AUTOMATYKA • 2010 • Tom 14 • Zeszyt 3/1

Sebastian Stoliński*, Wojciech Bieniecki*, Jacek Stańdo**

Automatic Detection and Evaluation of the Spline Function Plot

1. Introduction

Electronic marking is a new idea in the field of teaching which aim is to enable automatic checking of exam assessments with aid of artificial intelligence and computer recognition systems, especially OCR and image understanding.

E-marking software gives the examiner a possibility of checking and marking the answer sheets directly on a computer screen rather than reading paper documents. The advantage of the e-marking system are: better organization of the examination session, improvement of the security of the process and additional features as statistical processing and visualization of the results.

Studying the impact of changes in the process of evaluation to its quality has shown, that if only this change does not involve exam preparation, it can keep its original accuracy and increase its reliability [4, 5]. For example, Williams and van Lent [5] claim, that introducing e-evaluation can improve exam quality by:

- providing complete anonymity for task assessment;
- suppress the halo effect – a way of solving by one student does not affect the way later tasks are evaluated;
- steadily spreading potential errors in assessing – tasks to assess to the examiners are assigned at random.

The e-marking method has been widely introduced in Great Britain and the USA. The experience gained by Examination Boards like AQA, OCR and EDEXCEL in Great Britain and ETS in the USA suggests, that introducing e-marking improves the quality and reliability of the exams.

Since 2007 in Poland the project „Electronic marking” is carried out by the Central Examination Board [3].

* Computer Engineering Department, Technical University of Lodz, Poland
** Centrum Nauczania Matematyki i Fizyki, Politechnika Łódzka

In our previous research we worked out the requirements for our own e-marking system [1]. Our current interests are focused on the image processing algorithms that enable analyze the plot which is drawn by hand. In [2] we developed the algorithms for recognition of simple math function plots: linear and parabolic. In this article the algorithm for a spline function plot analysis is shown.

2. Image segmentation algorithm

The image processing algorithm works in several phases. The first phase relies on extraction of the function plot from the scanned examination sheet and it has been presented in detail in [2]. Before we describe the algorithm for spline function marking, let us look at the task that were given to the students.

Task 1. Draw the plot of the spline function given by $y = \begin{cases} -4 & \text{for} \quad x \le -4 \\ 0.5x + 3 & \text{for} \quad x \in (-4, 4) \\ -x + 9 & \text{for} \quad x \ge 4 \end{cases}$

The score for the task was between 0 and 2.

Task 2. Draw the plot of the spline function given by $y = \begin{cases} -4 & \text{for} \quad x \le -4 \\ -0.5x + 3 & \text{for} \quad x \in (-4, 4) \\ -x + 9 & \text{for} \quad x \ge 4 \end{cases}$

The solution was scored from 0 to 3.
The plots have to be drawn in the coordinate system shown in Figure 1.

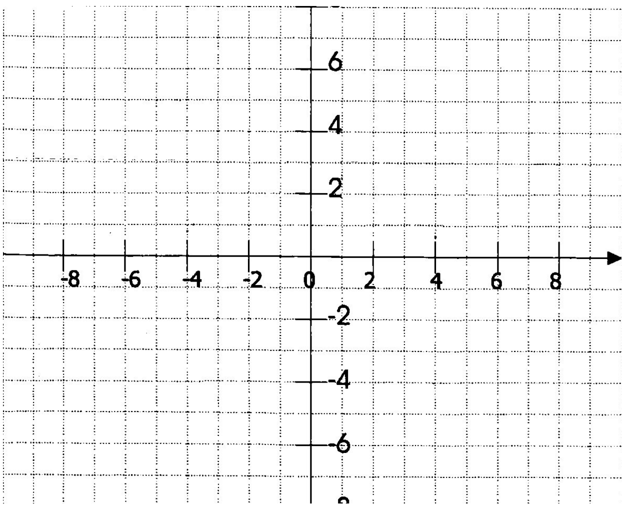

Fig. 1. Empty coordinate system

The algorithm for spline function plot extraction requires four or five input images:

I_1 – the image of empty coordinate system (Fig. 1),
I_2 – scanned image of the student's solution,
I_{t1} – scanned image of first part of model solution,
I_{t2} – scanned image of second part of model solution,
I_{t3} – scanned image of third part of model solution (only for task 2).

Steps of the algorithm:

With aid of the cross correlation method find the best match of I_1 and I_2 images. Obtain $I_r = I_1 - I_2$. Using mathematical morphology filters, remove the distortions (for instance, fragments of the coordinate system remained from imperfect scanner geometry) from I_r and leave only the plot curve (Fig. 2).

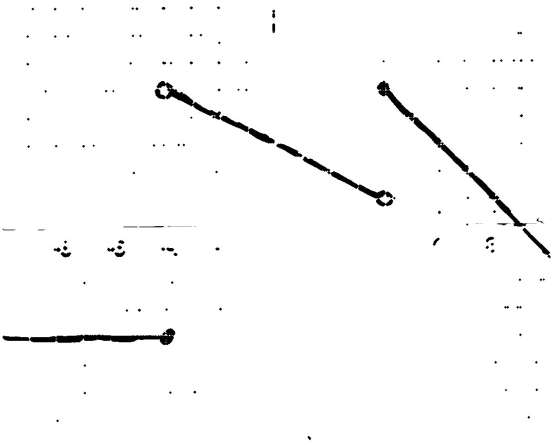

Fig. 2. Plot curves

For each part of student's solution define a region of interest (ROI), where the solution should be located. Then, do the following operations:

Find the best match of I_{tn} (Fig. 3) and I_2 (Fig. 4) images using the cross correlation method. Coordinates of point of maximum correlation are used to crop the expected solution image from the query image.

For I_{tn} do the dilation (the structuring element is a disk r = 10 pixel). Assign it as I_{rdn} and S_{rd} as a connected component in the image.

Calculate, how much pixels from reference plot S_{rd} matches the pixels of test plot S_t. using the formula:

$$P_1 = \left|\{I_{rdn} : I_{rdn} = 1\}\right|$$

$$P_2 = \left|\{I_{rdn} \,\&\, I_t : I_{rdn} \,\&\, I_t = 1\}\right| \tag{1}$$

$$P = P_2 / P_1$$

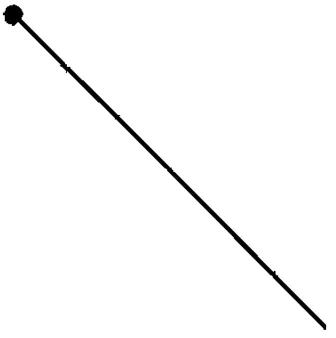

Fig. 3. Extracted query plot (I_{t3})

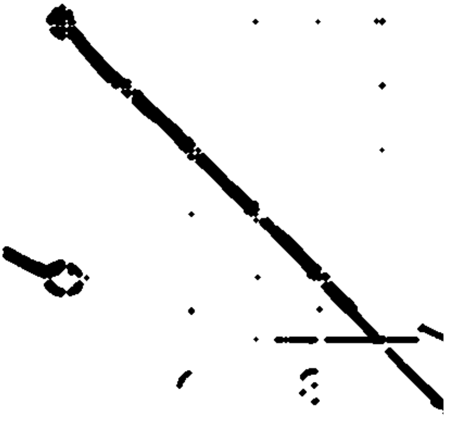

Fig. 4. Extracted student plot

It may happen, that the number of pixels P_2 is very small (the student drew only a few dots, not the whole plot). Compare the number of pixels in test and reference dilated plots. We set the condition experimentally:

$$P_1 \leq 100 \cdot P_2 \tag{2}$$

If the condition is not satisfied, the plot is not valid (so, put the negative number). In all other cases P is in range of (0, 1) and it illustrates the compliance rate between reference and query plots.

The next step is detection of end points of the line segment which have no ends in plus or minus infinity (fits inside the image).

The inspected bounding rectangles are located outside of the line segment, accordingly on right side of right end point of line segment and left side of left side of end point of line segment, end line of line segment is located in the middle of theirs height (Fig. 5). Each of them is 1 pixel wide, their height depends on angle of line segment. The difference between areas is 10 pixels (it is about 0.1 of the unit).

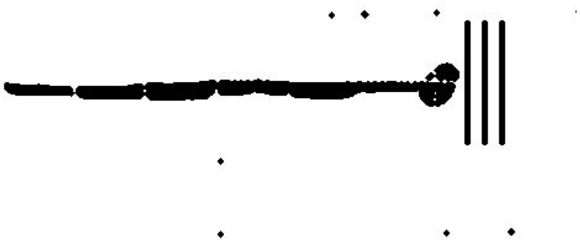

Fig. 5. Inspection of the end of the line segment

Check, which of these three vertical lines crosses the inspected line segment. If there is more than one cross, the solution is incorrect – the line exceeds the range.

Otherwise, if (2) is satisfied, the line segment is qualified as a correct one.

Finally, cut out all found and recognized line segments from I_2 (student's solution image) to detect other objects (3):

$$I_{e2} = I_2 - I_{t1} - I_{t2} - I_{t3} \tag{3}$$

Solution is classified as unrecognized, when the image I_{e2} has more than t_{e2} black pixels.

3. The experiment

The experiment has been carried out on two groups of students: 22 and 28 persons. Each student received a print-out with empty coordinate systems and had to complete Task1 and Task 2. All pages were collected and scanned with the resolution 300 DPI with 8-bit grayscale mode. Then the teacher assessed all the solutions, as during a normal exam. He also drew the plots that were used as reference data (the page was also scanned). Each grade have three possible values: 1 – line segment correct, 0 – line segment not correct, U – unrecognized image.

The exemplary comparison of the grades given by the teacher and calculated by the computer system is presented in Tables 1–2.

R is a percentage of black pixels of I_{e2} image.

For all samples the average error rates have been calculated (Tab. 3–4). The error occurs if the teacher's score is not the same as the computer's. In the case of real exam, if the grade is under the pass level the work is evaluated once again. The undesirable situation is when we accept the incorrect task solution (overestimation).

Table 1

Exemplary samples for the algorithm evaluation (task 1)

Sample code	Grade (teacher) Line segment		Compliance of images					Grade Line segment	
	1	2	Corr. 1	End 1	Corr. 2	End 2	R (%)	1	2
800075	1	0	0.88	1	0.93	1	0.057	1	1
800076	1	1	0.98	1	0.93	1	0.034	1	1
800077	1	1	0.98	1	0.94	1	0.026	1	1
800078	1	1	0.99	1	0.71	1	0.14	U	U

Table 2

Exemplary samples for the algorithm evaluation (task 2)

Sample code	Grade (teacher) Line segment			Compliance of images							Grade Line segment		
	1	2	3	Corr. 1	End 1	Corr. 2	End 2	Corr. 3	End 3	R (%)	1	2	3
800028	1	0	0	0.95	1	0.67	0	0.07	0	1.57	U	U	U
800029	1	1	1	0.96	1	0.82	1	0.71	1	0.46	1	1	1
800030	0	0	0	0.96	0	0.20	1	0.51	1	0.77	0	0	1
800031	1	1	1	0.99	1	0.12	1	0.77	1	0.20	1	1	1

Table 3

Rate error for task 1 (28 samples)

	Threshold 0.25		Threshold 0.5	
	Samples	Relative error (%)	Samples	Relative error (%)
Compatible results	18	64	18	64
Overestimated score	2	7	2	7
Underestimated score	1	4	1	4
Unrecognized	7	25	7	25

Table 4

Rate error for task 2 (22 samples)

	Threshold 0.25		Threshold 0.5	
	Samples	Relative error (%)	Samples	Relative error (%)
Compatible results	14	64	9	41
Overestimated score	2	9	2	9
Underestimated score	1	4	6	27
Unrecognized	5	23	5	23

4. Conclusion

The presented algorithm works properly in a case of spline function. Some errors occurred for solutions with strike-throughs and amendments or when the student drew some additional line by mistake. In these cases the images were rejected. Big amount of unrecognized plots has impact on the relative errors.

What should improve the results is changing the exam sheets and introducing new method for grading the solutions. During tests, we discovered that if the size of the individual line segment is large, it is more difficult to recognize the image.

In the process of electronic marking yet we did not check if the ends of ranges were properly drawn by a student (full or empty circles).

Our future research will include the analysis of a graphical solution of equation set.

References

[1] Bieniecki W., Stańdo J., Stoliński S., *Analiza wymagań dla systemu elektronicznego oceniania rozwiązań zadań egzaminacyjnych*. Automatyka (półrocznik AGH), t. 13, 2009, 1397–1406.

[2] Bieniecki W., Stańdo J., Stoliński S., *Automatic evaluation of examination tasks in the form of function plot.* Proc. of the 6th IEEE International Conference MEMSTECH 2010: Perspective technologies and methods in mems design, Lviv-Polyana, Ukraine, 140–143.

[3] e-ocenianie http://e-ocenianie.pl.

[4] Fowles D., Adams C., *Does assessment differ when e-marking replaces paper-based marking?* IAEA paper v3(cra+def).doc, 2005.

[5] Szaleniec H., Weziak-Bialowolska D., *Czy e-ocenianie może zapewnić większą rzetelność punktowania?* In Niemierko B., Szmigiel M.K. [red.] Uczenie się i egzamin w oczach nauczycieli. XIV Krajowa konferencja z „Diagnostyka Edukacyjna" PTDE 2008.