MECHANICS
Vol. 25 No. 4 2006

Marian JABŁOŃSKI*, Agnieszka OZGA**

ON STATISTICAL PARAMETERS CHARACTERIZING VIBRATIONS OF OSCILLATORS WITHOUT DAMPING AND FORCED BY STOCHASTIC IMPULSES***

SUMMARY

Our theoretical study aims at finding the statistical parameters characterizing the vibrations of an oscillator without damping and forced by stochastic impulses magnitude of the different distribution. We will derive the dependence of these parameters on rigidity and mass of the oscillator and on the stochastic distribution of the impulse magnitude. We will also carry out a numerical simulation verifying the derived mathematical model and interpret the differences between the results obtained in simulation and the mathematical calculations. This study is the first stage of research aimed at designing and calibrating a probe that will facilitate measuring of parameters determining the quality of a technological process.

***Keywords**: oscillator, stochastic impulses, stochastic process, expectation, variance*

STATYSTYCZNE PARAMETRY CHARAKTERYZUJĄCE DRGANIA OSCYLATORA BEZ TŁUMIENIA Z WYMUSZENIEM STOCHASTYCZNYM

Celem badań teoretycznych w tej pracy jest znalezienie statystycznych parametrów charakteryzujących drgania oscylatora bez tłumienia i będącego pod wpływem stochastycznych impulsów o różnych rozkładach. Zostanie wyprowadzona zależność tych parametrów od sztywności i masy oscylatora oraz od stochastycznego rozkładu wielkości impulsu. Przeprowadzimy również symulację numeryczną weryfikującą wyprowadzony model matematyczny oraz zinterpretujemy różnice pomiędzy wynikami otrzymanymi z symulacji a obliczeniami teoretycznymi. Badania te są pierwszym etapem prac mających na celu zaprojektowanie i skalibrowanie sondy, która umożliwi dokonywanie pomiarów parametrów określających jakość procesu technologicznego.

***Słowa kluczowe**: oscylator, stochastyczne impulsy, wartość oczekiwana, wariancja*

1. INTRODUCTION

The work was inspired by attempts at constructing a measuring device that would control the composition of the medium in a dust pipeline. The difficulties that arose then in connection with statistical interpretation forced us to search for a mathematical model that would explain its causes.

In the case of stochastic impulses forcing mechanical systems such as an oscillator, a string, a membrane, etc., the parameters of movement of particular elements in the system, including location, velocity and acceleration are random and thus are recognized as stochastic variables.

The first partial mathematical results regarding the vibration of oscillators forced by stochastic impulses can be found in the following works [2, 3, 4, 9, 11, 17]. In the study [19] by L. Takacs we can find the results that generalize the findings published in previous works. The works [11, 12, 13, 14, 15, 16, 17] consider vibrations of oscillators forced by stochastic impulses and suggest their possible technological applications. These results will help solve the problem of stochastic response of systems to the action of stochastic impulses. Monographs [8, 10, 18] introduce the foundations of the considered problems. Works by R. Iwankiewicz [5, 6, 7] include certain results concerning nonlinear systems subjected to stochastic forces that might not act continually. These systems are solved by stochastic equations with Ito integral.

In this study we will, first of all, present a mathematical reasoning and a theorem that will allow us to calculate the basic parameters of considered stochastic variables like mathematical expectation or variance. We will assume that the probability that an impulse will occur in a short time interval is proportional to its duration and the moments of impulse occurrences and their magnitude are not probabilistically interrelated.

Secondly, in this study, we will apply these results for the oscillator without damping, forced by impulses in the form $f(t) = \sum_{t_i<t} \eta_i \delta_{t_i}$. We will calculate mean value of the amplitude of this oscillator as well as its theoretical variance related to time and characteristic parameters of the oscillator. Further, we will carry out our numerical simulation, compare the results of the simulation with theoretical calculations and interpret the differences that may occur.

The obtained results will allow us to suggest possible ways of interpreting the statistical qualities influencing the system depending on statistical characteristics of measurements. They will also explain certain difficulties connected with the initial studies on designs of measurement devices mentioned at the beginning of this introduction.

* Jagiellonian University in Cracow, Poland, Faculty of Mathematics and Computer Science; mjablon@ii.uj.edu.pl
** AGH University of Science and Technology in Cracow, Poland, Department of Mechanic and Vibroacoustics; aozga@agh.edu.pl
*** Praca została wykonana w ramach prac statutowych 11.11.130.137

2. THEORICAL BACKGROUND

Let g_i: $[0, \infty) \to R$, $i = 1, 2, 3, ..., m$ be a sequence of continuous functions, A be a bounded connected Borel subset of R^p for some $p \in N$, h_i: $A \to R$, $i = 1, 2, 3, ..., m$ be a sequence of bounded and continuous functions, $\{\tau_i\}_{i=1}^{\infty}$ – a sequence of independent identically distributed (i.i.d.) random variables with exponential distribution $F(x) = 1 - \exp(-\lambda x)$ for $x > 0$ and $F(x) = 0$ for $x < 0$, $\{\eta_i\}_{i=1}^{\infty}$ – a sequence of i.i.d. random variables with finite expectation, $\{\zeta_i\}_{i=1}^{\infty}$ – a sequence of i.i.d. random variables with values in the set A and finally let $\{\alpha_i\}_{i=1}^{\infty}$ be a sequence of real numbers. Let us put $t_0 = 0, t_i = \sum_{j=1}^{i} \tau_j, i = 1, 2, 3, ...$ and

$$\xi(t) = \sum_{n=1}^{m} \alpha_n \sum_{0<t_j<t} \eta_j h_n(\zeta_j) g_n(t - t_j) \tag{1}$$

Denote by ϕ_ζ and ϕ_η distributions of ζ and η respectively. Let $A_i \subset C$ and $B_i \subset [0, \infty)$ $i = 1, 2, \ldots, m$ be Borel sets and $k(i, j)$, for every fixed i, be an increasing sequence of all natural numbers such that

$$\chi_{A_i}\left(\eta_{k(i,j)}\right)\chi_{B_i}\left(\xi_{k(i,j)}\right) = 1 \tag{2}$$

where $\chi_A(x) = 1$ if $x \in A$ and $\chi_A(x) = 0$ if $x \notin A$. Write $t_j^i = t_{k(i,j)}$ and $\tau_j^i = t_j^i - t_{j-1}^i$.

We will say that $\xi(t)$ is decomposable if for every $n \in N$, all Borel sets $A_i \subset A$ and $B_i \subset [0, \infty)$, $i = 1, 2, ..., n$ such that $A_i \times B_i$ are mutually disjoint

$$\bigcup_{i=1}^{\tau} A_i \times B_i = A \times B \tag{3}$$

τ_j^i are i.i.d random variables with exponential distribution

$$F(x) = 1 - \exp\left(-\lambda \Phi_\xi(A_i)\Phi_\eta(B_i)\right) \text{ for } x > 0 \text{ and } F(x) = 0 \text{ for } x < 0 \tag{4}$$

$$\xi_i(t) = \sum_{n=1}^{m} \alpha_n \sum_{0<t_i^j<t} \eta_{k(i,j)} h_n\left(\zeta_{k(i,j)}\right) g_n(t - t_j^i), \quad i = 1, 2, ..., m \tag{5}$$

are independent and

$$\xi(t) = \sum_{i=1}^{n} \xi_i \tag{6}$$

From the technical point of view, decomposability of the process $\xi(t)$ means that we can divide the acting stochastic forces in any way, and space onto which they acted (for a string, a membrane etc) can be divided into any areas. If we consider the processes corresponding with the acting forces, let's say group number i acting on the area number j, we will receive a series of processes. As regards these processes we assume that they are independent.

Theorem 1

If the above defined process $\xi(t)$ is decomposable, then

1) characteristic function of $\xi(t)$ is given by

$$\varphi(s) = \exp\left(\lambda t\left(\int_A \int_0^\infty \int_0^1 \exp\left(is\sum_{n=1}^{m} \alpha_n y h_n(z) g(ut)\right) du\phi_\eta(dy)\phi_\zeta(dz) - 1\right)\right) \tag{7}$$

2) the expectation of $\xi(t)$ is

$$E(\xi(t)) = \frac{\varphi'(0)}{i} = \lambda t E(\eta) \sum_{n=1}^{m} \alpha_n E(h_n(\zeta)) \int_0^1 g_n(tu) du \tag{8}$$

3) the variance of $\xi(t)$ is

$$D^2(\xi(t)) = E(\xi^2(t)) - E^2(\xi(t)) = \frac{1}{i^2}\left(\varphi''(0) - (\varphi')^2(0)\right) = \lambda t E\left(\eta^2\right) \sum_{n=1}^{m}\sum_{j=1}^{m} \alpha_n \alpha_j E(h_n(\zeta) h_j(\zeta)) \int_0^1 g_n(tu) g_j(tu) du \tag{9}$$

Lemma 2

Let g: $[0, \infty) \to R$ be an integrable in the Riemann sense function, $\{\tau_i\}_{i=1}^{\infty}$ be a sequence of i.i.d random variables with exponential distribution $F(x) = 1 - \exp(-\lambda x)$ for $x > 0$ and $F(x) = 0$ for $x < 0$.

Let us put $t_0 = 0$, $t_i = \sum_{j=1}^{i} r_j, i = 1, 2, 3, ...$ and

$$\xi(t) = \sum_{t_j<t} g(t - t_j) \tag{10}$$

1) characteristic function $\varphi(s)$ of $\xi(t)$ is given by the following formula

$$\varphi(s) = \exp(\lambda t\left(\int_0^1 \exp\left(isg(ut)du - 1\right)\right) \tag{11}$$

2) the expectation of $\xi(t)$ is

$$E(\xi(t)) = \frac{\varphi'(0)}{i} = \lambda t \int_0^1 g(tu) du \tag{12}$$

3) the variance of $\xi(t)$ is

$$D^2(\xi(t)) = E(\xi^2(t)) - E^2(\xi(t)) = \frac{1}{i^2}\left(\varphi''(0) - (\varphi')^2(0)\right) = \lambda t \int_0^1 g^2(tu) du \tag{13}$$

Proof. Let $N(t) = \max\{n: t_n < t\}$.It is well known that (see [1]) $N(t) = \sum_{i=1}^{\infty} \chi_{[0,t]}(t_i)$ is a Poisson process with intensity λ. For $n \in N$ and $k = 0, 1, 2, ..., n-1$ let us put

$$Y_k^n = \begin{cases} g\left(\frac{kt}{n}\right) & if \ \{t_n\}_{n=1}^{\infty} \cap \left(\frac{kt}{n}, \frac{(k+1)t}{n}\right) \neq 0 \\ 0 & if \ \{t_n\}_{n=1}^{\infty} \cap \left(\frac{kt}{n}, \frac{(k+1)t}{n}\right) = 0 \end{cases} \tag{14}$$

We can represent Y_k^n in the following way

$$Y_k^n = \chi\left(N\left((k+1)t/n\right) - N(kt/n)\right) - g(kt/n) \tag{15}$$

where $\chi(x) = 1$ if $x > 0$ and $\chi(x) = 0$ otherwise. This, since $N((k+1)t/n) - N(kt/n)$, $k = 0, 1, 2, 3, ..., n-1$ are mutually independent random variables, implies that Y_k^n are also mutually independent. Now let us set

$$X^n(t) = \sum_{k=0}^{n-1} Y_k^n \tag{16}$$

Since $\tau_i > 0$ almost everywhere and in any interval $(0, t)$ process $N(u)$ is finite with probability one, by 1 $X^n(t)$ is convergent to $X(t)$ almost everywhere. It is well known (see [1]) that

$$\begin{aligned} &P(N(t_2) - N(t_1) = 0) = 1 - \lambda(t_2 - t_1) + o(t_2 - t_1), \\ &P(N(t_2) - N(t_1) = 1) = \lambda(t_2 - t_1) + o(t_2 - t_1) \end{aligned} \tag{17}$$

and

$$P(N(t_2)) - N(t_1) > 1) = o(t_2 - t_1).$$

Therefore, characteristic function of Y_k^n is

$$\begin{aligned} \varphi_k^n(s) = &\left(1 - \frac{\lambda t}{n} + o(1/n)\right)\exp(is0) + \\ &+\left(\frac{\lambda t}{n} + o(1/n)\right)\exp(isg(kt/n)) \end{aligned} \tag{18}$$

And, consequently, characteristic function of $X^n(t)$ is given by

$$\begin{aligned} \varphi_n(s) &= \prod_{k=0}^{n-1} \varphi_k^n(s) = \\ &= \prod_{k=0}^{n-1}\left[\left(1 - \frac{\lambda t}{n} + o(1/n)\right) + \left(\frac{\lambda t}{n} + o(1/n)\right)\exp(isg(kt/n))\right] \end{aligned} \tag{19}$$

We can rewrite the above formula in the following way

$$\begin{aligned} \varphi_n(s) = &\left(1 - \frac{\lambda t}{n} + o(1/n)\right)^n + \left(1 - \frac{\lambda t}{n} + o(1/n)\right)^{n-1} \times \\ &\times\left[\frac{\lambda t}{n} + o(1/n)\right]\sum_{k=0}^{n-1}\exp(isg(tk/n)) + \left(1 - \frac{\lambda t}{n} + o(1/n)\right)^{n-2} \times \\ &\times\left[\frac{\lambda t}{n} + o(1/n)\right]^2 \sum_{k<k_2}\exp(is(g(tk_1/n) + g(tk_2/n))) + \\ &+\left(1 - \frac{\lambda t}{n} + o(1/n)\right)^{n-3}\left[\frac{\lambda t}{n} + o(1/n)\right]^3 \times \\ &\times \sum_{k<k_2<k_3}\exp(is(g(tk_1/n) + g(tk_2/n) + g(tk_3/n))) + ... = \\ &= I_0 + I_1 + I_3 + ... \end{aligned} \tag{20}$$

It is easy to see that

$$\lim_{n\to\infty} I_0 = e^{-\lambda t}$$

and

$$\lim_{n\to\infty} I_1 = e^{-\lambda t}\lambda t\int_0^1 e^{isg(ut)}du \tag{21}$$

Since for every function $f(t)$, $n \in$ N and $k_1 = k_2 = 0, 1, 2, ..., n-1$

$$\begin{aligned} &\sum_{k_1<k_2} f(k_1t/n)f(k_2t/n) = \\ &= \frac{1}{2}\sum_{k_1,k_2} f(k_1t/n)f(k_2t/n) - \sum_{k=0}^{n-1} f^2(kt/n) \end{aligned} \tag{22}$$

for I_2 we obtain

$$\lim_{n\to\infty} I_2 = e^{-\lambda t}\frac{(\lambda t)^2}{2}\left(\int_0^1 e^{isg(ut)}du\right)^2 \tag{23}$$

Without any difficulties, by the same reasoning as for I_2, for $p > 2$ we obtain

$$\lim_{n\to\infty} I_p = e^{-\lambda t}\frac{(\lambda t)^p}{p!}\left(\int_0^1 e^{isg(ut)}du\right)^p \tag{24}$$

Now, since for $n > p$ and some c

$$\begin{aligned} I_P &\leq \left(1 - \frac{\lambda t}{n} + o(1/n)\right)^{n-p}\left[\frac{\lambda t}{n} + o(1/n)\right]^P \binom{n}{p} \leq \\ &\leq \frac{(\lambda t + c)^P}{p!} \end{aligned} \tag{25}$$

after some computations we get

$$\lim_{n\to\infty} \varphi_n(s) = e^{-\lambda t} \sum_{k=0}^{\infty} \frac{(\lambda t)^k}{k!} \left(\int_0^1 e^{isg(ut)} du \right)^k = \\ = \exp(\lambda t \left(\int_0^1 e^{isg(ut)} du - 1 \right)) \tag{26}$$

Thus, by the Lévy–Cramér theorem we have 1. The remaining part of the thesis is a simple consequence of 1.

Proof of the theorem

The theorem we prove for $p = 1$, $A = [0, b]$ and at the additional assumption that η is bounded by number M. For $p > 1$ and η unbounded the proof is similar. Let us consider the following family of two dimensional intervals $A_{kn} \times B_{qn}$ where $A_{kn} = [bk/n, b(k+1)/n)]$ for $k = 0, 1, \ldots, n-1$, $A_{n-1,n} = [b(n-1)/n, bn/n]$, $n = 1, 2, 3, \ldots$, $B_{qn} = [Mq/n, M(q+1)/n)]$, $q = 0, 1, \ldots, n-1$, $B_{n-1,n} = [M(n-1)/n, bn/n]$, $n = 1, 2, 3, \ldots$

It is obvious that

$$\bigcup_{k=0}^{n-1} \bigcup_{q=0}^{n-1} A_{kn} \times B_{qn} = [0, b] \times [0, M] \tag{27}$$

Write

$$\xi_{kq}^n(t) = \sum_{r=1}^{m} \alpha_r \sum_{0<t_j^{nkq}<t} \frac{Mq}{n} h_r\left(\frac{kb}{n}\right) g_r(t - t_j^{nkq}) = \\ = \sum_{0<t_j^{nkq}<t} \sum_{r=1}^{m} \left(\alpha_r \frac{Mq}{n} h_r\left(\frac{kb}{n}\right)\right) g_r(t - t_j^{nkq}) \tag{28}$$

and

$$\xi^n(t) = \sum_{k=0}^{n-1} \sum_{q=1}^{n-1} \xi_{kq}^n(t) \tag{29}$$

where t_j^{nkq} is a subsequence of $\{t_j\}$ such that $\zeta_j \in A_{kn}$ and $\eta_j \in B_{kn}$. It is obvious (see [1]) that $\xi^n(t)$ is convergent to $\xi(t)$ as n tends to ∞. Since $t_j^{nkq} - t_{j-1}^{nkq}$ have the same exponential distribution $F(x) = 1 - \exp(-\lambda \Phi_\xi(A_{kn}) \Phi_\eta(B_{qn}))$, by lemma 2 the characteristic function φ_{kq}^n of $\xi_{kq}^n(t)$ is given by

$$\varphi^n{}_{kq} = \exp\Bigg(\lambda t \phi_\rho(A_{kn}) \phi_\eta(B_{qn}) \times \\ \times \left(\int_0^1 \exp is \left[\sum_{r=1}^{m} \left(\alpha_r \frac{Mq}{n} h_r\left(\frac{kb}{n}\right)\right) g_r(tu) \right] du - 1 \right)\Bigg) \tag{30}$$

and consequently, since $\xi_{kq}^n(t)$ are independent, the characteristic function φ_n of $\xi_n(t)$ is

$$\varphi_n(s) = \prod_{k=0}^{n-1} \prod_{q=0}^{n-1} \exp\Bigg(\lambda t \phi_\xi(A_{kn}) \phi_n(B_{qn}) \times \\ \times \left(\int_0^1 \exp is \left[\sum_{r=1}^{m} \left(\alpha_r \frac{Mq}{n} h_r\left(\frac{kb}{n}\right)\right) g_r(tu) \right] du - 1 \right)\Bigg) \tag{31}$$

It is obvious (see [1]) that $\varphi_n(s)$ tends to $\varphi(s)$ as $n \to \infty$. By the Lévy–Cramér theorem we obtain 1) of the theorem. The remaining part of the thesis of Theorem 1 is a simple consequence of 1.

Applications

Let us consider the differential equation of the forced harmonic oscillator without damping and with one degree of freedom

$$\frac{d^2 x}{dt^2} + a^2 x = f(t) \tag{32}$$

The solution of the above equation satisfying the following initial conditions

$$x(0) = 0 \tag{33}$$

and

$$\dot{x}(0) = 0 \tag{34}$$

has the form

$$x(t) = \frac{1}{a} \int_0^t f(u) \sin a(t-u) du \tag{35}$$

If η_i is any sequence of real numbers, t_i is any increasing sequence of real numbers and $f(t)$ is given by

$$f(t) = \sum_{t_i<t} \eta_i \delta_{t_i} \tag{36}$$

where δ_{t_i} are δ-Dirac distributions at t_i, then the solution of (32), (33) (34) takes the following form

$$x(t) = \frac{1}{a} \sum_{t_i<t} \eta_i \sin(a(t - t_i)) \tag{37}$$

Let us notice that the derivation of the above function is discontinuous at t_i and, consequently, the second derivative of this function does not exist in the classical sense. Fortunately, function (37) can be considered as a solution of (32), (33),(34) in the distribution sense and it is sufficient for our purposes.

If η_i $i = 1, 2, \ldots$ are independent and identically distributed random variables with finite expectation and $\tau_i = t_i - t_{i-1}$, $i = 1, 2, \ldots$, are also independent and identically distributed random variables with exponential distribution $F(u) = 1 - \exp(-\lambda u)$ for $u > 0$ and for some λ and $F(u) = 0$ for $u < 0$) then $x(t)$ is a stochastic process satisfying assumptions of theorem 1 with $m = 1$ and $h_1 = 1$.

Applying this theorem we get the following formulas for the expectation and variance of $x(t)$:

$$E\left(x(t)\right) = \lambda E(\eta)\frac{1}{a}\int_0^t \sin(au)du =$$

$$= \lambda E(\eta)\frac{1}{a^2}(1 - \cos at);$$

$$D^2\left(x(t)\right) = \lambda E(\eta^2)\frac{1}{a^2}\int_0^t \sin^2(au)du =$$

$$= \lambda E(\eta^2)\frac{1}{2a^3}(at - \cos\left(at\right)\sin\left(at\right)).$$

3. NUMERICAL SIMULATION

For numerical simulation we will consider stochastic process $x(t)$ given by (37) for $t \in [0, 20\pi/a]$. Here $2\pi/a$ is the period of the solution of (32), (33), (34) with $f(t) = 0$.

For simplicity of computations we assume that random variables η_i have only two values. One of them is 728 and the second 214. The probability that η_i assumes 728 is 2/3 and the probabilty that η_i assumes 214 is 1/3. The numbers 728, 214, and the probabilities 2/3, 1/3 are chosen by chance, but in such a way that disturbances of vibrations of the oscillator are clearly distinguishable. In a similar way we choose $\lambda = 10$.

To get a statistical sample with n elements we repeat the following procedure n times. First we choose randomly τ_i in accordance with exponential distribution for $\lambda = 10$ until $t_n > 20\pi/a$ for the first time. We remember that $t_n = \sum_{i=1}^{n}\tau_i$. After that we choose randomly the values of η_i in accordance with the distribution $P(\eta = 728) = 2/3$ and $P(\eta = 214) = 1/3$. We substitute these data into (37) and thus we obtain an element of our statistical sample. Elements of the sample are denoted by $x^k(t)$. Three such elements are given in the Figure 1. We will consider three different statistical samples.

Having n elements of the sample we can write

$$\tilde{E}_n\left(x(t)\right) = \frac{1}{n}\sum_{i=1}^{n} x^i(t)$$

and

$$\tilde{D}_n^{\,2}\left(x(t)\right) = \frac{1}{n}\sum_{i=1}^{n}\left(x^i(t) - E\left(x(t)\right)\right)^2.$$

$\tilde{E}_n\left(x(t)\right)$ and $\tilde{D}_n^{\,2}\left(x(t)\right)$ are estimators of the expectation and the variance of $x(t)$ respectively.

By the law of large numbers for every fixed $t \in [0, 20\pi/a]$ $\tilde{E}_n\left(x(t)\right)$ and $\tilde{D}_n^{\,2}\left(x(t)\right)$ are convergent to theoretical expectation $E(x(t))$ and theoretical variance $D^2(x(t))$, respectively, as n tends to infinity.

Figure 1 presents three different realisations of the process (37), showing three different elements of statistical sample.

Figure 2 shows three diagrams $\tilde{E}_n\left(x(t)\right)$ for three different n-element statistical samples where n = 10 000 and a diagram of theoretical expectations $E\left(x(t)\right)$. Differences between theoretical results and value of estimator of the expectation for three statistical saples in simulation are small and thus statistically insignificant.

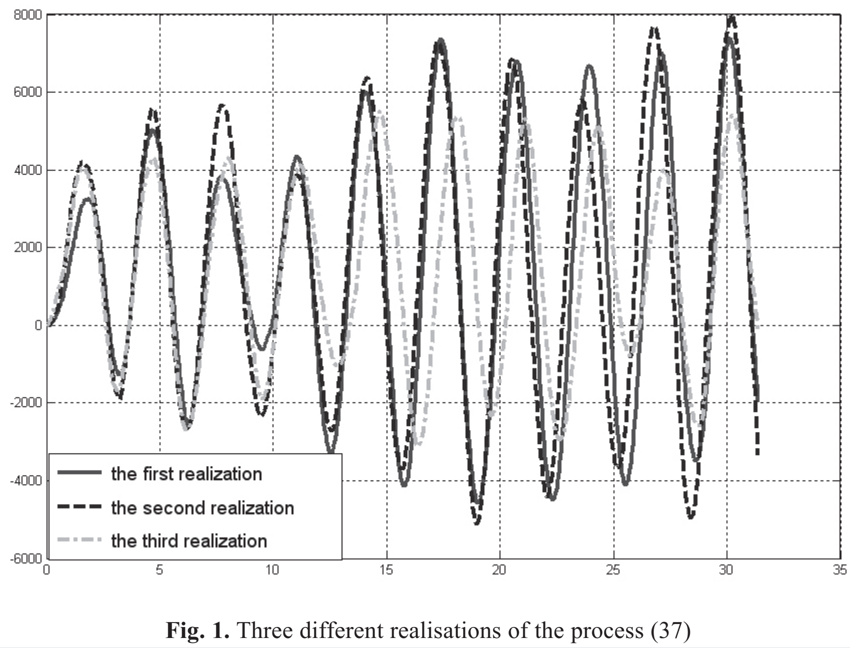

Fig. 1. Three different realisations of the process (37)

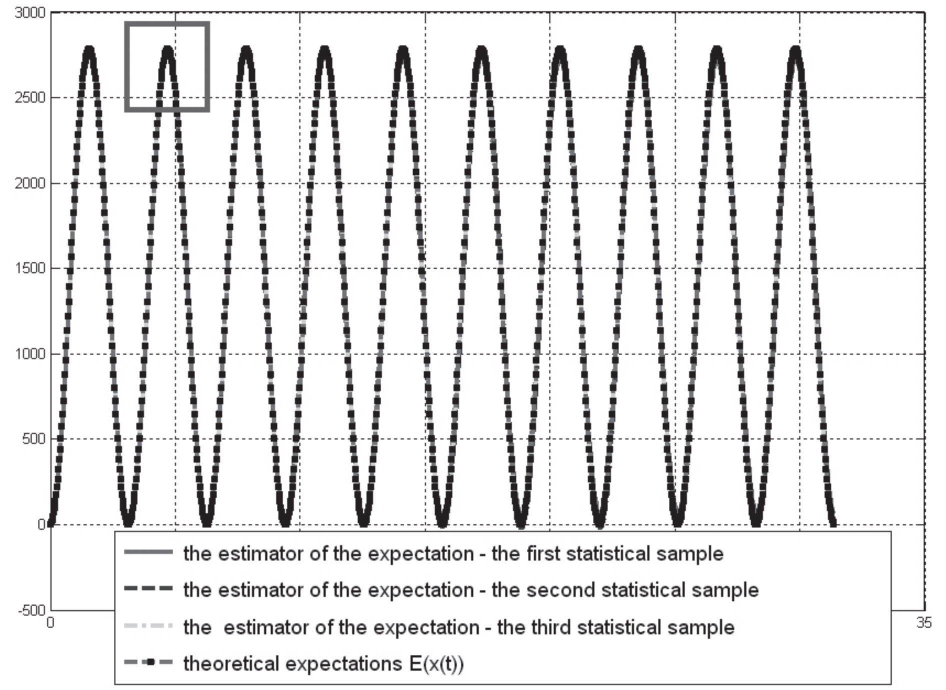

Fig. 2. Estimators of the expectation for the oscillations shown in Figure 1

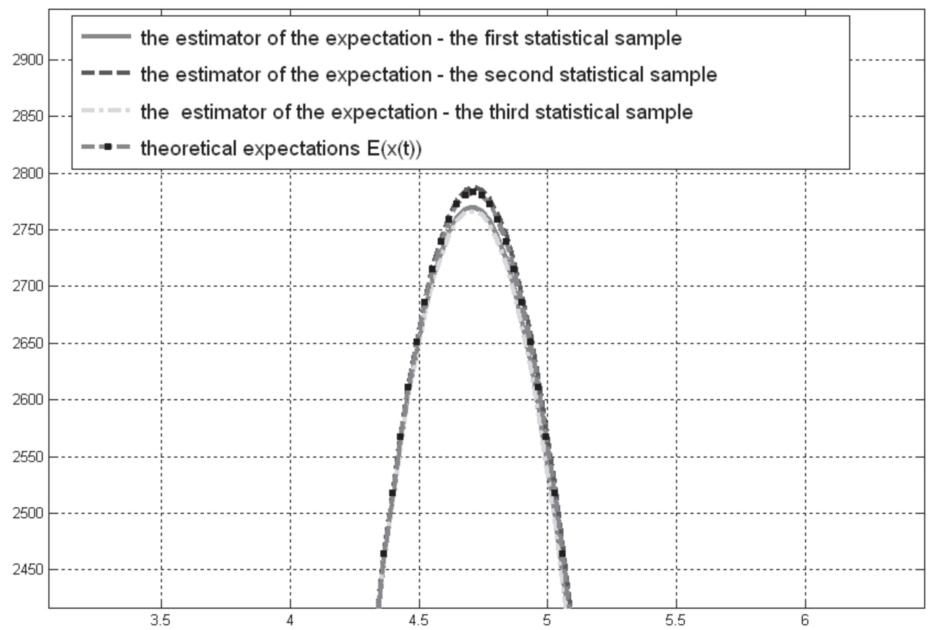

Fig. 3. The blow-up of the framed fragment of the diagram

In Figure 3 we show a blow-up of the framed fragment of the diagram.

Diagrams of the estimated variances $\tilde{D}_n^2\left(x(t)\right)$ for the same statistical samples as well as the diagram of theoretical variance are shown in Figure 4.

We can see high conformity between the simulation and the theoretical results for the initial values of t. The discrepancies at higher values of t are caused by the fact that variance is a steeply growing function, which is in accordance with statistic laws.

If the sample size is increased to 100 000 elements, the diagrams $\tilde{D}_n^2\left(x(t)\right)$ of estimators approximate theoretical variance $D^2\left(x(t)\right)$ much better, what is shown in Figure 5.

In Figure 6 we present diagrams of estimated variances when $E(\eta)$ is of the same value as before, but its variance is different $D^2(\eta)$.

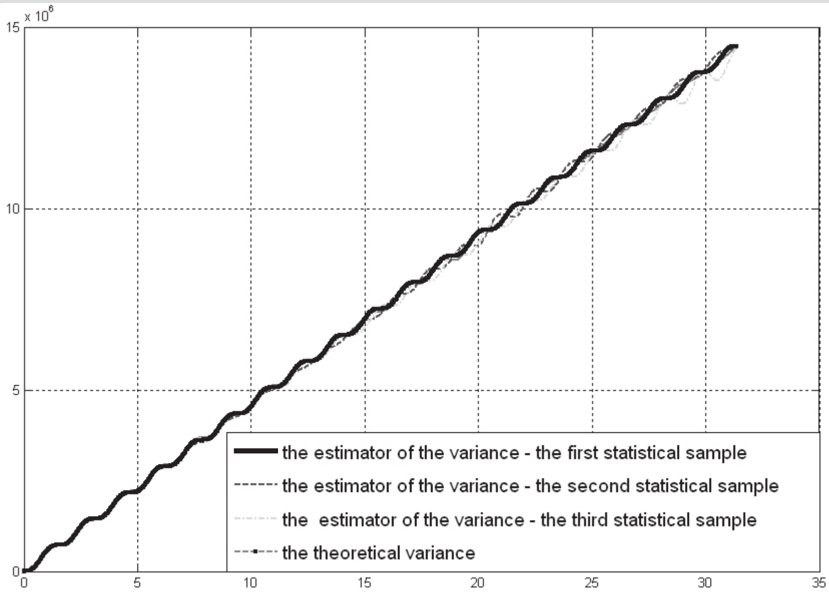

Fig. 4. Estimators of the variance for the oscillations shown in Figure 1

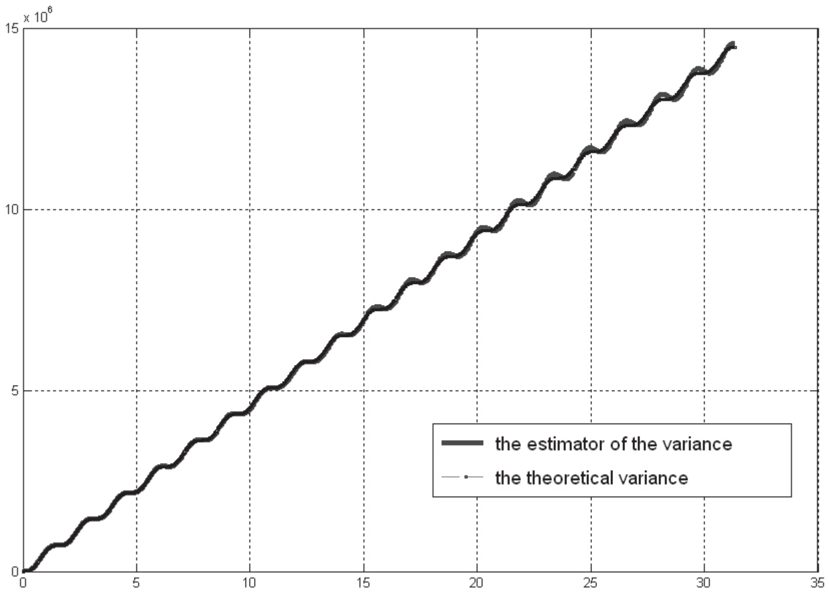

Fig. 5. The estimator of the variance for n = 100 000

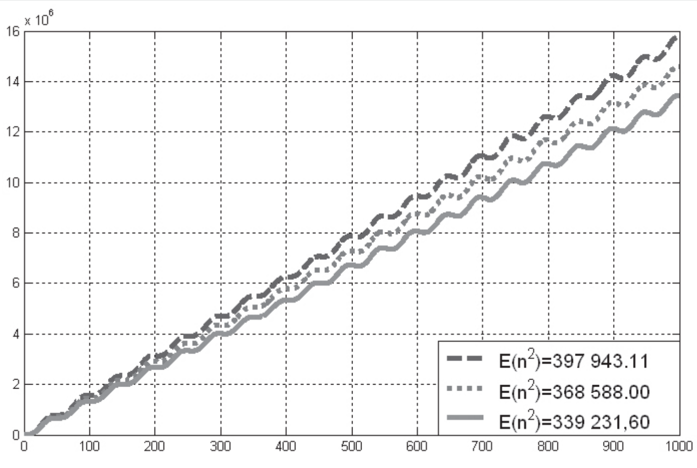

Fig. 6. Variances of $x(t)$ when $E(\eta)$ is of the same but $D^2(\eta)$ is different

4. CONCLUSIONS

The first part of computer simulation was to test whether the derived formulas for parameters characterizing the stochastically forced vibrations of oscillator were derived correctly. This stage was also meant to assess the size of the statistical sample, so that the estimators $\tilde{E}_n(x(t))$ i $\tilde{D}_n^{\,2}(x(t))$ could well approximate the theoretical solution.

The second stage of simulation was to visualize the dependence between the distribution of particle sizes with the same mean statistical value multiplied by the strike rate λ, but with different variances.

In an actual technological system the occurrence of large particles evoking movement of the oscillator at the same average mass of medium flowing through the dust pipeline in a time unit was an unfavourable phenomenon. Calculation of the mean value and the variance allows for detecting such occurrences.

The diagrams of variance estimators presented above suggest how we should conclude about the distributions of values of stochastic forces influencing an oscillator from the values of variance estimators and the mean value of the process $x(t)$ given by (37).

If λ increases, the maximum $E(\eta^2)$ at constant flow $\lambda E(\eta)$ = const must decrease, and thus, on the basis of the diagrams we may conclude about $D^2(x(t))$ that the less the $D^2(x(t))$, the closer the size of a falling particle to the mean value and the lesser the probability of a large particle strike.

To achieve the results of immediate technological significance, the problem requires further studies of more complex mechanical systems with damping, like a string, a membrane or a hollow log. Such systems placed within a dust pipeline will allow for maintenance of appropriate statistical parameters of the medium.

References

[1] Billingsley P.: P*rawdopodobieństwo i miara*. Warszawa, PWN 1987

[2] Campbell N.: *The study of discontinuous phencmena*. Proceedings of the Cambridge Philosophical Society, 15 (1909)

[3] Campbell N.: *Discontinuities in light emission*. Proc. Cambr. Phil Soc., 15(1909),

[4] Hurwitz H., Kac M.: *Statistical analysis of certain types of random functions.* Annals of Math. Stat., 15 (1944)

[5] Iwankiewicz R., Nielsen S.R.K.: *Dynamic response of non-linear systems to renewal impulses by path integration*. J. Sound Vib. Mech., 1996

[6] Iwankiewicz R.: *Dynamic systems under random impulses driven by a generalized Erlang renewal process*. In: Proc. of the 10th IFIP WIG 7.5 Working Conference on Reliability and Optimization of Structural Systems, 25–27 March 2002, Kansai University, Osaka, Japan. Eds. 2003

[7] Iwankiewicz R.: *Dynamic response of non-linear systems to random trains of non-overlapping pulses*. Meccanica, 37, 2002

[8] Kasprzyk S.: *Dynamika układów ciągłych*. Kraków, 1994

[9] Khintchine A.: *Theorie des abklingenden Spontaneffektes.* Bull. Acad. Sci. URSS, Ser. Math., 3 (1938)

[10] Plucińska A., Pluciński E.: *Probabilistyka*. Warszawa, WNT 2000

[11] Rice S.O.: M*athematical analysis of random noise I.* Bell. System Technical Journal, 23, 1944

[12] Roberts J.B.: *On the Harmonic Analysis of Evolutionary Random Vibrations*. J. Sound Vibr., 1965

[13] Roberts J.B.: *The Response of Linear Vibratory Systems to Random Impulses*. J. Sound Vibr., 2, 1965, 375–390

[14] Roberts J.B.: *System Response to Random Impulses*. J. Sound Vibr., 24, 1972

[15] Roberts J.B.: *Distribution of the Response of Linear Systems to Piosson Distributed Random Pulses.* J. Sound Vibr., 28, 1973

[16] Roberts J.B., Spanos P.D.: *Stochastic Averaging: an Approximate Method for Solving Random Vibration Problems*. Int. J. Non-Linear Mech., 21, (2), 1986

[17] Rowland E.N.: *The theory of mean square variation of a function formed by adding; known functions with random phases and applications to the theories of shot effect and of light.* Proc. Cambr. Phil. Soc., 32, (1936)

[18] Sobczyk K.: *Stochastyczne równania różniczkowe*. Warszawa, WNT 1996

[19] Takács L.: *On secondary processes generated by a Poisson process and their applications in physics*. Acta Math. Acad. Sci. Hungar., 5, 1954