On the Groundless Use of Mathematics Concerning the d'Alembert's Rule

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„*The essence of mathematics lies in its freedom.*"
Georg Cantor

ABSTRACT

The paper discusses the groundless use of mathematics. This question has been explained mainly based on example of the so called d`Alembert's rule which unfortunately still functions in science none the less it has been based on a fiction, contrary to the truth cognition as the fundamental purpose of science. That pejorative feature of the mentioned paradigm is marked very clearly in the paper to evaluate it negatively as the only one possible note. Next the adequate characteristics of variable body motion has been presented and the description of Atwood device given in view of explaining the essence of a real equilibrium of the system of material bodies where their real inertia is of importance. In conclusion the characteristics of real inertia force being the measure of this inertia is presented. The erroneous up-to-date view concerning the measure of body inertia, assuming mass as the measure of this magnitude enabling free manipulation of the acceleration value, has been revealed. At the end it is stressed that acceleration is always positive in its nature and is existent in each condition of the material reality.

Keywords: d`Alembert's rule; Fictional inertia force; Identity; Space-time; Time constant; Potential field; Atwood device; Co-existence state; Energetic state; Dynamics

1. INTRODUCTION

The effort has been undertaken on the ground of science to investigate continuously appearing tendency to mathematize physics at any price. Such an approach may bring about bad consequences. It is being manifested that many descriptions of natural phenomena are simply inadequate, i.e. not in agreement with the investigated reality.

There are many paradigms being a cognitive barrier which should be excluded and replaced by adequate ones. First of them is the so called d'Alembert's rule, uniting fiction and deformed reality. Both these elements disqualify it completely though it still in force and

remains in the literature. Even the latest reference [1] accepts this rule and develops it into improper direction just to make it more attractive: they are Lagrange and Hamilton equations.

The next problem is the approximation. One should be very careful in the proceeding to avoid some fatal effects. Here is one of the examples presenting destructive consequences. The example comes from the classical mechanics, and it is the course of velocity in the variable motion replaced by a straight line passing by the beginning of coordination system. Such an activity led to block the development of mechanics as that philosophy of approach to the quest enables determination of only three, moreover deformed, physical magnitudes: path/way length, velocity, and acceleration.

All those examples, to mention just a few, clearly indicate the need or necessity to formulate the description of reality in a proper or adequate form, expressing the most important thing with a physical sense. That is possible but an essential condition must be fulfilled, which is justifiable application of mathematics.

It is worth noting that empirical or statistic-experimental formulae do not reveal the physical sense of phenomenon. Their derivation is justified, mainly in technical sciences, with the conscience of negatively marked effect. One cannot introduce some artificially imagined interpretation.

Creation a variety of forms of description of a determined phenomenon is essential. However, all they should have one and common true content. To say even more, they should come out of a cognition source which is the truth on reality. These remarks are directed to all activities on the ground of physics. Mathematics, of which the essence lies in its freedom, does not have to observe the external world (see above aphorism by G. Cantor). However, if it does, one should take care of the adequacy of mathematical descriptions.

At the end of this introduction it is worth referring to the thought presented in the work [2] that treats of hidden and nipped kernel/heart of the truth. One may assume that (…) *hiding errors leads to stiffening of our knowledge. Our laws become injunctions/commandments and not the description of natural phenomena*.

2. ON THE FICTION IN SO CALLED THE d'ALEMBERT'S RULE

Literature [3] informs that in the 18th century there was a group of scientists which took a position of the highest caution and limited range of scientific search just to achieve a complete certainty. Facts were the only subject of science and philosophy. This position was initiated in the mid-18th century in France and next termed in the 19th century as a positivism. Its main representative was d'Alembert.

It is strange that the certainty, being the most important subject of science and philosophy, does not correspond with the so called d'Alembert's rule carrying still the name of paradigm. This rule, contrary to the higher mentioned facts, contains hardly a piece of reality with the kernel being a fiction. Thus it is unintelligible that this rule still forms a fundament on which the solutions of the problems of material body dynamics is built.

These solutions are artificial, fictional but consciously adopted and tolerated. And they cannot be different if based on the d'Alembert's rule which sounds as follows: *In the moving material system the real forces are balanced in each moment with inertia forces* [4]. In reference [5] this rule is presented in a different way: *Active forces acting on the moving material point are in equilibrium with a thought inertia force in each moment*. Anyway, the essence of the rule is always the same: it is about thought, artificial, imaginable equilibrium between a fiction and reality with a deformed description.

Fictional, thought, imaginable inertia force, or even an apparent force, all they are names of that strange force functioning on the ground of science. They have nothing in common with the reality. The work [6] further indicates: *One should have in mind that the apparent forces form an auxiliary construction and not real forces.*

That fictional force is characterized as follows [7]:

(1)　Inertia force is equal to the product of mass of a moving body and its acceleration in motion.

(2)　Inertia force is always directed against the acceleration of motion.

(3)　Inertia force of a moving body is an imaginable force.

(4)　In the uniform linear motion the inertia force equals zero (as the total acceleration is also zero).

It is time now to reveal the genesis of this rule. It was formed by a formal transformation of equation of motion resulting from the second Newton's law. The question arises here, why that real inertia force had been excluded from that record, and substituted by an apparent force (fictional, thought, imaginable). To say more, it is said that such mathematical, formal transformation of the mentioned Newton's law enabled obtaining the rule which has a great meaning from the point of view of mechanics as it allows to use the statics methods in solving the problems of dynamics.

Thus the primary link of the action chain which led to formation of the subject rule is the mentioned Newton's law having the following mathematical configuration:

$$B = m \cdot a \qquad (1)$$

where symbol B denotes the inertia force, m – body mass, a – the body acceleration (and not the motion acceleration, as it is presented in the first two points of the higher mentioned characteristics of inertia force).

Now, according to the scientific rules, let the right side of the equation be transferred to the left side. The result of this operation is the equation of type:

$$B - ma = 0 \qquad (2)$$

and that may be written as follows:

$$B + (-ma) = 0. \qquad (3)$$

It is said that B presents an external force or resultant of that kind of forces if there is more than one force. That part which is in paranthesis/brackets, i.e. $(-ma)$ represents the inertia force.

However, one may note that dependences (2) and (3) are simply the identity (identity equation). To obtain a proof it is enough the record (1) is substituted to the relationship (2). As a result one obtains the following:

$$ma - ma \equiv 0 \qquad (4).$$

After substituting (1) to (3) the following identity form appears

$$ma + (-ma) \equiv 0 \qquad (5).$$

Mathematical treatment of the reality could be the name of those actions which led to formation of the so called rule. Thus the d'Alembert investigation is speculative in character and detached/diverted from the reality. It has no connection with the reality. Experiment and practice do not confirm such behaviour of the body as suggested by the d'Alembert's rule.

In reference to the identity of (2) and (3) it is worth adding that they are just two forms of the record of inertia force, as it is in use in mathematics, where e.g. there is a quadratic equation on one side and the product form of this equation or its canonical form on the other. Therefore these two identities contain the inertia force and its open (definition) form.

In a dynamic reality there is no equilibrium: nor force neither work is in balance. The body which moves with a variable motion (accelerated or retarded) possesses its inertia, and that never looses it. That feature is characteristic also at its rest (static energetic state) and in the uniform motion corresponding with the kinematic energetic state. One cannot deprive the body of such essential feature by introducing instead a fictional inertia force acting this way just to form an apparent equilibrium in forces.

There are some works treating about these quests. They are of understanding the body inertia [8] and the second Newton's law [9]. The work [10] treats about the d'Alembert's rule the contents of which is permeated by fiction, that is imagination. The determined energetic states have been also marked in references on the background of considerations leading to explanation of the energy notion [11-13].

Thus if there is no equilibrium during body motion in the space-time (that is a place where the body moves dynamically, i.e. with a variable motion) so each of them should be described separately. First, the adequate primary characteristics of variable motion, which is the dependence of the path/way length on time, should be formulated. Creation of further descriptions of reality on that cognitive ground guarantees that all they will have analogous adequate nature.

A series of examples provided by the existent knowledge may be presented to illustrate the use of so called d'Alembert's rule. One does not have to do it as that is not scientific in character simply due to the fictional elements contained with. Therefore in the next section the considerations leading to a proper, adequate solution of these problems will be presented. Furthermore, it will be referred to a given instructive example which will explain finally all by indicating even more clearly the essence of d'Alembert's speculative thoughts.

3. CHARACTERISTICS OF THE VARIABLE BODY MOTION

The primary source cell of characteristics of variable body motion is the differential equation of the following form:

$$ds = \pm \frac{\partial s}{\partial t} dt \qquad (6)$$

where: ds – total differential of the path length, dt – total differential of time, $\partial s / \partial t$ – partial derivative being the coefficient of path length of the body moving with this variable motion. Symbols $(\pm)$ are the algebraic operators fulfilling determined functions. Operator $(+)$ has a formal character and confirms the physical character of determined description only. The second one, negative operator $(-)$ ascribes such a sense to determined record and refers here to retarded variable motion [14-20]. Thus the differential equation (6) may be developed by determining first the total differentials ds and dt. That allows introducing the quotient ds/dt in

the place of that partial derivative. After separating its total differentials, the differential equation is obtained with the following solution:

$$s = s_1 \left(e^{\frac{t}{T_1}} - 1 \right) \tag{7}.$$

That refers to the accelerated variable motion. For that kind of retarded motion the equation on the path/way length takes the following form:

$$s = 2s_2 \left(1 - e^{\frac{t}{T_2}} \right) \tag{8}.$$

The way of proceeding, leading to obtain the equations of such a structure, has been presented in our other works [16-21]. That means such an approach relying on creation of the proper/adequate function is necessary on the ground of different realities (real systems).

The magnitudes s_1, and T_1 are the space-time parameters (**Fig. 1**), i.e. sites where the variable body motion occurs. Parameter s_1 is the total path/way length in the first space-time (dotted area in Fig. 1) and with the same the distance between the neighbouring potential fields which limit the mentioned space-time. First, it is about the static potential field SPF on which the body remains still in the static energetic state. The second field, kinematic potential field KPF, closes the space-time and on this field the kinematic energetic state occurs. The body moves there with a uniform motion.

The time constant, T_1 is the transition time of a body on the neighbouring potential field with the initial velocity v_0. That means the velocity (initial coefficient of the body path/way length) is the ratio of s_1 to T_1, being the derivative of the path length in the initial point, that is

$$v_0 = tg\,\alpha_0 = \left(\frac{\partial s}{\partial t} \right)_{t=0} = \left(\frac{ds}{dt} \right)_{t=0} = \frac{s_1}{T_1} \tag{9}.$$

The analyzed course touches the accelerated variable motion described by formula (7). The curve of path/way length has an exponential nature, rising progressively. One may determine now the second coordinate of the final point, or t_1, by substituting $s = s_1$ to the formula (7). Thus one obtains

$$t_1 = T_1 \ln 2 \tag{10}.$$

For a retarded variable motion, described by formula (8), the graphical illustration is more complex. Here the time constant T_1 is placed on the nominal static potential field [SPF] and the transition to the next, second space-time (also dotted area in Fig. 1) proceeds with the initial velocity equaled to the body velocity on the kinematic potential field (the area marked by oblique square in Fig. 1), or

$$v_1 = tg\,\alpha_1 = \left(\frac{ds}{dt} \right)_{t_1} = \left(\frac{s_1}{T_1} e^{\frac{t}{T_1}} \right)_{t_1} = \left(v_0 e^{\frac{t}{T_1}} \right)_{t_1} = 2v_0 \tag{11}.$$
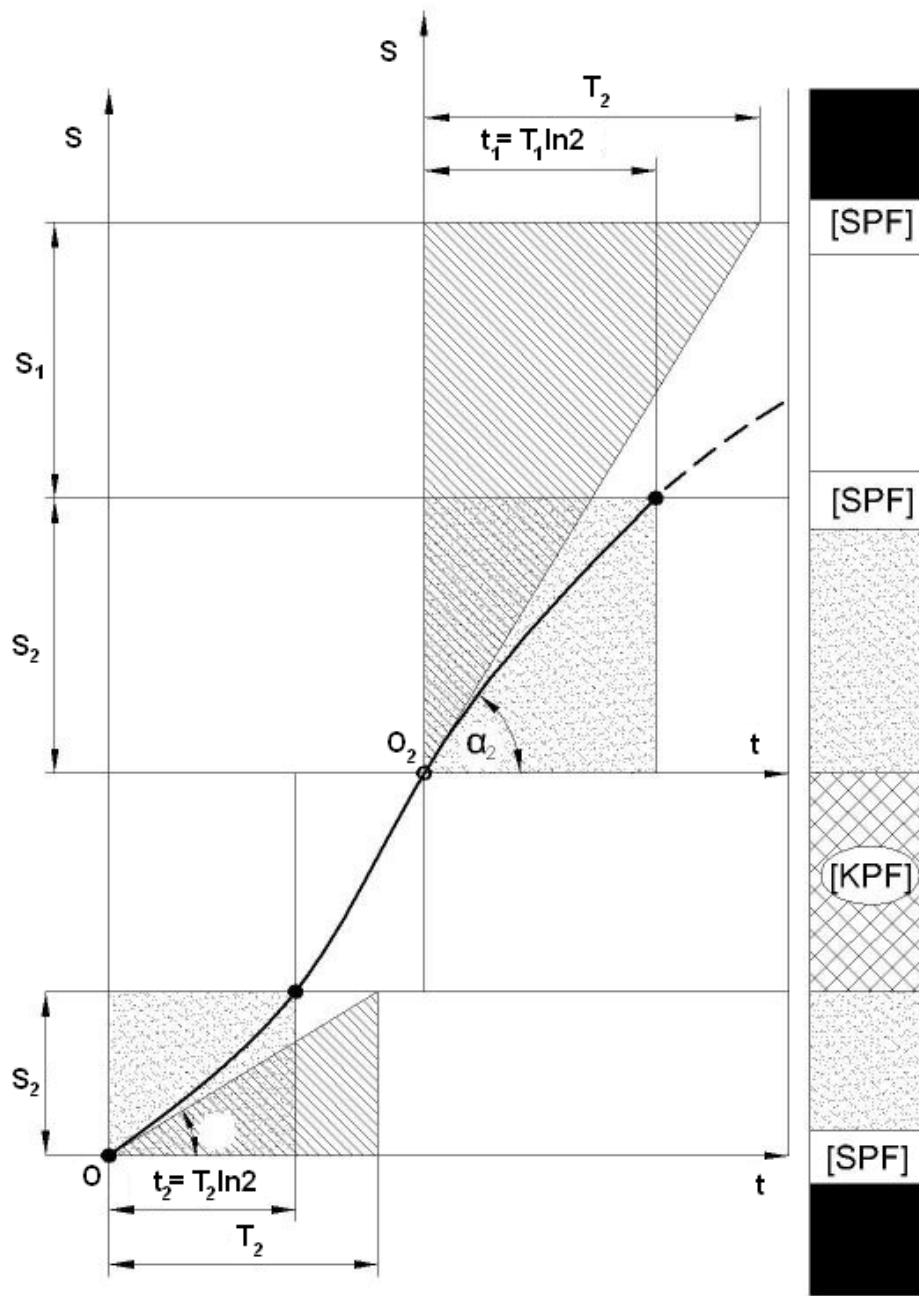
Fig. 1. Time characteristics of motions: accelerated, uniform, and retarded.

It is worth noting there are two space-times: the proper or real, and improper (auxiliary) one, presenting an apparent course of curve, the course without any physical sense (here the curve approaches asymptotically to the mentioned nominal potential field which is the asymptote). The real course of the path/way length also ends on the static potential field, and the coordinates of the final point are as follows: $t_2 = T_2\ln2$; s_2.

Clearly visible is the right use of mathematics relying first of all on separation the real and abstract spheres. In fact there is no infinite in time the body motion to the asymptote. That

way has to be real in character, reflecting the end in a determined time. Therefore introduced to the literature, e.g. [22,23] the notion of limitlessly long time has no physical grounds.

Other equally important thing is a proper formulation of the path/way length. It rises all the time. Such understood and presented the primary magnitude may be the basis to determine the consecutive magnitudes: velocity, acceleration, and impulse. The path/way length, treated sometimes as the coordinate of a body position, does not form any reference to the mentioned here the derivative magnitudes. None the less such formulation still remains, noted especially in the theory of vibrations occupying pages in such books like [24-27].

Equations of the path/way length, expressed by formulae (7) and (8) naturally enable to determine further physical magnitudes. They are not limited to the acceleration, being the second derivative of this maternal magnitude. Through the following differentiation further still unknown magnitudes may be extracted and that does not close the science development but rather opens it. The classical mechanics, even in the latest edition [1], ends this development on the third, unfortunately constant magnitude, i.e. the acceleration. The derivative of this magnitude equals zero. This is why further activities are concerned on creating consecutive forms of the classical description of reality. It results from these considerations there is a need or even necessity to return to the sources. The approaching this aim will allow to formulate an adequate knowledge of the considered parts of reality.

4. ADEQUATE DESCRIPTION OF MOTION OF THE ATWOOD DEVICE

It is worth focusing attention on the description of motion of the Atwood device as it used to be analyzed in this context. This device is an example the application of the d'Alembert's rule is often analyzed. Especially it has been exposed in the latest book [1]. The use of Lagrange equations proves of the importance.

As presented in the last paragraph of the second Chapter, first the adequate solution of this problem will be presented. On that cognitive background the dark quests, the known existent thought d'Alembert's speculations, will be marked more clearly.

The device, Atwood drop instrument, consists of two bodies attached to a string (**Fig. 2**) which fastens/wraps a roller supporting/carrying the whole system. One of these bodies possesses mass m_1, the second one − m_2, the third one, being this roller, possesses mass m, with $m_1 > m_2$. One has to add the geometric characteristics of the roller. It is the parameter being the radius r of this element.

The whole set of quests connected essentially with the dynamics of the body system in the Atwood device will be presented. They are as follows: energetic state of particular bodies, their force equilibrium in these states, courses of forces between these states, and accelerated free motion of the bodies in determined space-times.

It has been assumed the string is inextensible with the attached bodies and basis being the elements perfectly rigid. The performed assumptions do not change any essence nor adequacy of description of the considered reality. They *de facto* mark only limits of this description. First, let us consider the stable static energetic state of particular bodies (**Fig. 3**) taking place on the stable static potential fields SSPF. Particular states are reflected by corresponding equilibrium of all forces. The body of mass m_1 rests on the basis. The system it is in has been formed with the following bodies: subjected body, Earth, basis, and string. Thus there are to be four forces being the measures of reaction of the mentioned material bodies. They are as follows: inertia force $B_1^{(0)}$, gravity force Q_1, basis reactive force R, and a tension force of spring $S_1^{(0)}$.
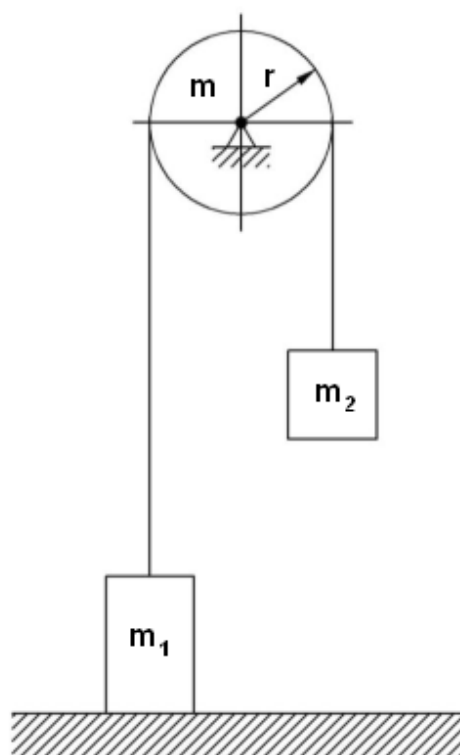
Fig. 2. Scheme of the Atwood device

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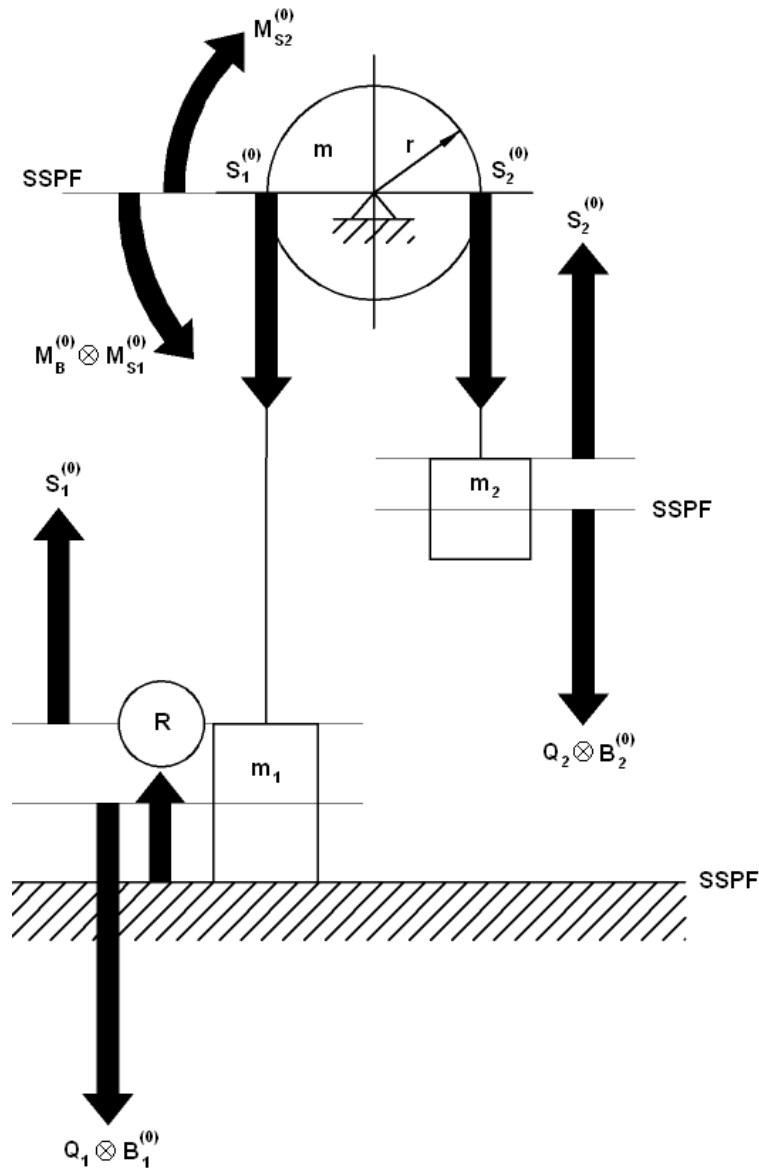
Fig. 3. Conditions of stable static energetic equilibrium of elements in the Atwood device.

The stable energetic state of the roller means in turn the equilibrium of moments. They are three moments, namely: inertia moment of the roller itself, or $M_B^{(0)}$, tension moments of both strings, i.e. $M_{S1}^{(0)}$ and $M_{S2}^{(0)}$, with the inertia moment $M_B^{(0)}$ and tension force $M_{S1}^{(0)}$ being in the state of co-existence, that means $M_B^{(0)} \otimes M_{S1}^{(0)}$.

Let us consider now the unstable static energetic equilibrium of elements of the Atwood device. The interference of a determined external stimulus is required to lead this solid body system to the mentioned equilibrium (**Fig. 4**). This stimulus acts on the system from the body side of less mass m_2. The measure of this action is the force F, the value of which corresponds at the beginning with the value of the basis reaction force R, then $F_0 = R$. Afterwards it increases respectively to the maximum value on the unstable static potential field ASPF. The symbol of this maximum force is F_1, with the indicator informing of the neighbouring first potential field (the indicator zero denotes the initial exit potential field).
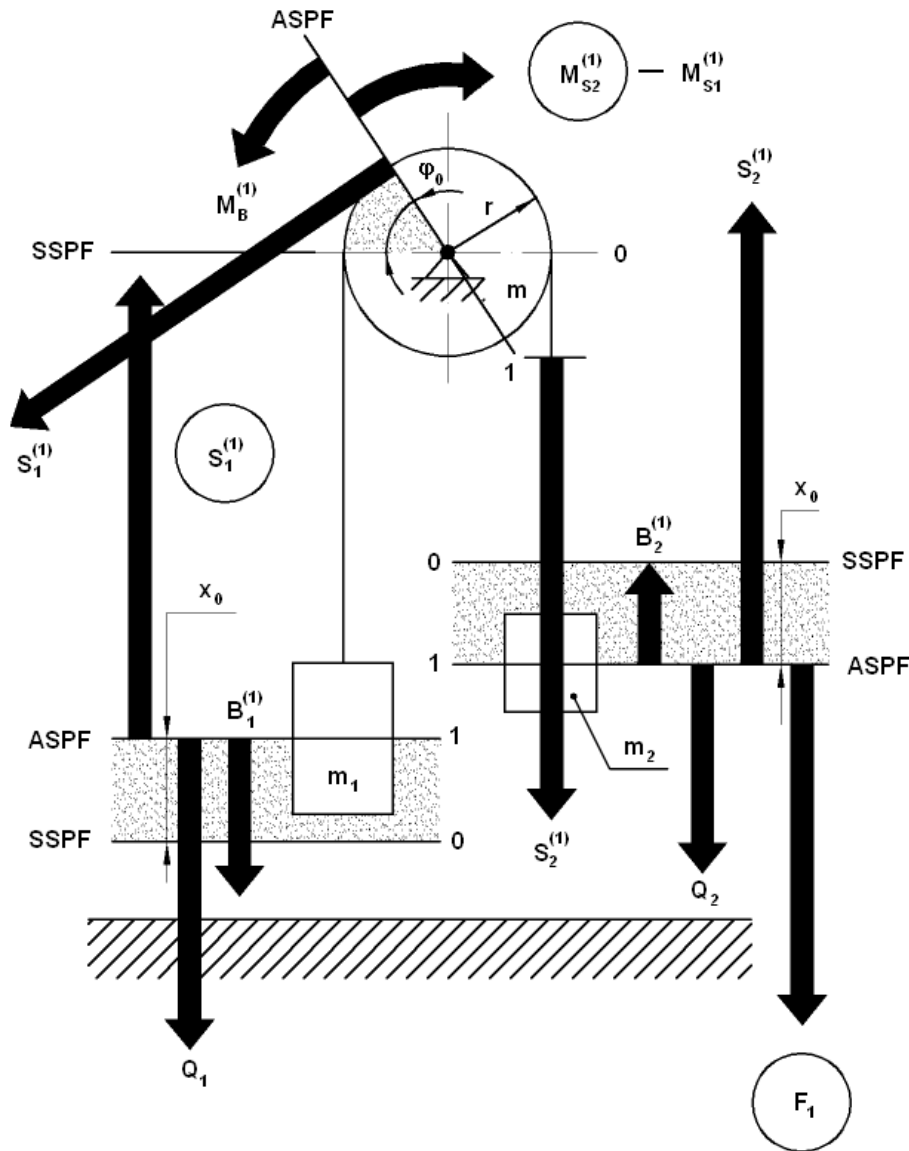
Fig. 4. Conditions of unstable static energetic equilibrium of elements in the Atwood device.

All these three bodies of the system have to pass through the corresponding space-times (dotted areas in Fig. 4). Two of the space-times ascribed to the bodies of masses m_1, and m_2, are linear; their lengths are denoted by a common symbol x_0. The space-time of the roller of mass m is angular with the length denoted by a symbol φ_0.

Unstable energetic equilibrium of a body of mass m_1 is presented as follows:

$$Q_1 + B_1^{(1)} = S_1^{(1)} \tag{12}$$

where the symbols $^{(1)}$ inform that the corresponding forces, explained above, refer to the neighbouring first potential field.

It is worth noting that the inertia force (see Fig. 4) is directed to the primary zero potential field where the body was in stable state. That is the rule from which the sense of acceleration results, connected with the inertia force.

The following relation

$$B = m \cdot a \tag{13}$$

proves of that which is the second Newton's law. Here the symbol a denotes just the acceleration.

The inertia force on the first potential field, $B_1^{(1)}$, is equal to the half of the gravity force Q_1, then $B_1^{(1)} = Q_1 / 2$, that after introducing to (12) gives the following relationship: $S_1^{(1)} = \frac{3}{2} Q_1$.

That relationship between the forces of inertia, and of gravity results from the equilibrium of both forces (at the same time also the accelerations: of inertia a and of gravity g) on the stable field (0), after transition of the body from the stable field (1) through the space-time.

However, first the source equation of the path/way length should be presented as follows:

$$x = x_0 \left(e^{\frac{t}{T}} - 1 \right) \tag{14}$$

with the derivation given in the work [28]. The content of the symbol x_0 is known. The symbol t denotes time of the variable body motion, and T – the so called time constant. The last magnitude is interpreted as the time of the body transition to the neighbouring potential field with the initial velocity v_0. From this it results

$$v_0 = \frac{x_0}{T} \tag{15}.$$

The acceleration is the second derivative of the path/way length, so

$$a = \frac{d^2 x}{dt^2} = \frac{x_0}{T^2} e^{\frac{t}{T}} = a_1 e^{\frac{t}{T}} \tag{16}.$$

The time t_0 of the body motion through the whole space-time, i.e. corresponding with its length x_0, results from the formula (14) after substituting to it the last coordinate, so

$$t_0 = T \ln 2 \tag{17}.$$

After substituting (17) to (16) one obtains the dependence

$$a_0 = 2a_1 \tag{18}$$

where a_0 denotes the acceleration on the next stable potential field (according to the notations introduced in the paper).

Comparison of the accelerations ($a_0 = 2a_1 = g$), as discussed above, results in

$$a_1 = \frac{g}{2} \tag{19}$$

corresponding also with the record

$$B_1 = \frac{Q}{2} \tag{20}.$$

In reference to the body of mass m_1 it will be the above mentioned relation

$$B_1^{(1)} = \frac{Q_1}{2} \tag{21}$$

which now should be introduced to the formula (12), that results in the following:

$$S_1^{(1)} = \frac{3}{2} Q_1 \tag{22}.$$

Now it is time for the energetic analysis of the roller equilibrium. Its basic characteristics is the mass m, and the derivative is the polar mass J_0, described by the formula:

$$J_0 = \frac{mr^2}{2} \tag{23}$$

where r denotes the roller radius. It is worth noticing that the name of this moment is not in agreement with the conventional popular name: the polar inertia moment which is quite different. Its adequate definition has the following form:

$$M_B = J_0 \cdot \varepsilon \tag{24}$$

being the product of that polar mass moment J_0 and the angular acceleration ε. That interpretation of the formula on inertia moment is now in agreement with the terminological rule of univocality [27] which requires one name to one notion.

The angular acceleration ε may be expressed in function of the linear acceleration a and the roller radius r. That dependence has the following form:

$$\varepsilon = \frac{a}{r} \tag{25}.$$

By introducing (23) and (25) to (24) one obtains:

$$M_B = \frac{mr^2}{2} \cdot \frac{a}{r} = \frac{mra}{2} \tag{26}.$$

Referring this to the first unstable static potential field ASPF and taking into account that $a = g/2$ one obtains the following record of the formula on the considered magnitude:

$$M_B^{(1)} = \frac{mrg}{4} = \frac{Q \cdot r}{4} \tag{27}$$

where Q is the roller weight.

Now the equation for the roller equilibrium may be written. Its form is as follows:

$$M_B^{(1)} = M_{S2}^{(1)} - M_{S1}^{(1)} \tag{28}$$

where $M_{S2}^{(1)}$ denotes the moment of the string tension force S_2 and $M_{S1}^{(1)}$ is the moment of tension force S_1 of this element, then

$$M_{S2}^{(1)} = S_2^{(1)} \cdot r \tag{29}$$

$$M_{S1}^{(1)} = S_1^{(1)} \cdot r \tag{30}.$$

From the equation (28) one may determine $M_{S2}^{(1)}$, so

$$M_{S2}^{(1)} = M_B^{(1)} + M_{S1}^{(1)} \tag{31}$$

and substituting to it (27) and (30) one obtains the following:

$$M_{S2}^{(1)} = \frac{Q \cdot r}{4} + S_1^{(1)} \cdot r \tag{32}$$

and after substituting to this the formula (22), one obtains:

$$M_{S2}^{(1)} + \frac{Q \cdot r}{4} + \frac{3}{2}Q_1 \cdot r \tag{33}.$$

The equation of equilibrium of the last, third body, possessing the mass m_2, takes the following form:

$$Q_2 + F_1 = B_2^{(1)} + S_2^{(1)} \tag{34}$$

and then

$$F_1 = B_2^{(1)} + S_2^{(1)} - Q_2 \tag{35}.$$

The string tension $S_2^{(1)}$ results from the formulae (29) and (33). Assuming this remark one obtains as follows:

$$S_2^{(1)} = \frac{Q}{4} + \frac{3Q_1}{2} = \frac{1}{2}\left(\frac{Q}{2} + 3Q_1\right) \tag{36}.$$

Further assuming that $B_2^{(1)} = Q_2/2$, which results from the demonstrated relationship between the acceleration ($a_1 = g/2$), one obtains finally:

$$F_1 = \frac{Q}{4} + \frac{3Q_1}{2} - \frac{Q_2}{2} = \frac{1}{2}\left(\frac{Q}{2} + 3Q_1 - Q_2\right) \tag{37}.$$
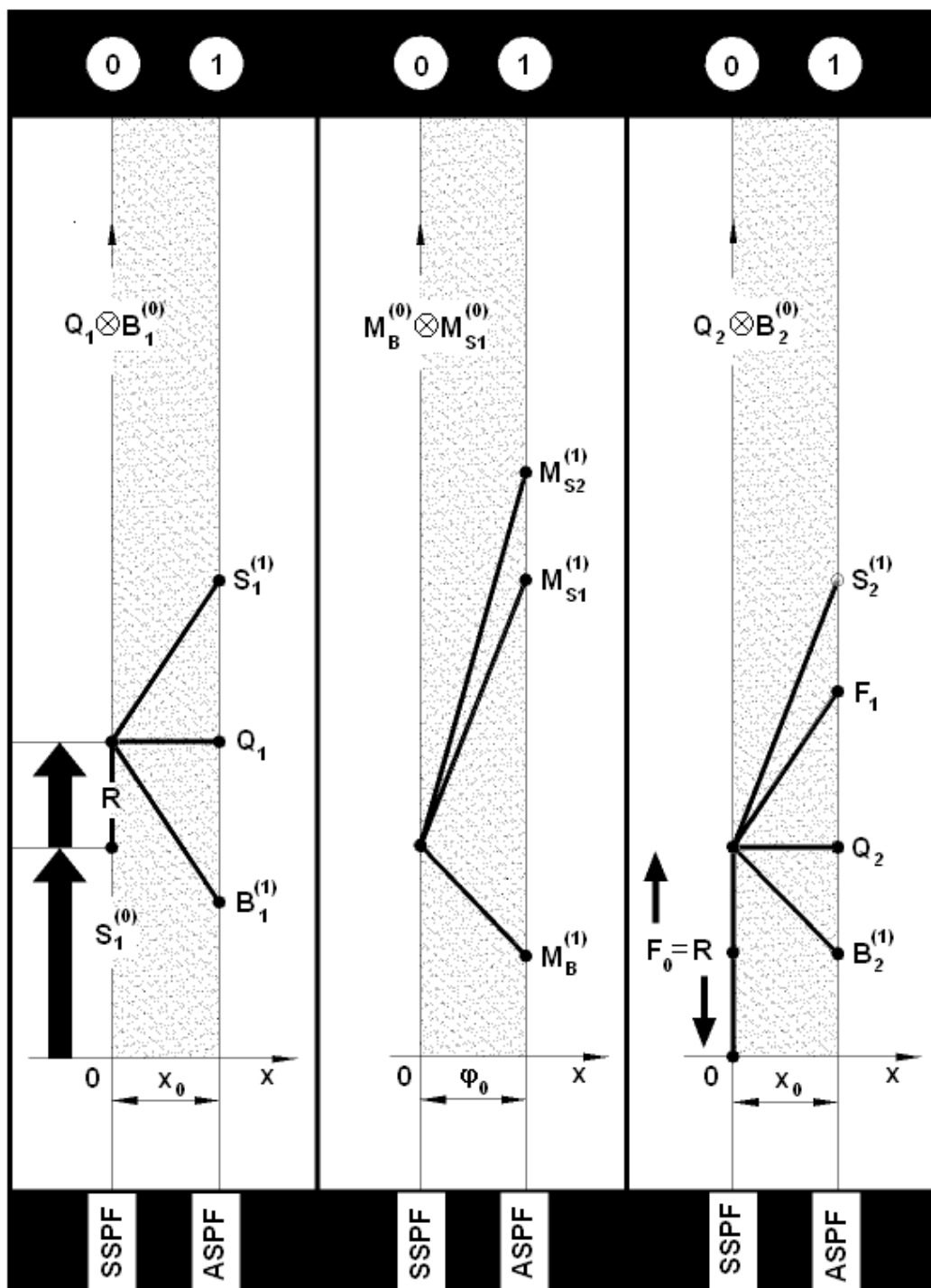
Fig. 5. Courses of forces and moments between stable and unstable potential fields.

The formulae on all unknowns, expressed in function of mass and acceleration, take the following forms:

$$R = Q_1 - Q_2 = g(m_1 - m_2) \tag{38}$$

$$S_1^{(1)} = \frac{3}{2}Q_1 = \frac{3}{2}m_1 g \qquad (39)$$

$$S_2^{(1)} = \frac{1}{2}\left(\frac{Q}{2} + 3Q_1\right) = \frac{g}{2}\left(\frac{m}{2} + 3m_1\right) \qquad (40)$$

$$F_1 = \frac{1}{2}\left(\frac{Q}{2} + 3Q_1 - Q_2\right) = \frac{g}{2}\left(\frac{m}{2} + 3m_1 - m_2\right) \qquad (41).$$

The courses of forces and moments between the stable and unstable potential fields (**Fig. 5**) allow to more clearly perceive/notice all these analyzed conditions of equilibria. Worth of noting is that here is no fiction, that was needed acc. to d'Alembert, to stop the motion of bodies of the considered system. The equilibrium, that real one, occurs on the potential fields only. There is no real equilibrium between them. This is why each of the forces need to be described separately which is a very simple task. To say more, one may analyze the free motion of each body of the system, by a sharp slowing down the action of external stimulus. All that may be realized though rather impossible in the frameworks of this single work.

5. CONCLUSION

In the summary the characteristics of real inertia force, being the measure of the material body inertia, is presented. This characteristics has been presented within three given below sections.

A material body, according to its nature, possesses inertia in each conditions, i.e. at rest, and in the motion. The motion may be of any type, a uniform or variable. From this results the characteristics of the inertia force. That force is equal to the product of body mass and its acceleration.

That means further the acceleration is such a kinetic magnitude which always has a positive value and is noticed not only in the variable motion. Therefore one cannot say about a lack of the body acceleration at the state of its rest, nor in the uniform linear motion.

Furthermore, one should say that the inertia force is always directed to the site where the body possesses a stable equilibrium state. That means during the body motion in that direction the sense of inertia force will be in agreement with the sense of acceleration. However, in case the body moves towards the unstable state so the senses of inertia force and acceleration will be correlated negatively.

The third thing which should be strongly underlined is the statement that the inertia force is a real force for the moving body. One cannot say about some imaginable force as it is apparent in the existent knowledge, by assuming the so called d'Alembert's rule.

From this characteristics of a real inertia force it results clearly that it is just the measure of the body inertia. A mass is the measure of matter.

The up-to-date approach referred to the measure of the body inertia, assuming the mass as the measure of this magnitude, allows for a free manipulation with the values of acceleration.

Thus it could be negative, zero, or positive. In fact, however, as presented above, the acceleration is always positive and marked in any conditions of the material reality.

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