THE ANGLE MEASUREMENT UNCERTAINTY EVALUATION BY MEANS OF INCREMENTAL ENCODER

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Abstract

The application of incremental encoders in the marine engines field was briefly described. The problem of the angle measurement uncertainty evaluation by means of an incremental encoder was presented. A theoretical assumption of experiment was discussed and a project of an experimental test allowing statistical evaluation of angle measurement uncertainty was proposed. The uncertainty estimation of a type A as well as the type B according to ISO standards was carried out. The numerical values of the results were presented. The proposition of improvement of test bed for encoders examination was proposed.

Keywords: marine engines, measurements, angle measurements, measurement uncertainty, incremental encoders,

1. Introduction

The incremental encoders have a practical application in automation since years. Last years one can meet them in a measurement technology. The modern marine engines manufacturers take advantage of them as a crankshaft angle position transmitter for electronically controlled engines [4,5]. The constructions of a measurement instruments utilizing signals from incremental encoders are in use today as well [2-7].

In the Laboratory of the Marine Engines in Maritime University of Szczecin there are two control and measurement systems utilizing incremental encoders. A problem of an angle measurement precision arose consequently.

No one of the manufacturers of the possessed encoders provided a calibration certificate. No one could specify the precision class of the instrument either. The necessity of an angle measurement uncertainty evaluation by means of an experiment arose subsequently.

In the experiment planning phase a suitable angle measurement standard had to be chosen. The encoders used in the laboratory are of the 3600 and 720 pulses per resolution. The most adequate standard should be of a high precision at a very small nominal value of the angle. Acquiring of such a standard and comparing with every position of the encoder might be as difficult as costly. Finally it was decided to utilize a direct measurement and a statistical work out of the measurement uncertainty. For a recognize experiment the 720 pulses per resolution encoder was chosen. The frequency of generated pulses should be lower and the experiment easier consequently.

At the beginning the assumption was made, that the manufacturer has made every effort to produce the instrument with the highest accuracy. Such an assumption seems to be burden with a

minima uncertainty and is essentials for further discussion. It was also assumed that the technology of the encoder production (which is not known to the author) consists in marking of the rotating disk circumference and subsequently filling in with equal marks in a nominal amount. For a tested encoder the nominal amount of marks is 720. Based on those two assumptions it can be assured that the encoder's shaft will rotate exactly 2π angle after nominal amount of pulses will be counted. Then the mean value of a measured angle will be equal to:

$$\bar{\alpha} = \frac{2\pi}{n} \tag{1}$$

If the encoder's shaft rotates uniformly with constant angular speed $\bar{\omega}$, the angle scale can be transformed into time scale. The advantage of this is that the time scale is much easier to measure than angle scale. In such a case, the angle of the shaft rotation by i pulses might be expressed as:

$$\alpha_i = T_i \cdot \bar{\omega} \tag{2}$$

Then the mean value of an angle $\bar{\alpha}$ can be determined by means of equation:

$$\bar{\alpha} = \frac{\bar{\omega}}{n} \cdot \sum_{i=1}^{i=n} T_i = \bar{\omega} \cdot \bar{T} \tag{3}$$

The difference between the value of a singular angle α_i, and the mean value $\bar{\alpha}$, can be recognized as the unknown value of the error:

$$\delta_i = |\alpha_i - \bar{\alpha}| \tag{4}$$

If the set of values α_i is treated as n independent observations of the expected value $\bar{\alpha}$, the experimental standard deviation of singular measurement can be determined:

$$s(\alpha_i) = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} (\alpha_i - \bar{\alpha})^2} \tag{5}$$

And the experimental standard deviation of a mean value can be expressed as:

$$s(\bar{\alpha}) = \sqrt{\frac{s^2(\alpha_i)}{n}} \tag{6}$$

Those two quantities carry direct information about uncertainty of the angle α_i measurement.

2. The experiment

The incremental encoders, manufactured in TTL standard, generate voltage pulses of 0 and 5VDC alternatively. Those two voltage levels refer to the logic states of 0 and 1 respectively. When the signal, measured at the output of the encoder, is presented on the diagram it forms a square waveform pattern. In the presented experiment the time between the consecutive rising edges of the square waveform was adopted as a period T_i which reflects the angle α_i. (Fig. 1).
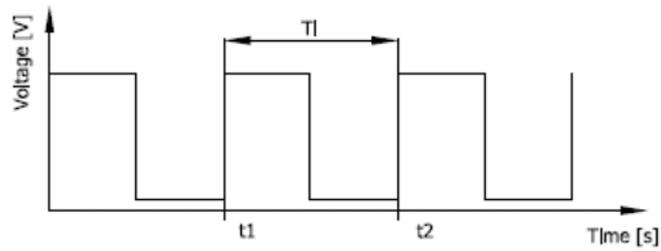
Fig. 1. The priod T_i determination methodology

The encoder was mounted on a driving assembly which consists of a squirrel-cage motor and a synchronous generator. The motor was supplied from a voltage inverter. The angular speed of the rotating masses is controlled by a variable supply voltage frequency acquired from the inverter (Fig. 2). For the tests a stand for a synchronous generators testing was utilized and the use of the generator is actually not essential for the experiment.
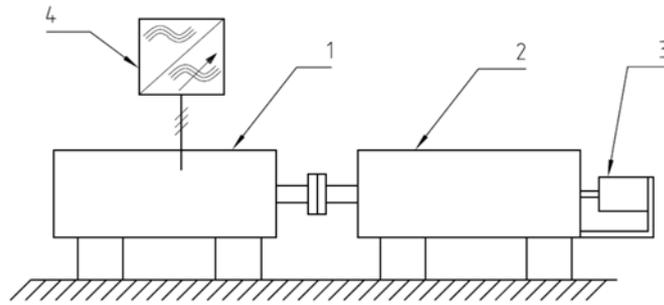
Fig. 2. The diagram of the encoder's drive; 1 – squirrel-cage motor, 2 – synchronous generator, 3 – tested encoder, 4 – inverter

In order to provide suitable accuracy of the period measurement it was necessary to reduce the angle speed of the rotating masses and/or increasing the sampling frequency. In the first case the increased influence of the unbalance of the masses has to be taken into account. The unbalance may cause increased irregularity of the angular speed. In order to determine the critical speed below the influence of the unbalance is essential that experiment was carried out at different speeds. The acquired signals were compared with the timer/counter of 80MHz frequency installed on board acquisition card (Fig. 3). In the table 1 the details of the measuring instruments were presented.
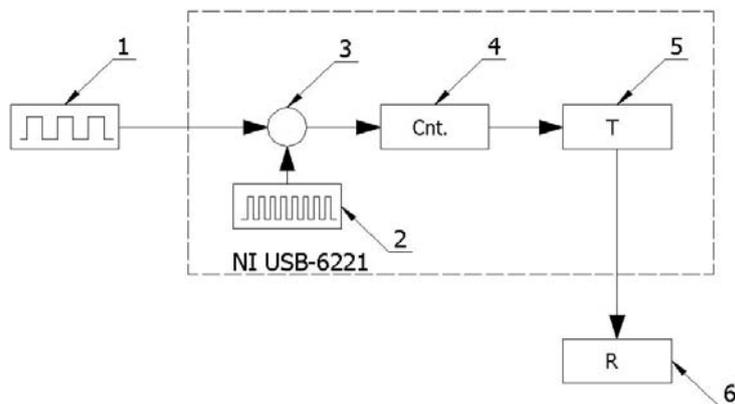
Fig. 3. Measurement diagram; 1 – Encoder's TTL signal filtered by a Schmitt trigger, , 2 – on board 80MHz timer/counter, 3 – comparator module, 4 – pulses counter, 5 – period calculation, 6 – recording module (PC).

In the measurement circuit a Schmitt trigger was engaged as a first filtering element. The Schmitt trigger is a semiconductor comparator circuit which generates logical 0 or 1 dependent on the input signal value. The applied Schmitt trigger has a very short latency of 20ns. The shortest recorded period T_i was 200 μs, which is 10000 times longer than the trigger latency. Assuming that the latency is constant, it has no significant influence on the results of experiment.

Tab. 1. Measurement instrumentation data

Instrument	Type
Measurement board	NI USB-6221, National Instruments, with 80MHz counter input gate
Encoder	PFI 60A0720TPT, Intron

3. Uncertainty determination

The measurements were conducted for 12 different mean angular speeds of the driving motor. For every mean angular speed at least 5 full revolutions was recorded. The recorded data contains 79 sets of 720 values of the period T_i. For every set the average was determined:

$$\bar{T} = \frac{1}{n} \sum_{i=1}^{i=n} T_i \tag{7}$$

As well as average angular speed:

$$\bar{\omega} = \frac{2\pi}{n \cdot \bar{T}} \tag{8}$$

Consequently, from equations (2) and (3) the values of the angles α_i and $\bar{\alpha}$ could be determined as well as the standard deviation of singular measurement and standard deviation of a mean value from (5) i (6). The calculated values of : $\bar{T}$, $\bar{\omega}$, $s(\alpha_i)$ and $s(\bar{\alpha})$, averaged for every angular speed, are presented in the table 2.

Tab. 2. Determined values averaged for every angular speed

$\bar{\omega}$ [rad/s]	$\bar{\alpha}$ [rad]	$s(\alpha_i)$ [rad]	$s(\bar{\alpha})$ [rad]	n [rpm]
7,82		1246,2E-6	46,6E-6	74,7
11,73		709,0E-6	26,6E-6	112,0
12,54		668,0E-6	25,1E-6	119,8
15,70		153,9E-6	5,76E-6	149,9
19,62		28,7E-6	1,07E-6	187,3
23,54	0,00872665	16,5E-6	0,63E-6	224,8
27,47		17,2E-6	0,65E-6	262,3
28,25		10,4E-6	0,39E-6	269,8
31,39		9,6E-6	0,36E-6	299,8
35,32		13,1E-6	0,49E-6	337,3
39,25		11,9E-6	0,44E-6	374,8
43,17		11,8E-6	0,44E-6	412,3

The above determined values of $s(\alpha_i)$ and $s(\bar{\alpha})$ presented in the function of average angular speed are shown on the graphs (fig. 4. and fig. 5.). One can learn that the values of $s(\alpha_i)$ and $s(\bar{\alpha})$ rapidly decrease until the speed of about 20 rad/s is reached. Above that speed the variation of the values is not so high. It can be noticed that the influence of the angular speed has no significant

effect. High values of $s(\alpha_i)$ and $s(\bar{\alpha})$ at low angular speed are most probably the consequences of the rotary speed irregularities. The irregularities come from the rotating masses unbalance and low power of the driving motor supplied from the inverter at low frequency. Only the values of $s(\alpha_i)$ and $s(\bar{\alpha})$ determined for the speeds above the critical 20 rad/s can be considered for evaluation of the encoder's angle measurement uncertainty.

Based on the ISO recommendations [1], the problem of the uncertainty evaluation was identified as a standard uncertainty type A. In such a case, the value of experimental standard deviation of the measurand's average $s(\bar{\alpha})$ determines directly the uncertainty of the angle $\bar{\alpha}$.

Finally the highest value of the standard deviation from the speed range above 20 rad/s was chosen as the value of standard uncertainty type A:

$$s(\alpha_i) = 17{,}2 \cdot 10^{-6}\ [rad]$$

$$s(\bar{\alpha}) = 0{,}65 \cdot 10^{-6}\ [rad]$$

After the results were rounded off to two significant digits one can express that the tested encoder was generating pulses of the rising edge corresponding to the angle of:

$$\bar{\alpha} = 0{,}00872665 \pm 0{,}00000065\ [rad]$$

$$a_i = 0{,}008727 \pm 0{,}000018\ [rad]$$
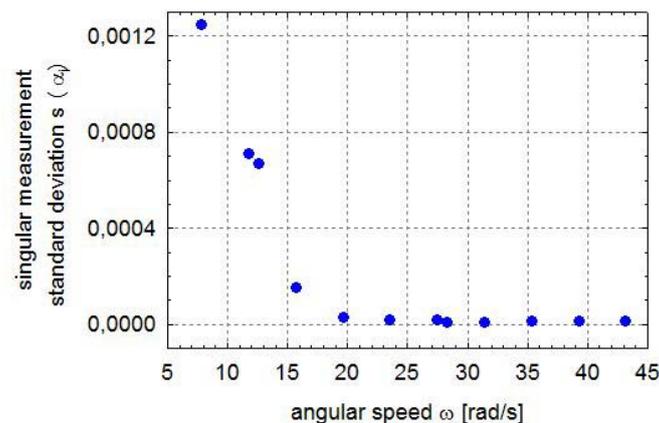
Fig. 4. The standard deviation of singular measurement –mean angular speed characteristic
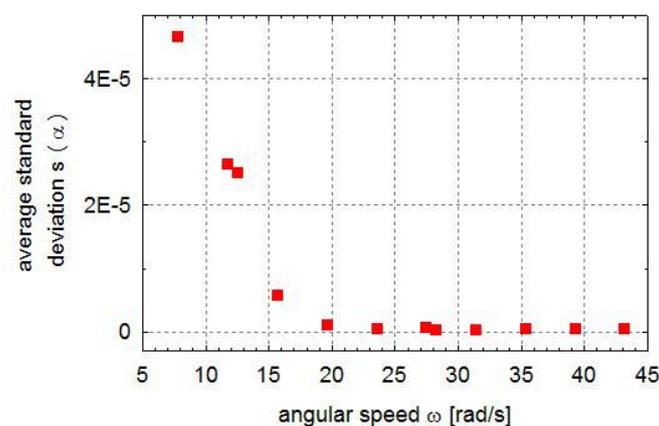
Fig. 5. The standard deviation of average –mean angular speed characteristic

In order to get the full view of the measurement uncertainty, the systematic uncertainty of the measurement should be evaluated. The ISO recommendations regarding determination of uncertainty Type B were applied.

The only physical quantity measured directly was period of singular pulse. That period was compared with the 80 MHz clock/timer. Consequently the scale interval of time measurement was:

$$\Delta T = \frac{1}{8 \cdot 10^7} \, s$$

According to propagation of uncertainty low the same scale interval refers to both, singular value measurement T_i and to the average $\bar{T}$. That enables the equation (3) to be differentiated with respect to T_i and $\bar{T}$:

$$u_\alpha = \sqrt{\left(\frac{\partial \bar{\alpha}}{\partial T_i} \cdot \Delta T\right)^2 + \left(\frac{\partial \bar{\alpha}}{\partial \bar{T}} \cdot \Delta T\right)^2} \tag{9}$$

For every one of 79 sets of n values of T_i, the type B uncertainty u_α was determined (fig 6.). Out of them the highest one was chosen as the value of final uncertainty:

$$u_\alpha = 0{,}77 \cdot 10^{-6} \, [rad]$$
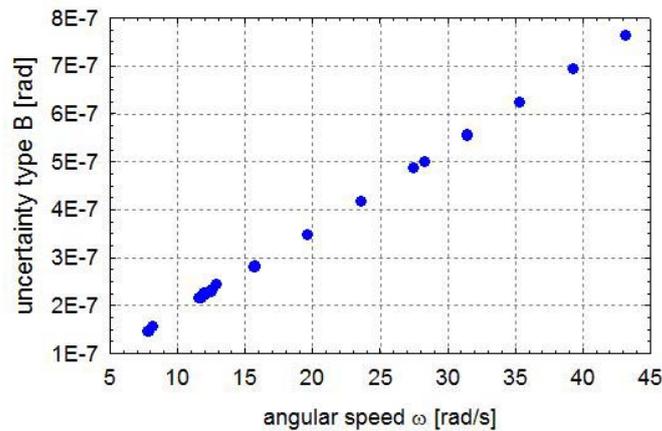
Fig. 6. The type B uncertainty –mean angular speed characteristic

The combined standard uncertainty of average was determined according to the formula:

$$u_{z\bar{\alpha}} = \sqrt{s(\bar{\alpha})^2 + \frac{u_\alpha^2}{3}}$$

$$u_{z\bar{\alpha}} = 0{,}79 \cdot 10^{-6} \, [rad]$$

$$\bar{\alpha} = 0{,}00872665 \pm 0{,}00000079 \, [rad]$$

$$\bar{\alpha} = 0{,}00872665 \, [rad] \pm 0{,}01 \, \%$$

And the combined standard uncertainty of singular measurement was adequately:

$$u_{z\alpha_i} = \sqrt{s(\alpha_i)^2 + \frac{u_\alpha{}^2}{3}}$$

$$u_{z\alpha_i} = 17,21 \cdot 10^{-6} \, [rad]$$

$$a_i = 0,008727 \pm 0,000018 \, [rad]$$

$$a_i = 0,008727 \, [rad] \pm 0,2 \, \%$$

The above shown uncertainty is specified for the level of confidence $p=68\%$. For level of confidence of $p=99\%$, a coverage factor $k=3$ was set. The expanded uncertainty of average then is equal to:

$$\bar{\alpha}_{p=99\%} = 0,00872665 \pm k \cdot u_{z\alpha} = 0,0087267 \pm 0,0000024 \, [rad]$$

$$\bar{\alpha}_{p=99\%} = 0,0087267 \pm 0,03 \, \% \, [rad]$$

And the expanded uncertainty of singular measurement:

$$a_{i\,p=99\%} = 0,008727 \pm k \cdot u_{z\alpha_i} = 0,008727 \pm 0,000054 \, [rad]$$

$$a_{i\,p=99\%} = 0,0087267 \pm 0,6 \, \% \, [rad]$$

4. Conclusion

It is essential to understand that the above determined uncertainty describes not the encoder alone but the en tire measurement chain. In that chain one can distinguish the following elements with the greatest influence on the uncertainty:

- The encoder accuracy itself,
- The unbalance of the rotating masses and the speed irregularity as consequence,
- Frequency of the clock/timer utilized for time domain measurement,
- Possible delays in the electric circuit caused by Schmitt trigger or other element.

Consequently the uncertainty of the angle measurement by an encoder itself can be determined as no lower than presented above. However in the scientific or engineering applications the overall uncertainty of the measurement chain is searched usually. So taking into account the entire measurement chain is justified. In order to limit the problem to the encoder only, the test bed has to be modified and the driving unit with better unbalance should be applied.

The systematic uncertainty of type B (Fig. 6.) depends linearly on the angular speed. This is the result of constant scale interval of time measurement ΔT at decreasing period T_i. During experiments it is important to take into account the systematic uncertainty at the angular speed corresponding to the speed in specific application.

Nomenclature

T_i – period of the singular pulse
$\bar{T}$ – average period of pulse T_i at one revolution
$\bar{\omega}$ – average angular speed of the encoder's shaft
α_i – angle of the shaft rotation during the period T_i

$\bar{\alpha}$ – mean value of a measured angle for one revolution,

n – nominal number of encoder pulses per revolution.

$s(\alpha_i)$ – experimental standard deviation of α_i singular measurement

$s(\bar{\alpha})$ – experimental standard deviation of a mean value $\bar{\alpha}$

δ_i – unknown value of the error

$\varDelta T$ – scale interval of time measurement

u_α – uncertainty Type B – systematic uncertainty

$u_{z\bar{\alpha}}$ – combined standard uncertainty of average

$u_{z\alpha_i}$ – combined standard uncertainty of singular measurement

$\bar{\alpha}_{p=99\%}$ - expanded uncertainty of average at level of confidence p=99%

$\alpha_{i\ p=99\%}$ - expanded uncertainty of singular measurement at level of confidence p=99%

References

[1] Główny Urząd Miar, *Wyrażanie Niepewności Pomiaru. Przewodnik. .* Warszawa, 1999.

[2] GmbH, Maridis, *Complex Diagnostic System Brochure*. [Online] [Cited: 20 June 2007.] http://www.maridis.de/Products/file_store/catalog/CDS_brochure.pdf.

[3] MAN B&W Diesel A/S, *PMI System Pressure Analyser*. [Online] czerwiec 2000. [Cited: 3 July 2007.] http://www.manbw.com/files/news/filesof2051/pmi.pdf.

[4] MAN B&W Diesel A/S, *50-108 ME Engines. Operation Manual*, Edition 01, January 2006

[5] Kowalak, Przemysław. *Speed Irregularity Characteristic of Low Speed, Two-Stroke Marine Diesel Engine Applied as Vessel's Main Propulsion. Journal of KONES.* Warsaw 2008, Tom IV, 15.

[6] Wimmer Andreas, Glaser Josef, *Indykowanie silnika.* Warszawa : Instytut Zastosowań Techniki, 2004. ISBN 83-88691-20-1.

[7] Wituszyński Krzysztof, *Prędkość kątowa i moment obrotowy, jako nośniki informacji o stanie silnika spalinowego.* Lubelskie Towarzystwo Naukowe, Lublin 1996, ISBN 83-85491-62-7