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# DESIGN OF VOLUME DISPLACEMENT SENSOR

*SUMMARY*

*This paper presents a new methodology for designing volume displacement sensors using shaped PVDF film on the basis of integration by parts. The design method proposed here does not require knowledge of the structural mode shapes. Through the design of the shape of the PVDF film, the charge output of PVDF sensor is proportional to the volume displacement of the beam. As shown the PVDF sensor shape is independent of the excitation (the type, position and frequency, etc.). Furthermore, the PVDF sensor proposed, is not sensitive to changes in the boundary conditions.*

***Keywords***: *volume displacement sensor; vibrating beam; PVDF sensor; boundary condition*

*PROJEKTOWANIE CZUJNIKA PRZESUNIĘCIA OBJĘTOŚCIOWEGO*

*Artykuł przedstawia nową metodę projektowania czujnika przesunięcia objętościowego przy użyciu materiału piezoelektrycznego PVDF. Metoda ta bazuje na całkowaniu przez części równania opisującego przesunięcie objętościowe drgającej struktury, mierzonego rozmieszczonym na strukturze materiałem PVDF. Dodatkowo prezentowana metoda nie wymaga znajomości drgań własnych struktury i pozwala znaleźć dokładny kształt czujnika oraz jego rozmieszczenie na strukturze drgającej, w prezentowanym przypadku na belce. Jak pokazano, czujnik dokładnie mierzy przesunięcie objętościowe belki w szerokim zakresie częstotliwości, niezależnie od warunków brzegowych belki i rodzaju przyłożonego wymuszenia.*

***Słowa kluczowe***: *czujnik przesunięcia objętościowego, materiał PVDF, czujnik akustyczny*

## 1. INTRODUCTION

Polyvinylidene fluoride (PVDF) strain sensors have attracted more and more attention in recent years for their use with active structural acoustic control (ASAC) techniques (Clark and Fuller 1992). Only structural actuator/sensors are employed to control the sound radiated by a structure. PVDF films are distributed sensors and thus avoid spatial aliasing problems, they give little additional loading on light structures, and are easy to cut into desired shapes. The width of a sensor strip can be varied along its length to achieve the required spatial sensitivity. In this way the output signal only requires a suitable amplifier, thus alleviating the need for further signal processing. The PVDF sensor bonded onto the host structure can be directly integrated into a smart structure for real-time structural-acoustical control. Since PVDF sensors measure structural vibration information, the sensors should be designed to take into consideration the relationship between the structural and acoustic response. The appropriately designed sensor should be able to mainly detect the strongly radiating vibration distributions. It has been widely accepted that PVDF sensors can be designed based on the volume displacement/velocity (Johnson and Elliott 1995; Gardonio *et al*. 2001; Henrioulle and Sas 2003; Mao *et al*. 2003; Mao and Pietrzko 2006; Pan *et al*. 1998; Zahui *et al*. 2001; Charette *et al*. 1998). At low frequencies, the volume displacement/velocity accounts for most of the radiated sound power. The strategy of cancelation the volume displacement strategy for an ASAC system has been shown to be very efficient for reducing sound radiation in the low frequency range. Ref. (Johnson and Elliott 1995; Gardonio *et al*. 2001) give an exhaustive literature survey on the design of volume velocity sensors for beam- and plate- type structures.

The theory used for designing volume displacement sensors with shaped PVDF films has been well documented. However, the implementation of shaped PVDF sensors is often difficult because the sensor design almost exclusively involves an orthogonality relationship with the structural mode shape function. This means the vibrational mode shape of the beam should be known accurately in advance. For given boundary conditions there are unique PVDF sensor shapes.

To alleviate these problems, this paper presents a new methodology for designing shaped PVDF sensors to detect the volume displacement from a vibrating beam. As opposed to the work of previous authors (using orthogonality relationships with the structural mode shape function, referred to as the modal approach), the PVDF shape design method proposed here is based on the integration by parts approach. The method proposed here does not require knowledge of the structural mode shapes of the beam. With the appropriate design of the shape of the PVDF film, the charge output of PVDF sensor is proportional to the volume displacement of the beam. Finally, numerical simulations have been conducted to verify the proposed volume displacement PVDF sensors.

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## 2. DESIGN OF VOLUME DISPLACEMENT SENSOR USING INTEGRATION BY PARTS APPROACH

Consider a beam of length $L$, width $b$ and thickness $h$. A piezoelectric film of uniform thickness is attached to the upper surface over the entire length of the beam, as shown in Figure 1. The volume displacement $D$ is defined as the integral of the displacement $w(x)$ over the surface of the beam, and can be represented as (Mao and Pietrzko 2006)

$$D = b\int_0^L w(x)\,dxa \tag{1}$$

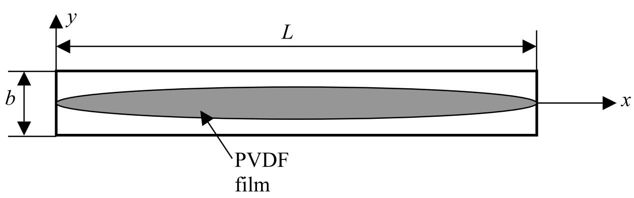


**Fig. 1.** Beam with a shaped PVDF film

Referring to Lee and Moon's work (Lee and Moon 1990), the output charge $Q$ of the shaped PVDF sensor can be expressed as follows,

$$Q = \frac{h+h_f}{2}\int_0^L F(x)\cdot\left(e_{31}\frac{\partial^2 w(x)}{\partial x^2}\right)dx \tag{2}$$

where $h_f$ is the PVDF sensor thickness, $F(x)$ is the shape function of the PVDF sensor, $e_{31}$ is the PVDF sensor stress/charge coefficient.

Integrating Eq. (2) by parts twice, we obtain

$$\begin{aligned} Q &= \frac{h+h_s}{2}e_{31}\left(F(x)\frac{\partial w(x)}{\partial x}\Bigg|_0^L - \int_0^L \frac{\partial F(x)}{\partial x}\frac{\partial w(x)}{\partial x}dx\right) = \\ &= \frac{h+h_s}{2}e_{31}\times\left(F(x)\frac{\partial w(x)}{\partial x}\Bigg|_0^L - \frac{\partial F(x)}{\partial x}w(x)\Bigg|_0^L + \right. \\ &\left. + \int_0^L \frac{\partial^2 F(x)}{\partial x^2}w(x)dx\right) \end{aligned} \tag{3}$$

To design shaped PVDF sensors that can accurately measure the volume displacement of the beam, the output charge $Q$ in Eq.(3) should be proportional to the volume displacement $D$. We can assume the PVDF shape function as

$$F(x) = Ax^2 + Bx + C \tag{4}$$

where $A$, $B$ and $C$ are unknown coefficients.

From Eq. (4), we can obtain $\partial^2 F(x) \big/ \partial x^2 = 2A$. Then substituting into Eq. (3),

$$\begin{aligned} Q &= \frac{h+h_s}{2}e_{31}\times \\ &\times\left(F(x)\frac{\partial w(x)}{\partial x}\Bigg|_0^L - \frac{\partial F(x)}{\partial x}w(x)\Bigg|_0^L + \int_0^L 2Aw(x)dx\right) = \\ &= \frac{h+h_s}{2}e_{31}\left(F(x)\frac{\partial w(x)}{\partial x}\Bigg|_0^L - \frac{\partial F(x)}{\partial x}w(x)\Bigg|_0^L\right) + \\ &+ A\frac{h+h_s}{b}e_{31}D \end{aligned} \tag{5}$$

From Eq. (5), it is simple to find that if the PVDF sensor shape function is satisfied,

$$F(x)\frac{\partial w(x)}{\partial x}\Bigg|_0^L - \frac{\partial F(x)}{\partial x}w(x)\Bigg|_0^L \equiv 0 \tag{6}$$

we obtain

$$Q = A\frac{h+h_s}{b}e_{31}D \tag{7}$$

Clearly, if the PVDF shape function $F(x)$ is satisfied in Eq. (4) and Eq. (6), the output charge $Q$ must be proportional to the volume displacement of the beam. For simplicity, we set $Q = D$ to obtain,

$$A = \frac{b}{(h+h_s)e_{31}} \tag{8}$$

The next step is to calculate the PVDF shape function coefficients $B$ and $C$ from Eq. (4) and (6). Since Eq. (6) depends on the boundary conditions of the beam, the PVDF shape function should be determined for different boundary conditions. It is found that the boundary conditions of the beam can be divided into two groups. For each group of boundary conditions, the volume displacement of the beam can be measured using a fixed shape of PVDF sensor. These two groups of boundary conditions are discussed in the following section.

### 2.1. Beam with one end clamped and the other end having arbitrary boundary conditions

First, we assume that the boundary condition of the beam is clamped at the left end and arbitrary at the right end. This boundary condition can be represented as

$$w(0) = \frac{\partial w(0)}{\partial x} = 0 \tag{9}$$

Substituting Eq. (9) into Eq. (6) yields

$$F(L)\frac{\partial w(L)}{\partial x} - \frac{\partial F(L)}{\partial x}w(x) = 0 \tag{10}$$

Clearly, Eq. (10) can be satisfied, only if

$$F(L) = \frac{\partial F(L)}{\partial x} = 0 \tag{11}$$

Using Eq. (11), The coefficients $B$ and $C$ in Eq. (4) can be obtained, namely:

$$B = -2AL \quad \text{and} \quad C = AL^2 \tag{12}$$

Substituting Eq. (12) into Eq. (4), the PVDF shape function can then be obtained

$$F(x) = A(x-L)^2 \tag{13}$$

Similarly, for the beam with the clamp at the right end (arbitrary boundary condition at left end), we obtain $w(L) = \frac{\partial w(L)}{\partial x} = 0.$ The PVDF shape function can be represented as

$$F(x) = Ax^2 \tag{14}$$

### 2.2. Beam with zero displacement at the each end

The boundary conditions for zero displacement at the each end can be represented as

$$w(0) = w(L) = 0 \tag{15}$$

Substituting Eq. (15) into Eq. (6), yields $F(L)\frac{\partial w(L)}{\partial x} - F(0)\frac{\partial w(0)}{\partial x} = 0,$ which can be satisfied by setting

$$F(L) = F(0) = 0 \tag{16}$$

Substituting Eq. (16) into Eq. (4), the PVDF shape function can be obtained

$$F(x) = A\left(x^2 - Lx\right) \tag{17}$$

If the first derivative of the charge output signal is taken, i.e. the current $I(t)$ is measured, then the sensor output is proportional to the volume velocity of the beam $V$, since $I(t) = \frac{dQ(t)}{dt}$ and $V(t) = \frac{dD(t)}{dt}$.

## 3. NUMERICAL CALCULATIONS

In order to verify the above PVDF volume displacement sensor design method, a beam with dimensions of 500 × 25 × 3.5 mm is considered (over a frequency range of 5 Hz to 800 Hz). The Young's modulus, density and damping ratio are $2 \times 10^{11}$ Pa, 8700 kg/m$^3$ and 0.01, respectively. The PVDF sensor shapes required for measurement of the volume displacement are shown in Figures 2 and 3 for two groups of boundary conditions, respectively.

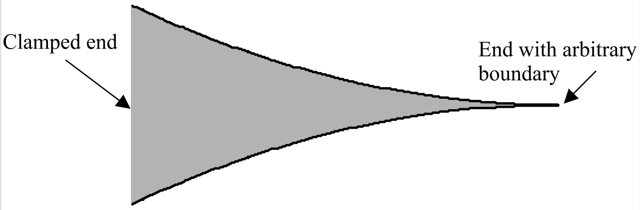


**Fig. 2.** The PVDF shape for a beam with one end clamped and the other end having arbitrary boundary conditions

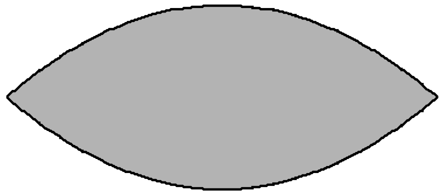

**Fig. 3.** The PVDF shape for a beam with the boundary conditions: zero displacement at the each end

In addition, Table 1 lists the PVDF shape function $F(x)$ for some typical boundary conditions. From Table 1, it is clearly seen that for the clamped-clamped beam, there are many possible sensor shapes to measure the volume displacement, because the PVDF shape function coefficients $B$ and $C$ are arbitrary. The PVDF shapes shown in Figures 2 and 3 can be used to measure the volume displacement of the clamped-clamped beam. Furthermore, it can be found that the PVDF shape for a clamped-free beam (as shown in Fig. 2) can be used to measure the volume displacement with the boundary conditions clamped-clamped, clamped-simply supported or clamped-free.

**Table 1**

PVDF shape function $f(x) = Ax^2 + Bx + C$ for some typical boundary conditions

| | Clamped | Simply supported | Free |
|---|---|---|---|
| Clamped | $B$, $C$ arbitrary | $C = -AL^2 - BL$<br>$B$ arbitrary | $F(x) = A(x-L)^2$ |
| Simply supported | $C = 0$, $B$ arbitrary | $F(x) = A\left(x^2 - Lx\right)$ | |
| Free | $F(x) = Ax^2$ | | |

For comparison, the shapes of the PVDF volume displacement sensors designed by the modal approach are shown in Figure 4. From Figure 4, it can be found that the modal approach yields unique sensor shapes for given boundary conditions. If the boundary condition is changed, the shaped of the PVDF sensor should be modified simultaneously. The main drawback of the modal approach is that the structural mode shape of the beam should be accurately known in advance. Any deviation of the mode shape of the beam may result in a large measurement error.

According to the above analysis, it can be found that the use of integration by parts seems to be more practical than the modal approach. For example, three unique shapes of PVDF sensors (as shown in Figs. 4b–d) designed by the modal approach are required to measure the volume displacement for the beam with clamped-clamped, clamped-simply supported and clamped-free boundary conditions. However, only one uniform sensor shape (shown in Fig. 2) designed by integration by parts can be used to measure the beam with all three of these boundary conditions.

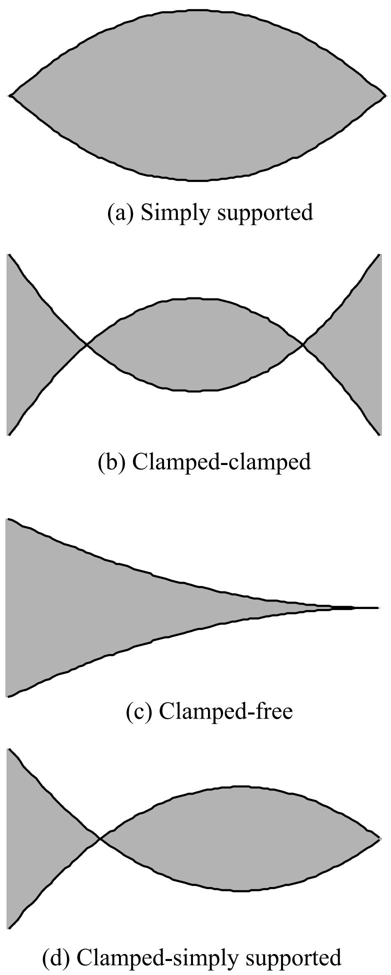


**Fig. 4.** The shape of the PVDF volume displacement sensors designed by the modal approach

Assume that the beam is excited by a point force located at $x_d$ = 0.11 m. Figures 5–7 show the signal outputs of the volume displacement PVDF sensor with the shapes shown in Figure 2 (designed by the integration by parts approach) measuring the beam with clamped-clamped, clamped-simply supported and clamped-free boundary conditions. It can be shown that the charge output curves agree well with analytical volume displacements for different boundary conditions, as expected. It should be noted that only one uniform shape of PVDF sensor is used to measure the volume displacements with these three boundary conditions.

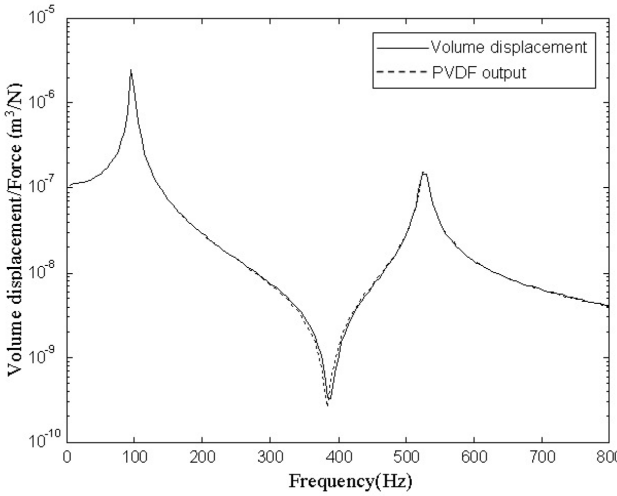


**Fig. 5.** The signal output of PVDF sensor (with shape in Fig. 2) for a Clamped-clamped beam

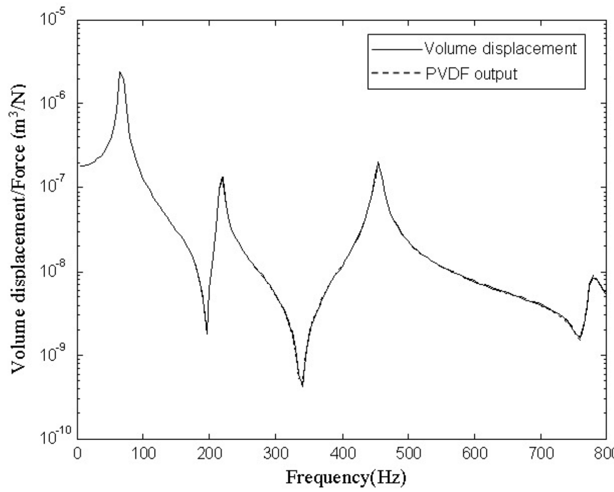


**Fig. 6.** The signal output of PVDF sensor (with shape in Fig. 2) for a clamped-simply supported beam

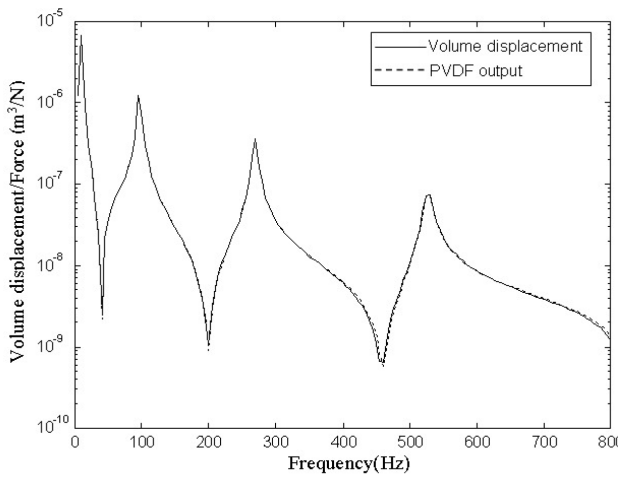


**Fig. 7.** The signal output of PVDF sensor (with shape in Fig. 2) for a clamped-free beam

## 4. CONCLUSIONS

This paper presents a technique for measuring the volume displacement from a vibrating beam using shaped PVDF films on the basis of the integration by parts approach. The main advantage of the method is that the shape function of the PVDF sensor is not sensitive to changes of the boundary conditions. The boundary conditions of a vibrating beam can be divided into two groups, that is, (1) one end is clamped and another end is arbitrary; (2) the displacement at each end is zero. For each group of boundary conditions, the volume displacement of the beam can be measured using only a simple fixed shape of a PVDF sensor. For example, for a beam with one clamped end, its volume displacement can be measured by using a PVDF sensor with the shape shown in Figure 2, no matter what type of boundary condition occurs at the other end. The other advantage is that the PVDF shape designed here is independent of the properties of excitation (the type, position and frequency, etc.). The numerical results show the feasibility of this new type of shaped PVDF sensor for the measurement of volume displacement.

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