



## Analysis of the use of a hexagonal tubular section in a beam using the finite element method

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### ABSTRACT:

A static and strength analysis of a simply supported steel beam with a special cross-section has been described in this work. The system has been subjected to a static load. On this basis the basic strength parameters of a beam with a hexagonal tubular section compared to a beam with a circular tubular section have been determined. The analysis consisted of assessing the possibility of using the proposed beam with a particular emphasis on the limitation of deflection compared to a beam with a circular tubular cross-section. The analytical calculations of the deflection and results from a program used for numerical calculations by the finite element method have been included. The values of reduced stresses and strains were compared according to the Huber von Mises hypothesis. Based on the results obtained, it was found that the use of a hexagonal cross-section is more effective than the comparison beam due to its smaller deflection. The conclusions also indicate the direction of further research.

### KEYWORDS:

deflection; hexagonal cross-section; simply supported beam; closed tubular profile; FEM

## 1. Introduction

One of the most frequently used construction materials is steel. Due to its high tensile strength, it is used both for solid-wall sections in structural systems and as a component of composite elements, e.g. in reinforced concrete. It is also used for structural reinforcement. Showing high tensile strength, but also the possibility of combining with other building materials, it is one of the most universal and irreplaceable of construction materials. The latest material solutions include composite materials made of synthetic fibers based on epoxy resins, glass and basalt fibers. The use of such substitutes brings many advantages, the most important of which are the reduced weight of the structure and often higher tensile strength than steel. Therefore, it is justified to use increasingly popular substitutes in composite materials.

However, there is no clear indication yet of the validity of replacing steel profiles with materials other than other metal alloys. Steel is a material that can be reprocessed and reused. In the era of searching for solutions leading to a reduction of carbon footprints and the use of material for recycling, as well as the implementation of Sustainable Development Goals, a rational solution is the use of steel in buildings.

The selection of the shape is a key element in the design of the structure. The cross-sections proposed in manufacturers' catalogues are most often used due to their availability and price. In this study, the possibility of using special steel sections in the working conditions of horizontal structural elements was assessed. Ensuring the proper functioning of the structure with a cross-section with the lowest possible deflection is the goal of static and strength analyzes. The cost of

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making such a cross-section was not taken into account in this work. Replacing solid-wall elements with tubular profiles is a common solution in places where tubular profiles can be built-in. It is not possible to clearly indicate a universal profile that will meet the operational requirements of the structure and ensure operational safety, taking into account the load-bearing capacity and use of each construction solution.

The selection of the appropriate section results from many factors. The analysis of the optimization of a steel cantilever beam using numerical methods has been described in [1]. In works [2, 3] the problem of optimizing the mass of a cantilever beam in order to obtain an I-section subjected to static forces has been presented. The problems of cantilever beams have also been described in [4], in which the beam was modeled and a numerical and graphical analysis of the deflection angle function and the beam deflection arrow function was performed using computer algebra programs. In [5], topological optimization was used in the design of the shape of statically loaded cantilever beams. The described method has been an introduction to constructing the shape and dimensions of beams. In [6] the distribution of stresses under the influence of static or fluctuating torsional moment loading has been analyzed. The tests have been carried out on the basis of a closed section in a cantilevered steel beam. Tests on beams with non-linear longitudinal sections have been described in [7]. On their basis, numerical tests have been carried out for various types of loads in order to analyze buckling. Beams with a non-linear longitudinal cross-section have been subjected to the analysis and optimization described in [8]. The volume of the cantilever beam was optimized under constraint of stress limited to the yield stress. In [9], on the example of a rectangular tubular cantilever beam, an analysis of shape optimization to minimize mass has been carried out using the finite element method. In [10] a universal design approach through an efficient finite element (FE)-based method of analyzing the cross-section of a longitudinally uniform beam, assuming any cross-section and material model, subjected to tension/compression, bending, torsion and shear has been presented. In [11] an optimization algorithm has been proposed for estimating the initial terms that depend on the boundary conditions. Design performance depends on a combination of cross-sectional shape, material and process. In [12], simple expressions for the Pareto optimal set of a beam with any cross-section shape under bending have been derived. This expression can be used at a very early design stage to select the possible cross-section shape and beam material from among the optimal solutions. The analysis of beam damage carried out in [13] based on numerical tests showed that the level of damage has a significant impact on the stiffness and final load-bearing capacity. A solution has been proposed to strengthen the beam with CFRP tapes. It has been proven that a reinforced steel beam with a damaged level can exceed a steel beam without damage by about 10% in ultimate load capacity. In the following work [14], the authors discussed the optimal shaping of a non-prismatic flat beam. The proposed model was based on the standard Timoshenko kinematics hypothesis (i.e. a plane cross-section remains flat due to deformation, but can rotate relative to the beam axis). The analytical solution for this type of beam is therefore used to obtain the beam deformations and stresses under various constraints when the load is assumed to be the sum of the overall external vertical variable and the self-weight. In the article [15] a stress analysis of elements made of cold-formed sigma steel, loaded uniformly in a plane parallel to the web and not passing through the shear center has been presented. Such load application occurs very often in engineering practice and corresponds to the load applied to the upper flange of the cross-section. This usually results in additional torque. Research presented in [16] assumed the selection of the cross-section taking into account the shift of the lower flange of the I-section. The authors presented the experimental and theoretical analysis of a cross-section that may be asymmetric, by observing the differences in the behavior of a monosymmetric and asymmetric cross-section. In [17] the research that involved analyzing a hybrid steel beam using ANSYS for various stiffening conditions has been described. The main benefit was to achieve savings and increase the load capacity of the beam through the use of stiffeners. The behavior of such beams was only considered under vertical static loads. In the article [18], the behavior of curved beams with a hexagonal cross-section has been analyzed to determine the resistance and response. The optimal design of the hexagonal cross-section of such curved beams has been obtained by observing the typical

crash responses of a series of specimens of thin-walled curved hexagonal beams. In [19] the compressive and flexural behavior of hexagonal concrete-filled steel tubular elements (CFST) have been described. The samples have been subjected to axial compression and bending. The type of damage, load-deformation relationships and deformation development have been analyzed. Parametric studies on the compressive and flexural behavior have been carried out to investigate the influence of various material properties such as steel ratio, steel yield strength and concrete strength. Open cross-sections are generally used in engineering practice. Even small geometric imperfections cause significant changes in the distribution of internal forces, which in turn reveals bending and torsional effects, which are very important in thin-walled profiles [20]. Closed profiles, in turn, are characterized by high resistance to torsion, many times greater than open profiles. Due to the high yield strength of steel, it is possible to shape structures with relatively small wall thicknesses and cross-sectional dimensions.

In this study, an attempt to assess the possibility of using a hexagonal tubular section, compared to a circular tubular section, due to the advantages of tubular sections in beam design has been proposed. According to design [21], circular tubular sections are not susceptible to buckling. In this work, a special hexagonal profile has been assessed based on static and strength analysis to determine its suitability for use in construction. A steel pipe with a special shape can better adapt to the specific working conditions and save material.

## 2. Model of the analysis

The static scheme adapted to the analysis of the internal loads of the system have been presented in Figure 1.

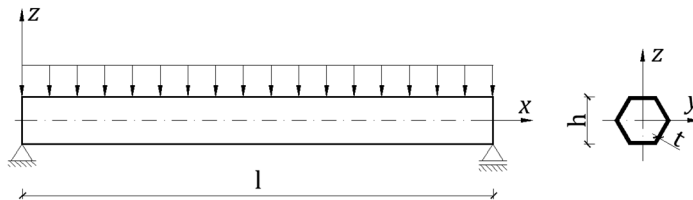


Fig. 1. Model of simply-supported beam with its cross-section

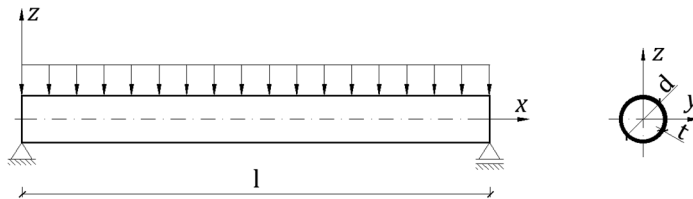


Fig. 2. Model of reference simple-supported beam with its cross-section

The static models of the systems have been described in a Cartesian  $xyz$  coordinate system. Both systems have been subjected to the same supports and loadings. The beams have been subjected to a continuous load  $q$ . The cross-section of the hexagonal beam (Fig. 1) has been defined by the dimensions of height  $h$  and thickness of the wall  $t$ . Since, the cross-section has been designed as a regular hexagon, the following dimensions were known.

The static scheme of the reference beam with circular tube is presented in Figure 2. The cross-section of the circular beam (Fig. 2) has been defined by the dimensions of the external diameter  $d$  (equal to height  $h$  in relative cross-section) and thickness of the wall  $t$  (the same as in the relative beam). The beam length has been denoted as  $l$ , likewise in both cases. Material properties for construction steel S355 in both analysed cases were as follows: Young's modulus  $E$  equals 210 GPa, Poisson's ratio  $\nu$  equals 0.3, the yield strength of steel  $f_y$  equals 355 MPa, and the tensile strength of steel  $f_u$  equals 490 MPa.

The properties of the static structure analysis have been determined analytically and numerically. In this paper, *SMath* software has been used to support calculation. The finite element method has been used in order to established values of reduced stresses and strains on the basis of the Huber von Mises hypothesis. The numerical model has been implemented in the *ANSYS Workbench* environment, where the static structural analysis has been performed. When the geometry of the system is made and the material properties are implemented, the boundary conditions and loads can be determined. The static calculations in the *Mechanical* module can then also be made. Due to the complex state of stresses, the value of reduced stresses was determined according to the Huber von Mises hypothesis. The most essential factor in understanding the behaviour of how the system will work is the deflection. The deflection in this work has been also determined analytically, according to the formula (eq. (1))

$$f = \frac{5}{384} \frac{ql^4}{EI} \tag{1}$$

where:  $q$  – continuous load,  $E$  – Young’s modulus,  $I$  – moment of inertia of the cross-section.

Based on the static analysis of loads, the results in the form of reduced stress and strain maximum values, gathered in the table, as well as with the maximum deflection of both cases have been included in the next chapter.

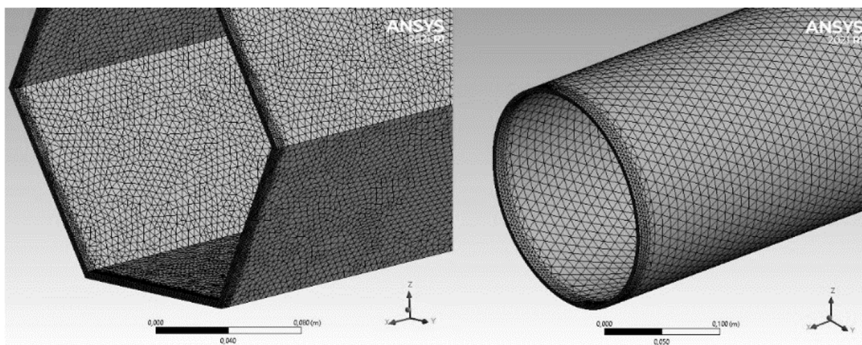
### 3. Results of the static analysis

In order to assess the effectiveness of the hexagonal cross-section the geometry was adopted based on the values in Table 1.

**Table 1**  
Initial parameters of the analysed beam made of steel S355

Parameter	Value	Unit
Cross-section height (Fig. 1) $h$	0.2	m
Cross-section diameter (Fig 2.) $d$	0.2	m
Thickness of the wall (both cases) $t$	0.004	m
Beam length (both cases) $l$	2.0	m
Continuous load (both cases) $q$	5	kN/m

In the finite element method, the importance is in adopting the element in mesh, and in refinement of areas, where the static influence may occur. In this work, the finite elements have been defined as solids, with size adjusted in supports area. Both of mesh divisions have been presented in Figure 3.



**Fig. 3.** Finite element mesh in hexagonal tube (left) and circular tube (right)

The hexagonal tube has been divided into smaller elements, but the circular tube has been divided in such a manner that there was no difference in results (both quality and quantity) between the presented models. After conducting a static strength analysis using the finite element method in the ANSYS program, the obtained values have been compared with the values of parameters determined by the analytical method. The values of the analyses have been presented in Table 2. As a result, the deflection of the simple-supported beam in the case of the hexagonal cross-section, as well as in the case of the cylindrical cross-section occurred in the middle section of both beams, according to the theory of strength of materials.

**Table 2**  
Results of static analyses

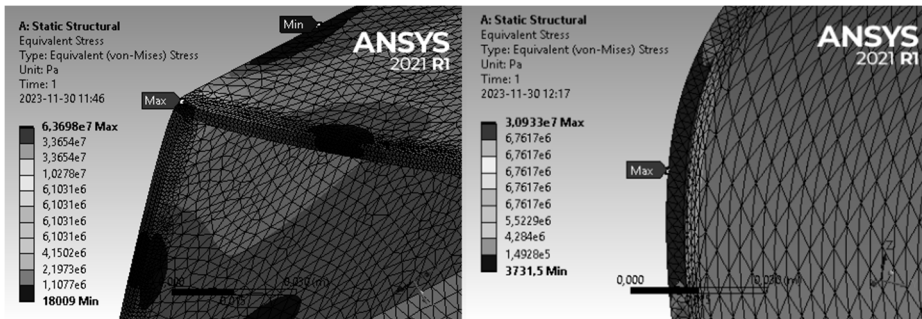
	Analytical	ANSYS fem	Relative error
Deflection hexagonal cross-section [m]	8.7855E-05	8.755E-05	0.26%
Deflection circular reference cross-section [m]	0.00010269	0.00010163	1.04%

The compliance of deflections with an accuracy of 0.26 and 1.04% has been demonstrated, because of the perfection of the calculations. A lower deflection value was obtained for the hexagonal cross-section due to the larger moment of inertia, obtaining a result that was 14.5% better than in the case of the reference beam. The reduced stress and strain results, according to the Huber von Mises hypothesis, have been presented in Table 3.

**Table 3**  
Results of the FEM analysis

	Equivalent stress [Pa]	Equivalent strain [m/m]
Hexagonal cross-section [m]	3.0933E+07	1.505E-04
Circular reference cross-section [m]	6.3699E+07	3.066E-04

The smaller values of displacement obtained for both cases resulted in increased values of stresses and maximum strains that occur in the support zones (Fig. 4). It is far from the limit values, including the yield strength of S355 steel and tensile strength.



**Fig. 4.** Support zones where the max. values of reduced stresses occur in hexagonal (left) and circular (right) tubes

The risk to the structure is the possibility of crushing when a concentrated load is applied, or when there is deflection above the permissible values specified in the standard ( $\sigma_{dop} < 408$  MPa, with safety coefficient  $\gamma_M = 1.2$ ).

## 4. Conclusions

The main goal of this work was to assess the use of a hexagonal cross-section in simple supported beam. The obtained results have been compared with a circular cross-section that is defined as one that will not buckle. The analysis showed that the value of the deflection arrow in the case of a hexagonal cross-section is smaller than in the case of the reference cross-section, which in turn increased the values of reduced stresses in the support zones. Based on the analysis performed, it can be concluded that the results are satisfactory due to the large reserve of unused plasticity. In order to clearly recommend the hexagonal cross-section as more effective, a larger group of cross-sections should be tested for different cross-section heights and the influence of dynamic loads should be taken into account. Tests of cross-sections taking into account dynamic loads will be the subject of subsequent works.

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## **Analiza zastosowania przekroju heksagonalnego rurowego w belce z wykorzystaniem metody elementów skończonych**

### **STRESZCZENIE:**

Przeprowadzono analizę statyczno-wytrzymałościową stalowej belki swobodnie podpartej o specjalnym przekroju. Poprzez dobór obciążenia statycznego możliwe było zbadanie podstawowych parametrów wytrzymałościowych belki o przekroju rurowym heksagonalnym w porównaniu do belki o przekroju rurowym okrągłym. Analiza polegała na ocenie możliwości stosowania proponowanej belki ze szczególnym wskazaniem ograniczenia ugięcia w porównaniu do belki o przekroju okrągłym rurowym. W pracy zawarto obliczenia analityczne strzałki ugięcia oraz wyniki z programu do obliczeń numerycznych z wykorzystaniem metody elementów skończonych. Porównano wartości zredukowanych naprężeń i odkształceń wg hipotezy Hubera von Misesa. Na podstawie otrzymanych wyników stwierdzono, że wykorzystanie przekroju heksagonalnego jest bardziej efektywne, ze względu na mniejsze ugięcie, niż w przypadku belki porównawczej. W podsumowaniu wskazano kierunek dalszych badań.

### **SŁOWA KLUCZOWE:**

ugięcie; przekrój heksagonalny; belka swobodnie podparta; profil rurowy zamknięty; MES