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RELATIVE ACCURACY OF DGPS

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ABSTRACT

It is important to know the accuracy of navigational parameters while analyzing them. This refers not only to position coordinates, although position determination or the assessment of a GPS system often involves navigational parameter accuracy.

Generally, accuracy is the degree of consistency between the measured or estimated value of a given quantity and its real value. Accuracy is usually interpreted as a statistical measure (or statistical-deterministic measure) of measurement errors. Relative accuracy is used in reference to radionavigational systems. According to the USCG Navigation Center Federal Radionavigation Plan, relative accuracy is the accuracy at which the user can determine a position relative to another user by means of the same system at the same time. Another approach is presented in geodesic and hydrographic publications, where both local accuracy and the relative ellipsis of errors are given. This article will attempt at combining and generalizing the two concepts in the form of relative position vector accuracy. The generalized definition of that accuracy is based on a theorem on the difference of random vectors and the concept of cross-covariance matrix of two random vectors. The concept of relative (differential) accuracy can be extended to cover any kind of measurements. When there is a significant positive correlation, differential measurements should be used, as is the case in hyperbolic systems. On the other hand, summing systems are more accurate in cases where negative correlation is significant. Elliptical systems would be the right ones for positioning systems.

Theoretical considerations are illustrated with practical examples of relative accuracy assessment in which AIS (DGPS) is used in ship-to-ship or ship-VTS communication. In the case of an AIS system, both positions are independently determined, but can be correlated through a common (D)GPS or another system. It is a classical example of relative accuracy application. It can also be used in a repeated assessment of hydrographic survey accuracy on condition that we possess recorded raw measurements of position coordinates or other measurements of position parameters.

Keywords – radionavigation, DPGS

INDRUCTION

The knowledge of the accuracy of measurements is important for their analysis as well as in practical navigation. Accuracy is herein understood [1, 2, 3, 4] as a degree of compatibility between measured or estimated parameter and its real value. Accuracy is generally interpreted as a statistical (or statistical-deterministic) measure of measurement errors.

One kind of accuracy used in reference to radionavigational systems is relative accuracy [3, 4]. According to publications, relative accuracy is the accuracy with which a user can measure position relative that of another user of the same navigation system at the same time.

Unfortunately, those publications do not specify any relationships that were used for calculating the relative accuracy of particular radionavigational systems, in spite of the fact that their characteristics include specific numerical values. A different approach is presented in geodesic and hydrographic publications [1, 5]. Although the concept of relative accuracy is not overtly expressed, such terms as local accuracy or relative ellipsis of errors are used.

Attention shall be given to a definition of relative accuracy that might be misleading and taken as relative error, the term that obviously has a different sense. The term local accuracy may also suggest the accuracy of navigational system (measurements) in a local environment considering the topography or propagation conditions.

This article presents the idea of combining and generalization of the two terms – relative and local accuracy – in the form of relative position vector accuracy. As an example of the calculation of relative accuracy, real DGPS static and dynamic measurements were used.

BASIC DEFINITIONS

Let us recall some terms from the probabilistic foundations of statistics, used further in the article for describing relative accuracy. For the random vector **x** with the expected value $\bar{\boldsymbol{x}}$ we can define the following covariance matrix:

$$\boldsymbol{P_x} = E[(\boldsymbol{x}-\bar{\boldsymbol{x}})(\boldsymbol{x}-\bar{\boldsymbol{x}})^{\boldsymbol{T}}] = \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1x_2} & \cdots & \sigma_{x_1x_k} \\ \sigma_{x_1x_2} & \sigma_{x_2}^2 & \cdots & \sigma_{x_2x_k} \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_{x_1x_k} & \sigma_{x_2x_k} & \cdots & \sigma_{x_k}^2 \end{bmatrix}. \tag{1}$$

For two random vectors x and y we can define the cross-covariance matrix as follows:

$$\boldsymbol{P_{xy}} = E[(\boldsymbol{x}-\bar{\boldsymbol{x}})(\boldsymbol{y}-\bar{\boldsymbol{y}})^{\boldsymbol{T}}] = \begin{bmatrix} \sigma_{x_1y_1} & \sigma_{x_1y_2} & \cdots & \sigma_{x_1y_k} \\ \sigma_{x_2y_1} & \sigma_{x_2y_2} & \cdots & \sigma_{x_2y_k} \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_{x_ky_1} & \sigma_{x_ky_2} & \cdots & \sigma_{x_ky_k} \end{bmatrix}. \tag{2}$$

Similarly to the equation (2), we can define the cross-covariance matrix of random vectors **y** and **x**:

$$\boldsymbol{P_{yx}} = E[(\boldsymbol{y}-\bar{\boldsymbol{y}})(\boldsymbol{x}-\bar{\boldsymbol{x}})^{\boldsymbol{T}}] = \begin{bmatrix} \sigma_{y_1x_1} & \sigma_{y_1x_2} & \cdots & \sigma_{y_1x_k} \\ \sigma_{y_2x_1} & \sigma_{y_2x_2} & \cdots & \sigma_{y_2x_k} \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_{y_kx_1} & \sigma_{y_kx_2} & \cdots & \sigma_{y_kx_k} \end{bmatrix} \tag{3}$$

The following equation occurs:

$$\boldsymbol{P_{xy}} = \boldsymbol{P_{yx}^T} \tag{4}$$

While evaluating the accuracy of sub-measurements we use the law of propagation of errors [2]. Its relevant theorem defines an expected value and the covariance matrix of linear combination of random vectors. When non-linear dependence relations exist between the vector **z** and random vectors $\mathbf{x}_i$, then we can take advantage of the theorem (approximately) after the linearization of particular dependences. Several remarks follow from the above theorem, of which only those pertaining to our considerations will be presented.

Remark 1
If $\boldsymbol{z} = \boldsymbol{x} \pm \boldsymbol{y}$, then:

$$\bar{\boldsymbol{z}} = \bar{\boldsymbol{x}} \pm \bar{\boldsymbol{y}} \text{ and } \boldsymbol{P_z} = \boldsymbol{P_x} + \boldsymbol{P_y} \pm \boldsymbol{P_{xy}} \pm \boldsymbol{P_{xy}^T} \quad (5)$$

The covariance matrix **Pz** defines the relative accuracy of position.

Remark 2
In particular, when $\boldsymbol{z} = \boldsymbol{Ax}$, then

$$\bar{\boldsymbol{z}} = \boldsymbol{A}\bar{\boldsymbol{x}} \text{ and } \boldsymbol{P_z} = \boldsymbol{AP_xA^T}. \quad (6)$$

THE ACCURACY OF VECTOR COORDINATED DIFFERENCES

We can determine relative accuracy as the accuracy of relative positions, that is the vector of coordinates differences. This situation is illustrated in Figure 1, in which mean ellipses of errors of particular positions are drawn.

From the remark (1) we can obtain the vector of coordinates differences equal to:

$$\Delta\boldsymbol{x} = \boldsymbol{x}_2 - \boldsymbol{x}_1 = \begin{bmatrix}\phi_2\\ \lambda_2\end{bmatrix} - \begin{bmatrix}\phi_1\\ \lambda_1\end{bmatrix} = \begin{bmatrix}\Delta\phi\\ \Delta\lambda\end{bmatrix} \quad (7)$$

Denoting particular matrixes

- covariance matrix of position (φ_1, λ_1):

$$\boldsymbol{P_{x_1}} = \begin{bmatrix}\sigma^2_{\phi_1} & \sigma_{\phi_1\lambda_1}\\ \sigma_{\phi_1\lambda_1} & \sigma^2_{\lambda_1}\end{bmatrix} \quad (8)$$

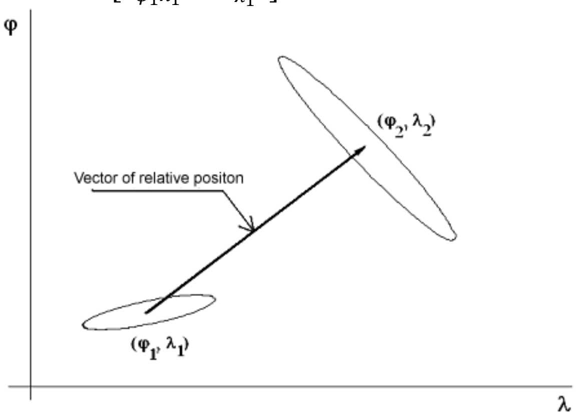

Fig. 1. Relative position of ships.

- covariance matrix of position (φ_2, λ_2):

$$\boldsymbol{P_{x_2}} = \begin{bmatrix}\sigma^2_{\phi_2} & \sigma_{\phi_2\lambda_2}\\ \sigma_{\phi_2\lambda_2} & \sigma^2_{\lambda_2}\end{bmatrix} \quad (9)$$

- cross-covariance matrix:

$$\boldsymbol{P_{x_1x_2}} = \begin{bmatrix}\sigma_{\phi_1\phi_2} & \sigma_{\phi_1\lambda_2}\\ \sigma_{\lambda_1\phi_2} & \sigma_{\lambda_1\lambda_2}\end{bmatrix}, \boldsymbol{P^T_{x_1x_2}} = \begin{bmatrix}\sigma_{\phi_1\phi_2} & \sigma_{\lambda_1\phi_2}\\ \sigma_{\phi_1\lambda_2} & \sigma_{\lambda_1\lambda_2}\end{bmatrix} \quad (10)$$

- and having denoted the covariance matrix of position coordinates differences (relative positions) as:

$$\boldsymbol{P_{\Delta x}} = \begin{bmatrix}\sigma^2_{\Delta\phi} & \sigma_{\Delta\phi\Delta\lambda}\\ \sigma_{\Delta\phi\Delta\lambda} & \sigma^2_{\Delta\lambda}\end{bmatrix} \quad (11)$$

we obtain

$$\boldsymbol{P_{\Delta x}} = \boldsymbol{P_{x_2}} + \boldsymbol{P_{x_1}} - \boldsymbol{P_{x_2x_1}} - \boldsymbol{P^T_{x_2x_1}} = \boldsymbol{P_{x_2}} + \boldsymbol{P_{x_1}} - \boldsymbol{P_{x_1x_2}} - \boldsymbol{P^T_{x_1x_2}} =$$

$$= \begin{bmatrix}\sigma^2_{\phi_2} & \sigma_{\phi_2\lambda_2}\\ \sigma_{\phi_2\lambda_2} & \sigma^2_{\lambda_2}\end{bmatrix} + \begin{bmatrix}\sigma^2_{\phi_1} & \sigma_{\phi_1\lambda_1}\\ \sigma_{\phi_1\lambda_1} & \sigma^2_{\lambda_1}\end{bmatrix} - \begin{bmatrix}\sigma_{\phi_1\phi_2} & \sigma_{\phi_1\lambda_2}\\ \sigma_{\lambda_1\phi_2} & \sigma_{\lambda_1\lambda_2}\end{bmatrix} - \begin{bmatrix}\sigma_{\phi_1\phi_2} & \sigma_{\lambda_1\phi_2}\\ \sigma_{\phi_1\lambda_2} & \sigma_{\lambda_1\lambda_2}\end{bmatrix}.$$

Therefore, the covariance matrix of coordinates differences (relative positions) equals:

$$\boldsymbol{P}_{\Delta x} = \begin{bmatrix} \sigma_{\phi_1}^2 + \sigma_{\phi_2}^2 - 2\sigma_{\phi_1\phi_2} & \sigma_{\phi_1\lambda_1} + \sigma_{\phi_2\lambda_2} - \sigma_{\phi_1\lambda_2} - \sigma_{\lambda_1\phi_2} \\ \sigma_{\phi_1\lambda_1} + \sigma_{\phi_2\lambda_2} - \sigma_{\phi_1\lambda_2} - \sigma_{\lambda_1\phi_2} & \sigma_{\lambda_1}^2 + \sigma_{\lambda_2}^2 - 2\sigma_{\lambda_1\lambda_2} \end{bmatrix} \tag{12}$$

There is often a need to determine the accuracy of relative positions in a local polar coordinates system: the origin of coordinates is a chosen point of own ship, while the coordinates are the distance D and the true bearing α. In this case the covariance matrix is expressed by the following relationship:

$$\boldsymbol{P}_{D\alpha} = \frac{1}{D^2}\begin{bmatrix} \Delta\phi^2 \cdot \sigma_{\Delta\phi}^2 + \Delta\lambda^2 \cdot \sigma_{\Delta\lambda}^2 + 2\Delta\phi \cdot \Delta\lambda \cdot \sigma_{\Delta\phi\Delta\lambda} & -\frac{\Delta\phi \cdot \Delta\lambda}{D} \cdot (\sigma_{\Delta\phi}^2 - \sigma_{\Delta\lambda}^2) + \frac{(\Delta\phi^2 - \Delta\lambda^2)}{D} \cdot \sigma_{\Delta\phi\Delta\lambda} \\ -\frac{\Delta\phi \cdot \Delta\lambda}{D} \cdot (\sigma_{\Delta\phi}^2 - \sigma_{\Delta\lambda}^2) + \frac{(\Delta\phi^2 - \Delta\lambda^2)}{D} \cdot \sigma_{\Delta\phi\Delta\lambda} & \frac{\Delta\lambda^2}{D^2} \cdot \sigma_{\Delta\phi}^2 + \frac{\Delta\phi^2}{D^2} \cdot \sigma_{\Delta\lambda}^2 - 2\frac{\Delta\phi \cdot \Delta\lambda}{D^2} \cdot \sigma_{\Delta\phi\Delta\lambda} \end{bmatrix} =$$

$$= \begin{bmatrix} \sigma_D^2 & \sigma_{D\alpha} \\ \sigma_{D\alpha} & \sigma_\alpha^2 \end{bmatrix} \tag{13}$$

Finally we obtain the following equations for the particular parameters:

- distance error

$$\sigma_D = \frac{1}{D}\sqrt{\Delta\phi^2\sigma_{\Delta\phi}^2 + \Delta\lambda^2\sigma_{\Delta\lambda}^2 + 2\Delta\phi\Delta\lambda\sigma_{\Delta\phi\Delta\lambda}} \tag{14}$$

- error of true bearing

$$\sigma_\alpha = \frac{1}{D^2}\sqrt{\Delta\lambda^2\sigma_{\Delta\phi}^2 + \Delta\phi^2\sigma_{\Delta\lambda}^2 - 2\Delta\phi\Delta\lambda\sigma_{\Delta\phi\Delta\lambda}} \tag{15}$$

- the covariance between distance and the true bearing

$$\sigma_{D\alpha} = -\frac{1}{D^2}\left(\frac{\Delta\phi \cdot \Delta\lambda}{D} \cdot (\sigma_{\Delta\phi}^2 - \sigma_{\Delta\lambda}^2) - \frac{(\Delta\phi^2 - \Delta\lambda^2)}{D} \cdot \sigma_{\Delta\phi\Delta\lambda}\right) \tag{16}$$

SURVEY RESULTS

Let us illustrate the above considerations with examples of real measurements. Seven series of synchronous measurements were analyzed. The data were acquired by two GPS/DGPS receivers. Statistical characteristics of measurements are presented in Table 1. The first three series were recorded in the Gulf of Szczecin by a pair of receivers. One was working in the DGPS mode, the other in the GPS mode. Gathered measurement data were used for calculations of relative accuracy of GPS-DGPS in ship-to-ship and ship-VTS relations. The fourth series of measurements made in the Bay of Gdansk presents a comparison of DGPS statistical measurements (short base receivers). All the other series (5-7) were executed by two DGPS receivers (also in the Bay of Gdansk). One receiver was stationary (at a VTS station), while the other was installed on board a ship.

Table. 1. Parameters of relative locations of two positions (DGPS/GPS and DGPS/DGPS).

No.	$\sigma_{\varphi 1}$ [m]	$\sigma_{\lambda 1}$ [m]	$\sigma_{\varphi 1\lambda 1}$ [m^2]	$\sigma_{\varphi 2}$ [m]	$\sigma_{\lambda 2}$ [m]	$\sigma_{\varphi 2\lambda 2}$ [m^2]	$\sigma_{\varphi 1\varphi 2}$ [m^2]	$\sigma_{\varphi 1\lambda 2}$ [m^2]	$\sigma_{\varphi 2\lambda 1}$ [m^2]	$\sigma_{\lambda 1\lambda 2}$ [m^2]	notes
1.	0.42	0.2	-0.0356	31.19	17.76	-172.8275	-1.6899	-0.2238	-0.3144	-1.0145	DGPS/GPS (static)
2.	1.61	0.56	-0.0613	91.39	25.46	-476.9918	34.5774	0.1640	0.9724	-0.5275	DGPS/GPS (static)
3.	1.41	0.61	-0.2056	64.92	21.11	-379.6178	20.5043	5.6554	35.9741	-4.5842	DGPS/GPS (static)
4.	0.713	0.534	0.004	1.183	1.240	0.122	-0.0855	0.19527	0.08545	-0.2068	DGPS/DGPS (static)

5.	1.098	0.597	-0.697	0.030	0.010	0.004	-0.1210	-0.0704	0.11975	0.03648	DGPS/DGPS (static)
6.	0.148	0.061	-0.076	0.016	0.005	-0.004	0.03258	-0.00384	-0.0237	0.00435	DGPS/DGPS (static)
7.	3.587	20.939	-7.274	0.086	0.068	-0.074	0.47171	-0.4238	-1.22183	1.12255	DGPS/DGPS (dynamic)

The variability of chosen positions is presented in the charts below. Figure 2 presents the fourth series of measurements – static measurement at a VTS station. Figure 3 presents the fifth series of static measurements at a VTS station and on board a ship.

The seventh series contains dynamic (ship) and static (VTS) measurements. However, to make their comparison possible, we calculated the linear regression of the ship's trajectory (on constant course; for variable courses the non-linear regression should be applied). Figure 4 presents the divergence of ship's position from the regression line and the positions determined by a VTS station.

Table 2 presents mean errors of estimated coordinates differences, their covariance, mean circle error of coordinates differences vector and errors of the estimation of distance and bearing between the receivers.

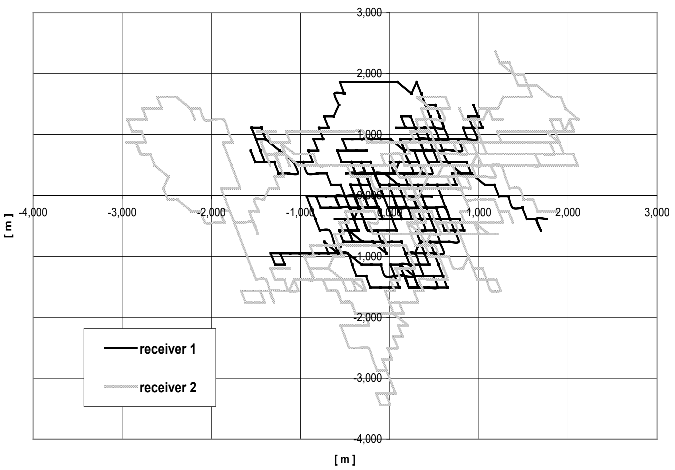

Fig. 2. Comparison of positions variability: static measurement at VTS (series 4)

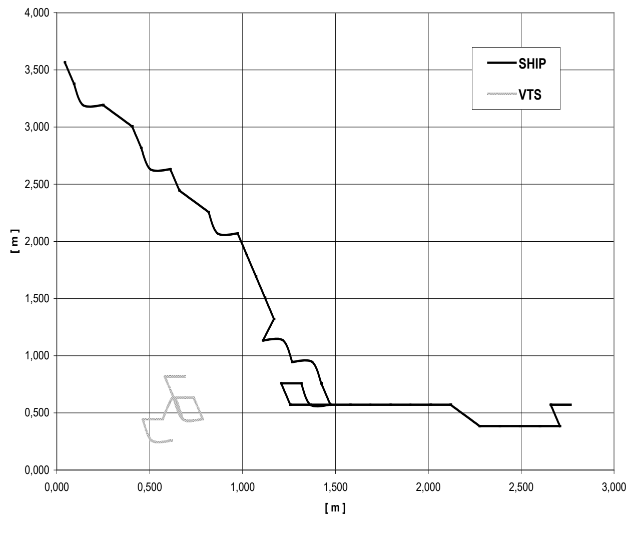

Fig. 3. Comparison of positions variability:
static measurements at VTS and on board ship (series 5)

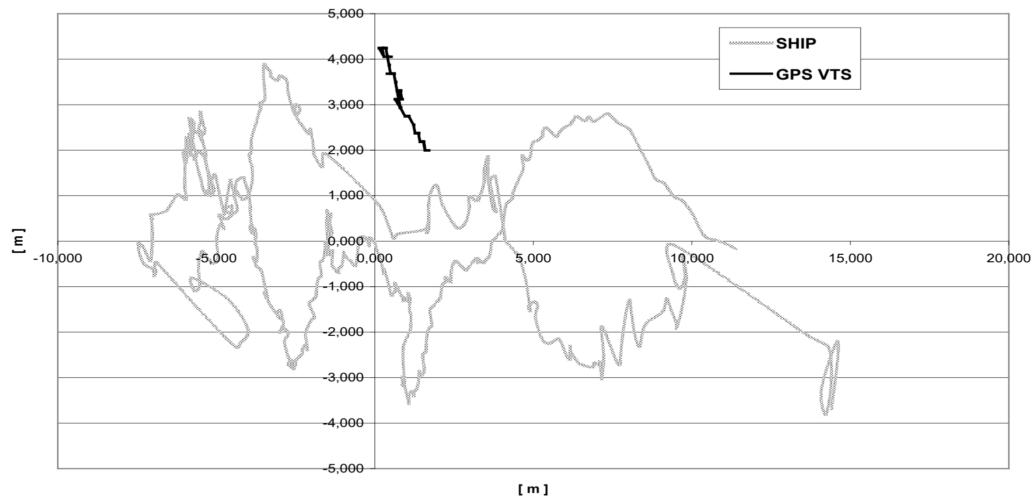

Fig. 4. Comparison of positions variability: dynamic measurements by VTS and ship (series 7)

Table. 2. Relative errors.

No.	σ_φ [m]	σ_λ [m]	$\sigma_{\varphi\lambda}$ [m^2]	M [m]	M_1 [m]	M_2 [m]	$M_\Sigma = \sqrt{M_1^2 + M_2^2}$ [m]	D [m]	σ_D [m]	σ_α [^{0}]
1.	31.247	17.818	-172.325	35.970	0.465	35.892	35.895	1 414.2	31.89	0.95
2.	91.025	25.487	-478.19	94.526	1.705	94.885	94.885	1 414.2	63.16	2.85
3.	64.619	21.335	-421.453	68.050	1.536	68.266	68.283	1 414.2	45.52	2.12
4.	2.066	2.188	-0.155	2.063	0.891	1.714	1.932	3.7	2.151	32.9
5.	1.370	0.534	-0.501	1.380	1.302	0.199	1.317	2 082.4	1.366	0.015
6.	0.099	0.058	-0.109	0.396	0.458	0.145	0.480	2 072.4	0.087	0.002
7.	2.730	18.762	-7.396	4.636	4.952	0.393	4.968	14 148.8	18.760	0.011

SUMMARY

The covariance matrix defined by the relation (15) fulfills the definitions of local accuracy [1, 5], and the mean ellipse of errors calculated from that matrix fulfills the conditions of relative ellipse of errors [5]. When measurements will be taken simultaneously ("at the same time") using the same radio-navigational system, then the definition of relative accuracy of radio-navigational system will be fulfilled as well [3, 4].

In the AIS system we have a situation in which two positions (those of own and another ship) are estimated independently, but they can be correlated by a common GPS or another system. It is a classical example for an application of relative accuracy. This can also be applied in reevaluating hydrographical surveys, provided that we possess registered raw measurements of position coordinates or other measured positional parameters. Besides, relative accuracy can be used in measurements taken by GPS/DGPS in situations such as maneuvering of two ships, hydrographic survey, calculating the corrections for relative position vector (VTS).

The concept of relative accuracy (as differential accuracy) can, be applied, in fact, to any measurements. Having a significant positive correlation, differential measurements should be used, as is the case in hyperbolic systems, whereas in the case of significant negative correlation summing systems are more accurate (equations 5). Finally, elliptical systems are most suitable for positioning systems.

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