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CALCULATION THE MOMENTS OF FORCE EQUATION, WITH USE METHOD T6, INCLUDING KINETIC INTERACTION

Abstract: This article presents the possibilities of the popular T6 method in robotics, bring this method to the designers of mechanics, technology and architects. Presented description of the method concerns the moments equation for different classes of robots, but it can be used wherever it occurs need to calculate the working force for arm. The T6 method can significantly accelerate and simplify calculations, which are also more reliable, with less the possibility of appearing an error. The T6 method also allows the calculation of much more complicated equations, like calculation the acting force of arm, which is not possibly making in the classic way.

1. Introduction

In time of creating software systems for robots with very different parameters, we often have a problem changing the parameters, when is change direction of movement of the robot arm. When arm is moving up, the moment of force is slow down movement, and during downward robot arm significantly speeds up. Not all systems make a solution of this problem equally well. Especially when designing a new system, solve of this problem can be taken to the robot arm operating program. We count the moments on the plane of the arm symmetry (OXZ), so we can omit the translation that determines rotation of the robot arm at the base (OXY) and does not flow on static moment of force (A1). Interesting is also, how the influence of the weight of object is flows on the problem described above. How changes parameters of move the load and unloaded arm.

2. Translations T6 method in moment of force calculation

When we calculate the robot's equation, we know all its input translations, and the result translation T6, which is his multiplication. Creation T6 equation is descript in “Essentials of robotics” [1].

$$T6 = A1 * A2 * A3 * A4 * .... * E \tag{1}$$

The same input translations can be used to calculate the moment equation. Acting force is equal:

$$F = m * g \tag{2}$$

The moment M1 for the arm section l1 is calculated as the product of half the arm, because the center the weight is in half the length. The force component must be also multiplied by $\cos \beta_1$ because is perpendicular to the arm.

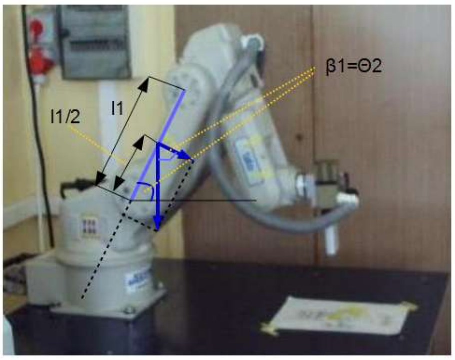

Fig. 1. Gravity force working on arm section l1

$$M_1 = \frac{l1}{2} * m_1 * g * \cos \beta_1 \tag{3}$$

In this case: $\beta_1 = \Theta 2$

$$M_1 = \frac{l1}{2} * m_1 * g * \cos \Theta 2 \tag{4}$$

A2 translation for the M1 moment can be written:

$$A2 = Rot(y,\ \Theta 2) * Trans(l1/2,\ 0,\ 0) \tag{5}$$

After saving in matrix form, we get:

$$A2 = \begin{bmatrix} \cos\theta 2 & 0 & -\sin\theta 2 & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta 2 & 0 & \cos\theta 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & l1/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

After multiplying the matrixes:

$$A2 = \begin{bmatrix} \cos\Theta 2 & 0 & -\sin\Theta 2 & 0.5 * l1 * \cos\Theta 2 \\ 0 & 1 & 0 & 0 \\ \sin\Theta 2 & 0 & \cos\Theta 2 & 0.5 * l1 * \sin\Theta 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

In the last column we get for a_{14} the coordinate x, for a_{24} the coordinate y, for a_{34} the coordinate z of the result function. In the formula for the moment M2, resultant is sum of the projection of the section l1 and the section l2 of robot arm on OX and OZ axes.

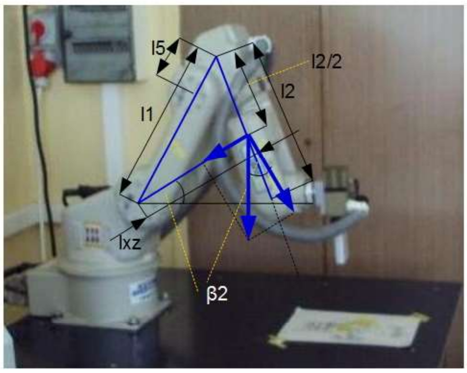

Fig. 2. Gravity force working on arm section l2

$$M_2 = l_{xz} * m_1 * g * \cos\beta_2 \tag{8}$$

Therefore, we can use A2 translation for the full length of the arm, in addition multiplied by A3 translation.

$$A3 = Rot(y, -\Theta 3) * Trans(l2/2, 0, 0) \tag{9}$$

In matrix notation:

$$A3 = \begin{bmatrix} \cos\theta 3 & 0 & \sin\theta 3 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta 3 & 0 & \cos\theta 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & l2/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

After multiplying the matrixes:

$$A3 = \begin{bmatrix} \cos\Theta 3 & 0 & \sin\Theta 3 & 0.5 * l2 * \cos\Theta 3 \\ 0 & 1 & 0 & 0 \\ -\sin\Theta 3 & 0 & \cos\Theta 3 & -0.5 * l2 * \sin\Theta 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

After multiplying A2*A3 translations, we get:

$$A2 * A3 = \begin{bmatrix} \cos\Theta 2 & 0 & -\sin\Theta 2 & l1 * \cos\Theta 2 \\ 0 & 1 & 0 & 0 \\ \sin\Theta 2 & 0 & \cos\Theta 2 & l1 * \sin\Theta 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos\Theta 3 & 0 & \sin\Theta 3 & 0.5 * l2 * \cos\Theta 3 \\ 0 & 1 & 0 & 0 \\ -\sin\Theta 3 & 0 & \cos\Theta 3 & -0.5 * l2 * \sin\Theta 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

To simplify the equation, we take C2= cosΘ2, C23=cos(Θ2-Θ3), S23= sin(Θ2-Θ3)

$$A2*A3=\begin{bmatrix} C23 & 0 & -S23 & 0.5*C23*l2+C2*l1 \\ 0 & 1 & 0 & 0 \\ S23 & 0 & C23 & 0.5*S23*l2+S2*l1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

Because the moment of force is working over result section

$$l_{xz}=\sqrt{l_x^2+l_z^2} \tag{14}$$

in the pattern

$$M_2=l_{xz}*m_1*g*\cos\beta_2 \tag{15}$$

Variables l_x and l_z calculated by matrix method, as resultant projection of arm sections l1 and l2 on the axis OX and OZ.

$$l_x=0.5*C23*l2+C2*l1 \tag{16}$$

$$l_z=0.5*S23*l2+S2*l1 \tag{17}$$

In next step we calculate β_2:

$$\beta_2=arctg(\frac{l_z}{l_x}) \tag{18}$$

Now we have all the data and can calculate the moment M2.

3. Universal formula for moment of force calculation

Similarly, we can calculate the moment M3 and generally Mj.

$$M_j=l_{xzj}*m_j*g*\cos\beta_j \tag{19}$$

To calculate arm of force for Mj, the translations product should be calculated:

$$T(M_j)=\prod_{i=2}^{j}Ai \tag{20}$$

$$T(M_j)=\prod_{i=2}^{j+1}(Rot(y,\ \Theta_i)*Trans(l_{i-1},\ 0,\ 0)) \qquad \text{for } l_j=l_i/2 \tag{21}$$

Then substitute for l_x and l_z corresponding element of the result matrix $l_x=a_{14}, l_z=a_{34}$. After calculation arm of force is equal:

$$l_{xzj} = \sqrt{l_{xj}^2 + l_{zj}^2} \quad (22)$$

$$\beta_j = arctg(\frac{l_{zj}}{l_{xj}}) \quad (23)$$

We can calculate moment of force Mj:

$$M_j = l_{xzj} * m_j * g * \cos\beta_j \quad (24)$$

The resultant moment is the sum of moments Mj:

$$M_c = \sum_{i=2}^{j} M_j \quad (25)$$

Resultant moment is calculated for corner Θ2, which is change from current position Θ2o to new position Θ2n. For calculation of time rotation is need mean moment Mc(Θ2) as a function of angle Q2. Initially, I tried to use the integral of the moment of force function in the limits from Q2o to Q2n, but integral was not take expected result. In next step I was use estimate of moment of force for corner Θ2 which is change from the angle Q2o to the angle Q2n with step 2°.

$$\overline{M_c(\Theta 2)} = \frac{1}{n} * \sum_{j=1}^{n \le \frac{\Theta 2_n - \Theta 2_o}{2}} M_c(\Theta 2_o + j * 2) \quad (26)$$

The formula for the time of rotation of the arm for angle Q2 with use estimate is therefore:

$$t_{obr} = \frac{\Delta\Theta 2_j}{\varpi} \pm k * \frac{\overline{M_c(\Theta 2)}}{l_k} * \sin\gamma \quad (27)$$

Note: These calculations apply to corner Θ2 between the robot base and the section arm l1. For the angle Θ3, the arm section l1 and A2 translation should be omitted in the calculations. For corner Θ4, similar l2 and A3 must be omitted.

4. Kinetic interaction in moment of force calculation

The resultant moment M_c is acting through a toothed wheel with a radius l_k on worm gear. Moment of force acting on the cogwheel is equal:

$$M_c = F * l_k \quad (28)$$

Then we can calculate the force:

$$F = \frac{M_c}{l_k} \quad (29)$$

It also acts on the worm gear with the angle γ:

$$F = \frac{M_c}{l_k} * \sin\gamma \tag{30}$$

To calculate dependence of time rotation robot arm section for a given increase the angle ΔΘj, we can took the proportional dependence of increase speed to working force.

$$t_{obr} = \frac{\Delta\Theta_j}{\varpi} \pm k * \frac{M_c}{l_k} * \sin\gamma \tag{31}$$

ω - middle angular speed.

In practice, was see that the value of coefficient k in this case is 1. In moving down direction, equation was working well. However, when arm was moving up, the equation of moments should take an additional kinetic component. Is effect of the conversion of potential energy into kinetic energy.

$$m_j * g * h = \frac{m_j * V^2}{2} \tag{32}$$

Then substitute for $h = l_{xyj} * \sin\beta_j$ and simplify m_j:

$$g * l_{xzj} * \sin\beta_j = \frac{V^2}{2} \tag{33}$$

Then we calculate V:

$$V = \sqrt{2 * g * l_{xzj} * \sin\beta_j} \tag{34}$$

At the same time, for calculate the speed in a circular motion I calculate the proportion:

$$\frac{\beta_j}{ł} = \frac{2 * \pi}{2 * \pi * r} = \frac{1}{r} = \frac{1}{l_{xzj}} \tag{35}$$

Arc length ł for the angle β_j is equal:

$$ł = \beta_j * l_{xzj} \tag{36}$$

Speed in move on the circle:

$$V = \frac{ł}{t} = \frac{\beta_j * l_{xzj}}{t} = \varpi_j * l_{xzj} \tag{37}$$

From the above and (32) equation we calculate increase angular velocity:

$$\Delta\varpi_j = \frac{\sqrt{2 * g * l_{xzj} * \sin\beta_j}}{l_{xzj}} \tag{38}$$

After simplifying:

$$\Delta\varpi_j = \sqrt{\frac{2 * g * \sin\beta_j}{l_{xzj}}} \tag{39}$$

After substituting to the general formula for the time of rotation arm section, we get:

$$t_{obr} = \frac{\Delta\Theta_j}{\varpi + \Delta\varpi_j} \pm k * \frac{\overline{M_c(\Theta)}}{l_k} * \sin\gamma \tag{40}$$

The obtained general formula takes all moments flows on the system in the time of arm rotation, and including the kinetic component occurring at upward movement.

5. Conclusion

The use of the T6 method significantly speeds up calculations, also with significant level of complications. It reduces the risk of error in calculations. For calculation the robot's moment of force equation, can be use fragments previous calculations of the robot arm equation descript in my article [2] “Robot SX-300 equation, with using New Gauss, and Parametric method” .

References

1. Szkodny T.: Essentials of robotics. Gliwice: Silesian University of Technology publishing house, 2011.
2. Godzwa M: Robot SX-300 equation, with using New Gauss, and Parametric method. Selected Engineering Problems, Vol. 7, (2016), pp. 9 – 14.