALYSES OF EXPERIMENTAL RESULTS

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Abstract

Evaluation of the fatigue load, the rivet squeezing force, the velocity of rivet close up and the hole diameter before sizing (after drilling) effect on the fatigue life of specimens with sized and riveted hole was presented in this paper. The work contains two parts. Design of experiment and the results of the fatigue tests performed by the described experiment plan was presented in the first part. The statistical analyses and examples of using the mathematical model of the experimental unit were presented in the second part.

Keywords: *riveted joints, fatigue life, rivet hole sizing, design of experiment*

1. Introduction

The second part of the work about evaluation of the selected factors effect on the fatigue life of specimens with sized and riveted hole was presented in this paper. The part contains the statistical analyses of experimental results and examples of using the mathematical model of the experimental unit.

2. Statistical analyses

2.1. Elimination of results with gross error

The Grubbs' test was used to elimination of results with gross error. The mean value of the dependent variable for each set of the independent variables *u* was determined by using following equation

$$\bar{y}_u = \frac{\sum_{i=1}^{r} y_{u/i}}{r} \tag{1}$$

and standard deviation

$$s_u = \sqrt{\frac{\sum_{i=1}^{r} \left(y_{u/i} - \bar{y}_u\right)^2}{r-1}} \tag{2}$$

where:
$r = 3$ – a number of repetition the same for each set,
$y_{u/i}$ – a result number i from r repetitions for set number u.

The test statistic was determined separately for the maximum and the minimum value of the dependent variable for each set of the independent variables. It was determined for the largest and for the least value of the dependent variable by using an equation

$$B_{\max} = \frac{\max\left(y_{u/i}\right) - \bar{y}_u}{s_u}, \qquad B_{\min} = \frac{\bar{y}_u - \min\left(y_{u/i}\right)}{s_u}. \tag{3}$$

The critical value of the test statistic $B_{\alpha;r} = 1.412$ was determined for the number of repetition $r = 3$ and the significance level $\alpha = 0.05$ [3]. Results of calculation were presented in tab. 1.

Because for each set of independent variables

$$B_{max} < B_{\alpha;r} \tag{4}$$

and

$$B_{min} < B_{\alpha;r}. \tag{5}$$

there is no reason to rejection the maximal and minimal values for each set as gross error results.

Tab. 1. Summary of calculation for results with gross error elimination

No. of set u	Coded independent variables				Dependent variable		Grubbs' statistics	
	$\breve{x}_1$	$\breve{x}_2$	$\breve{x}_3$	$\breve{x}_4$	$\bar{y} = \log \overline{N}$	s_u	B_{max}	B_{min}
1	-1	-1	-1	-1	5.785	0.007	0.852	1.101
2	1	-1	-1	-1	5.078	0.038	0.895	1.079
3	-1	1	-1	-1	5.796	0.013	1.104	0.846
4	1	1	-1	-1	5.083	0.044	0.914	1.068
5	-1	-1	1	-1	5.788	0.009	0.880	1.088
6	1	-1	1	-1	5.061	0.027	1.115	0.818
7	-1	1	1	-1	5.793	0.009	1.055	0.935
8	1	1	1	-1	5.094	0.033	0.819	1.115
9	-1	-1	-1	1	5.228	0.030	1.087	0.881
10	1	-1	-1	1	4.684	0.084	1.011	0.989
11	-1	1	-1	1	5.285	0.018	1.133	0.758
12	1	1	-1	1	4.734	0.060	0.793	1.123
13	-1	-1	1	1	5.212	0.022	0.853	1.101
14	1	-1	1	1	4.657	0.055	0.801	1.121
15	-1	1	1	1	5.269	0.029	0.858	1.098
16	1	1	1	1	4.711	0.052	1.121	0.801
17	$+\alpha$	0	0	0	4.680	0.087	1.022	0.977
18	$-\alpha$	0	0	0	6.065	0.036	0.823	1.113
19	0	$+\alpha$	0	0	5.284	0.042	1.079	0.895
20	0	$-\alpha$	0	0	5.234	0.026	1.022	0.976
21	0	0	$+\alpha$	0	5.291	0.027	0.886	1.084
22	0	0	$-\alpha$	0	5.298	0.026	1.073	0.907
23	0	0	0	$+\alpha$	4.636	0.048	0.959	1.036
24	0	0	0	$-\alpha$	5.562	0.033	1.119	0.806
25	0	0	0	0	5.319	0.031	0.976	1.023
26	0	0	0	0	5.282	0.023	0.897	1.078
27	0	0	0		5.306	0.029	1.026	0.972
28	0	0	0	0	5.271	0.038	0.862	1.096
29	0	0	0	0	5.279	0.038	0.980	1.019
30	0	0	0	0	5.312	0.018	1.127	0.782
31	0	0	0	0	5.274	0.025	1.084	0.887

2.2. Inter-row variance and standard deviation

Inter-row variance was calculated from equation [3]

$$s_u^2 = \frac{\sum_{i=1}^{r}\left(y_{u/i} - \bar{y}_u\right)^2}{r-1}, \tag{6}$$

and standard deviation from

$$s_u = \sqrt{s_u^2}\,. \tag{7}$$

Homogeneity of variance in a sample was checked with Cochran's C-test (the same number of repetition for each set of independent variables $r = 3$).

The test statistic was determined by equation

$$G = \frac{\max\left(s_u^2\right)}{\sum_{j=1}^{u} s_u^2} = 0.1610, \tag{8}$$

where $\max\left(s_u^2\right) = 0.007530$ and $\sum_{j=1}^{u} s_u^2 = 0.046\,778$ were determined by using results of equations (6) and (7).

The Cochran's test critical value $g_{\alpha;k;v} = 0.1940$ was determined for the significance level $\alpha = 0.05$ and degrees of freedom $k = N = 31$ and $v = r - 1 = 2$ [3].

Because

$$G < g_{\alpha;k;v}, \tag{9}$$

there is no reason to rejection the hypothesis about homogeneity of variance.

2.3. Determination of coefficients in regression function in coded form

Coefficients in regression function in coded form were determined by following equations:

$$k_0 = D \cdot (0y) + E \cdot \sum_{s=1}^{4}(ssy), \tag{10}$$

$$k_s = \frac{(s\bar{y})}{e} = (s\bar{y}) \cdot e^{-1}, \tag{11}$$

$$k_{(s-1)s} = \frac{((s-1)s\bar{y})}{n_k} = ((s-1)s\bar{y}) \cdot n_k^{-1}, \tag{12}$$

$$k_{ss} = (F - G) \cdot (ss\bar{y}) + G \cdot \sum_{s=1}^{3}(ss\bar{y}) + E \cdot (0\bar{y}), \tag{13}$$

where auxiliary factors values D= 0.1428, E = -0.0357, F = 0.035, G = 0.0037 oraz e^{-1} = 0.0416 were selected from [4] and with agree with the plan of the experiment n_k^{-1} = 0.0625.

Auxiliary factors values were calculated from equations:

$$(0y) = \sum_{u=1}^{N} y_u = 162.35, \tag{14}$$

$$(1y) = \sum_{u=1}^{N} \tilde{x}_{1u} \cdot y_u = -7.824, \tag{15}$$

$$(2y)=\sum_{u=1}^{N}\breve{x}_{2u}\cdot y_u\ = 0.372, \qquad (16)$$

$$(3y)=\sum_{u=1}^{N}\breve{x}_{3u}\cdot y_u\ = \text{-}0.104, \qquad (17)$$

$$(4y)=\sum_{u=1}^{N}\breve{x}_{4u}\cdot y_u\ = \text{-}5.549, \qquad (18)$$

$$(12y)=\sum_{u=1}^{N}\breve{x}_{1u}\cdot\breve{x}_{2u}\cdot y_u = 0.014, \qquad (19)$$

$$(13y)=\sum_{u=1}^{N}\breve{x}_{1u}\cdot\breve{x}_{3u}\cdot y_u = \text{-}0.025, \qquad (20)$$

$$(14y)=\sum_{u=1}^{N}\breve{x}_{1u}\cdot\breve{x}_{4u}\cdot y_u = 0.639, \qquad (21)$$

$$(23y)=\sum_{u=1}^{N}\breve{x}_{2u}\cdot\breve{x}_{3u}\cdot y_u = 0.024, \qquad (22)$$

$$(24y)=\sum_{u=1}^{N}\breve{x}_{2u}\cdot\breve{x}_{4u}\cdot y_u = 0.164, \qquad (23)$$

$$(34y)=\sum_{u=1}^{N}\breve{x}_{3u}\cdot\breve{x}_{4u}\cdot y_u = \text{-}0.075, \qquad (24)$$

$$(11y)=\sum_{u=1}^{N}\breve{x}_{1u}^{2}\cdot y_u = 126.236, \qquad (25)$$

$$(22y)=\sum_{u=1}^{N}\breve{x}_{2u}^{2}\cdot y_u = 125.328, \qquad (26)$$

$$(33y)=\sum_{u=1}^{N}\breve{x}_{3u}^{2}\cdot y_u = 125.614, \qquad (27)$$

$$(44y)=\sum_{u=1}^{N}\breve{x}_{4u}^{2}\cdot y_u = 124.049, \qquad (28)$$

Coefficients in regression function in coded form were determined by equations (10)÷(13):

$$k_0 = D\cdot(0\bar{y})+E\cdot\sum_{s=1}^{3}(ss\bar{y}) = D\cdot(0\bar{y})+E\cdot(11\bar{y}+22\bar{y}+33\bar{y}+44\bar{y}) = 5.290, \qquad (29)$$

$$k_1 = (1\bar{y})\cdot e^{-1} = -0.3255\,, \qquad (30)$$

$$k_2 = (2y)\cdot e^{-1} = 0.01546\,, \qquad (31)$$

$$k_3 = (3\bar{y})\cdot e^{-1} = -0.004326\,, \qquad (32)$$

$$k_4 = (4\bar{y})\cdot e^{-1} = -0.2308\,, \qquad (33)$$

$$k_{12} = (12\bar{y})\cdot n_k^{-1} = 0.0008602\,, \qquad (34)$$

$$k_{13} = (13\bar{y})\cdot n_k^{-1} = -0.001541\,, \qquad (35)$$

$$k_{14} = (14\bar{y})\cdot n_k^{-1} = 0.03996\,, \qquad (36)$$

$$k_{23} = (23\bar{y}) \cdot n_k^{-1} = 0.001480\,, \tag{37}$$

$$k_{24} = (24\bar{y}) \cdot n_k^{-1} = 0.01023\,, \tag{38}$$

$$k_{34} = (34\bar{y}) \cdot n_k^{-1} = -0.004681\,, \tag{39}$$

$$k_{11} = (F - G) \cdot (11\bar{y}) + G \cdot (11\bar{y} + 22\bar{y} + 33\bar{y} + 44\bar{y}) + E \cdot (0\bar{y}) = 0.009829\,, \tag{40}$$

$$k_{22} = (F - G) \cdot (22\bar{y}) + G \cdot (11\bar{y} + 22\bar{y} + 33\bar{y} + 44\bar{y}) + E \cdot (0\bar{y}) = -0.01858\,, \tag{41}$$

$$k_{33} = (F - G) \cdot (33\bar{y}) + G \cdot (11\bar{y} + 22\bar{y} + 33\bar{y} + 44\bar{y}) + E \cdot (0\bar{y}) = -0.009661\,, \tag{42}$$

$$k_{44} = (F - G) \cdot (44\bar{y}) + G \cdot (11\bar{y} + 22\bar{y} + 33\bar{y} + 44\bar{y}) + E \cdot (0\bar{y}) = -0.05862\,. \tag{43}$$

2.4. Statistical analysis of regression function

The Student's t-test was used for significance rating of coefficients in regression function. The inter-row variance in the centre of the experiment plan was determined by equations

$$s_e^2 = \frac{S_E}{f_E} = 0.000\,388\,985\,, \tag{44}$$

and

$$S_E = \sum_{u=n_k+n_\alpha+1}^{N} \left(\bar{y}_{0u} - \bar{\bar{y}}_0\right)^2\,, \tag{45}$$

$$\bar{\bar{y}}_0 = \frac{\sum_{u=n_k+n_\alpha}^{N} \bar{y}_{0u}}{n_0}\,, \tag{46}$$

$$f_E = n_0 - 1 = 6, \tag{47}$$

where:

$\bar{y}_{0u}$ – the mean values of the dependent variable in the centre of experiment plan,

$\bar{\bar{y}}_0$ – the mean value of the dependent variable values in the centre of experiment plan,

n_k – a number of sets for full factorial plan PS/DK-2^n,

n_α – a number of sets for axial points,

n_0 – a number of sets for central points,

N – a total number of sets in the plan of experiment,

f_E – degrees of freedom.

Auxiliary factors values for calculations were presented in tab. 2.

The variance of coefficients in regression function were calculated by following equations

$$s^2(k_0) = D \cdot s_e^2 \tag{48}$$

$$s^2(k_s) = \frac{s_e^2}{e} = s_e^2 \cdot e^{-1} \tag{49}$$

$$s^2\left(k_{(s-1)s}\right) = \frac{s_e^2}{n_k} = s_e^2 \cdot n_k^{-1} \tag{50}$$

$$s^2(k_{ss}) = F \cdot s_e^2 \tag{51}$$

Tab. 2. Auxiliary factors values for variance in the centre of experiment plan determination

No. of set u	Coded independent variables				Dependent variable	Auxiliary factors
	$\breve{x}_1$	$\breve{x}_2$	$\breve{x}_3$	$\breve{x}_4$	$\bar{y}_{0u}$	$\left(\bar{y}_{0u} - \bar{\bar{y}}_0\right)^2$
25	0	0	0	0	5.319	0.000 736
26	0	0	0	0	5.282	0.000 094
27	0	0	0	0	5.306	0.000 205
28	0	0	0	0	5.271	0.000 432
29	0	0	0	0	5.279	0.000 168
30	0	0	0	0	5.312	0.000 387
31	0	0	0	0	5.274	0.000 312
					$\bar{\bar{y}}_0 = 5.292$	$S_E = 0.002\ 334$

The standard deviation of coefficients in regression function were calculated by equations

$$s(k_0) = \sqrt{s^2(k_0)} \tag{52}$$

$$s(k_s) = \sqrt{s^2(k_s)} \tag{53}$$

$$s\left(k_{(s-1)s}\right) = \sqrt{s^2\left(k_{(s-1)s}\right)} \tag{54}$$

$$s(k_{ss}) = \sqrt{s^2(k_{ss})} \tag{55}$$

The test statistic was determined by equation

$$t_0 = t(k_0) = \frac{|k_0|}{s(k_0)} \tag{56}$$

$$t_s = t(k_s) = \frac{|k_s|}{s(k_s)} \tag{57}$$

$$t_{(s-1)s} = t\left(k_{(s-1)s}\right) = \frac{\left|k_{(s-1)s}\right|}{s\left(k_{(s-1)s}\right)} \tag{58}$$

$$t_{ss} = t(k_{ss}) = \frac{|k_{ss}|}{s(k_{ss})} \tag{59}$$

The significance rating of coefficients in regression function were presented in tab. 3.

The critical value of Student's t-test $t_{\alpha; f_E} = 2.45$ was determined for the significance level $\alpha = 0.05$ and degrees of freedom f_E.

If following equation is satisfied

$$t_i \geq t_{\alpha; f_E} \tag{60}$$

then there is no reason to reject null hypothesis about insignificance of a coefficient in regression function i.e. the coefficient is significant for dependent variable.

If following equation is satisfied

$$t_i < t_{\alpha; f_E} \tag{61}$$

then there is the reason to accept null hypothesis about insignificance of a coefficient in regression function in statistical sense with the significance level i.e. the coefficient is insignificant for dependent variable and it can be omit in regression function.

Tab. 3. The significance rating of coefficients in regression function

Coefficient	Variance of coefficient	Standard deviation	Test statistic	Student's t-test	Test results
k_i	$s^2(k_i)$	$s(k_i)$	$t_i = t(k_i)$		
k_0	0.000 055 547	0.007 453	709.76	$t_i \geq t_{\alpha; f_E} = 2.45$	significant
k_1	0.000 016 182	0.004 023	80.91	$t_i \geq t_{\alpha; f_E} = 2.45$	significant
k_2	0.000 016 182	0.004 023	3.84	$t_i \geq t_{\alpha; f_E} = 2.45$	significant
k_3	0.000 016 182	0.004 023	1.08	$t_i < t_{\alpha; f_E} = 2.45$	insignificant
k_4	0.000 016 182	0.004 023	57.38	$t_i \geq t_{\alpha; f_E} = 2.45$	significant
k_{12}	0.000 024 312	0.004 931	0.17	$t_i < t_{\alpha; f_E} = 2.45$	insignificant
k_{13}	0.000 024 312	0.004 931	0.31	$t_i < t_{\alpha; f_E} = 2.45$	insignificant
k_{14}	0.000 024 312	0.004 931	8.10	$t_i \geq t_{\alpha; f_E} = 2.45$	significant
k_{23}	0.000 024 312	0.004 931	0.30	$t_i < t_{\alpha; f_E} = 2.45$	insignificant
k_{24}	0.000 024 312	0.004 931	2.07	$t_i \geq t_{\alpha; f_E} = 2.45$	insignificant
k_{34}	0.000 024 312	0.004 931	0.95	$t_i < t_{\alpha; f_E} = 2.45$	insignificant
k_{11}	0.000 013 614	0.003 690	2.66	$t_i \geq t_{\alpha; f_E} = 2.45$	significant
k_{22}	0.000 013 614	0.003 690	5.04	$t_i \geq t_{\alpha; f_E} = 2.45$	significant
k_{33}	0.000 013 614	0.003 690	2.62	$t_i \geq t_{\alpha; f_E} = 2.45$	significant
k_{44}	0.000 013 614	0.003 690	15.89	$t_i \geq t_{\alpha; f_E} = 2.45$	significant

2.5. Significance rating of multivariate correlation coefficient

The multivariate correlation coefficient can be the fitting measure of regression function to experiment results

$$R = \sqrt{1 - \frac{\sum_{u=1}^{N} (\hat{y}_u - \bar{y}_u)^2}{\sum_{u=1}^{N} (\bar{y}_u - \bar{\bar{y}})^2}} = \sqrt{1 - \frac{0.024133}{3.996}} = 0.997\,, \tag{62}$$

where the mean value of dependent variable for set u with r number of repetition

$$\bar{y}_u = \frac{\sum_{i=1}^{r} y_{u/i}}{r}\,, \tag{63}$$

and the mean value of dependent variable for experimental unit for *N* sets

$$\bar{\bar{y}} = \frac{\sum_{u=1}^{N} \bar{y}_u}{N}. \tag{64}$$

The output variable values of mathematical model of experimental unit for set *u* of input variables were calculated by regression function (without insignificant coefficients rejection)

$$\begin{aligned} \hat{y} = k_0 + k_1 \cdot \hat{x}_1 + k_2 \cdot \hat{x}_2 + k_3 \cdot \hat{x}_3 + k_4 \cdot \hat{x}_4 + \\ + k_{11} \cdot \hat{x}_1^2 + k_{22} \cdot \hat{x}_2^2 + k_{33} \cdot \hat{x}_3^2 + k_{44} \cdot \hat{x}_4^2 + \\ + k_{12} \cdot \hat{x}_1 \cdot \hat{x}_2 + k_{13} \cdot \hat{x}_1 \cdot \hat{x}_3 + k_{14} \cdot \hat{x}_1 \cdot \hat{x}_4 + \\ + k_{23} \cdot \hat{x}_2 \cdot \hat{x}_3 + k_{24} \cdot \hat{x}_2 \cdot \hat{x}_4 + k_{34} \cdot \hat{x}_3 \cdot \hat{x}_4. \end{aligned} \tag{65}$$

The Snedecor's F-test was used for the multivariate correlation coefficient significance determination.

The test statistics was determined by equation

$$F = \frac{N-L}{L-1} \cdot \frac{R^2}{1-R^2} = 188.11, \tag{66}$$

where:
$N = 31$ – the total number of sets in experimental plane,
$L = 15$ – a number of coefficients in regression function.

The critical Snedecor's F-test value $F_{\alpha;r_1;r_2} = 2.37$ was determined for the significance level $\alpha = 0{,}05$ and degrees of freedom $r_1 = L - 1 = 14$ and $r_2 = N - L = 16$.

Because

$$F > F_{\alpha;r_1;r_2} \tag{67}$$

there is no reason to rejection the hypothesis about the multivariate correlation coefficient significance and simultaneously about correctness of coefficients in regression function.

2.6. Adequacy of the mathematical model of experimental unit

The adequacy of the mathematical model of experimental unit rating was determined by using Snedecor's F-test [4].

The test statistics was determined by equation

$$F = \frac{s^2(y)_a}{s^2(y)} = 3.74. \tag{68}$$

The adequacy variance characterised approximation accuracy

$$s^2(y)_a = \frac{SQ_a}{f_2} = 0.001\,453, \tag{69}$$

where $f_2 = N - L - 1 = 15$ and the tests accuracy variance was determined on the base of results for central point's sets $n_0 = 7$ from plane of experiment (tab. 2):

$$s^2(y) = \frac{\sum_{u=n_k+n_\alpha+1}^{N} \left(\bar{y}_{0u} - \bar{\bar{y}}_0\right)^2}{n_0 - 1} = \frac{S_E}{n_0 - 1} = 0.000\,389. \tag{70}$$

The numerator value in (54) was determined by equation

$$SQ_a = n_0 \cdot \left(\overline{\overline{y}}_{0u} - \hat{y}_{0u}\right)^2 + \sum_{u=1}^{N-n_0} \left(\overline{y}_u - \hat{y}_u\right)^2 = 0.021799 \ , \tag{71}$$

where $\overline{\overline{y}}_{0u} = 5.292$ (from tab. 4) and $\hat{y}_u = 5.290$ (from tab. 6).

The critical Snedecor's F-test value $F_{\alpha;f_1;f_2} = 3.94$ was determined for the significance level α = 0,05 and degrees of freedom $f_1 = n_0 - 1 = 6$ and $f_2 = N - L - 1 = 15$.

Because

$$F < F_{\alpha;r_1;r_2} \tag{72}$$

there is no reason to rejection the hypothesis about adequacy of the mathematical model of experimental unit to the experimental results from statistical point of view.

2.7. Decoding of coefficients in regression function

Decoding of coefficients in regression function was made by using following relations:

$$\begin{aligned} b_0 = {} & k_0 - k_1 \cdot \frac{x_{10}}{\Delta x_1} - k_2 \cdot \frac{x_{20}}{\Delta x_2} - k_3 \cdot \frac{x_{30}}{\Delta x_3} - k_4 \cdot \frac{x_{40}}{\Delta x_4} + k_{12} \cdot \frac{x_{10} \cdot x_{20}}{\Delta x_1 \cdot \Delta x_2} + \\ & + k_{13} \cdot \frac{x_{10} \cdot x_{30}}{\Delta x_1 \cdot \Delta x_3} + k_{14} \cdot \frac{x_{10} \cdot x_{40}}{\Delta x_1 \cdot \Delta x_4} + k_{23} \cdot \frac{x_{20} \cdot x_{30}}{\Delta x_2 \cdot \Delta x_3} + k_{24} \cdot \frac{x_{20} \cdot x_{40}}{\Delta x_2 \cdot \Delta x_4} + \\ & + k_{11} \cdot \left(\frac{x_{10}}{\Delta x_1}\right)^2 + k_{22} \cdot \left(\frac{x_{20}}{\Delta x_2}\right)^2 + k_{33} \cdot \left(\frac{x_{30}}{\Delta x_3}\right)^2 + k_{44} \cdot \left(\frac{x_{40}}{\Delta x_4}\right)^2 = -166.15, \end{aligned} \tag{73}$$

$$\begin{aligned} b_1 = {} & \frac{k_1}{\Delta x_1} - k_{12} \cdot \frac{x_{20}}{\Delta x_1 \cdot \Delta x_2} - k_{13} \cdot \frac{x_{30}}{\Delta x_1 \cdot \Delta x_3} - k_{14} \cdot \frac{x_{40}}{\Delta x_1 \cdot \Delta x_4} + \\ & - 2 \cdot k_{11} \cdot \frac{x_{10}}{\Delta x_1^2} = -0.11539, \end{aligned} \tag{74}$$

$$\begin{aligned} b_2 = {} & \frac{k_2}{\Delta x_2} - k_{12} \cdot \frac{x_{10}}{\Delta x_1 \cdot \Delta x_2} - k_{23} \cdot \frac{x_{30}}{\Delta x_2 \cdot \Delta x_3} - k_{24} \cdot \frac{x_{40}}{\Delta x_2 \cdot \Delta x_4} \\ & - 2 \cdot k_{22} \cdot \frac{x_{20}}{\Delta x_2^2} = -0.22485, \end{aligned} \tag{75}$$

$$\begin{aligned} b_3 = {} & \frac{k_3}{\Delta x_3} - k_{13} \cdot \frac{x_{10}}{\Delta x_1 \cdot \Delta x_3} - k_{23} \cdot \frac{x_{20}}{\Delta x_2 \cdot \Delta x_3} - k_{34} \cdot \frac{x_{40}}{\Delta x_3 \cdot \Delta x_4} \\ & - 2 \cdot k_{33} \cdot \frac{x_{30}}{\Delta x_3^2} = 10.867, \end{aligned} \tag{76}$$

$$\begin{aligned} b_4 = {} & \frac{k_4}{\Delta x_4} - k_{14} \cdot \frac{x_{10}}{\Delta x_1 \cdot \Delta x_4} - k_{24} \cdot \frac{x_{20}}{\Delta x_2 \cdot \Delta x_4} - k_{34} \cdot \frac{x_{30}}{\Delta x_3 \cdot \Delta x_4} \\ & - 2 \cdot k_{44} \cdot \frac{x_{40}}{\Delta x_4^2} = 127.94, \end{aligned} \tag{77}$$

$$b_{12} = \frac{k_{12}}{\Delta x_1 \cdot \Delta x_2} = 0.000031279, \tag{78}$$

$$b_{13} = \frac{k_{13}}{\Delta x_1 \cdot \Delta x_3} = -0.0020547, \tag{79}$$

$$b_{14} = \frac{k_{14}}{\Delta x_1 \cdot \Delta x_4} = 0.031969 \tag{80}$$

$$b_{23} = \frac{k_{23}}{\Delta x_2 \cdot \Delta x_3} = 0.044834, \tag{81}$$

$$b_{24} = \frac{k_{24}}{\Delta x_2 \cdot \Delta x_4} = 0.18592, \tag{82}$$

$$b_{34} = \frac{k_{34}}{\Delta x_3 \cdot \Delta x_4} = -3.1208, \tag{83}$$

$$b_{11} = \frac{k_{11}}{\Delta x_1^2} = 0.000015726, \tag{84}$$

$$b_{22} = \frac{k_{22}}{\Delta x_2^2} = -0.015359, \tag{85}$$

$$b_{33} = \frac{k_{33}}{\Delta x_3^2} = -10.735, \tag{86}$$

$$b_{44} = \frac{k_{44}}{\Delta x_4^2} = -23.448. \tag{87}$$

The mathematical model of the experimental unit has finally following form:

$$\begin{aligned} \hat{y} = {} & \underline{-166.15} \underline{-0.11539} \cdot x_1 \underline{-0.22485} \cdot x_2 + 10.867 \cdot x_3 + \underline{127.94} \cdot x_4 + \\ & + \underline{0.000015726} \cdot x_1^2 \underline{-0.015359} \cdot x_2^2 \underline{-10.735} \cdot x_3^2 \underline{-23.448} \cdot x_4^2 + \\ & + 0.000031279 \cdot x_1 \cdot x_2 - 0.0020547 \cdot x_1 \cdot x_3 + \underline{0.031969} \cdot x_1 \cdot x_4 + \\ & + 0.044834 \cdot x_2 \cdot x_3 + 0.18592 \cdot x_2 \cdot x_4 + 3.1208 \cdot x_3 \cdot x_4. \end{aligned} \tag{88}$$

The above-mentioned relations can be write with independent variables denatation:

$$\begin{aligned} \log N_c = {} & \underline{-166.15} \underline{-0.11539} \cdot S_{\max} \underline{-0.22485} \cdot P + 10.867 \cdot v + \\ & + \underline{127.94} \cdot d_w + \underline{0.000015726} \cdot S_{\max}^2 \underline{-0.015359} \cdot P^2 + \\ & \underline{-10.735} \cdot v^2 \underline{-23.448} \cdot d_w^2 + 0.000031279 \cdot S_{\max} \cdot P + \\ & - 0.0020547 \cdot S_{\max} \cdot v + \underline{0.031969} \cdot S_{\max} \cdot d_w + \\ & + 0.044834 \cdot P \cdot v + 0.18592 \cdot P \cdot d_w + 3.1208 \cdot v \cdot d_w. \end{aligned} \tag{89}$$

Underlined terms of equation (74) or (75) are significant in a statistical sense.

The equation (75) allows to estimate the fatigue life depending on the significant process parameters (the rivet squeezing force P, the hole diameter before sizing d_w and velocity of rivet close up v) and the nominal fatigue load in a hole section described by maximal tension stress in a cycle (the cycle asymmetry factor R=0) S_{max}.

3. Examples of using the mathematical model of experimental unit

Sample fatigue life plots obtained from the mathematical model of experimental unit (equation (75)) were presented in Fig. 1÷3. As it can be seen the fatigue load S_{max} has the largest influence on fatigue life of tested specimens. The hole diameter before sizing d_w is the second important factor (Fig. 1).

The rivet squeezing force P (in range of 8,5÷12,9 kN) has the least influence on fatigue life (Fig. 2) especially for large values of sizing degree (smaller hole diameter before sizing d_w). It can be connected with high work hardening around hole after sizing.

Velocity of rivet close up v has the significant influence on fatigue life too (Fig. 3). It may be concluded that the velocity of rivet close up, the higher the fatigue life.

4. Summary

The example of evaluation of the selected factors effect on the fatigue crack initiation in the area of sized rivet hole by using experimental design was presented in this paper. The analyses were performed by using results obtained with the static determined five level plan of the experiment called PS/DS-P:λ in Polish classification. The advantage of this plan is high efficiency factor that means that comparable information about experimental unit can be obtain from less than a 5% of the static determined full factorial plan sets. It has special importance in case of very long-lasting and expensive fatigue tests.

The statistical analyses were presented that there are no results with gross error. The most significant coefficients in the regression function is the fatigue load S_{max}. The least significant coefficient is the rivet squeezing force P. The multivariate correlation coefficient value R = 0.997 is significant for the regression function and finally the mathematical model of experimental unit is adequate to the experimental results from statistical point of view.

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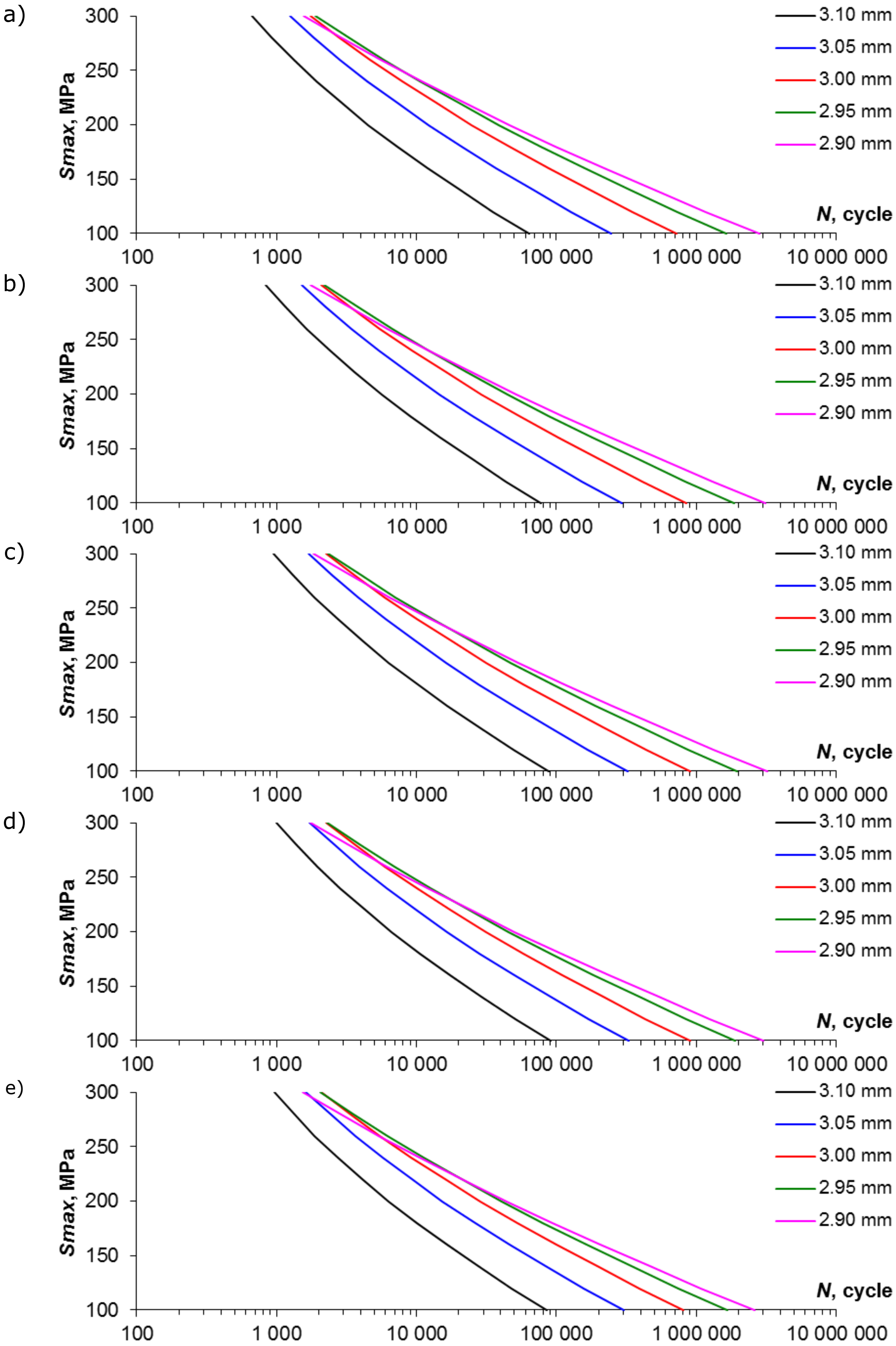

Fig. 1. Sample fatigue life plots with extrapolation outside of S_{max} range in plan of experiment obtained from the mathematical model of experimental unit depending on the hole diameter before sizing d_w and the rivet squeezing force P: a) 8.5 kN, b) 9.6 kN, c) 10.7 kN, d) 11.8 kN, e) 12.9 kN (velocity of rivet close up v = 0.08 mm/s = const)

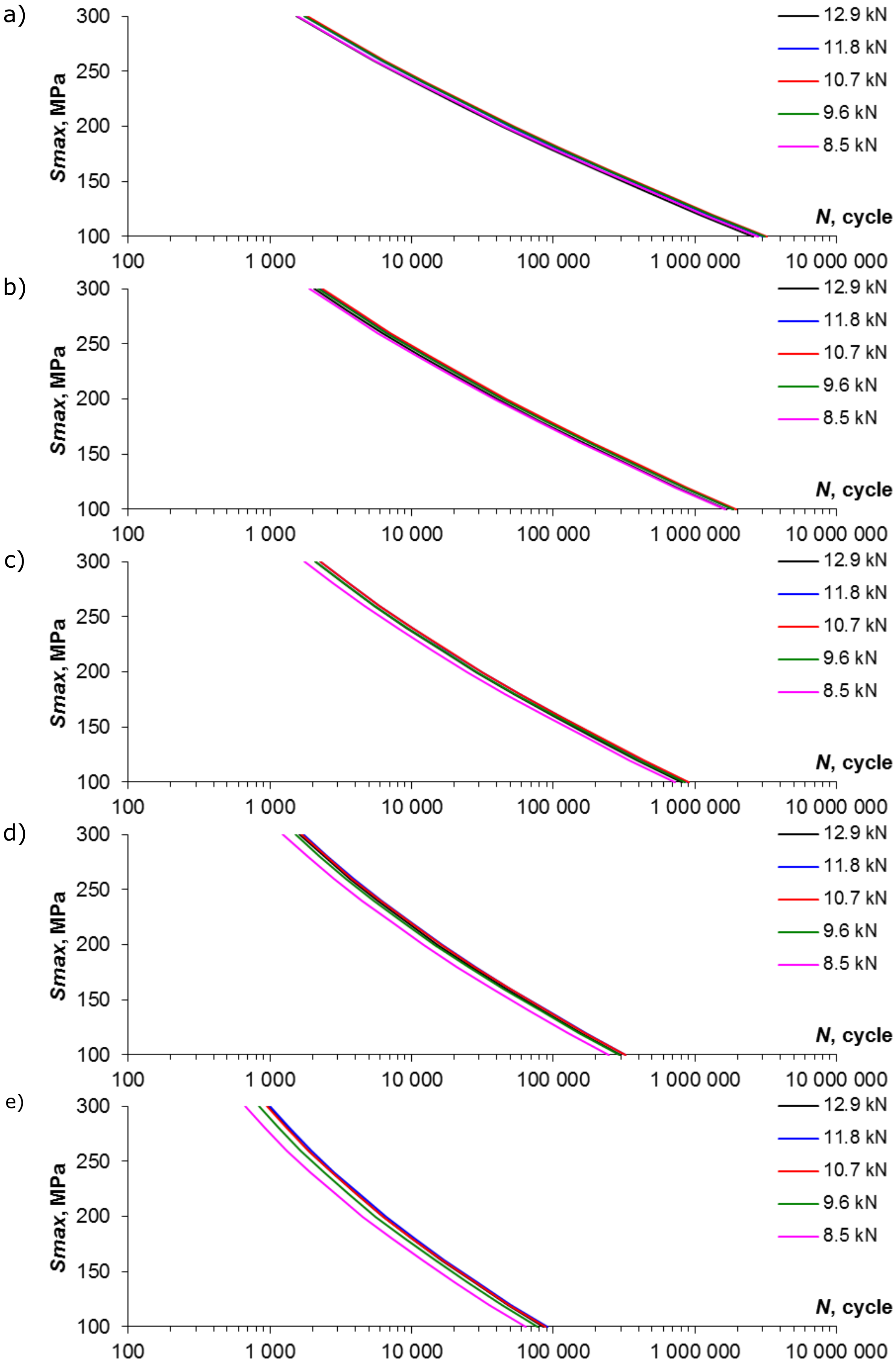

Fig. 2. Sample fatigue life plots with extrapolation outside of S_{max} range in plan of experiment obtained from the mathematical model of experimental unit depending on the rivet squeezing force P and the hole diameter before sizing d_w: a) 2.90 mm, b) 2.95 mm, c) 3.00 mm, d) 3.05 mm, e) 3.10 mm (velocity of rivet close up v = 0.08 mm/s = const)

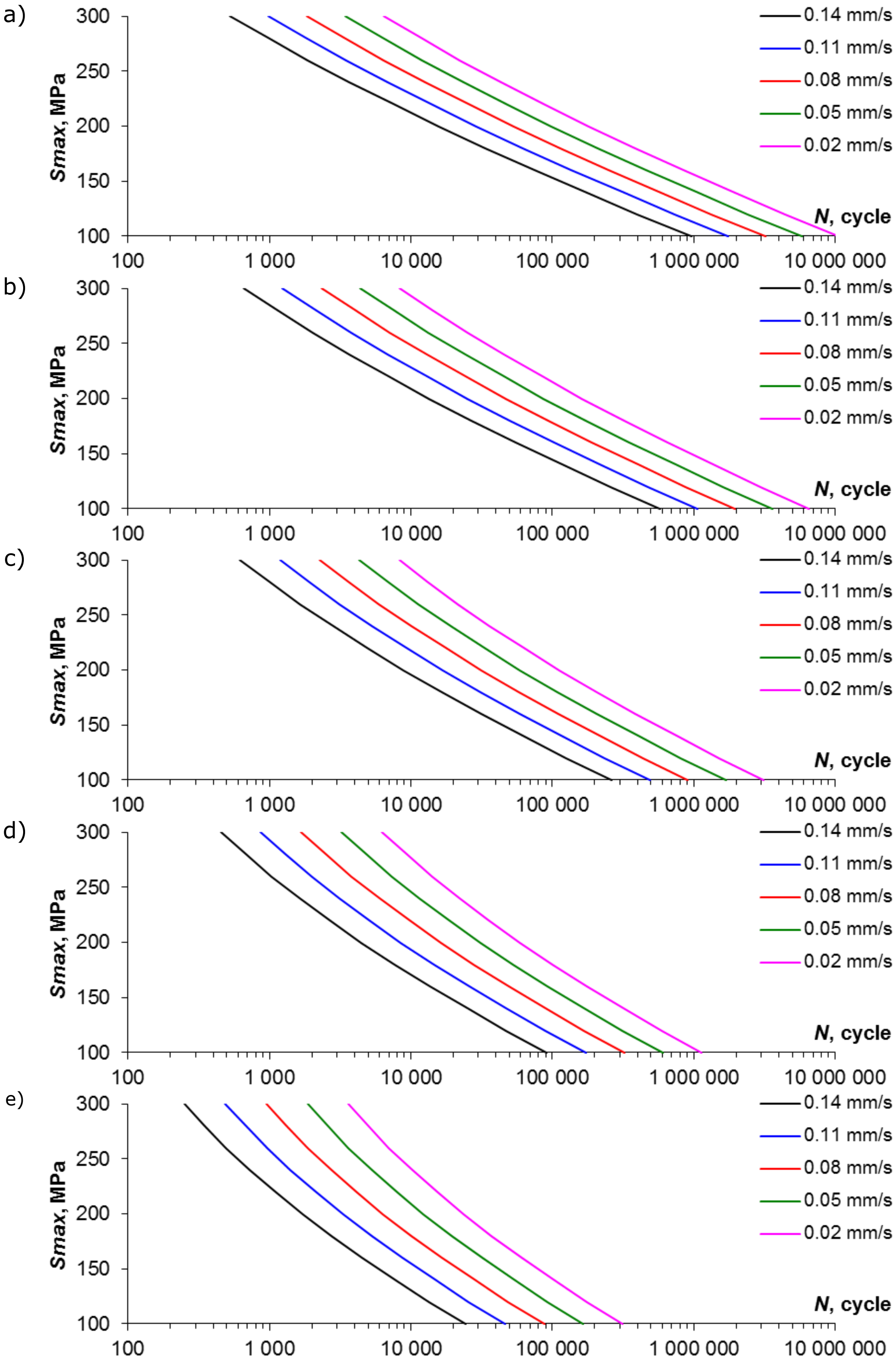

Fig. 3. Sample fatigue life plots with extrapolation outside of S_{max} range in plan of experiment obtained from the mathematical model of experimental unit depending on velocity of rivet close up v and the hole diameter before sizing d_w: a) 2.90 mm, b) 2.95 mm, c) 3.00 mm, d) 3.05 mm, e) 3.10 mm (the rivet squeezing force P = 10.7 kN = const)