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MASS OPTIMISATION OF THE VIBROINSULATING FRAME OF A SHORT VIBRATORY CONVEYER**

SUMMARY

Short vibratory conveyers placed on vibroinsulating frames were analysed in the paper. It has been shown, on the basis of the double-mass model, that conveyers can be placed on frames of masses smaller than the mass of the trough and the significant reduction of forces transmitted to the foundation can be achieved. The equation allowing to compare the force transmitted to the foundation by the conveyer without the frame placed directly on the system of leaf springs with the force transmitted by the vibroinsulated conveyer was determined in the paper.

***Keywords**: vibratory conveyer, vibroinsulating frame*

OPTYMALIZACJA MASY RAMY KRÓTKIEGO PRZENOŚNIKA WIBRACYJNEGO

W pracy przeanalizowano krótkie przenośniki wibracyjne posadowione na ramie wibroizolacyjnej. Wykazano na podstawie modelu dwumasowego, że przenośniki można sadowić na ramach o mniejszej masie niż masa rynny, uzyskując przy tym dużą redukcję sił przekazywanych na podłoże. W pracy wyznaczono wzór pozwalający porównać siłę przenoszoną na podłoże przenośnika bez ramy posadowionego na układzie listew resorujących z siłą przekazywaną przez przenośnik wibroizolacyjny.

***Słowa kluczowe**: przenośniki wibroizolacyjne, ramy wibroizolacyjne*

INTRODUCTION

Vibratory conveyers are applied in industry for a continuous transport of materials at short distances, usually up to 20 m. The main disadvantage of such transport is a transmission of significant dynamic forces to the foundation. Due to this feature the conveyers are often supported on expensive, heavy frames softly vibroinsulated from the foundation. In the case of long conveyers, of several meters length, the most difficult construction problem constitute natural vibrations of the trough – occurring as bilateral bending in the symmetry plane of the conveyer – disturbing the transportation procedure. The detailed analysis of long vibratory conveyers was given in papers [1, 2, 3].

The analysis of short conveyers, of the length of some meters only, placed on the frame vibroinsulated from the foundation – is presented hereby. It is generally assumed, at the designing stage of vibroinsulating frames, that the frames should be much heavier than the trough of the conveyer [4]. The fact that this assumption is not based on scientific grounds will be proved in the paper. Since the production cost as well as the cost of soft suspension of the frame significantly increases with the increase of its mass – optimisation of this parameter seems reasonable and economically justified.

To verify whether the vibroinsulating frames are performing properly the vibratory conveyer of typical parameters was analysed.

1. FORCES TRANSMITTED TO THE FOUNDATION BY THE VIBRATORY CONVEYER – WITHOUT THE INSULATING FRAME

Forces transmitted to the foundation by short vibratory conveyers situated directly – via the system of leaf springs – on the foundation are related to the deformation of elastic elements (Fig. 1). The total value of the foundation reaction, in the direction of vibrations $F_\tau(t)$ for the trough of mass m_1 supported by uniformly distributed system of springs – of the elasticity constant k_τ and damping constant b_τ – which forms β angle with the horizontal line and is excited for vibrations by the vibrator of a directed input force $P_w(t)=P_{wo}\sin(\omega t)$, (causing the vibrations of the trough of the amplitude A) equals in the steady state:

$$F_\tau(t) = A\sqrt{k_\tau^2 + b_\tau^2\omega^2}\cdot\sin(\omega t+\gamma) \tag{1}$$

Thus, the maximum value of the foundation reaction – in the steady state – equals:

$$F_{\tau\max} = A\sqrt{k_\tau^2 + b_\tau^2\omega^2} \tag{2}$$

In practice, in the steady state, when the system operates outside the resonance zones, the damping of the system has insignificant influence on the force transmitted to the foundation. Thus,

$$F_{\tau\max} \cong A\cdot k_\tau \tag{3}$$

For the parameters of the analysed typical vibratory conveyer, where the trough mass m_1=3700 kg, A = 0.00241 m, k_τ = 1 531 750 N/m, the maximum force transmitted to the foundation is:

$$F_{\tau\max} \cong 3700\ \text{N} \tag{4}$$

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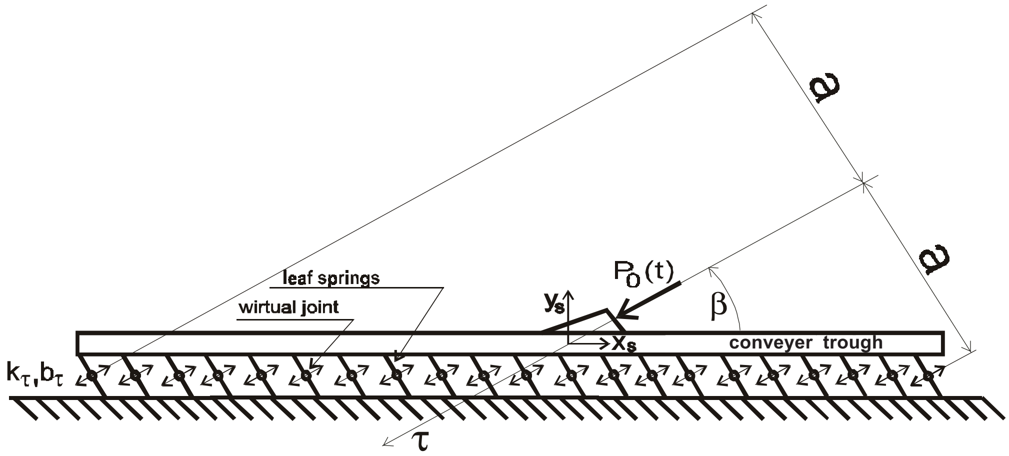

Fig. 1. Model of the vibratory conveyer situated directly on the foundation

2. VIBRATORY CONVEYER – SUPPORTED ON THE VIBROINSULATING FRAME

At the designing stage of the vibroinsulating frame the following aspects should be taken into account:

- Direction of the exciting force should pass through the centre of gravity of the trough and the frame as well as through the centre of elastic forces of suspension.
- The frame should be relatively massive in order not to be significantly excited in the working direction since that would disturb the transport of material.
- Amplitude of the dynamic force transmitted to the foundation should be much smaller than the one of the force transmitted by the conveyer situated directly on the foundation.
- The system – in the steady state – should not operate in the vicinity of resonance zones.
- Static bending due to the material weight should not disturb the "technological procedure".

2.1. Analysis of the system: trough – vibroinsulating frame

To estimate the effectiveness of the vibroinsulating frame applied in short conveyers the model of the system presented in Figure 2 was analysed.

The presented system has four degrees of freedom related correspondingly to coordinates x, y, τ and β. At the assumption that $k_x = k_y$ and the direction of exciting force passes through the centre of gravity as well as the centre of the suspension system, the system of the conveyer can be reduced to the one given in Figure 3.

Equations of motion for the scheme are as follows:

$$\begin{aligned} &m_1\ddot{x} + b_\tau\left(\dot{x} - \dot{y}\right) + k_\tau(x - y) = P_0(t) \\ &m_2\ddot{y} - b_\tau\left(\dot{x} - \dot{y}\right) + b\dot{y} - k_\tau(x - y) + ky = 0 \end{aligned} \tag{5}$$

Assuming monoharmonic excitement of the form $P_0(t) = m_w e\omega^2 \sin(\omega t)$ and limiting the calculations to the steady state, the differential equations system can be reduced to the formulae:

$$\begin{aligned} &-m_1\omega^2 \underline{X} + b_\tau i\omega\left(\underline{X} - \underline{Y}\right) + k_\tau(\underline{X} - \underline{Y}) = me\omega^2 \\ &-m_2\omega^2 \underline{Y} + b_\tau i\omega\left(\underline{Y} - \underline{X}\right) + bi\omega\underline{Y} + k_\tau(\underline{Y} - \underline{X}) + k\underline{Y} = 0 \end{aligned} \tag{6}$$

where:
- $\underline{X}$ – harmonic motion amplitude along coordinate x,
- $\underline{Y}$ – harmonic motion amplitude along coordinate y,
- ω – frequency of exciting force $F(t)$ [rad/s],
- i – imaginary unit,

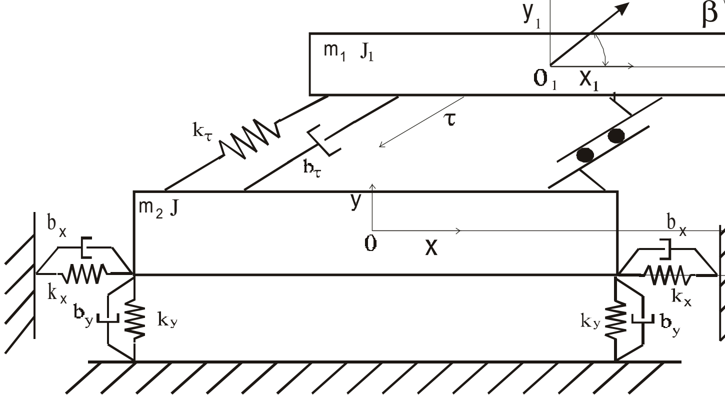

Fig. 2. Schematic presentation of a vibratory conveyer placed on a stiff frame suspended elastically

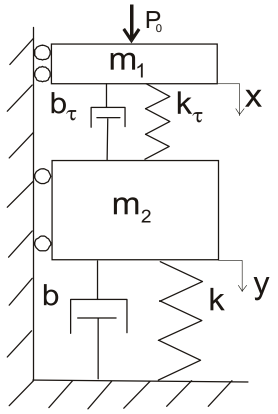

Fig. 3. Simplified scheme of the vibroinsulated conveyer

m_1 – mass of the conveyer trough,
m_2 – mass of the conveyer frame,
m – total unbalanced mass of the vibrator,
e – eccentric of mass m,
k_τ, b_τ – total elasticity and damping of the leaf springs in the working direction τ,
$k = 2k_x = 2k_y$ – elasticity coefficient of the suspension system of the vibroinsulating frame,
$b = 2b_x = 2b_y$ – damping coefficient of the suspension system of the vibroinsulating frame.

The solution of the above equation for unknowns X and Y provides the complex amplitude values:

$$\begin{aligned}
\underline{X} &= \frac{-me\omega^2\left(m_2\omega^2 - i\left(b_\tau - b\right)\omega - \left(k_\tau + k\right)\right)}{\left(m_1m_2\omega^4 - m_1\omega^2\left(k_\tau + k\right) - m_2\omega^2k_\tau + k_\tau k - b_\tau b\omega^2\right) - i\cdot\left(m_1\omega^3\left(b_\tau + b\right) - k_\tau b\omega + m_2\omega^3 b_\tau - b_\tau k\omega\right)}\\
\underline{Y} &= \frac{me\omega^2\left(ib_\tau\omega + k_\tau\right)}{\left(m_\tau m\omega^4 - m_\tau\omega^2\left(k_\tau + k\right) - m_2\omega^2k_\tau + k_\tau k - b_\tau b\omega^2\right) - i\cdot\left(m_1\omega^3\left(b_\tau + b\right) - k_\tau b\omega + m_2\omega^3 b_\tau - b_\tau k\omega\right)}
\end{aligned} \tag{7}$$

Modules of the complex amplitude values give the actual amplitude value.

$$X = \sqrt{\begin{aligned}&\left(\frac{me\omega^2\left(m_1\omega^4m_2 - m_1\omega^2k_1 - m_1\omega^2k_2 - m_2\omega^2k_1 + k_1k_2 - b_1\omega^2b_2\right)\left(m_2\omega^2 - k_1 - k_2\right)}{\left(m_1m_2\omega^4 - m_1\omega^2\left(k_1+k_2\right) - m_2\omega^2k_1 + k_1k_2 - b_1\omega^2b_2\right)^2 + \left(-m_1\omega^3\left(b_1+b_2\right) + k_1b_2\omega - m_2\omega^3b_1 + b_1k_2\omega\right)^2} + \frac{me\omega^2\left(-m_1\omega^3\left(b_1+b_2\right) + k_1b_2\omega - m_2\omega^3b_1 + b_1\omega k_2\right)\left(\omega\left(-b_1 - b_2\right)\right)}{\left(m_1m_2\omega^4 - m_1\omega^2\left(k_1+k_2\right) - m_2\omega^2k_1 + k_1k_2 - b_1\omega^2b_2\right)^2 + \left(-m_1\omega^3\left(b_1+b_2\right) + k_1b_2\omega - m_2\omega^3b_1 + b_1k_2\omega\right)^2}\right)^2 +\\ &+\left(\frac{-me\omega^2\left(-m_1\omega^3\left(b_1+b_2\right) + k_1b_2\omega - m_2\omega^3b_1 + b_1k_2\omega\right)\left(m_2\omega^2 - k_1 - k_2\right)}{\left(m_1m_2\omega^4 - m_1\omega^2\left(k_1+k_2\right) - m_2\omega^2k_1 + k_1k_2 - b_1\omega^2b_2\right)^2 + \left(-m_1\omega^3\left(b_1+b_2\right) + k_1b_2\omega - m_2\omega^3b_1 + b_1k_2\omega\right)^2} + \frac{me\omega^2\left(m_1\omega^4m_2 - m_1\omega^2k_1 - m_1\omega^2k_2 - m_2\omega^2k_1 + k_1k_2 - b_1\omega^2b_2\right)\left(\omega\left(-b_1 - b_2\right)\right)}{\left(m_1m_2\omega^4 - m_1\omega^2\left(k_1+k_2\right) - m_2\omega^2k_1 + k_1k_2 - b_1\omega^2b_2\right)^2 + \left(-m_1\omega^3\left(b_1+b_2\right) + k_1b_2\omega - m_2\omega^3b_1 + b_1k_2\omega\right)^2}\right)^2\end{aligned}} \tag{8}$$

$$Y = \sqrt{\begin{aligned}&\left(\frac{me\omega^2k_1\left(m_1\omega^4m_2 - m_1\omega^2k_1 - m_1\omega^2k_2 - m_2\omega^2k_1 + k_1k_2 - b_1b_2\omega^2\right)}{\left(m_1m_2\omega^4 - m_1\omega^2\left(k_1+k_2\right) - m_2\omega^2k_1 + k_1k_2 - b_1\omega^2b_2\right)^2 + \left(-m_1\omega^3\left(b_1+b_2\right) + k_1b_2\omega - m_2\omega^3b_1 + b_1k_2\omega\right)^2} + \frac{me\omega^3b_1\left(m_1\omega^3\left(b_1+b_2\right) - k_1b_2\omega + m_2\omega^3b_1 - b_1k_2\omega\right)}{\left(m_1m_2\omega^4 - m_1\omega^2\left(k_1+k_2\right) - m_2\omega^2k_1 + k_1k_2 - b_1\omega^2b_2\right)^2 + \left(-m_1\omega^3\left(b_1+b_2\right) + k_1b_2\omega - m_2\omega^3b_1 + b_1k_2\omega\right)^2}\right)^2 +\\ &+\left(\frac{me\omega^3b_1\left(m_1m_2\omega^4 - m_1\omega^2\left(k_1+k_2\right) - m_2\omega^2k_1 + k_1k_2 + b_1b_2\omega^2\right)}{\left(m_1m_2\omega^4 - m_1\omega^2\left(k_1+k_2\right) - m_2\omega^2k_1 + k_1k_2 - b_1\omega^2b_2\right)^2 + \left(-m_1\omega^3\left(b_1+b_2\right) + k_1b_2\omega - m_2\omega^3b_1 + b_1k_2\omega\right)^2} + \frac{-me\omega^2k_1\left(-m_1\omega^3\left(b_1+b_2\right) + k_1b_2\omega - m_2\omega^3b_1 + b_1k_2\omega\right)}{\left(m_1m_2\omega^4 - m_1\omega^2\left(k_1+k_2\right) - m_2\omega^2k_1 + k_1k_2 - b_1\omega^2b_2\right)^2 + \left(-m_1\omega^3\left(b_1+b_2\right) + k_1b_2\omega - m_2\omega^3b_1 + b_1k_2\omega\right)^2}\right)^2\end{aligned}}$$

Analytical solution of the equation of motion allows analysing the motion possibilities of the conveyer system in dependence on the mass of the vibroinsulating frame at constant frequency ω of the exciting force.

Further calculations were performed for the parameters of the typical vibratory conveyer where the mass of the trough was 3700 kg, working frequency 16.7 Hz, amplitude of the trough 2.4 mm and the amplitude of the exciting force P_{wo} = 90 kN. Partial frequency of the system: trough – leaf spring suspension system equalled

$$\omega_p = \sqrt{\frac{k_\tau}{m_1}} = 20.3 \text{ rad/s.}$$

The damping of the system was given by the dependency

$$b = \frac{\psi \cdot k}{2\pi\omega},$$

where:

$\psi = 0.06$ – for the trough suspension on leaf springs,
$\psi = 0.13$ – for the frame suspension on coil springs.

It should be mentioned here, that the obtained results can be generalised on the bases of the analysed conveyer structure for various masses of the trough, however at the condition that the partial frequency, working frequency and the working amplitude of vibrations remains the same as the analysed ones.

2.2. Selection of the mass of the vibroinsulating frame

The ratio of the vibration amplitude of the trough, in direction τ, to the vibration amplitude of the vibroinsulating mass as a function of the ratio of masses: the frame to the trough (in the steady state) – is presented in Figure 4. It should be noted that the elasticity coefficient k depended on the mass of the vibroinsulating frame – according to the formula

$$k = k_\tau \frac{m_2 + m_1}{m_1} \tag{9}$$

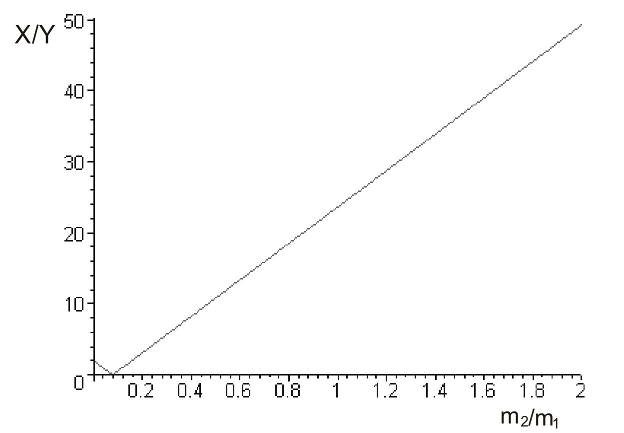

Fig. 4. Ratio of the vibration amplitude of the trough, in direction τ, to the vibration amplitude of the vibroinsulating mass as a function of the ratio of masses: the frame to the trough

As can be seen from Figure 4, the satisfactory ratio – being over ten – of the vibration amplitude of the trough to the one of the vibroinsulating frame is obtained already for the mass ratio equal 0.45, which means for the frame mass being over 1665 kg – when the mass of the trough equals 3700 kg.

The amplitude of the vibration of the vibroinsulating frame as a function of its mass drawn on the basis of equation (8), is presented in Figure 5. It can be seen that at small masses of the frame the amplitude of its vibration is large, due to the vicinity of the resonance zone. When that mass increases its vibration amplitude in the working direction reduces significantly, which in turn reduces the force $F_r = k{\cdot}X$ in N transmitted to the foundation via the suspension system of the vibroinsulating frame (Fig. 6).

As can be seen from the graph presented in Figure 6 the amplitude of the force transmitted to the foundation for the mass of the frame being 1665 kg equals 540 N, which constitutes 14.4% of the amplitude of the force transmitted to the foundation by the conveyer not placed on the vibroinsulating frame. When the mass of the frame exceeds 2600 kg, that being 70% of the mass of the trough, more than 10 times reduction of the force transmitted to the foundation is achieved.

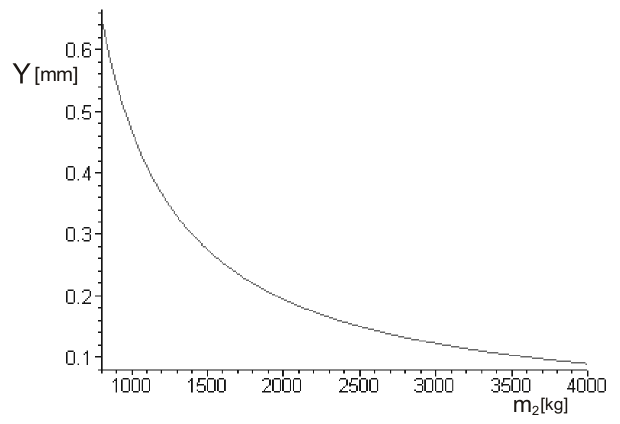

Fig. 5. Amplitude of the vibration of the vibroinsulating frame as a function of its mass

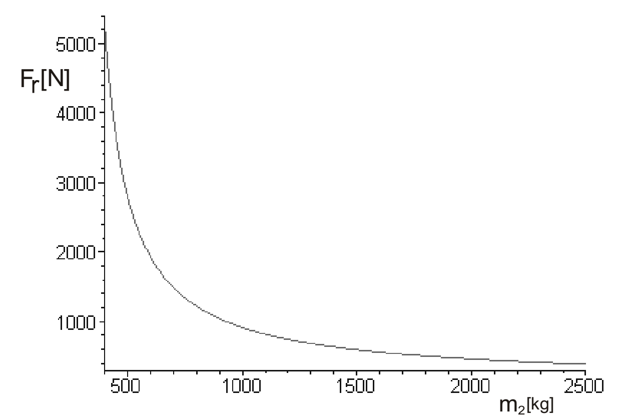

Fig. 6. Amplitude of the force transmitted to the foundation versus the mass of the frame

2.3. Amplitude of the vibration of the conveyer trough

The amplitude of the vibration of the trough – in the working direction – should not differ significantly from the amplitude required by the proper performance of the technological process. This condition has to be taken into account when designing the vibratory conveyer, supported on the vibroinsulating frame. Figure 7 presents the amplitude of vibration of the trough versus the ratio of the masses: frame to trough, drawn on the basis of formula (8). This amplitude is practically constant, regardless of the mass of the vibroinsulating frame, however, under the condition that the system operates outside the resonance zones.

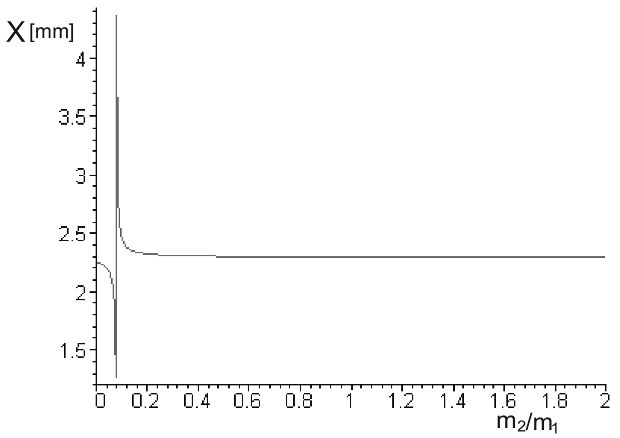

Fig. 7. Amplitude of the vibration of the trough versus the ratio: the mass of the frame to the mass of the trough

2.4. Static deflection of the vibroinsulated conveyer

When the vibratory conveyer is supported on the vibroinsulating frame the static deflection of the conveyer increases. It requires special consideration, when large amounts of material are being transported, since there might be problems with the height difference at the ends of the conveyer.

The static deflection caused by the mass of the transported material, m_n in the conveyer, in which leaf springs are directly fixed to the foundation (Fig. 1) – equals respectively:

– in x direction: $$A_{xstat} = \frac{m_n g \cdot \sin\alpha \cdot \cos\alpha}{k_\tau} \tag{10}$$

– in y direction: $$A_{ystat} = \frac{m_n g \cdot \sin^2\alpha}{k_\tau} \tag{11}$$

This is caused by the fact that the force related to the mass of the material can be projected in two directions – as shown in Figure 8. The force acting in η direction, which is the axial direction of leaf springs, does not cause any static deflection, while the force acting in the direction of movement τ causes – in that direction – the static deflection, $A_{\tau stst} = m_n g \cdot \sin\alpha / k_\tau$, which – after projecting on x and y directions – gives values A_{xstst} and A_{ystst}.

In the case of the conveyer supported on the vibroinsulating frame the static deflection in x direction remains unchanged, while in y direction equals

$$A_{ystatw} = \frac{m_n g \cdot \sin^2\alpha}{k_\tau} + \frac{m_n g \cdot m_1}{k_\tau \cdot (m_1 + m_2)} \tag{12}$$

The graph in Figure 9 presents the ratio of the static bending in y direction – caused by the mass of the transported material – of the conveyer placed on the vibroinsulating frame to the static bending of the conveyer without the supporting frame versus the mass of the frame. Stiffness coefficient of the frame support is given by equation (9).

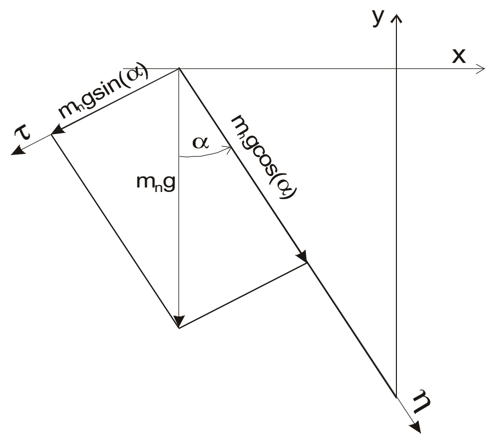

Fig. 8. Distribution of forces related to the mass of the material

As can be clearly seen from the graph the static bending of the conveyer placed on the vibroinsulating frame might cause problems at large loads of the transported material. For the conveyer being analysed – of the mass of the frame equal 1665 kg – the static bending in y direction is 3.7 times higher than the static bending of the conveyer without the vibroinsulation.

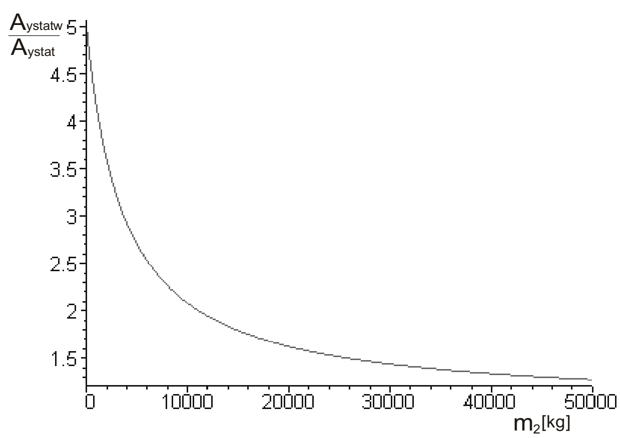

Fig. 9. Ratio of the static bending caused by the mass of the material to the static bending of the conveyer without a frame versus the mass of the frame

2.5. Vibration frequency

The system presented in Figure 3 has two natural – not damped – frequencies

$$\omega_{1,2} = \sqrt{\frac{1}{2}\left(\frac{k_1 + k_2}{m_2} + \frac{k_1}{m_1}\right) \pm \sqrt{\frac{1}{4}\left(\frac{k_1 + k_2}{m_2} + \frac{k_1}{m_1}\right)^2 - \frac{k_1 \cdot k_2}{m_1 \cdot m_2}}} \tag{13}$$

Taking into account equation (9) we obtain

$$\omega_{1,2} = \sqrt{\frac{1}{2}\left(\frac{k_1 + k_1 \dfrac{m_1 + m_2}{m_1}}{m_2} + \frac{k_1}{m_1}\right) \pm \sqrt{\frac{1}{4}\left(\frac{k_1 + k_1 \dfrac{m_1 + m_2}{m_1}}{m_2} + \frac{k_1}{m_1}\right)^2 - \frac{k_1^2 (m_1 + m_2)}{m_1^2 \cdot m_2}}} \tag{14}$$

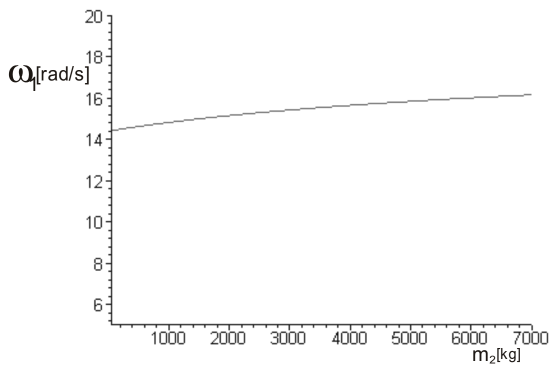

Fig. 10. The first natural frequency of the system as a function of the mass of the vibroinsulating frame

Figures 10 and 11 present the dependence of the frequency of vibrations of the system on the mass of the vibroinsulating frame based on the equation 14. It is seen – from the graphs – that the first natural frequency is not hazardous for the machine operation since it is much lower than the working frequency

$$\omega = 16.7 \cdot 2 \cdot \pi = 105 \text{ rad/s}.$$

The second natural frequency is high for small masses of frames. In the case of the parameters of the analysed conveyer, at the mass of the frame equal 1665 kg, the second natural frequency of free vibrations ω_2 is 49.5 rad/s, which means that it is sufficiently low to allow the system to pass both resonance zones at the proper selection of the vibrator drives.

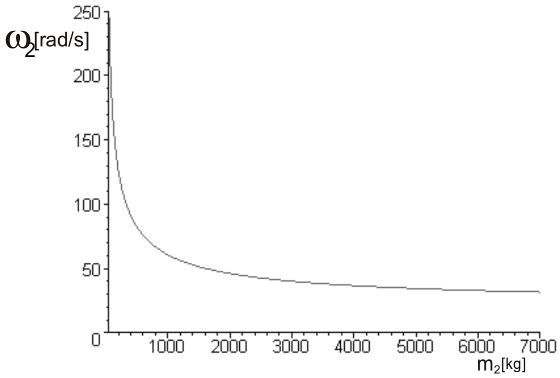

Fig. 11. The second natural frequency of the system as a function of the mass of the vibroinsulating frame

When the mass of the frame is going to infinity, frequencies ω_1 and ω_2 are going to the value of 20.3 rad/s, which is the value of the frequency of natural – not damped – vibrations of the system without the vibroinsulating frame.

2.6. Amplitude versus frequency graphs

Figures 12 and 13 present the amplitude frequency dependencies determined by means of equation (8) for the proposed mass of the vibroinsulating frame being 1665 kg while the mass of the trough was 3700 kg. The graphs allow noticing that the system of vibroinsulated conveyer – in the steady state at ω = 105 rad/s – operates behind two resonance zones – which are important at selecting the drive of the forcing system.

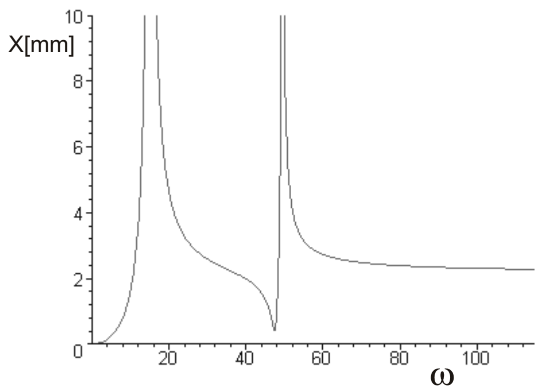

Fig. 12. Amplitude of the trough vibrations of the vibroinsulated vibratory conveyer versus the forced frequency

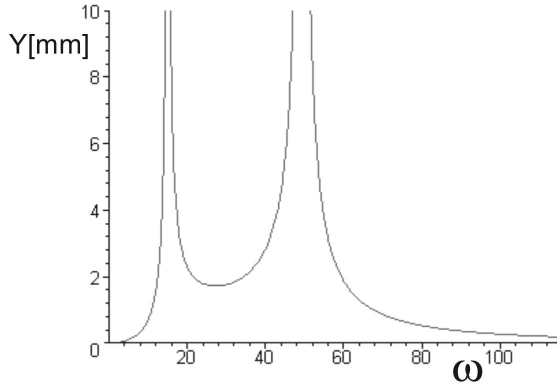

Fig. 13. Amplitude of the frame vibrations of the vibroinsulated vibratory conveyer versus the forced frequency

3. ESTIMATION OF THE COEFFICIENT OF THE FORCES TRANSMITTED TO THE FOUNDATION

In order to estimate the dimensionless ratio of forces transmitted to the foundation by the system with and without vibroinsulating frame the comparison of forces presented in Figures 1 and 3 was performed.

Determination and solution of the set of equations of motion of those systems – disregarding the damping phenomena – for the amplitudes of vibration allows to estimate the forces transmitted to the foundation. This – in turn – allows to determine the ratio of forces transmitted by the system in dependence of the ratio of the trough mass to the frame mass and the ratio of partial frequency of the trough supported by the leaf spring suspension to the frequency of the exciting force.

Formulating the equations of motion for systems presented in Figures 1 and 3 in the form:

– for the conveyer without a frame

$$\left\{ m_1 \ddot{\tau} + k_\tau \tau = P_0(t) \right.$$

– for the conveyer with a frame

$$\begin{cases} m_1 \ddot{x} + k_\tau (x - y) = P_0(t) \\ m_2 \ddot{y} - k_\tau (x - y) + ky = 0 \end{cases} \tag{15}$$

assuming that:

$$k_t = m_1 \omega_p^2,$$

$$k = k_t \left(\frac{m_2}{m_1} + 1 \right)$$ – in accordance with equation (9),

and assuming also the monoharmonic forcing of the form: $P_0(t) = m_w e \omega^2 \sin(\omega t)$, while limiting the considerations to the steady state. The differential equations system (15) can be written in the following form:

$$\left\{ -m_1 \omega^2 \mathrm{T} + m_1 \omega_p^2 \mathrm{T} = P_0 \right.$$

$$\begin{cases} -m_1 \omega^2 X + m_1 \omega_p^2 (X - Y) = P_0 \\ -m_2 \omega^2 Y + m_1 \omega_p^2 k_\tau (X - Y) + m_1 \left(\frac{m_2}{m_1} + 1 \right) \omega_p Y = 0 \end{cases} \tag{16}$$

Solution of equations (16) versus the amplitudes provides formulae (17) and (18).

$$\left\{ \mathrm{T} = \frac{P_0}{m_1 \omega^2 \left(\left(\frac{\omega_p}{\omega} \right)^2 - 1 \right)} \right. \tag{17}$$

$$\begin{cases} X = \dfrac{P_0 \left(2\left(\frac{\omega_p}{\omega} \right)^2 - \frac{m_2}{m_1} + \left(\frac{\omega_p}{\omega} \right)^2 \frac{m_2}{m_1} \right)}{m_1 \omega^2 \left(\left(\frac{\omega_p}{\omega} \right)^4 + \frac{m_2}{m_1} - 2\left(\frac{\omega_p}{\omega} \right)^2 - 2\left(\frac{\omega_p}{\omega} \right)^2 \frac{m_2}{m_1} + \left(\frac{\omega_p}{\omega} \right)^4 \frac{m_2}{m_1} \right)} \\ Y = \dfrac{P_0 \left(\frac{\omega_p}{\omega} \right)^2}{m_1 \omega^2 \left(\left(\frac{\omega_p}{\omega} \right)^4 + \frac{m_2}{m_1} - 2\left(\frac{\omega_p}{\omega} \right)^2 - 2\left(\frac{\omega_p}{\omega} \right)^2 \frac{m_2}{m_1} + \left(\frac{\omega_p}{\omega} \right)^4 \frac{m_2}{m_1} \right)} \end{cases} \tag{18}$$

On their basis forces transmitted to the foundation can be determined:

– for the conveyer without a frame $F_{r1} = k_t \cdot \mathrm{T}$

– for the conveyer with a frame $F_{r2} = k \cdot Y$ (19)

When these values are known their quotient can be calculated, which after several rearrangements takes the form

$$\frac{F_{r2}}{F_{r1}} = \left(\frac{m_2}{m_1} + 1 \right) \left(\frac{\left(\frac{\omega_p}{\omega} \right)^2 \left(\left(\frac{m_2}{m_1} \right)^2 - 1 \right)}{\left(\frac{\omega_p}{\omega} \right)^4 + \frac{m_2}{m_1} - 2\left(\frac{\omega_p}{\omega} \right)^2 - 2\frac{m_2}{m_1}\left(\frac{\omega_p}{\omega} \right)^2 + \frac{m_2}{m_1}\left(\frac{\omega_p}{\omega} \right)^4} \right) \tag{20}$$

where:

- F_{r1} – force transmitted to the foundation by the conveyer without the vibroinsulating frame,
- F_{r2} – force transmitted to the foundation by the conveyer with the vibroinsulating frame,
- $\frac{m_2}{m_1}$ – ratio of the frame mass to the trough mass,
- $\frac{\omega_p}{\omega}$ – ratio of the partial frequency of the trough supported by leaf spring suspension to the frequency of the exciting force P_0.

Equation (20) provides the possibility of estimating the effectiveness of the vibroinsulation regardless of the exciting force value and regardless of the frame mass, however at the assumption that equation (9) is satisfied.

Figure 14 presents the ratio of forces transmitted to the foundation by the system with and without the vibroinsulation versus the ratio of the frame mass to the trough mass for the analysed example where

$$\frac{\omega_p}{\omega} = \frac{\sqrt{k_\tau / m_1}}{\omega} = 0.19 \tag{21}$$

The graph from Figure 12 indicates that when the frame mass equals 70% of the trough mass the tenfold reduction of the forces transmitted to the foundation occurs, which verifies the conformity of the equation with the prior calculations.

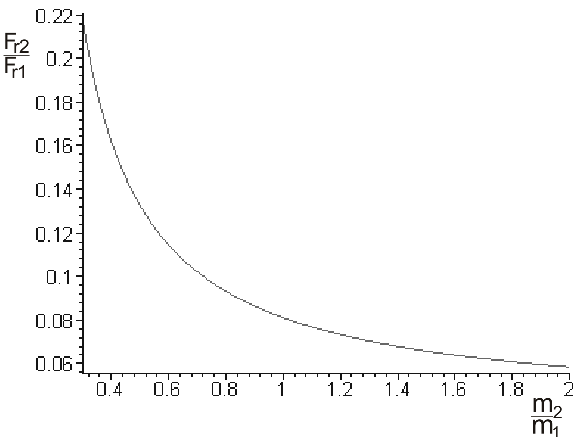

Fig. 14. Ratio of forces transmitted to the foundation by the system with and without the vibroinsulation versus the ratio of the frame mass to the trough mass

CONCLUSIONS

Taking into account the calculations performed for the typical vibratory conveyer of the mass of the trough being 3700 kg the practical possibilities of vibroinsulation by an application of a massive vibroinsulating frame, can be assessed as follows:

- The satisfactory ratio – being over 10 – of the vibration amplitude of the trough to the vibration amplitude of the vibroinsulating frame is achieved when the ratio of masses: frame to trough – equals 0.45, which means the mass of the frame being above 1665 kg.
- In practical cases, when the vibroinsulating frame was applied (even of a very small mass) the amplitude of the trough vibrations in the working direction did not differ significantly from the amplitude required by the technological process.
- Serious problem of vibroinsulating conveyers constitutes their stiffness bending caused by the mass of the transported material.
- Natural frequencies of the system, when the vibroinsulating frame of a very small mass is applied, appear dangerously close to the forced frequency. At the increasing mass of the frame the frequency of natural vibrations of the system decreases thus, moving away from the resonance zone. For the proposed frame of a mass of 1665 kg the second frequency of natural vibrations constitutes less than 50% of the forced frequency.
- When equation (9) is satisfied, the effectiveness of the two-mass system versus the single-mass system can be estimated from equation (20).

Thus, the hereby-presented research proves that there is no evident reason for applying extremely massive frames for the vibroinsulation of the vibratory conveyers.

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