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IDENTIFICATION OF PHYSICAL PARAMETERS OF RIGID ROTOR IN MAGNETIC BEARINGS

SUMMARY

Identification of physical parameters of unstable system, as magnetic bearing is, is an important problem, for example in the design of its diagnostic system. Most of the identification methods are applied for stable systems, without requiring feedback terms for identification purpose. However, for identification of marginally stable or unstable systems, feedback control is required to ensure the overall system stability. In many cases, a system, although stable, may be operated in the closed-loop, and it is impossible to remove the existing feedback controller for security or for production reasons. In diagnostic systems, there are usually observed trends in changes of physical parameters (e.g., mass, resistance, inductance, and so on). The physical realization is one of the possible system realizations. For any dynamic system, although Markov system parameters are unique, the realized state-space model is not unique. If one needs to compare the identified state-space model with the analytical model, both models have to be in the same coordinates.

An identification method of the state-space model resulting from the physics laws is presented in the paper. The method is a modification of the OKID method and it works exactly in the case when the sum of the input number and output number equals to the dimension of the state vector. There is no always possibility to measure the state vector, so for identification, it is estimated by deadbeat state observer. On the base of the reconstructed model it is possible to extract ARX model of observer/controller. This model is used to obtain the physical state-space model of the open-loop system for voltage controlled magnetic bearings. As a plant we consider a rigid rotor supported by magnetic bearings. The described method in easy way let us to identify physical parameters (e.g., mass, resistance, inductance, and so on) of the plant, which could be used in diagnostic system.

***Keywords**: identification, diagnostics, control*

IDENTYFIKACJA PARAMETRÓW FIZYCZNYCH WIRNIKÓW SZTYWNYCH W ŁOŻYSKACH MAGNETYCZNYCH

Identyfikacja parametrów fizycznych obiektów niestabilnych, jakimi są łożyska magnetyczne, jest ważnym problemem, na przykład przy projektowaniu systemu diagnostycznego. Większość istniejących metod identyfikacji systemu jest wystarczająca dla systemów stabilnych bez tworzenia sprzężenia zwrotnego dla potrzeb identyfikacji. Jakkolwiek, do identyfikacji warunkowo stabilmych i niestabilnych systemów sprzężenie zwrotne jest potrzebne do zapewnienia systemowi stabilności. W wielu przypadkach, pomimo iż system jest stabilny, pracuje on w układzie zamkniętym i niemożliwe jest otwarcie pętli ze względów bezpieczeństwa lub ograniczeń produkcyjnych. Przy diagnozowaniu zazwyczaj są obserwowane trendy w zmianach parametrów fizycznych (np. masy, rezystancji, indukcyjności, itp.). Realizacja fizyczna jest jedną z możliwych realizacji. Dla dowolnego systemu dynamicznego, mimo że parametry systemu Markova są niepowtarzalne, realizacji w przestrzeni stanu jest wiele. Jeśli chcemy porównać zidentyfikowany model w przestrzeni stanu z modelem analitycznym, wystarczy, aby oba modele miały te same współrzędne. W literaturze opisywane są dwa specyficzne przypadki, w których jest możliwe otrzymanie modelu fizycznego przestrzeni stanu na podstawie sygnałów wejścia-wyjścia.

W niniejszym artykule zaprezentowana jest metoda identyfikacji modelu przestrzeni stanu, której wynikiem są prawa fizyczne. Metoda jest zmodyfikowaną metodą OKID i działa wtedy, kiedy liczba wejść i liczba wyjść jest równa wymiarowi wektora stanu. Nie zawsze jest możliwe określenie wektora stanu, więc do identyfikacji sygnał wyjściowy jest estymowany przez obserwator deatbead. Na podstawie określonego modelu jest możliwe wydobycie modelu AXR obserwatora/regulatora. Na podstawie tego modelu otrzymujemy fizyczny model w przestrzeni stanu łożyska magnetycznego sterowanego napięciowo. Opisywana metoda jest łatwiejszym sposobem identyfikacji parametrów parametrów fizycznych (np. masy, rezystancji, indukcyjności, itp.) wirników sztywnych, może być wykorzystana w systemach diagnostycznych.

***Słowa kluczowe**: identyfikacja, diagnostyka, kontrola*

1. INTRODUCTION

System identification is the process of mathematical model construction for a tested dynamic system based on its input and output data. In the past few decades, a great variety of system identification methods have been studied extensively, for example [1, 2]. The choice of an identification method depends on the nature of the system and on the identification aim. From diagnostics point of view there is a need for a model of an open-loop system, since

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the feedback loop frequently compensates the changes in the physical parameters. Consequently, the open-loop system identification has to be performed on the closed-loop system.

Many frequency and time domain methods have been formulated for the calculation of an open-loop system realization. In particular, a time domain method called Observer/Kalman Filter Identification (OKID) [3] was considered to design the state-space realization {**A**, **B**, **C**, **D**} of linear systems. The Eigensystem Realization Algorithm (ERA) [4] used in OKID was the first formal technique directly applied for modal parameter identification in the form of state-space matrices, where modal parameters are eigenvalues (frequencies and modal damping) and eigenvectors, respectively. In diagnostic systems, there are usually observed trends in changes of physical parameters (e.g., mass, resistance, inductance, and so on). Therefore, it is desirable to identify the physical (analytical) model, which results from the physics laws.

The physical realization is one of the possible system realizations. For any dynamic system, although Markov system parameters are unique, the realized state-space mo-del is not unique. If one needs to compare the identified state-space model with the analytical model, both models have to be in the same coordinates. In the literature, there have been known three special cases in which the physical state-space model can be obtained from the input/output data (Markov parameters) used in ERA. For example, in [5] a unique transformation matrix was derived to transform any realized state-space model to be in a form, which is usually employed for structural dynamic system, so that any identified system parameter can be compared with the corresponding one. This unique transformation matrix exists only when one-half of the states can be measured directly. If this condition is not satisfied, the other transformation matrices may exist, but they are usually not unique. In [6] it is assumed that there exists a full state sensor. In this case, the measurement matrix is an identity (or diagonal) matrix and the inversion of this matrix realization can be used as a transformation matrix to transform the obtained system realization into its physical form. In [7] it is used observer/controller model for the identification purposes.

In the presented paper we use the last method for the identification of magnetic bearing physical parameters. It is assumed that the sum of inputs and outputs equals to the state vector dimension. In this case, the OKID method can be modified. The deadbeat observer is used to design the observer/controller model of the closed-loop system. In our case the Markov parameters are not calculated from the observer/controller system realization but the ARX model of the observer/controller is identified.

From this model we can directly calculate:

– the open-loop physical system realization,
– the observer gain physical realization.

Such approach was used to obtain the physical state-space model of the open-loop system for voltage controlled magnetic bearing. By inspection of identified physical parameters we can diagnose the system.

2. MODEL OF THE OPEN-LOOP SYSTEM

To control magnetic bearing in many applications one uses averaged values of currents: control current $i = (i_1 - i_2)/2$, and operation point current $i_o = (i_1 + i_2)/2$, where i_1, i_2 are currents in the opposite coils. In the voltage control there are usually two feedback loops to control these two currents. Such approach is not useful for diagnostics purposes, when we have to indicate the fault coil. Therefore we introduce another model.

For the mass supported by the two opposite coils we have the well known equations:

$$
\begin{aligned}
& m\ddot{x} = F_1 - F_2 + F_z \\
& u_1 = R_1 i_1 + L_{s1}\frac{d}{dt}i_1 + \frac{K_1}{2}\frac{d}{dt}\left(\frac{i_1}{x_o - x}\right) \\
& u_2 = R_2 i_2 + L_{s2}\frac{d}{dt}i_2 + \frac{K_2}{2}\frac{d}{dt}\left(\frac{i_2}{x_o - x}\right) \\
& F_1 = \frac{K_1}{4}\left(\frac{i_1}{x_o - x}\right)^2, \quad F_2 = \frac{K_2}{4}\left(\frac{i_2}{x_o - x}\right)^2
\end{aligned}
\tag{1}
$$

where:

- F_1, F_2 – forces generated by $\{_1\}$ and $\{_2\}$ opposite coils, respectively,
- F_z – external force,
- x_o – clearance,
- x – mass displacement from the operation point,
- K – constants,
- u – voltages,
- R – coil resistances,
- L_s – leakage inductances,
- L_o – air-gap inductances; while indices $\{_1\}$, $\{_2\}$ indicate the proper coil.

Let us linearize the above equations at the points: $x = 0$, $i_1 = i_o$, $i_2 = i_o$. This leads to the state space model of the open loop system:

$$
\begin{aligned}
& \dot{\mathbf{x}}(t) = \mathbf{A}_c\mathbf{x}(t) + \mathbf{B}_c\mathbf{u}(t) + \mathbf{B}_F\mathbf{F}_z \\
& \mathbf{y} = \mathbf{C}\mathbf{x}(t)
\end{aligned}
\tag{2}
$$

The above matrices are as follows:

$$\mathbf{x}=\begin{bmatrix}x\\ \dot{x}\\ i_1\\ i_2\end{bmatrix},\quad \mathbf{u}=\begin{bmatrix}u_1\\ u_2\end{bmatrix},\quad \mathbf{y}=\begin{bmatrix}x\\ i_1\\ i_2\end{bmatrix},\quad \mathbf{C}=\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$\mathbf{A}_c=\begin{bmatrix}0&1&0&0\\ \frac{k_{s1}+k_{s2}}{m}&0&\frac{k_{i1}}{m}&-\frac{k_{i2}}{m}\\ 0&-\frac{k_{i1}}{L_{s1}+L_{o1}}&-\frac{R_1}{L_{s1}+L_{o1}}&0\\ 0&\frac{k_{i1}}{L_{s2}+L_{o2}}&0&-\frac{R_2}{L_{s2}+L_{o2}}\end{bmatrix} \tag{3}$$

$$\mathbf{B}_c=\begin{bmatrix}0&0\\0&0\\ \frac{k_{w1}}{L_{s1}+L_{o1}}&0\\ 0&\frac{k_{w2}}{L_{s2}+L_{o2}}\end{bmatrix},\quad \mathbf{B}_F=\begin{bmatrix}0\\ \frac{1}{m}\\0\\0\end{bmatrix}$$

or in the shorter form:

$$\mathbf{A}_c=\begin{bmatrix}0&1&0&0\\ v_1&0&v_2&-v_3\\ 0&-v_4&-v_5&0\\ 0&v_6&0&-v_7\end{bmatrix},\quad \mathbf{B}_c=\begin{bmatrix}0&0\\0&0\\v_8&0\\0&v_9\end{bmatrix} \tag{4}$$

where:

k_{wj} – the amplifier gains,

$k_{si} = (K_i i_o^2)/(2x_o^3)$,

$k_{ij} = (K_j i_o^2)/(2x_o^2)$,

$K_j = N_j^2 A\mu_o$, $j = 1, 2$.

In the last expression there is:

N – active coil number in the electromagnet,
A – electromagnet pole cross section,
μ_o – magnetic permeability.

Thus, the open loop system is a plant with two inputs and three outputs and set values: $x = 0$, $i_1 = i_o$, $i_2 = i_o$. It means that the control errors are: $x_b = -x$, $i_{1b} = i_o - i_1$, $i_{2b} = i_o - i_2$.

The main aim of the control system is to bring the rotor to the center of the bearing ring, where $x = 0$. Therefore we should add the integral action to the controller.

The identification of matrices $\mathbf{A_c}$, $\mathbf{B_c}$ should facilitate the system diagnostics. All or a part of the elements in matrices are linear functions of physical parameters. For example, identified values of v_2, v_3 can give an information about short circuits in coils and information about number of working coils N_1, N_2, identified values of v_8, v_9 can give information about amplifiers (their gains k_{w1}, k_{w2}), and so on.

3. PHYSICAL SYSTEM REALIZATION

Consider an *n*th-order, *m*-input, *q*-output continuous-time linear model of the open-loop system resulting from the physics principles:

$$\mathbf{A}_c=\begin{bmatrix}0&1&0&0\\ v_1&0&v_2&-v_3\\ 0&-v_4&-v_5&0\\ 0&v_6&0&-v_7\end{bmatrix},\quad \mathbf{B}_c=\begin{bmatrix}0&0\\0&0\\v_8&0\\0&v_9\end{bmatrix} \tag{5}$$

where:
$\mathbf{A_c} \in R^{\mathrm{nxn}}$,
$\mathbf{B_c} \in R^{\mathrm{nxm}}$,
$\mathbf{C} \in R^{\mathrm{qxn}}$.

The state of the system is denoted by vector $\mathbf{x}(t)$, the control input by $\mathbf{u}(t)$, and the output by $\mathbf{y}(t)$, where: $\mathbf{x} \in R^{\mathrm{nx1}}$, $\mathbf{u} \in R^{\mathrm{mx1}}$, $\mathbf{y} \in R^{\mathrm{qx1}}$, respectively.

For the computer analysis or digital control purposes, the signals are sampled and the system (1) is discretized. It is assumed that the system is ideally sampled (with period Δt) by A-D converter and extrapolated by zero-order C-A converter. This leads to the following discrete-time state-space model:

$$\begin{aligned}\mathbf{x}(k+1)&=\mathbf{A}\mathbf{x}(k)+\mathbf{B}\mathbf{u}(k)\\ \mathbf{y}&=\mathbf{C}\mathbf{x}(k)\end{aligned} \tag{6}$$

where the integer k is the sample indicator and:

$$\mathbf{A}=e^{\mathbf{A}_c\Delta t},\quad \mathbf{B}=\int_0^{\Delta t} e^{\mathbf{A}_c\tau}d\tau\mathbf{B}_c \tag{7}$$

In the case of discrete-time system, the elements of matrices {**A**, **B**, **C**} are no longer (except of some special cases) any linear functions of physical parameters. Of course, one may convert such realized discrete-time system back to the continuous-time system {$\mathbf{A}_c$, $\mathbf{B}_c$, **C**} by relations:

$$\begin{aligned}\mathbf{A}_c&=\frac{1}{\Delta t}\ln\mathbf{A}\\ \mathbf{B}_c&=(\mathbf{A}-\mathbf{I})^{-1}\mathbf{A}_c\mathbf{B}\end{aligned} \tag{8}$$

Therefore, the matrices {**A, B, C**} from Eqs. (6) can be also called a discrete-time physical realization of the system.

For zero initial conditions the solution of Eqs. (6) for output $\mathbf{y}(k)$, in terms of inputs $\mathbf{u}(i)$, is in the form

$$\mathbf{y}(k)=\sum_{i=1}^{s}\mathbf{Y}_i\mathbf{u}(k-i) \tag{9}$$

where: $\mathbf{Y}_i = \mathbf{C}\mathbf{A}^{i-1}\mathbf{B}$, $i = 1, 2, 3, ...$, are known as the system Markov parameters, and s is sufficiently large. The Markov parameters are elements of Hankel matrices, which are used in ERA to calculate the system realization $\{\tilde{\mathbf{A}}, \tilde{\mathbf{B}}, \tilde{\mathbf{C}}\}$. Since the Markov parameters sequence is simply the pulse response of the system, they must be unique for a given system. This may be shown by noting that any coordinate transformation of the state vector, say $\mathbf{x}(k) = \mathbf{Tz}(k)$, which leads to the state-space model of the system in new coordinates:

$$\mathbf{z}(k+1) = \tilde{\mathbf{A}}\mathbf{z}(k) + \tilde{\mathbf{B}}\mathbf{z}(k), \mathbf{y} = \tilde{\mathbf{C}}\mathbf{z}(k) \tag{10}$$

with matrices:

$$\tilde{\mathbf{A}} = \mathbf{T}^{-1}\mathbf{AT}, \quad \tilde{\mathbf{B}} = \mathbf{T}^{-1}\mathbf{B}, \quad \tilde{\mathbf{C}} = \mathbf{CT} \tag{11}$$

yield the same Markov parameters

$$\mathbf{Y}_i = \tilde{\mathbf{C}}\tilde{\mathbf{A}}^{i-1}\tilde{\mathbf{B}} = \mathbf{CT}\left(\mathbf{T}^{-1}\mathbf{AT}\right)^{i-1}\mathbf{T}^{-1}\mathbf{B} = \mathbf{CA}^{i-1}\mathbf{B}$$
$$i = 1, 2, 3, ... \tag{12}$$

There are an infinite number of coordinate transformation matrices **T** that produce the same Markov parameters. Therefore, the ERA gives one of the infinite numbers of system realizations $\{\tilde{\mathbf{A}}, \tilde{\mathbf{B}}, \tilde{\mathbf{C}}\}$. It is evident that to obtain the physical realization {**A**, **B**, **C**} one should find the proper transformation matrix **T**. Then, according to Eq. (3), the physical realization can be obtained using relations:

$$\mathbf{A} = \mathbf{T}\tilde{\mathbf{A}}\mathbf{T}^{-1}, \quad \mathbf{B} = \mathbf{T}\tilde{\mathbf{B}}, \quad \mathbf{C} = \tilde{\mathbf{C}}\mathbf{T}^{-1} \tag{13}$$

It can be noticed there are two cases where the proper transformation matrix **T** can be obtained in a simple way. The first, when the state is completely controlled (**B** is square, nonsingular, and known), i.e.

$$\mathbf{T} = \tilde{\mathbf{B}}\mathbf{B}^{-1} \tag{14}$$

or the second one, when the state is completely measured (i.e., **C** is square, nonsingular and known), then

$$\mathbf{T} = \mathbf{C}^{-1}\tilde{\mathbf{C}} \tag{15}$$

In dynamic systems the complete state can be estimated by the full-order state observer, Kalman filter, or neural network. If the true measurement matrix **C** is known, then the transformation matrix **T** can be calculated based on Eq. (15). Because this matrix can be used only for perfectly known model of the system, therefore, in the next section another approach to obtain known, square, nonsingular matrices **C** or/and **B** is given.

4. SYSTEM WITH STATE FEEDBACK CONTROLLER

We assume a full state feedback controller with a gain matrix **F**, $\mathbf{F} \in R^{m\times n}$. The full state is measured or estimated by a state observer or Kalman filter. As it is shown in Figure 1 the control signal $\mathbf{u}(k)$ is a sum of persistent (usually pseudo-random) excitation signal $\mathbf{r}(k)$ and feedback signal $\mathbf{u}_f(k)$. Thus, the input signal $\mathbf{u}(k)$ and the control law $\mathbf{u}_f(k)$ are in the form:

$$\mathbf{u}(k) = \mathbf{u}_f(k) + \mathbf{r}(k), \quad \mathbf{u}_f(k) = -\mathbf{F}\hat{\mathbf{x}}(k) \tag{16}$$

In the OKID method [3] the controller gain and the open-loop system dynamics are assumed to be unknown. The closed-loop system is excited by a known (measured) excitation signal $\mathbf{r}(k)$, and the closed-loop system response $\mathbf{y}(k)$ and the feedback control signal $\mathbf{u}_f(k)$ are measured. The input signal $\mathbf{u}(k)$ can also be considered as a known one, based on the first equation of (16). It follows from the control scheme in Figure 1, where dynamics of an observer is described by equations:

$$\hat{\mathbf{x}}(k+1) = (\mathbf{A} + \mathbf{GC})\hat{\mathbf{x}}(k) + \mathbf{Bu}(k) - \mathbf{Gy}(k)$$
$$\hat{\mathbf{y}} = \mathbf{C}\hat{\mathbf{x}}(k) \tag{17}$$

where **G** is an observer gain.

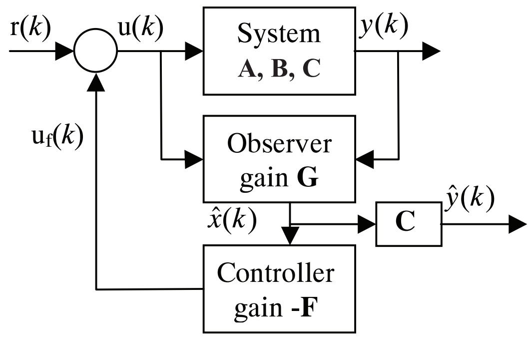

Fig. 1. Identified (effective) control system with state feedback controller

Combining Eqs. (16) and Eqs. (17), we have the state space model for the system in Fig.1 in the following form:

$$\hat{\mathbf{x}}(k+1) = \bar{\mathbf{A}}\hat{\mathbf{x}}(k) + \bar{\mathbf{B}}\begin{bmatrix}\mathbf{u}(k)\\ \mathbf{y}(k)\end{bmatrix}$$
$$\bar{\mathbf{y}}(k) = \begin{bmatrix}\hat{\mathbf{y}}(k)\\ \mathbf{u}_f(k)\end{bmatrix} = \bar{\mathbf{C}}\hat{\mathbf{x}}(k) \tag{18}$$

where:

$$\bar{\mathbf{A}} = \mathbf{A} + \mathbf{GC},$$
$$\bar{\mathbf{B}} = [\mathbf{B} \quad -\mathbf{G}],$$
$$\bar{\mathbf{C}} = \begin{bmatrix}\mathbf{C}\\ -\mathbf{F}\end{bmatrix}.$$

When the existing observer is asymptotically stable then it can be assumed that after some steps, for example s steps, one has $\hat{\mathbf{x}}(k) \cong \mathbf{x}(k), \hat{\mathbf{y}}(k) \cong \mathbf{y}(k)$. After s steps the input/output relations can be expressed in terms of finite numbers of the Markov parameters $\bar{\mathbf{Y}}_i$ of the effective observer/controller system described by Eqs. (18) as

$$\mathbf{y}_u(k) = \sum_{i=1}^{s} \bar{\mathbf{Y}}_i \mathbf{v}(k-i) \tag{19}$$

where:

$$\mathbf{y}_u(k) = \begin{bmatrix} \mathbf{y}(k) \\ \mathbf{u}_f(k) \end{bmatrix},$$

$$\mathbf{v}(k-i) = \begin{bmatrix} \mathbf{u}(k-i) \\ \mathbf{y}(k-i) \end{bmatrix},$$

$$\bar{\mathbf{Y}}_i = \bar{\mathbf{C}}\bar{\mathbf{A}}^{i-1}\bar{\mathbf{B}}, \quad i = 1, 2, 3, \; ... \; p,$$

p – number of identified observer/controller Markov parameters.

5. IDENTIFICATION PROCEDURE

Consider the case when the existing observer is asymptotically stable, so that for some sufficiently large s, $\bar{\mathbf{Y}}_i »\mathbf{0}$ for all time steps $i \geq s$. This means that $(\bar{\mathbf{A}})^i = (\mathbf{A}+\mathbf{GC})^i »\mathbf{0}$ for $i \geq s$. The matrix $\mathbf{G}$ can be manipulated to reduce the number of identified observer/controller Markov parameters, and one can replace the existing state estimator with a gain matrix $\mathbf{G}$, $\mathbf{G} \in R^{nxq}$, by the deadbeat observer with gain matrix $\mathbf{G_d}$, $\mathbf{G_d} \in R^{nxq}$, which converges after p steps and $s > p$. The input-output description of the system, see Eq. (19), for l data samples, after the existing observer has converged in s time steps, becomes

$$\bar{\mathbf{y}} = \bar{\mathbf{Y}}\mathbf{V} \tag{20}$$

where:

$$\bar{\mathbf{Y}} = \left| \bar{\mathbf{C}}\bar{\mathbf{B}}_d \quad \bar{\mathbf{C}}\bar{\mathbf{A}}_d\bar{\mathbf{B}}_d \quad ... \quad \bar{\mathbf{C}}\bar{\mathbf{A}}_d^{\,p-1}\bar{\mathbf{B}}_d \right|,$$

$$\bar{\mathbf{y}} = \begin{bmatrix} \mathbf{y}(s+1) & \mathbf{y}(s+2) & ... & \mathbf{y}(l) \\ \mathbf{u}_f(s+1) & \mathbf{u}_f(s+2) & ... & \mathbf{u}_f(l) \end{bmatrix},$$

$$\mathbf{V} = \begin{bmatrix} v(s-1) & v(s) & \cdots & v(l-1) \\ v(s-2) & v(s-1) & \cdots & v(l-2) \\ \vdots & \vdots & \ddots & \vdots \\ v(s-p) & v(s-p+1) & \cdots & v(l-p) \end{bmatrix},$$

for $s > p$, and matrices $\bar{\mathbf{A}}_d, \bar{\mathbf{B}}_d$ are described by formula below Eqs. (18) with matrix $\mathbf{G}$ replaced by matrix $\mathbf{G}_d$. The least-squares solution of the above equation leads to the following observer/controller Markov parameters

$$\bar{\mathbf{Y}} = \bar{\mathbf{y}}\mathbf{V}^T \left[\mathbf{V}\mathbf{V}^T \right]^{-1} \tag{21}$$

From the observer/controller Markov parameters, one design the Hankel matrices to implement the ERA algorithm. As a result of ERA, we have the observer/controller realization $\{\bar{\mathbf{A}}_d, \bar{\mathbf{B}}_d, \bar{\mathbf{C}}\}$.

Using Eqs. (13) and Eqs. (18) we have:

$$\begin{aligned} &\bar{\mathbf{A}}_d = \mathbf{A} + \mathbf{G}_d\mathbf{C} = \mathbf{T}\tilde{\bar{\mathbf{A}}}_d\mathbf{T}^{-1} \\ &\bar{\mathbf{B}}_d = \left[\mathbf{B} \quad -\mathbf{G}_d \right] = \mathbf{T}\tilde{\bar{\mathbf{B}}}, \quad \bar{\mathbf{C}} = \begin{bmatrix} \mathbf{C} \\ -\mathbf{F} \end{bmatrix} = \tilde{\bar{\mathbf{C}}}\mathbf{T}^{-1} \end{aligned} \tag{22}$$

Case I. Let $\bar{\mathbf{C}}$ is known and nonsingular. It means that $\mathbf{C}$ and $\mathbf{F}$ are known and $\bar{\mathbf{C}}$ is square and nonsingular. In this case the transformation matrix has the form

$$\mathbf{T} = \tilde{\bar{\mathbf{C}}}\bar{\mathbf{C}}^{-1} \tag{23}$$

According to Eqs. (22) the physical matrices of the open-loop system are calculated from formula:

$$\left[\mathbf{B} \quad -\mathbf{G}_d \right] = \mathbf{T}\tilde{\bar{\mathbf{B}}}_d, \quad \mathbf{A} = \mathbf{T}\tilde{\bar{\mathbf{A}}}_d\mathbf{T}^{-1} - \mathbf{G}_d\mathbf{C} \tag{24}$$

Case II. Let $\bar{\mathbf{B}}$ is known and nonsingular. It means that $\mathbf{B}$ and $\mathbf{G}_d$ are known and $\bar{\mathbf{B}}_d$ is square and nonsingular. In this case the transformation matrix has the form

$$\mathbf{T} = \bar{\mathbf{B}}_d\tilde{\bar{\mathbf{B}}}_d^{-1} \tag{25}$$

According to Eqs. (22) the physical matrices of the open-loop system are calculated from formula:

$$\mathbf{A} = \mathbf{T}\tilde{\bar{\mathbf{A}}}_d\mathbf{T}^{-1} - \mathbf{G}_d\mathbf{C}, \quad \begin{bmatrix} \mathbf{C} \\ -\mathbf{F} \end{bmatrix} = \tilde{\bar{\mathbf{C}}}\mathbf{T}^{-1} \tag{26}$$

In the diagnostic systems, one usually starts with a good defined "nominal" model of the diagnosed system. Unfortunately, the matrix $\mathbf{G}_d$ changes with moving system from its nominal parameters. Therefore, the Case II cannot be considered for a diagnostic purpose. It means that sensor diagnostics should be carried out in another way.

6. ARX MODEL IDENTIFICATION

Observer/controller Markov parameters can be calculated from ARX model parameters of the observer/controller system (18). The ARX model is in the form

$$\mathbf{y}_u(k) = \sum_{i=1}^{p} \mathbf{a}_i \mathbf{y}_u(k-i) + \sum_{i=1}^{p} \mathbf{b}_i \mathbf{v}(k-i) \tag{27}$$

where: $\mathbf{a}_i, \mathbf{b}_i$ are ARX model parameters. Stacking up Eq. (27) for different k, one can form a matrix equation:

$$\mathbf{y}_V(k) = \mathbf{P}\mathbf{V}_V(k-1), \quad \mathbf{y}_\mathbf{v}(k) = \mathbf{P}\mathbf{V}_\mathbf{v}(k-1) \tag{28}$$

where:

$$\mathbf{P} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{b}_1 & \mathbf{a}_2 & \mathbf{b}_2 & \ldots & \mathbf{a}_p & \mathbf{b}_p \end{bmatrix},$$

$$\mathbf{y}_V(k) = \begin{bmatrix} y_u(1) & y_u(2) & \ldots & y_u(p) & \ldots & y_u(k) \end{bmatrix},$$

$$\mathbf{V}_V(k-1) = \begin{bmatrix} y_u(0) & \cdots & y_u(p-1) & \cdots & y_u(k-1) \\ v(0) & \cdots & v(p-1) & \cdots & v(k-1) \\ 0 & \cdots & y_u(p-2) & \cdots & y_u(k-2) \\ 0 & \cdots & v(p-2) & \cdots & v(p-2) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & y_u(0) & \cdots & y_u(k-p) \\ 0 & \cdots & v(0) & \cdots & v(k-p) \end{bmatrix}.$$

Exciting the closed-loop by known (measured) signal $\mathbf{r}(k)$ and measuring the output $\mathbf{y}(k)$ and control $\mathbf{u}_f(k)$ signals, it should be possible for one to calculate the ARX model parameters by the batch least-squares method

$$\mathbf{P}(k) = \mathbf{y}_V(k)\mathbf{V}_V^T(k-1)\left[\mathbf{V}_V(k-1)\mathbf{V}_V^T(k-1)\right]^{-1} \tag{29}$$

Using $\mathcal{Z}$-transform to Eq. (19), separating signals $\mathbf{y}_u(z), \mathbf{v}(z)$ and applying long division, we have

$$\mathbf{y}_u = \left\{\mathbf{b}_1 z^{-1} + (\mathbf{b}_2 + \mathbf{a}_1\mathbf{b}_1)z^{-2} + \left[\mathbf{b}_3 + \mathbf{a}_1(\mathbf{b}_2 + \mathbf{a}_1\mathbf{b}_1) + \mathbf{a}_2\mathbf{b}_1\right]z^{-3} + \ldots\right\}\mathbf{v}(z) \tag{30}$$

Based on the above equation and the $\mathcal{Z}$-transform of Eq. (19), the observer/controller Markov parameters can be recursively calculated from the estimated ARX model parameters

$$\bar{\mathbf{Y}}_k = \mathbf{b}_k + \sum_{i=1}^{k} \mathbf{a}_i \bar{\mathbf{Y}}_{k-i} \tag{31}$$

Now, the ARX model will be calculated from the observer/controller state-space model (18). In the considered case, the matrix $\bar{\mathbf{C}}$ is square and nonsingular. It means the state vector can be calculated directly from the output

$$\mathbf{x} = \bar{\mathbf{C}}^{-1}\bar{\mathbf{y}} \tag{32}$$

Multiplying from the left the first equation of (18) by matrix $\bar{\mathbf{C}}$, and inserting Eq. (32), yields

$$\mathbf{y}_u(k+1) = \bar{\mathbf{C}}\bar{\mathbf{A}}_d\bar{\mathbf{C}}^{-1}\mathbf{y}_u(k) + \bar{\mathbf{C}}\bar{\mathbf{B}}\mathbf{v}(k) \tag{33}$$

The comparison of Eq. (27) to Eq. (33) leads to simple formula:

$$\mathbf{a}_1 = \bar{\mathbf{C}}\bar{\mathbf{A}}_d\bar{\mathbf{C}}^{-1} \tag{34}$$

$$\mathbf{b}_1 = \bar{\mathbf{C}}\bar{\mathbf{B}}_d \tag{35}$$

and $p = 1$. From Eq. (31) it can be concluded that there is only one observer/controller Markov parameter that has the matrix form

$$\bar{\mathbf{Y}}_1 = \mathbf{b}_1 = \bar{\mathbf{C}}\bar{\mathbf{B}}_d \tag{36}$$

Note, that Hankel matrix $\mathbf{H}(1)$ needs at least nonzero $\bar{\mathbf{Y}}_2$. Therefore, it cannot be calculated and one cannot obtain the realization of the matrix $\mathbf{A}$ in the way described in the previous section. Fortunately, for a known matrix $\bar{\mathbf{C}}$ the physical matrices can be calculated immediately from Eqs. (34) and (35), i.e.

$$\bar{\mathbf{A}}_d = \mathbf{A} + \mathbf{G}_d\mathbf{C} = \bar{\mathbf{C}}\mathbf{a}_1\bar{\mathbf{C}}^{-1} \tag{37}$$

$$\bar{\mathbf{B}}_d = \begin{bmatrix} \mathbf{B} & -\mathbf{G}_d \end{bmatrix} = \bar{\mathbf{C}}\mathbf{b}_1 \tag{38}$$

7. COMPUTER SIMULATION

The following nominal parameters of the rotor and magnetic bearings system are assumed: m = 6 kg, i_o = 1 A, x_o=3.5·10^{-4} m, $R_1 = R_2$ = 19 Ω, $k_{s1} = k_{s2}$ =586 355 N/m, $k_{i1} = k_{i2}$ = 4.1·10^2 A/m, $k_{w1} = k_{w2}$ = 1, $L_{s1} = L_{s2}$ = 0.057 mH, $L_{o1} = L_{o2}$ = 0.143 mH. The nominal matrices of the open-loop system are:

$$\mathbf{A}_{Ce} = \begin{bmatrix} 0 & 1.00 & 0 & 0 \\ 195451.6 & 0.12 & 68.4 & -68.4 \\ 0.07 & -2040.8 & -49.7 & 0 \\ 0 & 2040.8 & 0 & -49.7 \end{bmatrix},$$

$$\mathbf{B}_{Ce} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 4.97 & 0 \\ 0 & 4.97 \end{bmatrix}.$$

For this model, we designed the actual observer and controller. The same actual observer gain and controller gain are used in all examples.

Example 1. Identification of nominal model. After the identification procedure of the model with nominal parameters we obtain the following matrices:

$$\mathbf{A}_{Ce} = \begin{bmatrix} 0 & 0.99 & 0 & -0.00 \\ 195451.6 & 0.12 & 68.4 & -68.4 \\ 0.07 & -2040.8 & -49.7 & 0.01 \\ 0 & 2040.8 & 0 & -49.7 \end{bmatrix},$$

$$\mathbf{B}_{Ce} = \begin{bmatrix} 0.0038 & 0.0038 \\ -0.6348 & 0.6348 \\ 4.9722 & 6.0410 \\ 6.0410 & 4.9722 \end{bmatrix}.$$

To evaluate the accuracy of the identification procedure the matrix percentage indexes are introduced with array division:

$$\mathbf{A}_p = \frac{\mathbf{A}_{Ce} - \mathbf{A}}{\mathbf{A}_C} \cdot 100\%,$$

$$\mathbf{B}_p = \frac{\mathbf{B}_{Ce} - \mathbf{B}}{\mathbf{B}_C} \cdot 100\%.$$

In this example one obtains:

$$\mathbf{A}_p = \begin{bmatrix} \times & 0.12 & \times & \times \\ 0.14 & \times & 0.13 & 0.15 \\ \times & 0.02 & 0.27 & \times \\ \times & 0.02 & \times & 0.31 \end{bmatrix}\%,$$

$$\mathbf{B}_p = \begin{bmatrix} \times & \times \\ \times & \times \\ 0.004 & \times \\ \times & 0.005 \end{bmatrix}\%.$$

By inspection of elements v_i, $i = 1, \ldots, 9$, in above matrices we can notice that the biggest identification error does not cross 0.32%. The impulse response of mass displacement for the open-loop system is presented in Figure 2, while – for the closed-loop system – in Figure 3. In both figures there are given the displacement for simulated model and for identified model. They cover each other with high accuracy. Therefore, they are seen as a single line. The answer of the open-loop system (Fig. 2) is typical for an unstable system.

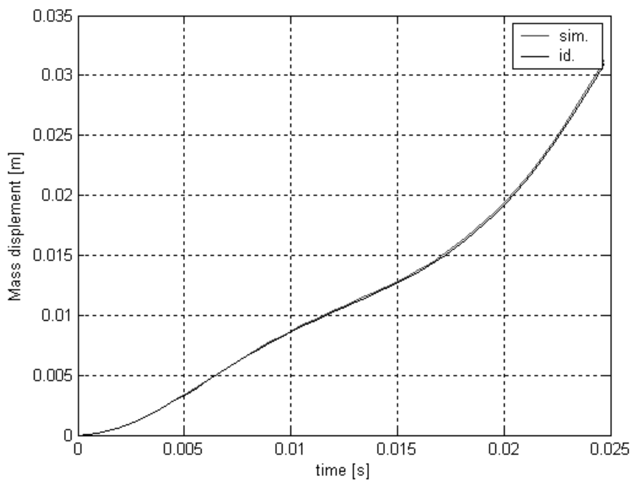

Fig. 2. Impulse response of mass displacement for the simulated and identified unstable open-loop system

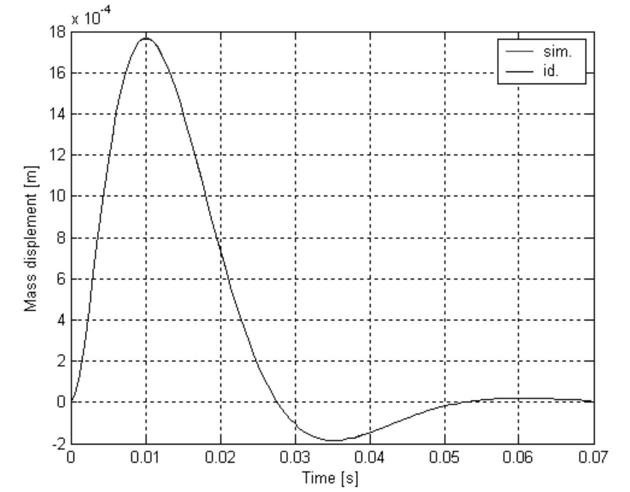

Fig. 3. Impulse response of mass displacement for the simulated and identified closed-loop system

Example 2. We assume 10% increase of the amplifier gain k_{w2} over nominal value. It changes element v_9 in the input matrix. In this case, the matrix percentage index is

$$\mathbf{B}_p = \begin{bmatrix} \times & \times \\ \times & \times \\ 0.004 & \times \\ \times & 10.05 \end{bmatrix}\%.$$

We can notice that the element v_9 in the identified input matrix increased about 10% while the percentage change of element v_8 is very small.

Example 3. We assume 20% increase of the resistance R_2 in the first coil over nominal value. It changes element v_7 in the state matrix. In this case, the matrix percentage index is

$$\mathbf{A}_p = \begin{bmatrix} \times & 0.21 & \times & \times \\ 0.54 & \times & 0.42 & 0.40 \\ \times & 0.12 & 0.57 & \times \\ \times & 0.12 & \times & 20.39 \end{bmatrix}\%.$$

We can see that the element v_7 in the identified state matrix increased about 20% while the changes of other elements are below 0.57%.

8. CONCLUSIONS

The identification method of the open-loop state-space model resulting from the physics laws is presented in this paper. We assumed that the sum of the inputs and outputs equals to the state vector dimension. In this case, there exists a simple solution of considered problem. At first, we designed observer/controller system of which ARX model was identified. Open-loop physical state-space model (matrices: **A**, **B**) and observer gain are calculated from ARX model parameters.

One should notice that for the main part of systems, the elements of matrices **A**, **B** are simpler expressions than coefficients in transfer function. Therefore, the true physical parameters are often easy calculated from the identified elements of state-space model's matrices. Thus, this method can be used in the diagnostics systems.

For the diagnostics purposes, we should use the model of magnetic bearing, which is presented in the paper. In the model, we avoided averaged values of the current and voltage in the opposite electromagnet coils, which lead to averaged parameters in the identified matrices of the state-space model.

References

[1] Ljung L.: *System Identification Theory for the User.* Englewood Cliffs, NJ, Prentice-Hall 1987, ISBN 0138816409

[2] Soderstrom T., Stoica P.: *System Identification.* Prentice-Hall Inc., UK, 1994, ISBN 0131276069

[3] Juang J.-N.: *Applied System Identification.* Englewood Cliffs, NJ, Prentice-Hall 1994, ISBN 013079211X

[4] Juang J.-N., Pappa R.S.: *An Eigensystem Realization Algorithm for Modal Parameter Identification and Model Reduction.* Journal of Guidance, Control and Dynamics, vol. 8, No. 5, September-October 1985, 620–627

[5] Lee H.C., Hsiao J.K., Huang J.K., Chen C.-W.: *Identification of Stochastic System and Controller via Projection Filters.* Proc. of American Control Conference (Baltimore, MD), 1994, 921–925

[6] Huang J.-K., Hsiao M.-H., Cox D.E.: *Indirect Identification of Linear Stochastic Systems with Known Feedback Dynamics.* Journal of Guidance, Control and Dynamics, vol. 19, No. 4, July-August 1996, 836–841

[7] Gosiewski Z., Paszowski M.: *Identification of Physical Parameters of Unstable Systems: Theoretical Background.* Systems Analysis Modelling Simulation, vol. 43, No. 3, March 2003, 301–311