Journal of POLISH CIMAC
Faculty of Ocean Engineering & Ship Technology
GDAŃSK UNIVERSITY OF TECHNOLOGY

INVESTIGATION OF THE OPERATIONAL MODAL ANALYSIS APPLICABILITY IN COMBUSTION ENGINE DIAGNOSTICS

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Abstract

The paper contains application of operational modal analysis in use of combustion engine chosen technical state identification. The combustion engine No. 138C.2.048 with 1.4l. swept capacity, power 55 kW, generally applied to Fiat was the investigation object. It makes possible to introduce generated vibration signals as well as the investigation of his adjustment influence on the combustion engine vibration signals change. Conducted researches of combustion engine depended on delimitations of vibroacoustics measures for fit engine and comparison of this measures, with measures appointed for damaged engine (e.g. damaged injector) and accomplishment the assessment of received results influence on engine state by operational modal analysis. The present research use vibration methods as Operational Modal Analysis to recognize the technical state of the system.

Keywords: *operational modal analysis, diagnostic inference, combustion engine*

1. Introduction

Owing to the complicated character of vibrations, technical diagnostics of combustion engines is very difficult and only a few of proposed methods of diagnosis can have a wider technical application. Application of the modal analysis as one of the vibroacoustics tools is the new approach to the technical analysis of the combustion engine. Following the changes of modal model parameters as a result of engine maladjustment, waste, damages or its failure is the main idea of operational modal analysis. The modal parameters are [4,5,6]:

- modal frequency,
- modal damping,
- mode shape.

Modal analysis is the process of determining the modal parameters of a structure for all modes in the frequency range of interest. The ultimate aim is to use these parameters to construct a modal model of the response.

Operational modal is the name for the technique to do modal analysis on operational data - cases where we do not excite the structure artificially but just allow the natural operating loads to excite the structure.

2. Operational modal analysis – LSCE method theory

In this paper Least Squares Complex Exponential method was used to determine the modal model parameters, by which the correlation function is approximated by the sum of exponentially decaying harmonic functions. This method, applied to impulse response of system is a well known method in modal analysis yielding global estimators of system poles – the root of the transfer function denominator. It can be proved that the cross correlation function can be used to identify modal system parameters in identical way as does the impulse response of the system. [4,5]

The dynamic equation of the system motion can be expressed by formula [4,5]:

$$M\ddot{x} + C\dot{x} + Kx = F(t)\,, \tag{1}$$

where:

M,C,K – mass, damping and stiffness matrices,
$\ddot{x}, \dot{x}, x$ – acceleration, velocity and displacement vectors,
F(t) – vector of exciting forces.

The next step was transformation of formula (1) to principal coordinates applying the transformation expressed by the formula [4,5]:

$$x(t) = \Psi q(t) = \sum_{r=1}^{n} \Psi_r q_r(t)\,, \tag{2}$$

where:

Ψ – matrix of modal vectors, the columns of which are eigenvectors corresponding to the given free vibration frequency,
q_r – principal (modal) coordinate,
n – number of vibration forms included in the model of vibration forms.

Assuming that damping is small and proportional, on substituting relation (2) into formula (1) and multiplying by Ψ^T, de-coupled equation set is obtained in the form [4,5]:

$$\ddot{q}_r(t) + 2\xi_r\omega_{nr}\,\dot{q}(t) + \omega_{nr}^2 q_r(t) = \frac{1}{m_r}\Psi_r^T f(t)\,, \tag{3}$$

where:

ω_{nr} – is the r-th free vibration frequency,
ξ_r – is the modal damping coefficient for the r-th vibration form,
m_r – is the modal mass.

Assuming zero initial conditions for arbitrary excitation, the solution of equation (3) can be written in the form of convolution [4,5]:

$$q_r(t) = \int_{-\infty}^{t} \psi_r^T f(\tau) g_r(t-\tau)\,d\tau\,, \tag{4}$$

where:

$g_r(t) = 0 \quad \text{for} \quad t < 0,$

$g_r(t) = \dfrac{1}{m_r\omega_{rd}} \exp(-\xi_r\omega_{nr}t)\sin\omega_{nr}t \quad \text{for} \quad t \geq 0,$

$\omega_{rd} = \omega_{nr}(1-\xi_r^2)^{\frac{1}{2}}$ – is the frequency of damped free vibrations.

Making use of solution formula (4) for modal coordinates to determine the solution in generalised coordinates x(t), we obtain the formula [4,5]:

$$x(t) = \sum_{r=1}^{n} \psi_r \int_{-\infty}^{t} \psi_r^T f(\tau) g_r(t-\tau) d\tau \,, \tag{5}$$

where:

n – number of vibration forms taken into account in the solution.

Cross correlation function for two response signals at point i and j, resulting from an excitation applied at point k in the form of white noise has form [4,5]:

$$R_{ijk}(T) = E_o[x_{ik}(t+T) x_{ik}(t)] \,, \tag{6}$$

where:

E_o – denotes the expected value operator.

If we know relation between cross correlation function, having the form of a sum of exponentially decaying harmonic functions and impulse transition function for direct application to do modal analysis tests, correlation function can be transformed to the form [4,5]:

$$R_{ij}(T) = \sum_{r=1}^{n} \frac{\psi_{ir} g_{jr}}{m_r \omega_{rd}} \exp(-\xi_r \omega_{nr} T) \sin(\omega_{rd} T + \vartheta_r) \,, \tag{7}$$

where:

ϑ_r – the New phase angle,
G_{jr} – constant.

3. Model of diagnostics signal generation

The investigations object was a combustion engine no. 138C.2.048 applied to the Fiat and Lancia cars that is shown on figure 1. Basis on this system during investigations was created model of diagnostics signal generation [2,3]. The proposed model of combustion engine diagnostic signal generation is shown on figure 2.

The received signals in the any point of engine body are the sum of the answer at all elementary events u_n (t, Θ), outputs in individual partial dynamic arrangements with the pulse function of input h_n (t, Θ). These influences after passing by proper dynamic arrangements are sum up on the engine body, on chosen points was measured by the vibration transducers. As a result of conducted measurements output signals was used to estimation. By n (t, Θ) was marked accidental influence stepping out from presence of dynamic micro effects such as friction [2,3,6].

Conducted investigations of combustion engine depended on delimitations of vibroacoustics measures for fit engine and comparison them with measures appointed for damaged engine (eg. damaged injector) and accomplishment the assessment of received results influence on engine state by operational modal analysis methods.

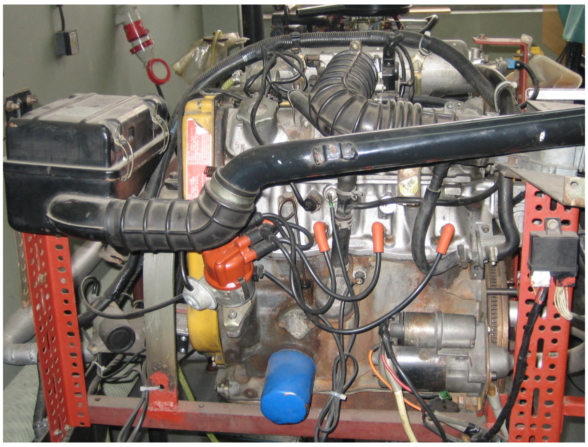

Fig. 1. The investigation object – combustion engine No. 138C.2.048

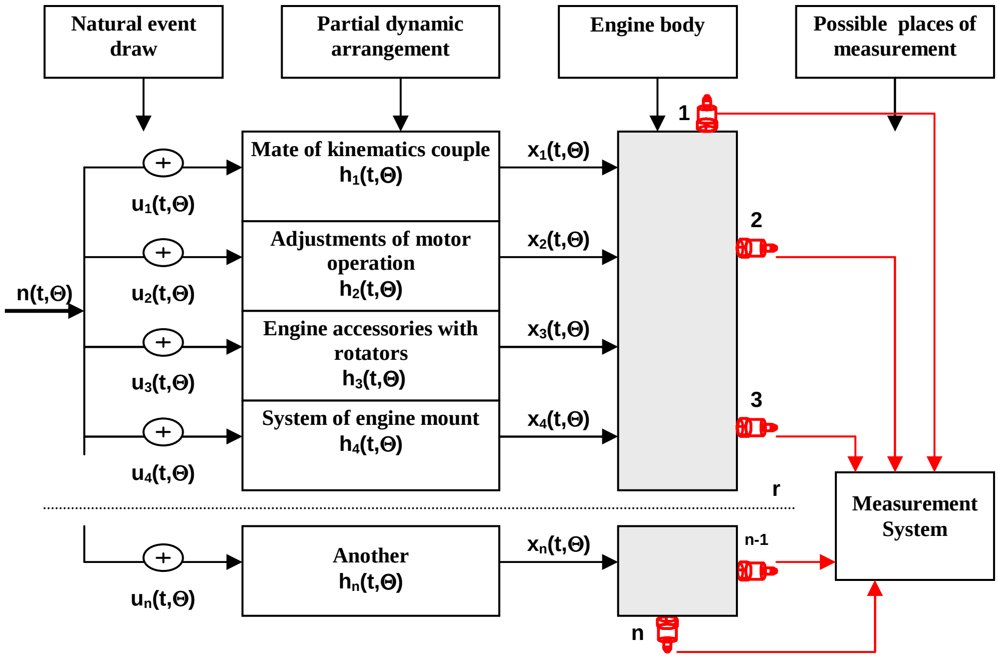

Fig.2. Combustion engine diagnostics signal generation model[2,3]

4. The investigations results

The modal model of combustion engine was created for put dynamic states on the basis of received measuring results. During investigations have been done vibroacoustics measures for fit engine and for engine with damaged injector and spark plug for each cylinder.

As a results of engine modal tests was created the stabilisation diagrams for each technical state. Basis on the stabilisation diagrams was created the modal model includes modal order, natural frequency and damping [2,4,5]. Figure 3 display the window with the stabilization diagram of engine in fit state. Table 1 present the results of modal investigations – modal model for put engine technical states. Basis on modal model parameters and estimators of vibroacoustics signal received during investigations in table 2 was shown the main observation matrix for engine performance. The final observation matrix of engine performance described 13 symptoms. The matrix have six modal symptoms (ω_1 - first natural frequency, rząd1 - modal order of first natural frequency, ξ_1 - modal damping coefficient of first natural frequency, ω_2 - second natural frequency, rząd2 - modal order of second natural frequency, ξ_2 - modal damping coefficient of second natural frequency) and the last seven symptoms are vibration process (H(f) – real part of transfer function, H(f)L – imagine part of transfer function, γ^2_{xy} – coherence function, $A_{RMS(t)}$ – Root Mean Square in time domain, β_{kurt} – Kurtosis, C_s - Crest factor, I - Impulse factor).

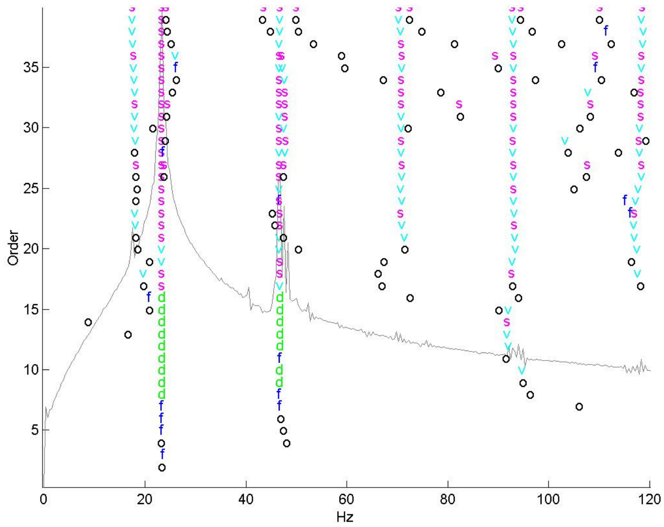

Fig. 3. Operational Modal Analysis stabilization diagram of investigated engine in fit technical state: s – stable pole, v – the frequency of vibration and modal vector is stabilized, d – the frequency of vibration and the stifling is stable, f – only the frequency of the vibration is stable, o – the pole is unstable

Table 1. Parameters of modal model received during investigations for put of 9 technical states of combustion engine: ω – is the free vibration frequency, Order – order of the model, ξ – is the modal damping coefficient

Technical state	**Parameters of modal model**						
1- fit engine	ω (Hz)	23,27	46,96				
	Order	18	17				
	ξ (%)	0,67	1,34				
2 - damaged injector on 4th cylinder	ω (Hz)	16,62	21,82	38,09			
	Order	20	19	20			
	ξ (%)	4,08	0,68	4,33			
3 - damaged injector on 3th cylinder	ω (Hz)	17,81	22,57	27,94	39,74		
	Order	19	17	28	18		
	ξ (%)	4,81	1,47	4,82	2,00		
4 - damaged injector on 2th cylinder	ω (Hz)	16,33	22,13	27,99	38,59	49,13	
	Order	29	18	31	17	23	
	ξ (%)	7,05	3,11	7,13	4,09	5,51	
5 - damaged injector on 1th cylinder	ω (Hz)	17,36	22,82	29,24	40,03	50,87	91,64
	Order	23	19	34	27	25	16
	ξ (%)	6,69	1,18	6,17	3,21	2,90	2,24
6 - damaged spark plug on 4th cylinder	ω (Hz)	20,13	22,05	39,08	49,60		
	Order	18	24	23	23		
	ξ (%)	1,93	7,93	6,98	4,80		
7 - damaged spark plug on 3th cylinder	ω (Hz)	16,52	20,70	25,51	41,43	47,43	
	Order	19	17	29	27	26	
	ξ (%)	10,11	2,48	6,73	4,61	4,07	
8 - damaged spark plug on 2th cylinder	ω (Hz)	16,50	21,89	37,74	46,34		
	Order	23	18	24	20		
	ξ (%)	11,47	1,21	6,33	1,78		
9 - damaged spark plug on 1th cylinder	ω (Hz)	17,59	23,58	45,93			
	Order	25	17	18			
	ξ (%)	3,83	0,71	1,27			

Table 2. The main observation matrix for engine performance

State	ω 1	rząd 1	ξ 1	ω 2	rząd 2	ξ 2	H(f)	H(f)L	γ^2_{xy}	$A_{RMS(t)}$	β_{kurt}	C_s	I
1	23,27	18	0,67	46,96	17	1,34	68,56	-2,18	108,18	0,2177	1,5567	1,7239	1,9268
2	21,82	19	0,68	38,09	20	4,33	47,08	30,59	100,22	0,1392	1,8989	2,1204	2,4456
3	22,57	17	1,47	39,74	18	2,00	36,42	8,84	104,40	0,2040	1,7532	1,8656	2,1198
4	22,13	18	3,11	38,59	17	4,09	31,34	-15,28	91,11	0,1769	1,9245	2,0762	2,3992
5	22,82	19	1,18	40,03	27	3,21	46,16	-75,94	101,15	0,2312	1,7148	2,0982	2,3673
6	20,13	18	1,93	39,08	23	6,98	42,24	-8,50	83,73	0,1702	2,5205	2,8157	3,3986
7	20,70	17	2,48	41,43	27	4,61	38,76	22,77	82,34	0,1363	2,2943	2,2926	2,7564
8	21,89	18	1,28	46,34	20	1,78	40,51	-19,29	83,29	0,1726	1,7401	2,0176	2,2929
9	23,58	17	0,71	45,93	18	1,27	19,45	-23,34	99,63	0,1904	1,6144	1,8260	2,0527

5. Results validation

As a results of investigation in this paper is shown presentation of singular value decomposition (SVD) method usage for combustion engine technical state results validation. The SVD method is the appropriate tool for analyzing a mapping from one vector space into another vector space, possibly with a different dimension [1]. The first step of SVD procedure is to centre and normalization all symptoms given in table two relative to the initial value of symptom vector. The observation matrix of transformate symptoms relative to the initial value is shown on figure 4.

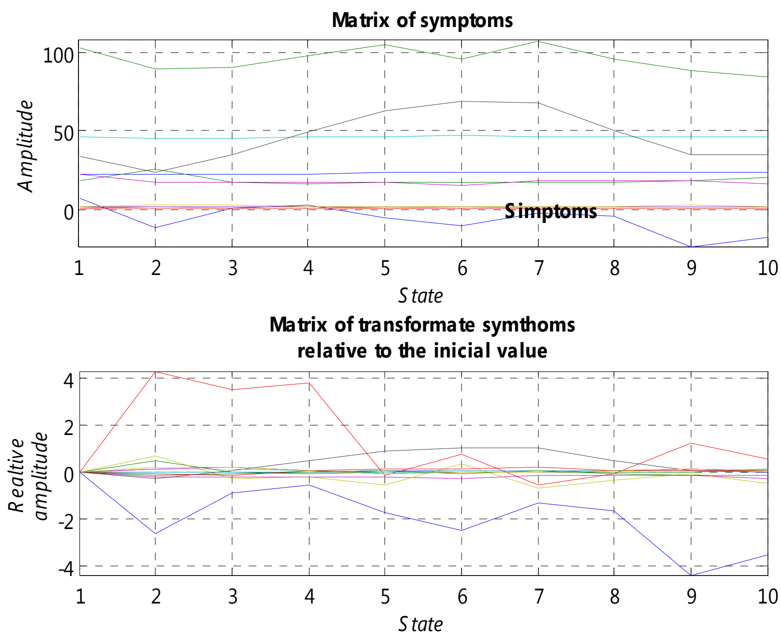

Fig.4. Matrix of symptoms before and after transformation

The second step of SVD procedure is to calculate the first generalized damage and evolution of damage. Graphical interpretation of this calculations is given in figure 5. Making data analysis in SVD method as a result we got the line up of symptoms together with the proportional description of given individual symptom of combustion engine technical state. Thanks to SVD methods we could decide which symptom given in observation matrix is the best to recognize a set of combustion engine technical state [1].

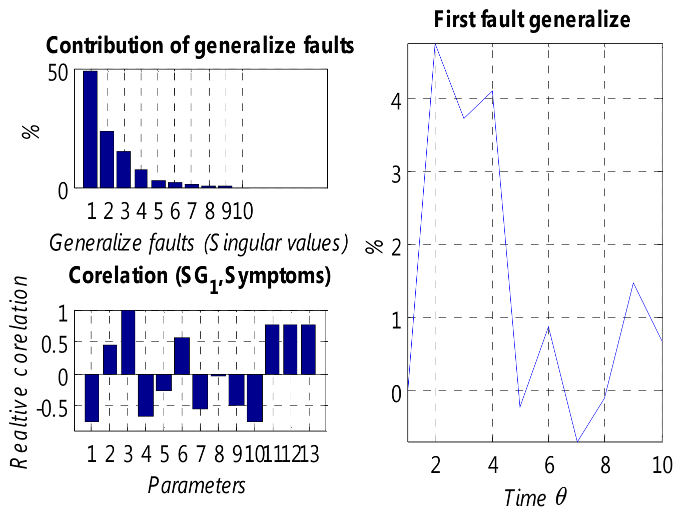

Fig.5. Graphical interpretation of first generalized damage and evolution of damage

In SVD procedure as a result we got a line up of five best symptoms given in table 3 that are most important in description of set technical state of combustion engine.

Table 3. Results of SVD method with five best symptoms for set of engine technical state

State	1 symptom	2 symptom	3 symptom	4 symptom	5 symptom
1	ξ_1	ω_1	$A_{RMS(t)}$	β_{kurt}	C_s
2	H(f)L	rząd 1	ξ_2	H(f)	γ^2_{xy}
3	ξ_1	H(f)L	rząd$_1$	ω_1	H(f)
4	rząd$_2$	H(f)L	ω_2	ξ_2	ξ_1
5	ξ_1	rząd$_1$	β_{kurt}	$A_{RMS(t)}$	γ^2_{xy}
6	H(f)L	ξ_1	$A_{RMS(t)}$	H(f)	rząd$_2$
7	ξ_1	C_s	I	rząd$_1$	γ^2_{xy}
8	H(f)L	ξ_1	ξ_2	γ^2_{xy}	ω_1
9	H(f)L	ξ_1	β_{kurt}	C_s	I

Relationships cause - consecutive expressing quantitative relation between studied variable symptoms results in this work were qualified using the function of the multiple regression. Basis on SVD results as a best symptoms in multiple regression were given: ω_1 – first natural frequency, ξ_1 - modal damping coefficient of first natural frequency, ξ_2 - modal damping coefficient of second natural frequency, H(f)L – imagine part of transfer function, γ^2_{xy} – coherence function. The equation of multiple regression is obtained in the form:

$$y = -1{,}44923\omega_1 - 0{,}61558\xi_1 - 0{,}35989\xi_2 - 0{,}06520H(f)L + 0{,}14424\gamma^2_{xy} + 35{,}9994, \qquad (8)$$

Graphical interpretation of this calculations for first dependent variable ω_1 is given in figure 6. The red line present real data received during investigations, the blue line – estimated model for dependent variable.

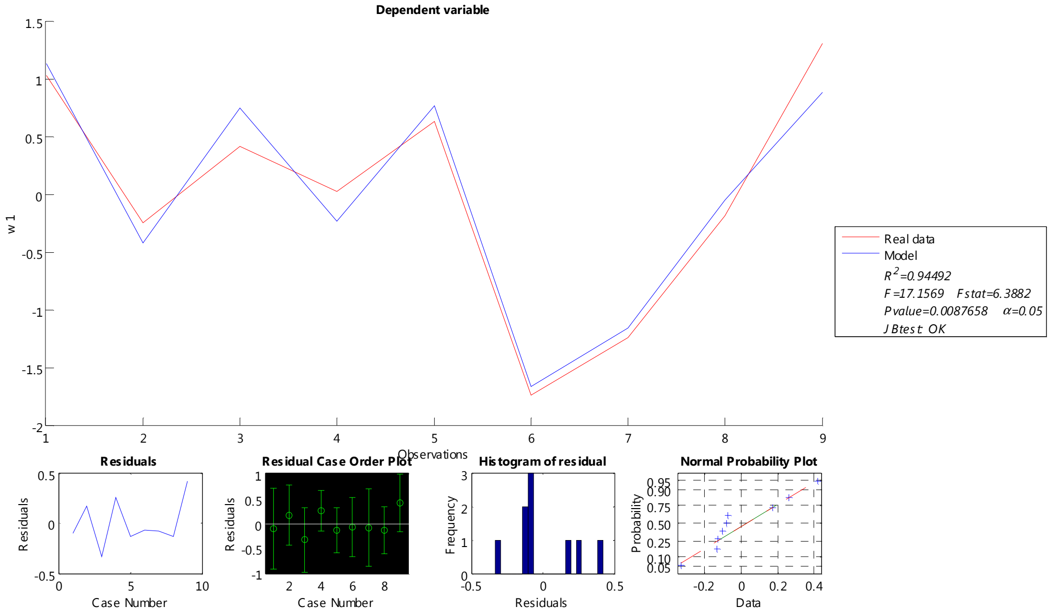

Fig. 6. Graphical interpretation of multiple regression for first dependent variable ω_1

6. Conclusion

Received in the experiment modal parameters and numerical estimators of vibroacoustics signal unambiguously show that the previously assumed conditions of the combustion engine's state reflect themselves in modal as well as other parameters characterising the vibrations and they are possible to be identified.

The use of the operational modal analysis in diagnostic investigations finds its use as one of many methods of marking the actual technical state of studied object. To complete the analysis process a SVD method and multiple regression were used. SVD methods marked most important symptom in description of engine technical state.

On the basis of the results, it is possible to determine the actual technical state of an object of the same type by means of comparison of the achieved results with the model ones and assigning them to the particular model's state, which answers to a particular damage, or its loss, in the object.

The introduced in paper results of investigations are the part of realized investigative project and they do not describe wholes of the investigative question, only chosen aspects.

References

[1] Cempel, Cz., *SVD Decomposition Of Symptom Observation Matrix As The Help In A Quality Assessment Of A Group Of Applications*, Diagnostyka v.35, PTDT Warszawa 2005.
[2] Łukasiewicz, M., *Próba odwzorowania modelu modalnego stanu technicznego silnika spalinowego w zastosowaniu do badań diagnostycznych*, Diagnostyka v.33, PTDT, Warszawa 2005.
[3] Łukasiewicz, M., *Badania napędu zasilania awaryjnego eksploatacyjną analizą modalną*, Inżynieria i Aparatura Chemiczna 1/2007, pp.83-84, Bydgoszcz 2007.
[4] Uhl, T., Lisowski, W., *Eksploatacyjna analiza modalna i jej zastosowanie*, AGH, Kraków 1999.
[5] Uhl, T., Kurowski P., *Vioma – instrukcja użytkownika*, AGH, Kraków 2002.
[6] Żółtowski, B., *Badania dynamiki maszyn*, ATR, Bydgoszcz, 2002.