

## Optimized Shapley value cost allocation model for carriers' collaboration in road haulage transportation

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**Abstract:** Transportation carriers can achieve significant profit or cost savings if they collaborate rather than engage in wasteful competition among themselves. However, the challenge in cooperative game theory is finding the optimal cost allocation methods to support pecuniary expectations of coalition members. In this paper, we determine cost allocation model that supports horizontal collaboration among transportation carriers involved in downstream distribution of packaged cement from shipper's processing plant to customer locations in selected states in Nigeria. The study focuses on the relationship between the shipper and haulage carriers that service the transport needs of its geographically distributed customers. A cost allocation mechanism based on game theory is proposed to implement win-win collaboration among the carriers. We applied a Shapley value cost allocation model to fairly distribute the cost savings from operation of five grand coalitions (S) formed by the carriers. The Shapely values were then optimized with mixed integer programming model to realize optimal cost savings from the coalition. The result revealed that the coalitions: S3 (N165,173,700.00) and S4 (N27,200,960.00) contributed significantly to the optimal savings apart from their initial contributions. The path that corresponds to S3 (X3) is the

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coalition providing service from Calabar to Jos while the path that corresponds to S4 (X4) is the coalition providing service from Calabar to Owerri and the optimal savings is N48,286,760,000.00. Based on these results, we therefore encourage horizontal collaboration among haulage transport providers in their overall interest, that of the shipper and hence ensure supply or distribution chain cost efficiency.

**Keywords:** Shapley value, mixed-integer programming, haulage transportation carriers, transport distribution chain, cost allocation

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## 1. Introduction

Collaboration and competition among haulage transportation industries are encouraged to achieve cost reduction/savings for the collaborators and improve overall efficiency in the supply chain distribution of cargoes. When resources are pooled and deployed, average costs drop due to total costs being shared across collaborating units. Thus, instead of transportation carriers competing with one another, they are encouraged to collaborate to achieve cost savings and enhance service delivery in their overall interest and the supply chain network. This collaboration, devoid of competition among rational decision-makers (transport providers in this case), is called a cooperative game. This is in contrast with a zero-sum game, otherwise known as a competitive game, where each player tries to take advantage of the other to win the market. We call it a zero-sum game because a loss to one is a gain to the other. In a collaborative game theory, each player contributes to the overall profit made by the collaborators, the grand coalition. The sharing of the profit in a cooperative game (Wang, 2023) can be achieved using the Shapley value method. The Shapley value method is efficient and distributes the average profit from the outcome of the collaboration, and the cooperative game provides a mathematical model for this kind of problem (Malawski et al., 2006). However, it has been observed that though the Shapley value method ranks higher than the proportional allocation and decomposition method (Kayikci, 2020), it still has its shortcomings. One of the shortcomings is that it tends to favour those who contributed less than those who contributed more in the collaboration (Zaremba, L., Zaremba, C.S. & Suchenek, 2017).

In this paper, we seek to apply the optimization method to maximize the savings for the collaborators and to determine the best combination of distribution routes that will be beneficial to the collaborators. Shapley value method offers the most transparent platform to encourage the collaborators to remain in the coalition, even though it needs to be optimized with other models. In this paper, the Shapley value-based cost allocation method in a transportation supply chain distribution network is a stepping stone to our proposed optimization method. Therefore, we are proposing an optimization (using mixed integer programming) of savings from Shapley value-based cost allocation in horizontal transportation carrier collaboration. The method will reflect the actual contribution of each collaborator to the grand coalition, thereby helping the decision-makers (haulage transport providers) optimize their savings (profit). Another innovation we are bringing here is that the costs or profit made by the collaboration will be optimized using a mixed integer programming model, and the optimal profit (savings) obtained for the collaborators. A mixed integer linear programming problem allows some of its feasible solutions to be integers while others assume fractional values (Dantzig, 2002). This model is appropriate for transportation/supply chain distribution since the distributed cargoes do not necessarily need to be integers.

We organize this paper in the following order: Section One is devoted to a general introduction, Section Two contains the review of some related literature, Section Three treats material and methods, Section Four handles data presentation and analysis and Section Five treats discussion and conclusion.

## 2. Literature review

Various cost/profit allocation models for transportation carriers' collaboration in game theoretic settings exist in literature. The pros and cons of deploying these models have been discussed extensively in the literature. The choice or decision to use one or a combination of these models is a function of the problem statement and context. In this section, we present some reviews of works done in the area of transportation carrier collaboration and then focus our reviews on the desirable features of the Shapely value method optimized with the Linear programming method.

Akyol and Sarisakal (2023) proposed a cost allocation model which supports the joint use of subcontractors by textile companies in a cooperative collaboration. In practice, textile producers use subcontractors to keep their product specifications confidential. In this scenario, the joint use of vehicles obtainable in a typical cooperative game theory framework would not apply. Therefore, to accommodate this peculiar feature of the textile producers, a cost allocation framework that enables coalition members to choose subcontractors closest to their facility (which they can trust with information) is proposed. Three generic cost allocation models (Shapley value, nucleolus, and weighted relative savings method) were applied to compare cost savings obtainable when companies operate individually and in a coalition. Although all the cost models proved the advantage of operating in a coalition, the weighted relative savings method provided more computationally stable results. The contribution of the research work to literature lies in the novel application of cooperative game theory models to textile companies with peculiar operational features. The work by Tatarczak (2018) sought to overcome the challenges associated with finding a cost allocation model that favours individual member's preferences in a fourth-party logistics supply chain coalition. As noted by Vanovermeire, Vercruysse, and Sörensen (2014), each coalition has its own set of preferences and comprises partners with different characteristics; this characteristic marks the application of the existing cost allocation method. Thus, a two-step solution was proposed to achieve fair allocation of common costs among the member enterprises. The author first applied generic cost allocation methods, such as the Weighted Relative Savings Model, Proportional allocation, Shapley value, and Nucleolus, which awarded positive benefits to all coalition members. In the second phase, the coalition satisfaction degree (CSD) model was then introduced to measure the allocation plan's fairness at the coalition members' level. Although the author's model was added to the literature by proposing a mechanism that could improve existing models in supply chain partners' collaboration, the major drawback of CSD is that its application and subsequent results were not based on real-world data. The work by Vanovermeire, et al., (2014) evaluated different cost allocation methods obtainable in practice and in academic literature, namely: Shapley, Nucleolus, Equal Charge Method (ECM), Alternative Cost Avoided Method (ACAM) and Cost Gap Method (CGM). Others include the Stand Alone (SA) model, Weighted Relative Savings Model (WRSM) and Equal Profit method (EPM).

Under a set of simulated conditions, the suitability of these models was tested to determine their stability in different collaborative transport settings. The authors found that their usefulness and limitations depend on certain characteristics of the coalition members: Number of partners, Total number of pallets, Flexibility and Strong sub-coalitions. The researchers noted that when companies have asymmetric properties, the cost allocations from the different methods diverge. The usefulness of their work is the assessment of cost allocation methods in terms of their suitability for every given situation. However, Liu, Wu and Xu (2010) investigated cost savings in game theoretic collaboration of Less Than Truck Load (LTL) transportation carriers. In two transportation scenarios involving back hauling and lane request/exchange, cost allocation to the coalition was computed using three existing cost-sharing methods: Proportional allocation, shapely value, and nucleolus. It was found that the methods did not generate satisfactory results in all the test cases as a coalition member who contributed more did not gain commensurate profit as expected than the other two members; hence, as a remedy, the authors proposed the Weighted Relative Savings model. Thus, implementation of the model proved that collaboration among LTL carriers is beneficial as it would result in cost savings in the range of 7.3% to 18.7%. In the proceeding section, we examine the diverse applications of the Shapely value method optimized with Linear programming models but with limited applications in cooperative collaboration among transportation carriers.

The Shapley value method is derived from the axioms of efficiency, symmetry, dummy player axiom, and additivity, thus providing a particular distribution that is computationally efficient in

transportation carriers' collaboration. However, this method does not guarantee long-term partner cooperation (Liu et al., 2010; Dai & Chen, 2012; Frisk, Gothe-Lundgren, Jornsten & Ronnqvist, 2010). Several empirical papers investigate cost/profit allocation using Shapely values in a cooperative game setting. Notable examples include Aziz, Cahan, Gretton, Kilby, Mattei, & Walsh (2014), who proposed some procedures to calculate the cost of serving each location in a single-vehicle transport setting. They opined that Shapley value is one of the most essential normative division schemes in cooperative games, giving all involved principled and fair allocation. This finding is consistent with Masimli (2023) and Rifki, Danloup, Guo & Allaoui (2023).

However, Ivanov, Pavlov and Sokolov (2014) developed a multi-period and multi-commodity distribution (re)planning problem for a multi-stage centralized upstream network with structure dynamics. They reduced the problem to a Linear programming problem (LP) as their starting point and finally transformed the problem into a multi-objective optimization problem. Since our interest is in optimizing the savings derived from applying the Shapley value method, we shall briefly review some literature on mixed integer Linear programming. Rappos and Thompson (2008) used integer programming to retrieve information and allocate houses in London. They used an integer programming approach to tackle the problem faced by the Department for works and Pensions analysts working on housing benefit data. However, using this data set is not straightforward, and the time needed to access and retrieve the data from the HMBS server can be significant, as its database consists of nearly one billion rows.

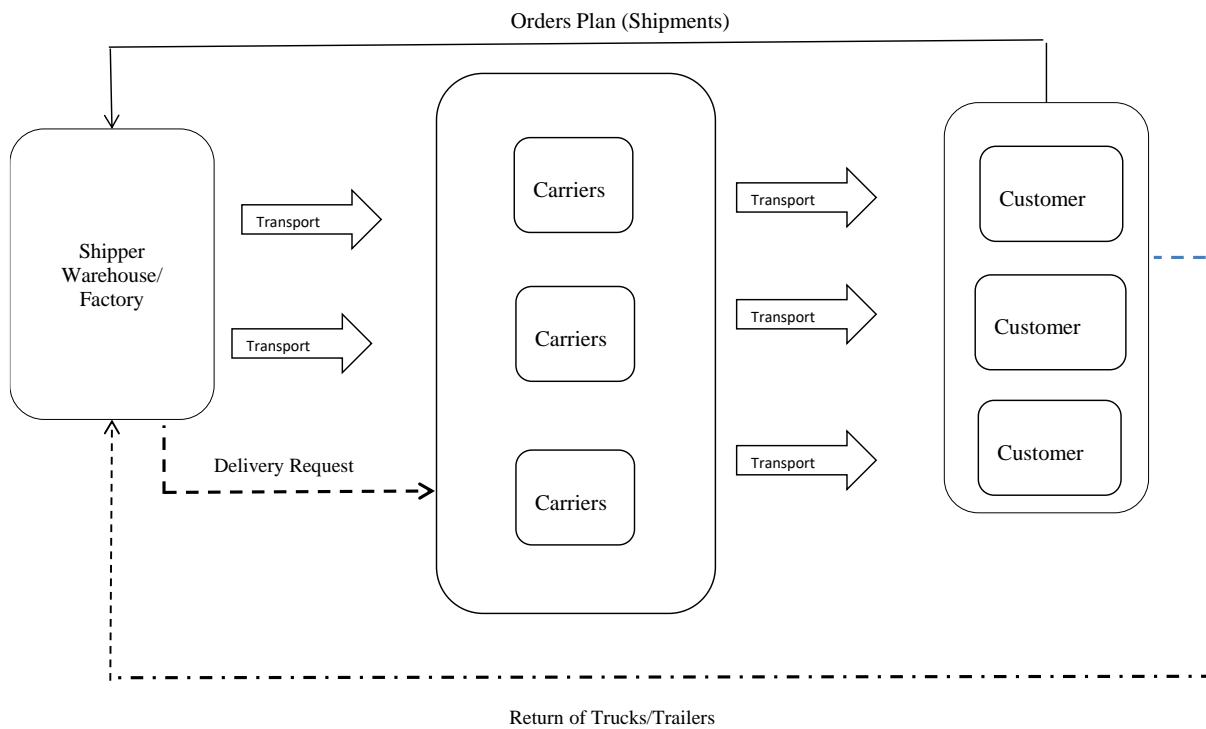
For this reason, they developed an integer programming model that significantly improved the efficiency of retrieving HB data from the data server. However, Darby & Wilson (2002) observed the developments in integer programming and, using Branch and Bound (B & B) adopted a tree search in which the tree development process is characterized by two operations that perform branching and bounding of the solution space  $S$ . Also, Liittschwager & Wang (1978) used integer programming to model a classification problem. A classification problem is presented in which it is desired to assign a new individual or observation with  $K$  characteristics to one or two distinct populations based on historical sets of samples from the two populations. However, Nazario (1995) used integer programming to minimize labour costs. He observed that the conventional methods for solving integer linear programming problems include the branch-and-bound, cutting-plane, interior point, Lagrangian relaxation and neural network methods. They observed that the branch-and-bound algorithm is the most widely used approach to solving pure and mixed integer programming problems.

Mixed integer programming occurs where all essential variables (including slack or surplus) can take non-negative real (continuous) or fractional values. The reason is that in many real-life scenarios, it is quite possible and appropriate to have fractional solutions. For example, it is possible to transport or use 5.7 kg of raw material, 4.28 person-hours and 12.18-meter length of a sheet in a project, etc. This is a case where mixed integer programming comes in. However, there are many problems, especially in business and industry, in which only integer values for the variables in optimal solutions make sense. However, mixed integer programming is not far from the normal linear programming. The distinguishing characteristic of mixed integer programming is that at the optimal solution, the non-basic variables that entered the basis can assume either integer or fractional values in the same feasible solution space while satisfying the optimal objective function, unlike pure integer programming that must not allow any fractional feasible solutions. The savings (gains or profit) we seek to optimize in this paper assume different values, including integers and fractional values and hence, mixed integer programming is appropriate for modelling such a problem. Kayikci (2020) opined that Linear programming can be viewed as part of a significant revolutionary development which has given mankind the ability to state general goals and to lay out a path of detailed decisions to take in order to achieve its goals when faced with practical situations of great complexity. The Simplex method is one of the easiest methods of finding solutions to Linear programming. However, Rao (2009) observed that the Simplex method requires computing and recording an entirely new tableau at each iteration, although much of the information contained in the tableau is not used. This is one of the disadvantages of linear programming via the simplex method.

Also, David & Yinyu (2008) thought that the extensive experience with the Simplex procedure applied to problems from various fields indicated that the method can be expected to converge to an optimum solution in about  $m$  pivot operations, but Prem & Hira (2008) noted that the Simplex method is another efficient method developed by G.B Dantzig for solving LP problems; it is efficient in the

sense that it is easy to compute values of all the variables. In a related development, Asa & Syerre (1990) used Linear programming to model a classification problem. A classification problem is presented in which it is desired to assign a new individual or observation with K characteristics to one or two distinct populations based on historical sets of samples from the two populations. The resulting classification problem is formulated as a linear programming problem, but Ugwuayi (2007) opined that decision makers are interested in finding a solution that optimizes the stated aims and objectives. They go into the solution space (feasible region), containing all possible alternatives and employ a technique that enables them to select the best optimal solution.

**Figure 1: Model problem of the study**



Source: Adapted from Wang (2023)

### 3. Materials and methods

In this paper, we model the freight transport distribution network involving a bulk cement shipper that patronizes a group of trailer and mini-truck operators (or haulage carriers) that transport bagged cement from the shipper’s warehouse/packaging plant to customers’ warehouses in specific states in Nigeria. The shipper operates a cement packaging plant located at seaport terminals and then distributes bagged cement according to customers’ requests using the road haulage transportation carriers. The conceptual model of the cement distribution chain described here is shown in Figure 1. For supply chain efficiency and cost savings, these carriers opted to form a grand coalition of five. The collaborators were classified into two groups, one providing a trailer and the other providing mini trucks with loading capacities of 4,300 tons and 453 tons, respectively. The bagged cement distribution network is within Nigeria, see Appendix 1. The expected cost savings (profit) were to be shared among the collaborators according to their contribution to the grand coalition. According to Malawski et al. (2006), Cooperation Games Theory provides a mathematical model for solving this decision problem. However, we go a little beyond what we know about cooperative games because we seek to optimize the savings (profit) for the members' interest.

Let a pair  $(N, v)$ , be a finite set of players, and  $v: 2^N \mapsto \mathbb{R}$ , be the function assigning a real-valued payoff  $v(S)$  to each coalition  $S \subseteq N$  with  $v(\emptyset) = 0$ . Let  $|S|$  be the number of members in coalition  $S$  and  $N \setminus \{k\}$  be the set  $N$  except element  $k$ , where  $N$  is the grand coalition. Let  $x_i$  denote the share of the grand coalition’s payoff that a player  $i \in N$  receives. Then, the Shapley value of player  $k$  is defined as:



$$\varphi_k(N, v) = C_i = -\frac{1}{N!} \sum_{S \subseteq N \setminus \{k\}} (|S| - 1)! (|N| - |S|)! [v(S \cup \{k\}) - v(S)] \tag{1}$$

See (Liitschwager & Wang, 1978). Then, we can write equation (1) as equation (2)

$$Z_k(N, v) - C_i = y_i \tag{2}$$

where  $Z_k(N, v)$  is the initial total cost of transportation,  $C_i$  is the new cost from the Shapley value allocation method and  $y_i$  is the savings for the  $i_{th}$  partner of the coalition. However, our interest is to maximize the savings (profit) from the collaboration.

Now,

Let  $y_i = Z$ ; hence, we have

$$\text{Max } Z = \sum_{j=1}^n C_j X_j \tag{3}$$

Subject to

$$\sum_{i=1}^m \sum_{j=1}^n a_{ij} X_j \leq b_i \tag{4}$$

$$X_j \geq 0; \quad b_i \geq 0 \tag{5}$$

where  $Z$  is the objective function which maximizes savings per leg of each coalition, and  $X_j$  is the decision variable. These decision variables represent the collaborators;  $a_{ij}$  is the individual collaborator's profit and  $b_i$  is the capacity of trailers / Trucks that must not be exceeded.

## 4. Data presentation and analysis

### 4.1. Data presentation

The data on routes and combinations of trailers and trucks obtained from different road haulage transport providers are presented in Appendix 1. Computation of savings on each leg of the coalitions is presented in Appendix 2, and from Appendix 2, we present the data in Table 1.

**Table 1: Coalitions, cost savings, total savings per leg and capacity of trailers/trucks**

	Leg1	Leg2	Leg3	Leg4	Total	Maximum Tones
S1	12.94	13.0	18.68	0	44.62	9,035
S2	12.87	12.94	18.41	0	44.22	9,035
S3	67.80	66.40	66.90	84	285.1	13,335
S4	12.87	12.90	18.18	0	43.96	9,035
S5	22.37	22.44	22.47	28.05	95.34	13,335
S6	39.4	39.34	54.70	0	133.4	9,035

### 4.2. Analysis

From Table 1, S1, . . . , S6 are the respective coalitions' routes. Each route has leg1 to leg3 or leg4. The number of legs represents the number of coalitions per route. Our interest is to maximize the total savings from each coalition subject to the maximum tonnes (volume) each coalition must not exceed.

Hence, from equation (3) and equation (4), we have:

$$\text{Maximize } Z = 44.62x_1 + 44.22x_2 + 285.1x_3 + 43.96x_4$$

Subject to

$$12.94x_1 + 13.0x_2 + 18.68x_3 + 0x_4 + 0x_5 + 0x_6 \leq 9,035$$

$$12.87x_1 + 12.94x_2 + 18.4x_3 + 0x_4 + 0x_5 + 0x_6 \leq 9,035$$

$$67.8x_1 + 66.4x_2 + 66.9x_3 + 84x_4 + 0x_5 + 0x_6 \leq 13,335$$

$$12.87x_1 + 12.9x_2 + 18.18x_3 + 0x_4 + 0x_5 + 0x_6 \leq 9,035$$

$$22.37x_1 + 22.44x_2 + 22.47x_3 + 28.05x_4 + 0x_5 + 0x_6 \leq 13,335$$

$$39.4x_1 + 39.34x_2 + 54.70x_3 + 0x_4 + 0x_5 + 0x_6 \leq 9,035$$

$$x_1, x_2, \dots, x_6 \geq 0$$

Where  $x_1, \dots, x_6$  are the decision variables corresponding to routes S1, . . . , S6 in the objective function. We put the above problem in a standard form to have:

$$\text{Maximize } Z - 44.62x_1 - 44.22x_2 - 285.1x_3 - 43.96x_4 - 95.34x_5 - 133.4x_6 = 0$$

Subject to

$$\begin{aligned}
 12.94x_1 + 13x_2 + 18.68x_3 + 0x_4 + 0x_5 + 0x_6 + T_1 &= 9,035 \\
 12.87x_1 + 12.94x_2 + 18.4x_3 + 0x_4 + 0x_5 + 0x_6 + T_2 &= 9,035 \\
 67.8x_1 + 66.4x_2 + 66.9x_3 + 84x_4 + 0x_5 + 0x_6 + T_3 &= 13,335 \\
 12.87x_1 + 12.9x_2 + 18.18x_3 + 0x_4 + 0x_5 + 0x_6 + T_4 &= 9,035 \\
 22.37x_1 + 22.44x_2 + 22.47x_3 + 28.05x_4 + 0x_5 + 0x_6 + T_5 &= 13,335 \\
 39.4x_1 + 39.34x_2 + 54.7x_3 + 0x_4 + 0x_5 + 0x_6 + T_6 &= 9,035
 \end{aligned}$$

where  $T_1, \dots, T_6$  are the slack variables.

We solve the problem as presented in Table 2 to Table 4;

**Table 2: First computational table for optimal profit**

BASIC	Z	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>	x <sub>5</sub>	x <sub>6</sub>	T <sub>1</sub>	T <sub>2</sub>	T <sub>3</sub>	T <sub>4</sub>	T <sub>5</sub>	T <sub>6</sub>	bj
Z	1	-44.62	-44.22	-285.1	-43.96	-95.34	133.4	0	0	0	0	0	0	0
T <sub>1</sub>	0	12.94	13	18.68	0	0	0	1	0	0	0	0	0	9035
T <sub>2</sub>	0	12.87	12.94	18.4	0	0	0	0	1	0	0	0	0	9035
T <sub>3</sub>	0	67.8	66.4	66.9	84	0	0	0	0	1	0	0	0	13335
T <sub>4</sub>	0	12.87	12.9	18.18	0	0	0	0	0	0	1	0	0	9035
T <sub>5</sub>	0	22.37	22.44	22.47	28.05	0	0	0	0	0	0	1	0	13335
T <sub>6</sub>	0	39.4	39.3	54.7	0	0	0	0	0	0	0	0	1	9035

**Table 3: Second computational table for optimal profit**

BASIC	Z	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>	x <sub>5</sub>	x <sub>6</sub>	T <sub>1</sub>	T <sub>2</sub>	T <sub>3</sub>	T <sub>4</sub>	T <sub>5</sub>	T <sub>6</sub>	bj
Z	1	160.7354	160.6142	0	-43.96	-95.34	133.4	0	0	0	0	0	5.212066	47091.01
T <sub>1</sub>	0	-0.51506	-0.42091	0	0	0	0	1	0	0	0	0	-0.3415	5949.556
T <sub>2</sub>	0	-0.38338	-0.27974	0	0	0	0	0	1	0	0	0	-0.33638	5995.804
T <sub>3</sub>	0	19.61243	18.33473	0	84	0	0	0	0	1	0	0	-1.22303	2284.881
T <sub>4</sub>	0	-0.22492	-0.16168	0	0	0	0	0	0	0	1	0	-0.33236	6032.143
T <sub>5</sub>	0	6.185027	6.296106	0	28.05	0	0	0	0	0	0	1	-0.41079	9623.548
X <sub>3</sub>	0	0.720293	0.718464	1	0	0	0	0	0	0	0	0	0.018282	165.1737

**Table 4: Final computational table for optimal profit**

BASIC	Z	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>	x <sub>5</sub>	x <sub>6</sub>	T <sub>1</sub>	T <sub>2</sub>	T <sub>3</sub>	T <sub>4</sub>	T <sub>5</sub>	T <sub>6</sub>	bj
Z	1	170.9992	170.2094	0	0	-95.34	133.4	0	0	0.523333	0	0	4.572014	48286.76
T <sub>1</sub>	0	-0.51506	-0.42091	0	0	0	0	1	0	0	0	0	-0.3415	5949.556
T <sub>2</sub>	0	-0.38338	-0.27974	0	0	0	0	0	1	0	0	0	-0.33638	5995.804
X <sub>4</sub>	0	0.233481	0.218271	0	1	0	0	0	0	0.011905	0	0	-0.01456	27.20096
T <sub>4</sub>	0	-0.22492	-0.16168	0	0	0	0	0	0	0	1	0	-0.33236	6032.143
T <sub>5</sub>	0	-0.36412	0.173616	0	0	0	0	0	0	-0.33393	0	1	-0.00239	8860.561
X <sub>3</sub>	0	0.720293	0.718464	1	0	0	0	0	0	0	0	0	0.018282	165.1737

## 5. Discussion and conclusion

### 5.1. Discussion

The savings were optimized, and the optimal savings Z was N48,286,760,000.00; (forty-eight billion, two hundred and eighty-six million, seven hundred and sixty thousand naira only). This is reasonable and encourages the collaboration to continue. We also observed that the decision variable X<sub>3</sub> = S<sub>3</sub> contributed N165,173,700.00, (one hundred and sixty-five million, one hundred and seventy-three thousand, and seven hundred naira in excess of the optimal profit. Also, X<sub>4</sub> = S<sub>4</sub> contributed N27,200,960.00, which is twenty-seven million, two hundred thousand and nine hundred and sixty naira over the optimal profit. S<sub>1</sub> and S<sub>2</sub>, S<sub>5</sub> and S<sub>6</sub> do not contribute significantly to the optimal profit other than what they contributed earlier since they did not enter the basis. Though each of the routes contributed to the total savings, further analysis using a mixed integer optimization method revealed that the coalition should concentrate more on the routes S<sub>3</sub> and S<sub>4</sub>, that is, “CALABAR – JOS” and “CALABAR – OWERRI” because they generate extra profit to the grand coalition.

Further analysis also revealed that the Calabar to Jos route has four coalitions while the Calabar to Owerri has three. Shapley value method of cost allocation could not explain which routes and number of coalitions would optimize the total savings. Hence, we recommend a mixed integer

optimization method with the Shapley value method for optimal cost/profit allocation in transportation/supply chain distribution.

## 5.2. Conclusion

In this paper, we propose a coalition of road transportation carriers who operate a fleet of mini trucks and trailers for distributing packaged cement to customers' warehouses in some selected routes in Nigeria. Five (5) transport providers collaborated to provide the services. This kind of collaboration falls under the purview of cooperative game theory, where each contribute to the running of the haulage operation and share in the cost and gains that accrued to the coalition. We proposed two methods (models) for dealing with the coalition problem. Since it is a cooperative game theory problem, we developed and applied Shapley value-based model to obtain the gains from the collaboration for the grand coalition. We further modeled the gains using an optimization- mixed inter programming method to determine the best combination of resources and route that will optimize the savings (gains). After the analysis, we observed that the savings were optimized.

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### Conflicts of Interest

The authors declare no conflict of interest.

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Amuji, H.O., Onwuegbuchunam, D.E., Ogwude, I.C., Okeke, K.O., Ojutalayo, J.F., Nwachi, C.C., & Mustapha, A.M. (2024). Optimized Shapley value cost allocation model for carriers' collaboration in road haulage transportation. *Journal of Sustainable Development of Transport and Logistics*, 9(1), 19-29. doi:10.14254/jsdtl.2024.9-1.2.

### References

- Aziz, H., Cahan, C., Gretton, C., Kilby, P., Mattei, N., & Walsh, T. (2016). A study of proxies for shapley allocations of transport costs. *Journal of Artificial Intelligence Research*, 56, 573-611. <https://doi.org/10.48550/arXiv.1408.4901>
- Dai, B., & Chen, H. (2012). Profit allocation mechanisms for carrier collaboration in pickup and delivery service. *Computers & Industrial Engineering*, 62(2), 633-643. <https://doi.org/10.1016/j.cie.2011.11.029>
- Dantzig, G. B. (2002). Linear programming. *Operations research*, 50(1), 42-47. <https://doi.org/10.1287/opre.50.1.42.17798>
- Darby-Dowman, K., & Wilson, J. M. (2002). Developments in linear and integer programming. *Journal of the Operational Research Society*, 53(10), 1065-1071. <https://doi.org/10.1057/palgrave.jors.2601435>
- David, G.L., & Yinyu, Y. (2008). *Linear and Non-linear Programming*. 3<sup>rd</sup> ed. Springer, New York.
- Eren Akyol, D., & Sarisakal, O. (2023). Horizontal cooperation in subcontracting: A case study in home textile industry. *Mathematical Problems in Engineering*, 12, 1-13. Available: <https://doi.org/10.1155/2023/8256116> Accessed 04/04/24
- Frisk, M., Göthe-Lundgren, M., Jörnsten, K., & Rönnqvist, M. (2010). Cost allocation in collaborative forest transportation. *European Journal of Operational Research*, 205(2), 448-458. <https://doi.org/10.1016/j.ejor.2010.01.015>



- Gupta, P. K., & Hira, D. S. (1991). *Problems in operations research: principles and solutions*. S. Chand and Company Ltd, New Delhi.
- Hallefjord, Å., & Storøy, S. (1990). Aggregation and disaggregation in integer programming problems. *Operations Research*, 38(4), 619-623. <https://doi.org/10.1287/opre.38.4.619>
- Ivanov, D., Pavlov, A., & Sokolov, B. (2014). Optimal distribution (re) planning in a centralized multi-stage supply network under conditions of the ripple effect and structure dynamics. *European Journal of Operational Research*, 237(2), 758-770. <https://doi.org/10.1016/j.ejor.2014.02.023>
- Kayikci, Y. (2020). Analysis of Cost Allocation Methods in International Sea-Rail Multimodal Freight Transportation. *Yaşar Üniversitesi E-Dergisi*, 15(57), 129-142. <https://doi.org/10.19168/jyasar.568692> (in English).
- Liittschwager, J. M., & Wang, C. (1978). Integer programming solution of a classification problem. *Management Science*, 24(14), 1515-1525. <https://doi.org/10.1287/mnsc.24.14.1515>
- Liu, P., Wu, Y., & Xu, N. (2010). Allocating collaborative profit in less-than-truckload carrier alliance. *Journal of Service science and Management*, 3(01), 143-149. <https://doi.org/10.4236/jssm.2010.31018>.
- Malawski, M., Wiecek, A., & Sosnowska, H. (2012). *Konkurencja i kooperacja: teoria gier w ekonomii i naukach społecznych*. Państwowe Wydawnictwo Naukowe PWN.
- Masimli, A. (2023). Shapley Value for Shortest Path Routing in Dynamic Networks. *Preprints.org*. <https://doi.org/10.20944/preprints202304.1115.v1>
- Ramirez-Beltran, N. D. (1995). Integer programming to minimize labour costs. *Journal of the Operational Research Society*, 46(2), 139-146. <https://doi.org/10.1057/jors.1995.20>
- Rao, S.S. (2009). *Engineering Optimization: Theory and Practice*. 4<sup>th</sup> ed. John Wiley & Sons Inc. New Jersey. <https://doi.org/10.1002/9780470549124>
- Rappos, E., & Thompson, E. B. (2008). An integer programming approach to optimize Housing Benefit data retrieval. *Journal of the Operational Research Society*, 59(2), 182-185. <https://doi.org/10.1057/palgrave.jors.2602558>
- Rifki, O., Danloup, N., Guo, Y., & Allaoui, H. (2023). An aided decision framework based on optimisation and game theory for a green and shared vehicle routing problem. *IFAC-PapersOnLine*, 56(2), 2552-2557. <https://doi.org/10.1016/j.ifacol.2023.10.1337>
- Tatarczak, A. (2018). A Framework to Support Coalition Formation in the Fourth Party Logistics Supply Chain Coalition. *Acta Universitatis Lodzianis. Folia Oeconomica*, 5(338), 195-212. <https://doi.org/10.18778/0208-6018.338.12>
- Ugwuanyi, G.O.C. (2007). *Quantitative Techniques: A Managerial*. Ranco Ventures, Enugu Nigeria.
- Vanovermeire, C., Vercruyssen, D., & Sörensen, K. (2014). Analysis of different cost allocation methods in a collaborative transport setting. *International Journal of Engineering Management and Economics*, 4(2), 132-150. <https://doi.org/10.1504/IJEME.2014.066576>
- Wang, Y. (2023). A collaborative approach based on Shapley value for carriers in the supply chain distribution. *Heliyon*, 9(7), e17967. <https://doi.org/10.1016/j.heliyon.2023.e17967>
- Zaremba, L., Zaremba, C. S., & Suchenek, M. (2017). Modification of shapley value and its implementation in decision making. *Foundations of Management*, 9(1), 257-272. <https://doi.org/10.1515/fman-2017-0020>

### Appendix 1: Route and combination of trailers and trucks from different transport providers, cost and frequency of supply per month

	Origin Port Terminals- Destination warehouse. (O-D)	Road Haulage transport route from terminal - customer warehouse	Transport providers	No. of operators	Unit Cost(m)	Frequency	Unit Cost(m)	Frequency	Unit Cost(m)	Frequency	Unit Cost(m)	Frequency	Total Cost (b)
S1	Lagos - Kano	Lagos(+Tr) - Kaduna(+Tr) - Kano(+Tk)	Admiral Trucker, Country Service Solution, Vennis Truck	3	284	3	282	4	110	8			676
S2	Lagos - Enugu	Lagos(+Tr)- Onitsha(+Tr) - Enugu(+Tk)	Eccnosy Inte. Solution, Admiral Trucker, Maverick Int. Soltn.	3	280	4	278	5	112	7			670
S3	Calabar - Jos	Calabar(+Tr)- Owerri(+Tr) - Makrudi(+Tr) - Jos(+Tk)	Admiral Trucker, Country Service Solution, Eccnosy Int. Soln, Vennis Truck	4	274	3	283	4	283	3	112	5	952
S4	Calabar - Owerri	Calabar(+Tr) - Uyo(+Tk)- Owerri(+Tr)	Admiral Trucker, Country Service Solution, Maverick Int. Soln.	3	276	4	275	5	115	6			666
S5	Lagos - Sokoto	Lagos(+Tr) - Kaduna(+Tr) -- Skoto(+Tk)	Admiral Trucker, Country Service Solution, Eccnosy Int. Soln.	3	285	5	283	3	282	2	113		963
S6	Calabar - Abakiliki	Calabar(+Tr) - Onitsha(+Tr) - Enugu(+Tr)- Abakiliki(+Tk)	Admiral Trucker, Eccnosy Int. Soln., Maverick Int. Stn.. Vennis Truck	4	273	5	274	7	120	6			667

Tr: Trailers, Tk: Trucks

**Appendix 2: Computation of savings on each leg of the coalition**

	Total unit cost (TL)	Unit cost (U)	TL - U	Wi	Savings (yi)
S1	676	284	392	0.033	12.94
	676	282	394	0.033	13
	676	110	566	0.033	18.68
<b>Total</b>					<b>44.62</b>
S2	670	280	390	0.033	12.87
	670	278	392	0.033	12.94
	670	112	558	0.033	18.41
<b>Total</b>					<b>44.22</b>
S3	952	274	678	0.1	67.8
	952	288	664	0.1	66.4
	952	283	669	0.1	66.9
	952	112	840	0.1	84
<b>Total</b>					<b>285.1</b>
S4	666	276	390	0.033	12.87
	666	275	391	0.033	12.9
	666	115	551	0.033	18.18
<b>Total</b>					<b>43.96</b>
S5	963	285	678	0.033	22.37
	963	283	680	0.033	22.44
	963	282	681	0.033	22.47
	963	113	850	0.033	28.05
<b>Total</b>					<b>95.34</b>
S6	667	273	394	0.1	39.4
	667	274	393	0.1	39.3
	667	120	547	0.1	54.7
<b>Total</b>					<b>133.4</b>



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