

## STREAMLINING DIGITAL CORRELATION-INTERFEROMETRIC DIRECTION FINDING WITH SPATIAL ANALYTICAL SIGNAL

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**Abstract.** This study investigated a search-free digital method for radio direction-finding that utilizes spatial analytical signal reconstruction. The research focused on optimizing the method's accuracy for estimating the direction of a radio source. The analysis identified the separation distance and number of chosen antenna elements for signal reconstruction as the key parameters influencing accuracy. Analytical optimization and simulations were employed to determine the optimal values for these parameters. The research successfully modeled the impact of antenna element selection on direction-finding errors under specific signal-to-noise conditions. This model can be used to guide further development and optimization of this radio direction-finding method.

**Keywords:** streamlining, direction finding, digital correlation-interferometric method, spatial analytical signal

### USPRAWNIENIE CYFROWEGO KORELACYJNO-INTERFEROMETRYCZNEGO USTALANIA KIERUNKU ZA POMOCĄ PRZESTRZENNEGO SYGNAŁU ANALITYCZNEGO

**Streszczenie.** W niniejszym opracowaniu zbadano cyfrową metodę znajdowania kierunku radiowego bez wyszukiwania, która wykorzystuje przestrzenną rekonstrukcję sygnału analitycznego. Badania koncentrowały się na optymalizacji dokładności metody szacowania kierunku źródła radiowego. Analiza zidentyfikowała odległość separacji i liczbę wybranych elementów anteny do rekonstrukcji sygnału jako kluczowe parametry wpływające na dokładność. W celu określenia optymalnych wartości tych parametrów zastosowano optymalizację analityczną i symulacje. Badania z powodzeniem modelowały wpływ wyboru elementów anteny na błędy znajdowania kierunku w określonych warunkach stosunku sygnału do szumu. Model ten może być wykorzystany do kierowania dalszym rozwojem i optymalizacją tej metody znajdowania kierunku radiowego.

**Słowa kluczowe:** usprawnienie; ustalenie kierunku; cyfrowa metoda korelacyjno-interferometryczna; przestrzenny sygnał analityczny

### Introduction

Modern radio monitoring, navigation, and communication systems face the challenge of operating in complex and dynamically changing electromagnetic environments. Search-free digital correlation-interferometric direction finding (DF) offer a promising solution for such scenarios [8, 12]. However, the effectiveness of DF systems hinges on a delicate balance between noise immunity, processing speed, accuracy, and the associated hardware costs. This necessitates further research and optimization of search-free digital correlation-interferometric DF specifically for complex electromagnetic environments.

While a search-free digital method for correlation-interferometric DF with spatial analytical signal reconstruction was proposed in [1], the study did not delve into its optimization. Existing research (e.g., [1, 3]) has focused on optimizing efficiency and parameters for modern DF techniques that rely on antenna arrays (AA) to estimate the direction of a radio source (RS). These studies have identified optimal ratios within key parameters of partial directional patterns (DP) for effective spatial selection. However, they haven't addressed the optimization of search-free methods that involve complex analytical signal reconstruction.

This gap in research necessitates a focus on optimizing the search-free digital method of correlation-interferometric DF with spatial analytical signal reconstruction, as presented in [1]. This article aims to investigate and optimize this specific method to enhance its effectiveness in complex electromagnetic environments.

### 1. Materials and methods

Let us optimise the search-free digital method of correlation-interferometric DF with reconstruction of the spatial complex analytical signal and use of linear AA [8], which provides maximum noise immunity.

Let a useful random stationary signal  $S(t)$  with uniform power spectrum  $S^2(\omega)$  of a linear AA from  $Z$  identical DF radio channels be received. The AA radio channels have their own additive stationary normal noise  $n_z(t)$  with zero mathematical expectation and the same two-sided power spectral density  $N$  constant within the band  $[\omega_{S,L}; \omega_{S,H}]$  of the analysis frequencies. We will assume that the intrinsic noise of the AR radio channels has no inter-channel correlation and no correlation with the signal. We will also assume that there are no phase fluctuations on the signal propagation path. Thus, the initial conditions of the research can be represented as follows:

$$U_z(t) = S_z(t - \tau_z) + n_z(t) \quad (1)$$

where  $U_z(t)$  is the mixture received by the  $z$ -th DF channel;  $S_z(t - \tau_z)$  – useful signal received by the  $z$ -th DF channel;  $\tau_z$  – delay of the useful signal in the  $z$ -th channel relative to the reference channel, depending on the direction to the RS;  $n_z(t)$  – additive Gaussian noise with uniformly distributed two-sided  $N$  power density within the simultaneous analysis bandwidth of the  $z$ -th channel.

We will assume that frequency and spatial selections are applied in the reconstruction implementation.

For the specified conditions as the main indicator of direction finding immunity it is reasonable to use the dispersion of  $\sigma_\theta^2$  error of direction estimation on RS [5, 12], which is determined for the investigated method according to the equation [8]:

$$\sigma_\theta^2 = \frac{K_{Wt} \cdot K_{W\theta} \cdot m_S \cdot (c/d)^2}{\mu_{in} \cdot \Delta f_k \cdot Z \cdot W_\theta(z_1) \cdot W_\theta(z_2) \cdot \omega_0^2 \cdot T_a \cdot (z_2 - z_1)^2 \cdot \cos^2 \theta} \quad (2)$$

where  $K_{Wt}$  is the noise band coefficient of the weight function  $W_t(n)$  of the window for temporal spectral analysis;



$K_{W_0}$  – noise band coefficient of the weight function  $W_0(z)$  of digital spatial spectral analysis;  $m_S$  – number of spectral-spatial samples of the signal group;  $c$  – the propagation velocity of electromagnetic radiation in free space;  $d$  – step of the linear AA;  $\mu_m$  – input signal-to-noise ratio;  $\Delta f_k$  – frequency bandwidth of the DF radio channel;  $Z$  – number of identical DF radio channels of the AA;  $W_0(z_1), W_0(z_2)$  – values of the window weight function in the selected points  $z_1$  and  $z_2$  at spatial spectral analysis and formation of the multiblade DP;  $z_1, z_2$  – numbers of selected AA antenna elements for which the complex analytical signal is determined to estimate the value of its spatial frequency and DF;  $\omega_0 \in [\omega_{S,L}; \omega_{S,H}]$  – average frequency of the analysis band or carrier frequency of the DF radiation of the RS;  $T_a$  – duration of the process of analysing radio emissions;  $\theta$  – RS direction.

To optimise the method under study, let us determine the type of the target function and coupling functions [7]. For this purpose, on the basis of equations (1) and (2), let us analyse the features of the optimised DF method.

The analysis of equations (1) and (2) has shown that dispersion  $\sigma_\theta^2$  of DF error essentially depends on the way of realisation of the procedure of reconstructing the complex analytical signal within the AA aperture. At the same time, such parameters of the direction finder as the number  $Z$  of DF radio channels of the AA, the width  $\Delta f_k$  of their analysis frequency bandwidth and sensitivity  $\mu_{bx}$  are limited during the optimisation by the possibilities of modern technical implementation and requirements to compactness and price of the direction finder [6].

In this case, such parameters as  $K_{W_i}$ ,  $K_{W_0}$ ,  $m_S$  are determined by the requirements for noise immunity DF in a complex electromagnetic environment to ensure effective frequency and spatial selection of station interference and interference reflections. Therefore, the possibility of their variation in order to optimise DF immunity is also significantly limited.

In turn, the parameters of the radiation of the DF RS, such as the average or carrier frequency  $\omega_0$  of the time energy spectrum, and the direction  $\theta$  of arrival of radio radiation, do not affect the DF algorithm and have only a global limitation on the range of operating frequencies and the width of the DF sector. Duration  $T_a$  of the process of analysis of radio emissions received simultaneously within the frequency band  $\Delta f_k$  analysis of radio channels, significantly affects the noise immunity and speed of DF, but the implementation of the algorithm as a whole and the procedure for reconstructing the complex analytical signal does not affect.

The analysis of equation (2) shows that the absolute values of  $z_1$  and  $z_2$  numbers of the selected AA elements and their separation in space  $\Delta z = (z_2 - z_1)$ , as well as the values of  $W_0(z_1)$  and  $W_0(z_2)$  of the spatial weight function of the window, which determine the way of implementation of the procedure of reconstructing the complex analytical signal, have a significant impact on the DF immunity and dispersion  $\sigma_\theta^2$  of the DF error. The analysis showed that the dependence  $\sigma_\theta^2 = f(z_1, z_2, \Delta z)$  is determined by two factors. The first factor  $(z_2 - z_1)^2$  of the influence of separation in space  $\Delta z$  causes a significant quadratic decrease of the error  $\sigma_\theta^2$  of DF with

increasing  $\Delta z$ . The second factor  $W_0(z_1) \cdot W_0(z_2)$ , on the contrary, causes a significant increase in the error of  $\sigma_\theta^2$  DF when increasing  $\Delta z$ .

Therefore, it is reasonable to optimise the investigated DF method for maximum noise immunity by taking into account the parameters  $(z_2 - z_1)^2$ ,  $W_0(z_1)$  and  $W_0(z_2)$ .

## 2. Experiment and results

Based on the analysis of equation (2), we define the type of the target function  $F(z_1, z_2)$  and the optimality criterion as follows:

$$F(z_1, z_2) = K_\sigma \cdot W_0(z_1) \cdot W_0(z_2) \cdot (z_2 - z_1)^2 = \max \quad (3)$$

where  $K_\sigma$  is the proportionality factor independent of the variables  $z_1$  and  $z_2$ .

The streamlining problem taking into account (3) will take the form:

$$(z_1, z_2)_{opt} = \arg \max \{F(z_1, z_2)\} \quad (4)$$

The analysis of equations (3) and (4) shows that it is reasonable to solve the streamlining problem on the basis of scalar multivariate methods with the search for an extremum of the type [7].

Let us determine the type of link functions  $f_i(z_1, z_2)$ , their number  $L$  and the type of the sought extremum necessary for the implementation of the subsequent optimisation.

Firstly, correlation-interferometric DF should in general be performed in a search-free manner, that is, provided that the number of  $N_C$  cycles of correlation estimation of the direction to the RS is equal to one:

$$N_C = 1 \quad (5)$$

Second, the variables  $z_1$  and  $z_2$  can vary throughout the AA aperture:

$$\begin{aligned} (Z-1) > z_1 &\geq 0 \\ (Z-1) &\geq z_2 > 0 \\ z_2 &> z_1 \end{aligned} \quad (6)$$

From relations (5) and (6) we can conclude that the optimisation should be carried out considering four coupling functions, i.e.  $L=4$ , and searching for a global conditional extremum of the type  $\max$ .

In the general case, the function  $W_0(z)$  is nonlinear, so it is reasonable to use the methods of nonlinear programming to solve the set optimisation problem [7]. To simplify the following calculations, let us analyse the peculiarities of the distribution of samples of the spatial complex analytical signal  $S_a(jz)$  and its signal  $S_{aS}(jz)$  and additive noise  $S_{aN}(jz)$  components within the aperture of the AA when reconstructing it on the basis of the selected signal group of  $m_S$  samples [8]:

$$S_a(jz) = S_{aS}(jz) + S_{aN}(jz) \quad (7)$$

The analysis of equation (2) of dispersion  $\sigma_\theta^2$  of DF error shows that the level of the signal component  $S_{aS}(jz)$  of the analytical signal within the AA aperture varies significantly and is determined by the weight function  $W_0(z)$  of multiblade DP synthesis. The maximum value of the modulus of the signal component  $S_{aS}(z)$  corresponds to the midpoint  $z_C = Z/2$  of the AA aperture, i.e.  $\max[S_{aS}(z)] = S_{aS}(z_C)$ . At displacement to the left and to the right from the centre

point  $z_C$  the level of the signal component of the analytical signal symmetrically monotonically decreases, i.e.  $S_{aS}(z_C \pm \Delta z) < S_{aS}(z_C)$  at  $S_{aS}(z_C + \Delta z) = S_{aS}(z_C - \Delta z)$ , and is determined according to equation:

$$S_{aS}(z) = S_{aS}(z_C) \cdot W_\theta(z) \quad (8)$$

This dependence of the modulus of the signal component  $S_{aS}(z)$  is determined by the coherence of the arguments  $m_S$  of the signal components of the selected signal group and the peculiarities of the weight function  $W_\theta(z)$ .

Taking into account relations (3), (7) and (8) it is reasonable to distinguish two particular variants of realisation of the target function  $F(z_1, z_2)$ . The first variant  $F_1(z_1, z_2)$  is characterised by symmetrical distribution of the numbers of selected antenna elements  $z_1$  and  $z_2$  relative to the central element  $z_C$  of the AA:

$$\begin{aligned} F_1(z_1, z_2) &= K_\sigma \cdot W_\theta(z_1) \cdot W_\theta(z_2) \cdot (z_2 - z_1)^2 = \\ &= K_\sigma \cdot W_\theta(z_C - \Delta z / 2) \cdot W_\theta(z_C + \Delta z / 2) \cdot \Delta z^2 = \\ &= K_\sigma \cdot W_\theta^2(z_C - \Delta z / 2) \cdot \Delta z^2 \end{aligned} \quad (9)$$

The expediency of the first variant  $F_1(z_1, z_2)$  of realisation of the target function  $F(z_1, z_2)$  according to (9) is determined by the fact that all values of the weight function  $W_\theta(z)$  of the window are positive and do not exceed unity, and the local maximum values of the product  $W_\theta(z_1) \cdot W_\theta(z_2) \cdot (z_2 - z_1)^2$  are determined from the condition of symmetrical distribution of the numbers  $z_1$  and  $z_2$  relative to the chosen point  $z_h$  of symmetry on the AA aperture [9]:

$$\begin{aligned} W_\theta(z_h - \Delta z / 2) \cdot W_\theta(z_h + \Delta z / 2) \cdot \Delta z^2 < \\ < W_\theta(z_C - \Delta z / 2) \cdot W_\theta(z_C + \Delta z / 2) \cdot \Delta z^2 \end{aligned} \quad (10)$$

The analysis of equations (9) and (10) shows that when choosing a symmetrical distribution of the numbers of the selected AA antenna elements  $z_1$  and  $z_2$  relative to the selected central element  $z_h$ , in order to obtain the global maximum of the target function  $F_1(z_1, z_2)$ , it is reasonable to select the midpoint as the central element  $z_h = z_C$  of the AA aperture.

The second variant  $F_2(z_1, z_2)$  of the target function implementation  $F(z_1, z_2)$  is characterised by asymmetric distribution of the numbers  $z_1$  and  $z_2$  of the selected antenna elements within the AA aperture: the value  $z_1$  corresponds to the middle point  $z_C$  of the AA aperture, where the level of the signal component  $S_{aS}(z)$  is maximum, and the value  $z_2$  corresponds to the placement of the peripheral points of the AA aperture, that is:

$$\begin{aligned} F_2(z_1, z_2) &= K_\sigma \cdot W_\theta(z_C) \cdot W_\theta(z_2) \cdot (z_2 - z_C)^2 = \\ &= K_\sigma \cdot W_\theta(z_2) \cdot (z_2 - Z / 2)^2 \end{aligned} \quad (11)$$

where,  $z_1 = z_C$ ,  $W_\theta(z_C) = 1$ ,  $|\Delta z| = |z_2 - z_C| \leq Z / 2$ .

The analysis of equation (11) shows that the asymmetric distribution of the values of the numbers  $z_1$  and  $z_2$  within the AA aperture allows us to reduce the number of variables of the target function to one, as well as to use the optimal conditions for estimating the difference  $\Delta \psi_a$  of the arguments of the complex analytical signal  $S_a(jz)$  on the signal-to-noise ratio at the point  $z_1$ .

Let us analyse the distribution features of the noise component value  $S_{aN}(jz)$  of the complex analytical signal.

The noise component  $S_{aN}(jz)$  is formed as the sum of harmonic noise components  $S_{aN,m}(jz)$  of the signal group, that is:

$$S_{aN}(jz) = \sum_{m=1}^{m_S} S_{aN,m}(jz) \quad (12)$$

Each noise component  $S_{aN,m}(jz)$  is a narrowband random process with a normal probability density distribution of the probability density of samples [4, 11]. Therefore, the noise component  $S_{aN}(jz)$  of the analytic signal as a whole will also be a normal narrowband random process. The power  $P_{aN}$  of the noise component  $S_{aN}(jz)$  is equal to the sum of the powers of  $m_S$  its components  $S_{aN,m}(jz)$ :

$$P_{aN} = \sum_{m=1}^{m_S} P_{aN,m} \quad (13)$$

The noise component  $S_{aN}(jz)$  of the complex analytical signal has a certain interval  $\Delta z_{cor}$  of correlation, which is determined by the number  $m_S$  of its harmonic noise components of the signal group [4]:

$$\Delta z_{cor} = \frac{d \cdot Z}{m_S} \quad (14)$$

The analysis of equation (14) shows that for effective statistical estimation of the difference of the analytical signal arguments  $S_a(jz)$ , it is necessary that the separation  $\Delta z$  between the selected AA elements is not less than the interval of the correlation [11]:

$$\Delta z \geq \Delta z_{cor} \quad (15)$$

Requirement (15) ensures maximum uncorrelatedness and statistical independence of the noise components of the samples of the analytical signal argument difference  $S_a(jz)$ , which, in turn, ensures maximum efficiency of their subsequent statistical processing in dispersion-correlation analysis for a large antenna base [10].

Thus, to the set of coupling functions, in addition to conditions (5) and (6), it is reasonable to include the following condition:

$$z_2 - z_1 \geq \Delta z_{cor} \quad (16)$$

Considering the obtained expressions for the target functions  $F_1(z_1, z_2)$  and  $F_2(z_1, z_2)$ , as well as the link functions (5), (6) and (16), the corresponding optimisation equations will be of the form:

$$\begin{aligned} (z_1, z_2)_{opt1} &= \arg \max \{F_1(z_1, z_2)\} \\ (z_2)_{opt2} &= \arg \max \{F_2(z_2)\} \end{aligned} \quad (17)$$

We define the optimal values of the numbers  $z_1$  and  $z_2$  taking into account equations (17) as the solution of the corresponding differential equations:

$$\begin{aligned} (z_1, z_2)_{opt1} &= (\Delta z)_{opt1} = \arg \left\{ \frac{\partial F_1(z_1, z_2)}{\partial \Delta z} = 0 \right\} \\ (z_2)_{opt2} &= (\Delta z)_{opt2} = \arg \left\{ \frac{\partial F_2(z_2)}{\partial z_2} = 0 \right\} \end{aligned} \quad (18)$$

To solve equations (18) we use the MathCad software package. To illustrate the solution of equations (18), using equations (9) and (11), we plot analytical dependences of the target functions  $F_1(z_1, z_2)$  and  $F_2(z_2)$  on the separation

in space value  $\Delta z = (z_2 - z_1)$  for asymmetric (Fig. 1, row 1) and symmetric (Fig. 1, row 2) variants of separation of AA elements with numbers  $z_1$  and  $z_2$  at the number of DF channels  $Z = 64$ . In this case, we use the weight function  $W_\theta(z)$  Blackman of the 3rd order [2]:

$$W_\theta(z) = 0.42 + 0.5 \cdot \cos\left(\frac{2\pi \cdot (z - Z/2)}{Z}\right) + 0.08 \cdot \cos\left(\frac{4\pi \cdot (z - Z/2)}{Z}\right) \quad (19)$$

The analysis of equations (9) and (11) shows that the target functions  $F_1(z_1, z_2)$  and  $F_2(z_2)$  have a bell-shaped form with one maximum. With symmetric optimal separation in space  $\Delta z = 22$  with respect to the central  $z_C = 32$  AA element, the maximum value of the target function  $F_1(z_1, z_2)$  will be achieved at  $(z_1, z_2)_{opt1} = 21; 43$ .

At asymmetric optimal separation in space  $\Delta z = 15$ , the maximum value of the target function  $F_2(z_2)$  will be provided at  $z_1 = 32$  and  $(z_2)_{opt2} = 47$ , determined by equations (9) and (11). At the same time, since  $\arg\{F_1(z_1, z_2)\}_{(z_1, z_2)_{opt1}} > \arg\{F_2(z_2)\}_{(z_2)_{opt2}}$ , we can conclude that it is reasonable to use the symmetric separation  $\Delta z = 22$  to maximise the given target function  $F(z_1, z_2)$  and, accordingly, to minimise the variance  $\sigma_\theta^2$  of the error in estimating the direction to the RS.

Thus, the problem of streamlining of the search-free digital method of correlation-interferometric DF with reconstruction of the spatial complex analytical signal has been solved. At the same time  $(z_1, z_2)_{opt1}$ ,  $(z_2)_{opt2}$  and values of separation in space  $\Delta z$  between the selected AA elements, in which the complex analytical signal is calculated, are determined.

Using the developed software model in the MathCad environment, the direction finder operation was simulated according to the developed algorithm of the search-free digital method of correlation-interferometric DF with reconstruction of the spatial analytical signal [8]. The modelling was performed for the following initial conditions: signal type – continuous with linear frequency modulation:  $S(t) = A \cdot \sin(\omega_0 t + bt^2)$ ; signal spectrum width  $\Delta f_S = 0.6$  MHz; DF radio channel analysis bandwidth  $\Delta f_k = 10$  MHz; signal carrier frequency  $f_S = 2$  GHz; sampling frequency  $f_d = 2\Delta f_k = 20$  MHz; analysed number of time samples of the signal  $N_S = 2048$ ; duration of the analysis process  $T_a = 0.1$  ms; linear AA with the number of receiving channels  $Z = 64$ ; AA step  $d = 0.05$  m; selected number of harmonics of the signal group for reconstruction of the spatial analytical signal  $m_S = 6$ ; type of windows  $K_{Wt}$ ,  $K_{W\theta}$  – Blackman windows of the 3rd order, determined by equation (19).

As a result of modelling the family of dependences of methodical error  $\Delta\theta$  of DF estimation on the value of separation in space  $(z_2 - z_1)$  of the selected AA elements without taking into account the effect of noise was obtained (Fig. 1).

Figure 1 shows:

row 1 – for the condition of using symmetric with respect to the centre  $z_C = 32$  of the AA the separation of selected AA elements at a given direction to the RS  $\theta = 60^\circ$  ;  
row 2 – for the condition of using asymmetric relative to the AA centre spacing of selected AA elements: element  $z_1 = 32$  is selected in the AA centre, and  $z_2$  is shifted with a unit step to the AA edge;  
row 3 – for the condition of using symmetric with respect to the AA centre spacing of selected AA elements at a given direction to the RS  $\theta = 45^\circ$  .

The analysis of Fig. 1 shows that the dependence of the methodological error  $\Delta\theta$  of DF estimation has an oscillating unstable character when changing the given direction  $\theta$  on RS, but monotonically decreases up to a certain value of separation in space  $(z_2 - z_1)$  of the selected AA elements. Minimum methodical error  $\Delta\theta$  of DF estimation is achieved at separation in space  $(z_2 - z_1) = 16$  for the condition of using asymmetric with respect to the AR centre separation, and at separation in space  $(z_2 - z_1) = 16$  – for the condition of using symmetric with respect to the AA centre separation.

The dependence of the root mean square error (RMSE)  $\sigma_\theta$  of the DF estimation on the separation in space value  $(z_2 - z_1)$  of the selected AA elements at a given signal-to-noise ratio  $\mu_I = 0$  dB at the input of DF radio channels is investigated.

During modelling we obtained a family of dependences of RMSE  $\sigma_\theta$  of DF estimation on the value of separation in space  $(z_2 - z_1)$  of selected AA elements and direction  $\theta$  on RS for two variants of separation in space  $z_1$  and  $z_2$  at  $\mu_I = 0$  dB (Fig. 2). Number of experiments for estimation of one direction  $\sigma_\theta = 50$ .

Figure 2 shows:

row 1 – for the condition of using symmetric with respect to the AA centre separation of selected AA elements at a given direction to the RS  $\theta = 60^\circ$  ;  
row 2 – for the condition of using asymmetric relative to the AA centre spacing of selected AA elements: element  $z_1 = 32$  is selected in the AA centre, and  $z_2$  is shifted with a unit step to the AA edge at  $\theta = 60^\circ$  ;  
row 3 – for the condition of using symmetric with respect to the AA centre separation of selected AA elements at a given direction to the RS  $\theta = 45^\circ$  .

The analysis of Fig. 2 shows that for symmetric separation in space (Fig. 2, row 1, row 3) the value of the RMSE of DF estimation has a monotonic smooth dependence with one minimum at the value  $(z_2 - z_1) = 28$  regardless of the given direction to the RS. At asymmetric separation in space (Fig. 2, row 2) the value of RMSE of DF estimation changes insignificantly with increasing separation  $(z_2 - z_1)$  and remains twice as large as at symmetric separation, which confirms its lower efficiency.

It can be concluded that the analytical results of optimisation and simulation results agree well with a small deviation of practical results of minimising the RMSE of DF estimation at low signal-to-noise ratio towards increasing the value of symmetric separation in space  $(z_2 - z_1)$  from 22 to 28 steps  $d$  AA.

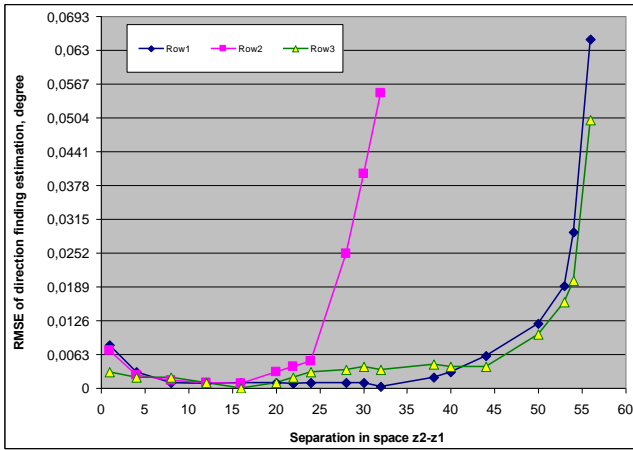


Fig. 1. Dependence of direction finding estimation error on the separation value of the selected AA elements without taking into account the effect of noise

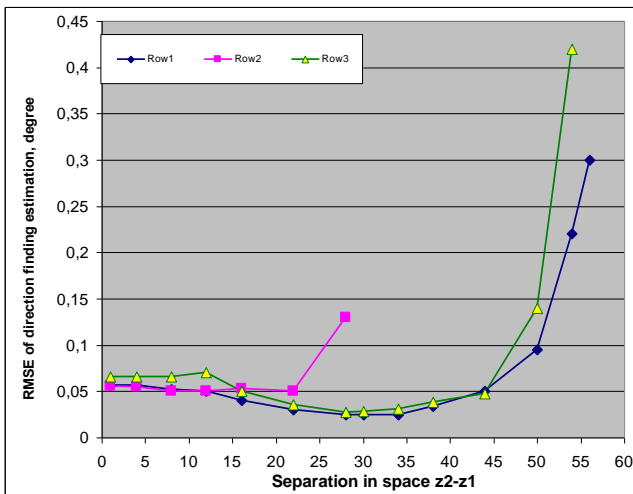


Fig. 2. Dependence of direction finding estimation error on the separation value of selected AA elements at zero value of signal to noise ratio

### 3. Conclusions

As a result of the research it was determined that the main parameters to be optimised, which are included in the equation of variance of the RS direction estimation error for the search-free digital method of correlation-interferometric DF with reconstruction of the spatial complex analytical signal, are: the value of separation in space  $\Delta z = (z_2 - z_1)$  between the selected AA elements, for the spatial positions of which the complex analytical signal is reconstructed, and the type of separation in space – symmetrical or non-symmetrical relative to the center  $z_C = 32$  of AA. Analytical optimisation of parameters of the investigated method has shown that  $\arg \{F_1(z_1, z_2)\}_{(z_1, z_2)_{opt1}} > \arg \{F_2(z_2)\}_{(z_2)_{opt2}}$ , therefore

for minimisation of dispersion  $\sigma_\theta^2$  of error of an estimation of direction on RS and maintenance of the maximum noise immunity of search-free DF it is expedient to use symmetric separation  $\Delta z = 28$ . The results of analytical optimisation and simulation results agree with a small deviation of the optimal value of separation  $\Delta z = 28$  obtained at the ratio signal/noise  $\mu_f = 0$  dB as a result of simulation from the optimal analytical value of separation in space  $\Delta z = 22$  steps  $d$  of AA.

Future research should focus on characterizing the noise distribution across the AA aperture. This analysis will inform

the refinement of signal and noise models for the spatial complex analytical signal. Consequently, improved expressions for estimating the noise immunity of the investigated direction finding (DF) method can be established.

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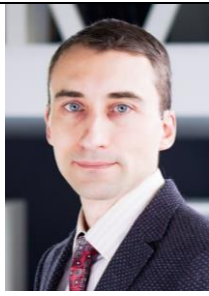
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