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THE METHOD OF APPLICATION OF SEA IN STRUCTURAL DESIGN PROCESS FOR REDUCING STRUCTURE-BORNE NOISE IN MACHINERY PART 2: STRUCTURAL OPTIMIZATION OF SUBSYSTEMS

SUMMARY

This paper presents the formulation of the structural optimization method on the basis of statistical energy analysis. A4 laser beam printer consisting of eight finite elastic steel plates is considered as an example, where the mass is taken as a constraint function. Consequently, taking one CLF as the objective function, an optimization of the thickness of the shell elements is performed showing the efficiency of the structural optimization method.

***Keywords**: statistical energy analysis, method of vibration analysis, vibration of continuous system, modeling, vibration control, noise control, identification, FEM*

METODY REDUKCJI HAŁASU MASZYN
Z WYKORZYSTANIEM STATYSTYCZNYCH METOD ANALIZY PRZEPŁYWU ENERGII.
CZĘŚĆ 2. OPTYMALIZACJA STRUKTURALNA PODUKŁADU

W pracy zaprezentowano sformułowanie metody optymalizacji struktury bazującej na analizie statystycznego rozkładu energii. Jako przykład obiektu wybrano drukarkę laserową, której elementami jest osiem stalowych płyt. W przyjętej metodzie optymalizacji masy płyt pełnią rolę ograniczeń. Przyjmując CLF jako funkcję celu, przeprowadzono optymalizację grubości elementów powłokowych, pokazując efektywność metod optymalizacji strukturalnej.

***Słowa kluczowe**: statystyczna analiza energii, metoda analizy drgań, drgania układów ciągłych, modelowanie, sterowanie drganiami, sterowanie dźwiękiem, identyfikacja, MES*

1. INTRODUCTION

When attempting to reduce structure-borne noise radiated from machinery, it is difficult to examine how the energy flow changes between structural subsystems with conventional structural optimization methods, in which the objective function is the natural frequency or the peak value of the frequency response function. On the other hand, Statistical Energy Analysis (SEA) (Lyon 1979) is a method for vibro-acoustic analysis which regards the system as composed of high modal density and focuses on the power equilibrium between the subsystems. In SEA, the coupling loss factor denotes the energy flow between the subsystems. Therefore, it is considered that setting the coupling loss factor to the objective function can result in structural optimization which considers the energy flow between the subsystems.

On the basis of experimental SEA (Norton and Karczub 2001), the authors developed a structural design process for reducing structure-borne sound (Yamazaki *et al*. 2007; Yamazaki and Kuroda 2009), whose efficiency was subsequently verified by applying it to various machines. This process focuses on determining the SEA subsystems in the entire SEA subsystem. In other words, the SEA parameters (damping loss factors and coupling loss factors) which need to be changed in order to effectively reduce structure-borne sound are specified. Thus far, the subsystem structures examined for the purpose of adjusting the coupling loss factor appropriately have been studied with the analytical formula for the coupling loss factors used in analytical SEA. However, detailed examination of the subsystem structures have been impossible to perform until now with analytical SEA, as the latter does not depend on the specific shape of the subsystem (Yamazaki and Kuroda 2008).

For this reason, in pursuit of a structural optimization method based on FEM, which is capable of accounting for the details of the structure, the authors studied the FEM-SEA method, where the evaluation of the coupling loss factors was performed with respect to the target subsystem instead of the entire system (Yamazaki *et al*. 2008).

Accordingly, this paper proposes the formulation of a structural optimization method for SEA subsystems for which the realization of the desired value of the loss factors is necessary. This method is based on the FEM-SEA, targeting only subsystems which include the subsystem determined in the implementation of the structural design process for reducing structure-borne sound.

Regarding previous research in which SEA has been employed in optimization algorithms, there are case studies concerning the minimization of in-cabin sound pressure level by using analytical SEA, where the thickness of the subsystem and the damping loss factors (Aran and Dhanesh 2004; 2008) or loss factors (Bartosch and Eggner 2007) are taken as design variables. However, since analytical SEA has been used in these cases, it has been impossible to study the structure of the SEA subsystems in detail. Until now,

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there have been no studies similar to the one presented here, in which the aim is to conduct a detailed study of the subsystem structure and to examine the structural optimization method based on a combination of SEA and FEM calculations together with an optimization procedure.

This paper begins with an outline of the fundamental SEA formula and the structural design process for reducing structure-borne sound, followed by the proposition and the formulation of the structural optimization method on the basis of SEA. Subsequently, an A4 laser beam printer consisting of eight finite elastic steel plates is considered as an example, where the mass is taken as a constraint function. Consequently, taking one CLF as the objective function, an optimization of the thickness of the shell elements is performed, thus showing the efficiency of the structural optimization method.

2. BASIC THEORY

2.1. Power balance equation for SEA

In SEA, the system is regarded as an assembly of subsystems. The energy of the i-th subsystem is written as E_i. The system is considered to be resonant, and therefore the total energy E_i, which is the sum of the kinetic and the potential energy, is equal to twice the kinetic energy. The external power input to subsystem i is denoted as P_i. Considering that the power balance leads to a set of equations, and the SEA equation is written in the following matrix form:

$$\begin{Bmatrix} P_1 \\ \vdots \\ \vdots \\ P_N \end{Bmatrix} = \begin{bmatrix} \eta_1 + \sum_{i \neq 1}^{N} \eta_{1i} & -\eta_{21} & \cdots & -\eta_{N1} \\ -\eta_{12} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ -\eta_{1N} & \cdots & \cdots & \eta_N + \sum_{i \neq N}^{N} \eta_{Ni} \end{bmatrix} \begin{Bmatrix} E_1 \\ \vdots \\ \vdots \\ E_N \end{Bmatrix}$$

$$\text{or } \mathbf{P} = \omega \mathbf{L} \mathbf{E} \qquad (1), (2)$$

Here, **P** is the external power input vector, **E** is the vector of the subsystem energies, ω is the angular center frequency of the band, and **L** is the loss factor matrix, which is composed of the damping loss factors (DLF) η_i of the i-th subsystem and the coupling loss factors (CLF) η_{ij} from the i-th to the j-th subsystem. The estimation of the damping and the coupling loss factors is referred to as the construction of the SEA model.

2.2. Structural design process for reducing structure-borne sound

The structural design process for reducing structure-borne sound (Yamazki *et al.* 2007; Yamazaki and Kuroda 2009) developed by the authors follows the procedure.

1. Constructing an experimental SEA model.
2. Evaluating the scaling factors, which are taken as the contribution of the energies of the structural subsystems toward the sound pressure levels at the target evaluation locations.
3. Identifying the external power input during operation.
4. Specifying the loss factors which should be changed in order to reduce the sound pressure levels.
5. Examining the structural design in order to change the desired values of the loss factors.

In specifying the loss factors which need to be changed, the loss factors which can effectively reduce the acoustical or the vibrational energy of the subsystem were determined using the perturbation method, followed by the definition of the structural specifications needed for adjusting the coupling loss factors, which were determined with analytical SEA.

2.3. FEM-SEA

In the structural design process for reducing structure-borne sound described in Section 2.2, the loss factors to be adjusted were specified using the perturbation method. Subsequently, the SEA subsystems adjacent to the junctions corresponding to these loss factors were determined and the loss factors were evaluated using the FEM model (the partial model) of the SEA subsystems adjacent to the junctions. The method used for evaluating the loss factors by using FEM calculation is referred to as "FEM-SEA" (Yamazaki *et al.* 2008) and follows the procedure described below:

1. Constructing a partial FE model from the SEA subsystems connected by the target junctions.
2. Performing an eigenvalue analysis on the partial model and calculating the natural frequencies and the mode shapes.
3. Calculating the energy of each subsystem by applying rain-on-the-roof excitation (Mace and Shoter 2000).
4. Calculating the SEA parameters on the basis of the Power Injection Method (PIM) (Bies and Hamid 1980).

FEM-SEA contains information regarding the shape of the subsystems, and if the simulation is performed under boundary conditions which are close to the connected state of the entire system, the range of applicable frequencies is wider for FEM-SEA in comparison to analytical SEA.

3. OPTIMIZATION OF STRUCTURAL DESIGN FOR SEA SUBSYSTEMS

The structural optimization method was implemented in accordance with the structural design process for reducing structure-borne sound. First, the SEA parameters which can effectively reduce the acoustical and the vibrational energy of the subsystem were determined using by the structural design process. Next, the partial model simulating the boundaries of the entire system was constructed using

FEM-SEA. Finally, the structure with the desired values of the loss factors under arbitrary constraints was obtained by applying a combination of FEM-SEA and the optimization procedure.

3.1. Structural optimization method

The flowchart of the developed structural optimization method is shown in Figure 1. Steps 3. and 4. in the flowchart are related to FEM-SEA as described in Section 2.3. The procedure of the structural optimization method as described in the flowchart in Figure 1 is as follows:

Step 1: Constructing the partial FE model for estimating the target loss factors on the basis of the results obtained with the structural design process.

Step 2: Setting the initial values of some parameters, such as the objective functions, the design variables, and the constraint functions. The objective functions are the SEA parameters at the arbitrary frequency bands. The design variables are the density, Young's modulus, and the damping values associated with the material properties, as well as the thickness of the plate elements, the shape, and the coupling between the subsystems related to the structures, and so on.
The constraint functions are the mass, the stiffness (displacement), the stress, the buckling load, and the natural frequency, and so on.

Step 3: Calculating the normalized energy for the subsystem by applying rain-on-the-roof excitation. All modes in the target frequency range are excited, inducing individual excitations at multiple points.

Step 4: Calculating the SEA parameters on the basis of PIM.

Step 5: Calculating the constraint functions by performing static analysis or dynamic analysis.

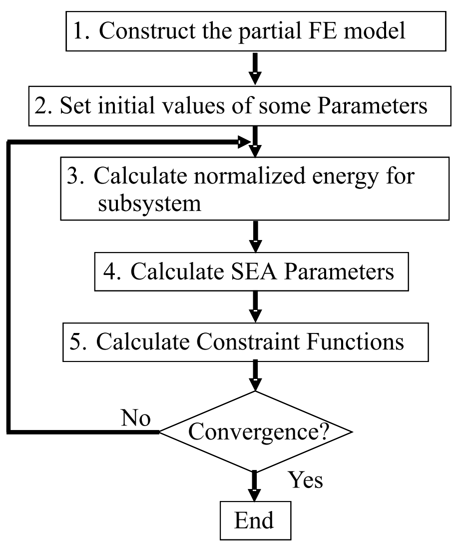

Fig. 1. Flowchart of optimization procedure

Within each optimization loop from Steps 3. to 5., the optimization algorithm defines new values for the design variables, and a new set of SEA parameter calculations is performed. The optimization algorithm depends on the classification of the design variables, the constraint functions, and the objective functions.

3.2. Formulation of the structural optimization problem

The formulation of the optimization problem by taking into account the subsystem structure is considered together with past structural optimization problems. The structure for which the objective function is maximized (minimized) or satisfies the target value is generated using a numerical method, such as the finite element method. The objective function is assumed to be CLF at an arbitrary frequency band and is used to formulate the minimization of the objective function.

In the case of the minimization of the objective function $CLF_i(\{x_j\})$ at multiple frequency bands (i = 1, …, n) on the basis of the constraint function $g(\{x_j\})$ in a feasible region D, the following equations can be written:

$$Minimize \sum_{i=1}^{n}\left(CLF_i\left(\left\{x_j\right\}\right)\right) \tag{3a}$$

$$Subject\ to\ g\left(\left\{x_j\right\}\right)-g_{\max} \le 0 \tag{3b}$$

$$\left\{x_j\right\}^L \le \left\{x_j\right\} \le \left\{x_j\right\}^U \ (j=1, ..., n) \tag{3c}$$

where $\{x_j\}$ is the vector of design variables, $g_{\max}$ is the upper limit of the constraint function $g(\{x_j\})$. $\{x_j\}^L$ ($\{x_j\}^U$) is for lower bound (upper bound) on $\{x_j\}$.

4. APPLICATION OF THE STRUCTURAL OPTIMIZATION METHOD

In this section, the validity of the structural optimization method proposed in the previous section is verified through its application to an actual machine. First, the structural design process described in Section 2.2 is applied to the machine, after which a partial model created on the basis of the results obtained with the structural design process is constructed. Next, the structural optimization method is applied to the partial model and a trial model is constructed on the basis of the optimization results. Finally, the sound pressure level and the SEA parameters are measured before and after the implementation of the countermeasure and the effects of the countermeasure are verified by comparing these values before and after its implementation.

In the application of the structural optimization method, it is necessary to use a combination of finite element analysis software for constructing the partial model of FEM-SEA and numerical programming software which performs inverse matrix calculations for computing the SEA parameters, as well as optimization software for obtaining the optimization results.

4.1. Structural design process

Here, the structural design process was applied to a laser printer designed for A4 monochrome printing at a speed of 14 pages per minute, and the printer without the outer casing was considered, as shown in Figure 2a. The structure of the laser printer in this paper is different from that of the laser printer used in the first report (Yamazaki and Kuroda 2009). The experimental method is the same as the first report.

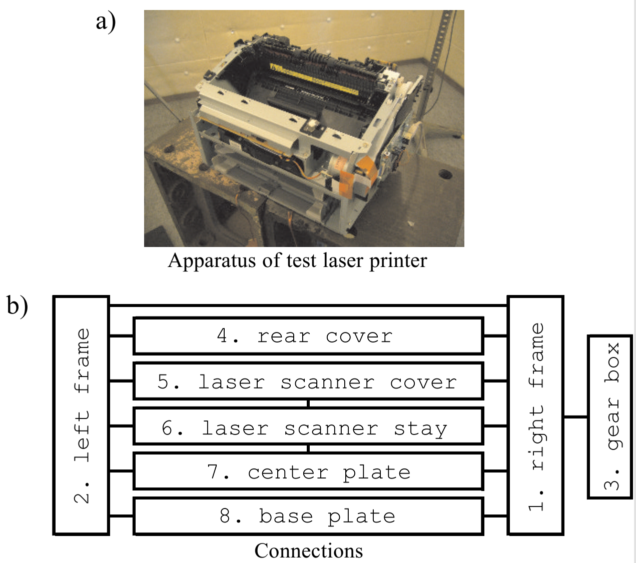

Fig. 2. Test laser printer and subdivision

The sound pressure level was measured at four points corresponding to the position of a standing person (1.5 m height and 1 m from front, back, left and right), as based on the ISO-7779 standard for free-field conditions. The sound pressure level was averaged over the values at the four positions, and the conditions for real-world operation were set to continuous printing of 14 pages. The sound pressure was regarded as the sound spectrum averaged over the time needed to print pages 2 through 13.

The sound pressure level measured in 1/3 octave bands is shown in Figure 3. The main sources of noise were the rotating motor, whose fundamental frequency was 504 Hz, and the sound produced by the photoconductive drum, which was around 1.2 kHz during the electrification and around 1.8 kHz during the development. Consequently, Figure 3 presents the noise level peak at the 500 Hz, 1.25 kHz, and 2 kHz in 1/3 octave bands, where the noise at 500 Hz was particularly high. It was impossible to reduce noise at 1.25 kHz and 2 kHz by means of structural modification since it was airborne sound.

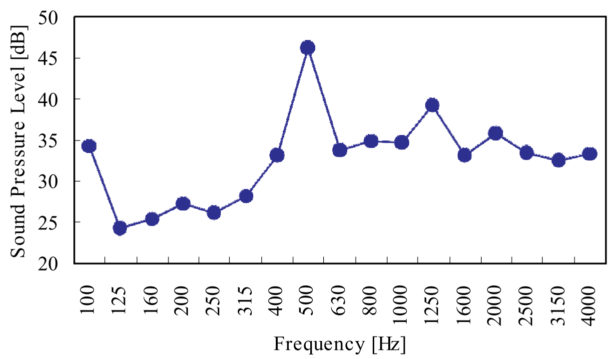

Fig. 3. Result of sound pressure levels of the test printer

The printer consisted of eight subdivisions, as shown in Figure 2b, where the steel plates were taken into consideration. The lower bound frequency in the effective SEA model was higher than 315 Hz.

The power inputs to the subsystems during printing were identified by measuring the energies of the respective subsystems. The power inputs to subsystems 1 and 3 were mostly recognized to be at the 500 Hz band.

Figure 4 shows the sensitivity of the sound pressure level at the evaluation locations. For example, from Figure 4a, the damping loss factors with the highest sensitivity were η_3 and η_6, which related to subsystems 3 and 6. Considering the coupling loss factors in Figure 4b, the coupling loss factors η_{13}, η_{16}, η_{26}, and η_{62} had larger sensitivity, and it was clear that η_3, η_6, η_{13}, η_{16}, and η_{26} should be larger while η_{62} should be smaller.

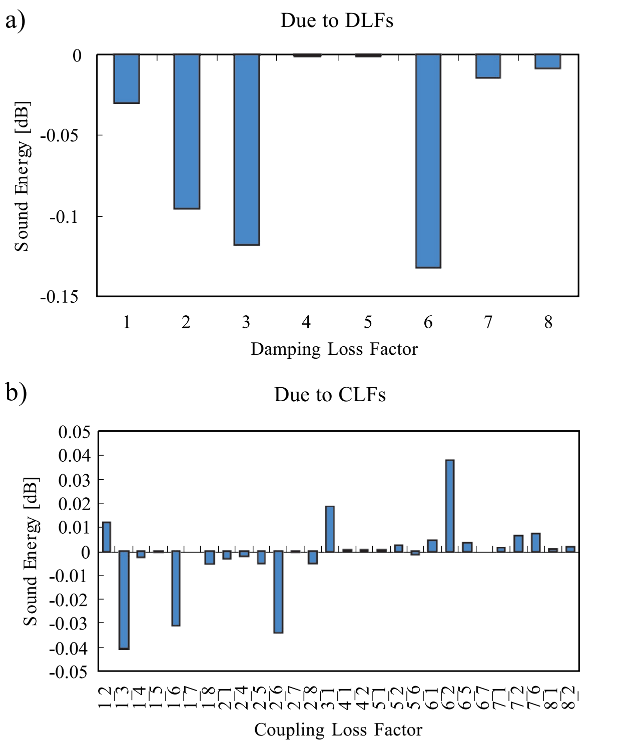

Fig. 4. Sound pressure levels due to the fluctuation of loss factors at 500 Hz in1/3 octave band during printing

From these results, the target frequency band was 500 Hz in the 1/3 octave band, and the objective function was η_{26}, taking into account the added workability for constructing the trial model.

4.2. Structural optimization method

The structural optimization method applied to the partial model focuses on subsystems 1, 2, and 6 in accordance with the results as described in Section 4.1, as well as by taking into account the adjustments in the experiment.

Here, ANSYS Ver. 11.0 was used for constructing the partial model together with APDL (ANSYS Parametric Design Language). The SEA parameters were calculated using MATLAB, and the optimization results were obtained using OPTIMUS 5.3, which is software for automation, integration, and optimization.

Step 1: Constructing the partial FE model

In constructing the FE model of the structure, which is shown in Figure 5, the material density and Poisson's ratio were set to 7860 kg/m^3 and 0.3, respectively. The thickness of the plate for each subsystem was 0.8 mm for subsystems 1 and 2 and 1 mm for subsystem 6. The total mass of the three subsystems was 0.648 kg. Furthermore, the element type was an elastic shell element (shell63) consisting of 4 nodes, with three translational and three rotational degrees of freedom per node. The element mesh was sufficiently dense to contain five nodes per wavelength for bending waves with frequencies of up to 1 kHz. There were a total of 1363 nodes and 1171 elements. The number of FE elements was 460 for subsystem 1, 449 for subsystem 2, and 262 for subsystem 6. All edges of the plate were freely supported in accordance with the boundary condition, where the adjustments in the experiment were taken into account.

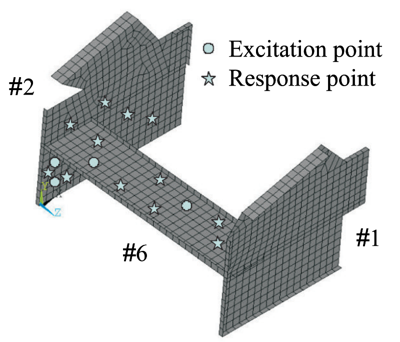

Fig. 5. A Partial finite element model

Step 2: Setting the initial values of some parameters

Various functions, such as objective functions, design variables, and constraint functions were set in accordance with the structural optimization method described in Section 3.1.

The objective function was η_{26} at 500 Hz in the 1/3 octave band, and the objective function was set within unity. The design variable was taken as the thickness of the FEM element, which is a commonly manipulated design variable in optimization problems regarding shell elements. Subsystem 2 was selected as a structural element which should be changed by taking into account the additional workability for constructing the trial model. Since the effects of the countermeasure were verified by comparing the values of the SEA parameters after constructing the trial model, it was decided that the design variables should take a discrete value (in the form of either a puncture or original thickness) instead of a continuous value by taking into account the additional workability for constructing the trial model. The value of the puncture was 1×10^{-6} m, the value of the original thickness was 8×10^{-4} m.

The elements considered for design variables were limited to the part by taking into account the additional workability for constructing the trial model, with the exception of geometrical limitations such as holes, screw holes, and uneven parts. Thus, there were 127 design variables.

In the case of considering the entire model, it is considered that the maximum stress and buckling load related to the strength are taken as the constraint functions. However, it is difficult to set the constraint functions to the partial model. Therefore, the total mass was taken as a constraint function by taking into account the adjustments in the experiment. The upper limit for the design variable was the original value (0.648 kg). These functions are described in terms of mathematical principles as follows.

$$Maximize\ CLF_{500\,\mathrm{Hz}}\left(\left\{x_j\right\}\right) \tag{4a}$$

$$subject\ to\ CLF_{500\,\mathrm{Hz}}\left(\left\{x_j\right\}\right) \le 1 \tag{4b}$$

$$total\ mass \le 0.648 \tag{4c}$$

$$x_j = 8\times10^{-4}\ or\ 1\times10^{-6} \tag{4d}$$

Step 3: Calculating normalized energy for subsystem

In the evaluation of the normalized energy for the subsystem, it is desirable to use rain-on-the-roof excitation as the excitation method, as explained in Section 2.3. However, considering the priority of the adjustments in the experiment, the same locations of the excitation points (2 positions per subsystem) shown in Figure 5 were selected. Furthermore, the response points (6 positions per subsystem) were also selected at the same locations as in the experiments.

The normalized energy of the subsystem was calculated for the range between 355 Hz and 710 Hz at 5 Hz steps by employing a modal superposition procedure. All modes within the frequency range of interest (0–2 kHz) were used. The loss factor was assumed to be 0.1 for all modes.

Step 4: Calculating SEA Parameters

Regarding the SEA parameters, the 1/3 octave frequency band characteristics were calculated in the range between 400 Hz and 630 Hz.

Step 5: Calculating Constraint Functions

The total mass was calculated in static analysis.

After the setup of the above mentioned Steps 1. to 5., the optimization algorithms were set in the OPTIMUS software. The Self-Adaptive Evolution (SAE) method, which is a kind of global optimization method applicable to discrete problems, was chosen here. The population size was chosen to be 5 times the number of design variables. That is, 635 values of the population size were generated in each iteration. Since the time required for obtaining the optimization results was rather long as compared to the local optimization method, the number of iterations was set to eight.

4.3. Optimization results

The iteration history of the objective function η_{26} is shown in Figure 6. Here, only the maximum value is indicated in each iteration. Figure 7 shows the structure in the optimization results, which is the result obtained after the forth iteration, which reaches the maximum value from eight iterations. In addition, η_{26} increased by about 400% and became 4.10×10^{-2} as compared with the initial value of 8.42×10^{-3}, while the total mass was reduced by about 6% of its initial value. The optimization results in this case indicate that the target value was achieved in the 500 Hz band. We plan to consider the objective function for multiple frequency bands in the near future.

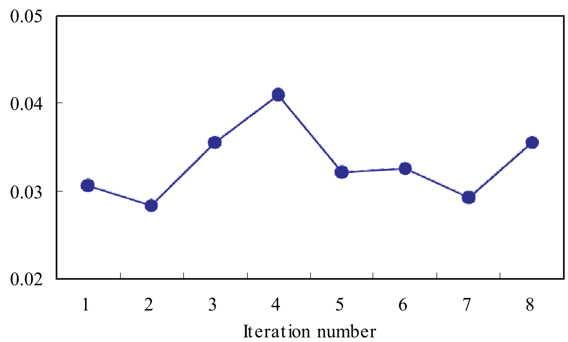

Fig. 6. Iteration history for the objective functions

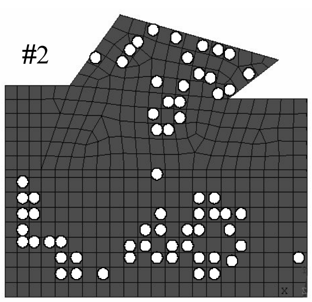

Fig. 7. Results of the optimization for subsystem configuration (o designates the location to be cut)

4.4. Verification of the optimization results with the trial model

Constructing a trial model on the basis of the optimization results obtained in the preceding section, the sound pressure level and the SEA parameters were measured before and after the implementation of the countermeasure, and the effects of the countermeasure were verified by comparing these values before and after its implementation. The structural changes applied to the parts described in Figure 7 were implemented by cutting or puncturing the plate as shown in Figure 8, Figures 9 and 10 show the sound pressure level and the SEA parameters before and after the implementation of the countermeasure, respectively. It can be seen that the reduction of noise by 4.3 dB was achieved at the 500 Hz 1/3 octave band in Figure 9. The value of the sound pressure level at the 1.25 kHz band, which is related to the cause of electrification noise, was 2.5 dB larger than original one. Since the plate was cut, the sound leakage increased. Figure 10 indicates that the implementation of the countermeasure increased the coupling loss factor η_{26} to about 300% of its original value.

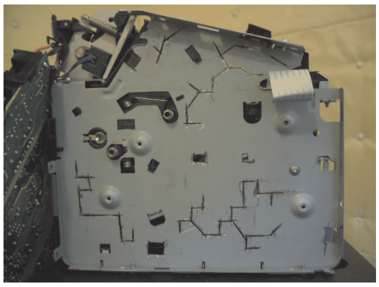

Fig. 8. Structural modification

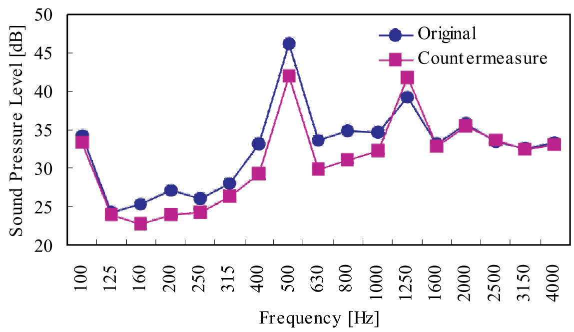

Fig. 9. Comparison of sound pressure levels with and without countermeasure

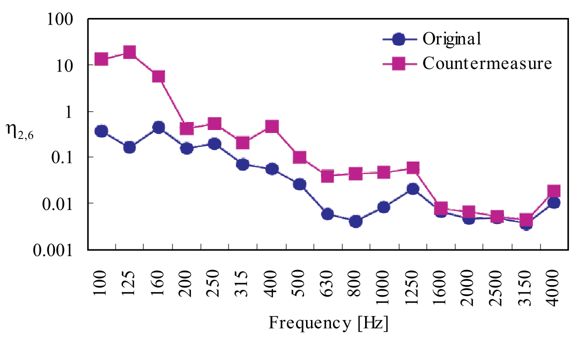

Fig. 10. Comparison of coupling loss factor from subsystem 2 to 6 between original and countermeasure

Therefore, a subsystem structure with the desired values of the loss factors under arbitrary constraints was constructed, and the efficiency of the proposed structural optimization method for SEA subsystems was verified using a combination of SEA and FEM calculations together with the optimization procedure.

Future plans include setting up the constraint functions to the partial model, the application of the objective function to multiple frequency bands, as well as the examination of the difference and the convergence of the optimization results for different optimization algorithms.

5. CONCLUSIONS

In this paper, with regard to the necessary countermeasures determined using the structural design process for reducing structure-borne sound, a structural optimization method which yields the desired values of SEA parameters was proposed and subsequently applied to a laser printer. The structural optimization method is based on SEA with FEM calculations, and the calculation of the optimal parameters is reiterated until a reasonable result is obtained for arbitrary constraints.

As a result of applying the proposed method to an actual machine, a subsystem structure with the desired values of CLF for the target frequency band under arbitrary constraints was constructed. Furthermore, by constructing a trial model on the basis of the optimization results, the sound pressure level and the SEA parameters were measured before and after the implementation of the countermeasure. The effects of the countermeasure were verified by comparing these values before and after the implementation of the countermeasure.

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