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Modelling oil spill layer thickness and hydro-meteorological conditions impacts on its domain movement at sea area

Keywords

oil spill, stochastic modelling, oil layer thickness, domain movement, hydro-meteorological conditions

Abstract

The general model of oil spill domain movement forecasting dependent on the thickness of oil spill layer based on a probabilistic approach considering the influence of the hydro-meteorological conditions at sea area is proposed. A semi-Markov model of the process of changing hydro-meteorological conditions is constructed. A two-dimensional stochastic process is used to describe the oil spill domain central point position movement. Parametric equation of oil spill domain central point drift trend curve considering the initial thickness of oil spill layer at the oil spill central point is used. Next, the method of oil spill domain determination dependent on the thickness of oil spill layer for various hydro-meteorological conditions is presented. The generalization of the presented approach assuming that the thickness is changing with time is also proposed. At the end, the research further perspective is given.

1. Introduction

In the era of globalisation of the economy, we can observe a negative impact of chemical releases in all the kinds of world waters. The significant issue is to forestall the oil slicks and mitigate its costs to the nations and biological life. For that reason, there is a requirement for strategies and methods decreasing the water contamination (Bogalecka, 2020; Bogalecka & Kołowrocki 2015a, 2015b, 2018; Fingas, 2016). One of the significant and essential ways of satisfying this need is the strategy for fast and accurate determining the oil slick area, thickness of a layer and the domain movement.

The oil spill central point drift trend, the oil spill domain shape and its random position fixed for changing different hydro-meteorological conditions allow us to construct the model of determination of the area in which, with the fixed probability, the oil spill domain is placed (Dąbrowska 2021; Dąbrowska & Kołowrocki, 2019). This model is considered in this chapter with taking into account the thickness of oil layer.

This chapter is composed of Introduction, Sections 1–5 and Conclusion.

First in Section 2, a semi-Markov model of the process of changing hydro-meteorological conditions is defined and its parameters and characteristics are introduced. Next in Section 3, stochastic modelling trend of spill central point drift with considering thickness of oil layer is presented. In Section 4, identification of oil spill drift trend with considering thickness of oil layer is considered. Next in Section 5, a theoretical background of oil spill domain movement with considering thickness of oil layer is presented. In conclusion, the research further perspective is given.

2. Process of changing hydro-meteorological conditions

Let $A(t)$ denote the process of varying hydro-meteorological conditions at the sea water area (where the oil spill happened) and let $A = \{1,2,...,m\}$, be the set of all possible states of $A(t)$ in which it may stay at the moment t, $t \in \langle 0,T \rangle$, where $T > 0$. Further, we assume a semi-Markov model (Grabski, 2014; Kołowrocki, 2014) of the process $A(t)$ and denote by θ_{ij} its con-

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ditional sojourn time in state i while its next transition will be done to state j, where $i, j \in \{1,2,...,m\}$, $i \neq j$.

Under these assumptions, the process of changing hydro-meteorological conditions $A(t)$ is completely described by the following parameters (Dąbrowska, 2021; Kołowrocki, 2014; Limnios & Oprisan, 2001; Xue, 1985; Xue & Yang, 1985, 1995):

- the vector of probabilities of its initial states at the moment $t = 0$

$$[p(0)] = [p_1(0), p_2(0),..., p_m(0)], \quad (1)$$

- the matrix of probabilities of its transitions between the particular states

$$[p_{ij}] = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix}, \quad (2)$$

where $\forall\ i = 1,2,...,m$, $p_{ii} = 0$,

- the matrix of distribution functions of its conditional sojourn times θ_{ij} at the particular states

$$[W_{ij}(t)] = \begin{bmatrix} W_{11}(t) & W_{12}(t) & \cdots & W_{1m}(t) \\ W_{21}(t) & W_{22}(t) & \cdots & W_{2m}(t) \\ \vdots & \vdots & \ddots & \vdots \\ W_{m1}(t) & W_{m2}(t) & \cdots & W_{mm}(t) \end{bmatrix}, \quad (3)$$

where $\forall\ i = 1,2,...,m$, $W_{ii}(t) = 0$,

- the expected values (mean values) of its conditional sojourn times θ_{ij} at the particular states

$$M_{ij} = E[\theta_{ij}] = \int_0^\infty t dW_{ij}(t), \quad (4)$$

$i, j = 1,2,...,m$, $i \neq j$,

- the variances of its conditional sojourn times θ_{ij} at the particular states

$$V_{ij} = D[\theta_{ij}] = \int_0^\infty (t - E[\theta_{ij}])^2 dW_{ij}(t), \quad (5)$$

$i, j = 1,2,...,m$, $i \neq j$.

Using the above defined parameters, we can determine the following characteristics of the considered process $A(t)$, $t \in \langle 0,T \rangle$, $T > 0$ (Kołowrocki, 2014; Kuligowska, 2018; Torbicki, 2018):

- the distribution functions of the unconditional sojourn time θ_i of the process of changing hydro-meteorological conditions at the particular states i, $i = 1,2,...,m$

$$W_i(t) = \sum_{j=1}^{m} p_{ij} W_{ij}(t),\ i = 1,2,...,m, \quad (6)$$

- the mean values of the unconditional sojourn time θ_i of the process of changing hydro-meteorological conditions at the particular states i, $i = 1,2,...,m$

$$M_i = E[\theta_i] = \sum_{j=1}^{m} p_{ij} E[\theta_{ij}],\ i = 1,2,...,m, \quad (7)$$

- the variances of the unconditional sojourn time θ_i of the process of changing hydro-meteorological conditions at the states i, $i = 1,2,...,m$

$$V_i = D[\theta_i] = E[\theta_i^2] - (E[\theta_{ij}])^2, \quad (8)$$

$i = 1,2,...,m$,

- the limit values of the transient probabilities of a process of changing hydro-meteorological conditions at the particular states

$$p_i(t) = P(W(t) = i), \quad (9)$$

$t \in \langle 0,+\infty)$, $i = 1,2,...,m$,

given by

$$p_i = \lim_{t\to\infty} p_i(t) = \frac{\pi_i M_i}{\sum_{j=1}^{m} \pi_j M_j},\ i = 1,2,...,m, \quad (10)$$

where M_i, $i = 1,2,...,m$, are given by (7), while the steady probabilities π_i of the vector $[\pi_i]_{1xm}$ satisfy the system of equations

$$\begin{cases}[\pi_i][p_{ij}]=[\pi_i]\\ \sum_{i=1}^{m}\pi_i=1.\end{cases} \quad (11)$$

which in the case of a periodic process of changing hydro-meteorological conditions, are the long term proportions of this process sojourn times at the particular operation states i, $i = 1,2,...,m$;

- the total sojourn times $\hat{\theta}_i$ of the process at the particular hydro-meteorological states i, $i = 1,2,...,m$, during the fixed time θ, that have approximately normal distributions with the expected value given by

$$\hat{M}_i = E[\hat{\theta}_i] = p_i\theta,\ i = 1,2,...,m, \quad (12)$$

where p_i, $i = 1,2,...,m$, are given by (10).

3. Stochastic modelling trend of spill central point drift with considering thickness of oil layer

First, for each fixed state k, $k \in \{1,2,\dots,m\}$, of the process $A(t)$ of changing hydro-meteorological conditions, introduced in Section 2, and time $t \in \langle 0,T\rangle$, $T > 0$where T is time we are going to model the behaviour of the oil spill domain we define the central point with considering thickness τ of oil layer of this oil spill domain as a point $(x^k(t,\tau), y^k(t,\tau))$, $t \in \langle 0,T\rangle$, $\tau \in \langle \tau_1,\tau_2\rangle$, τ_1, $\tau_2 > 0$, $k \in \{1,2,\dots,m\}$, on the plane Oxy that is the centre of the smallest circle, with the radius $r^k(t,\tau)$, $t \in \langle 0,T\rangle$, $\tau \in \langle \tau_1,\tau_2\rangle$, $k \in \{1,2,\dots,m\}$, covering this domain (Figure 1). We assume that the thickness τ is not changeable, i.e. $\tau = const$. Thus, for the fixed oil spill domain $\overline{D}^k(t,\tau)$, we have

$$x^k(t,\tau) = \left|\frac{x_1^k(t,\tau) + x_1^k(t,\tau)}{2}\right|, \quad (13)$$

$$y^k(t,\tau) = \left|\frac{y_1^k(t,\tau) + y_1^k(t,\tau)}{2}\right|, \quad (14)$$

for $t \in \langle 0,T\rangle$, $\tau \in \langle \tau_1,\tau_2\rangle$, $k \in \{1,2,\dots,m\}$, where the $P_1(x_1^k(t,\tau), y_1^k(t,\tau))$ and $P_2(x_2^k(t,\tau), y_2^k(t,\tau))$ are the most distant points of the oil spill domain $\overline{D}^k(t,\tau)$, $t \in \langle 0,T\rangle$, $\tau \in \langle \tau_1,\tau_2\rangle$, $k \in \{1,2,\dots,m\}$, and the radius $r^k(t,\tau)$, called the radius of the oil spill domain $\overline{D}^k(t,\tau)$, is given by

$$r^k(t,\tau) = \frac{1}{2}\sqrt{[x_1^k(t,\tau)-x_2^k(t,\tau)]^2+[y_1^k(t,\tau)-y_2^k(t,\tau)]^2}, \quad (15)$$

$t \in \langle 0,T\rangle$, $\tau \in \langle \tau_1,\tau_2\rangle$, $k \in \{1,2,\dots,m\}$.

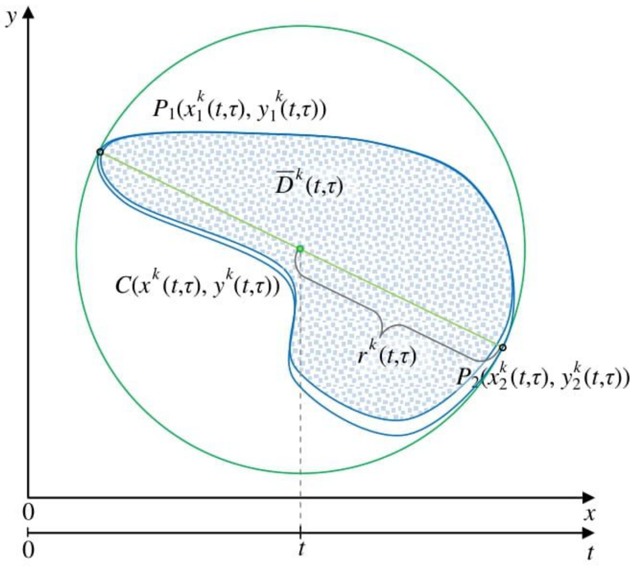

Figure 1. Interpretation of central point of oil spill definition.

Further, for each fixed state k, $k = 1,2,\dots,m$, of the process of changing hydro-meteorological conditions $A(t)$ and time t, $t \in \langle 0,T\rangle$, we define a two-dimensional stochastic process

$$(X^k(t,\tau), Y^k(t,\tau)),\ t \in \langle 0,T\rangle,\ \tau \in \langle \tau_1,\tau_2\rangle,$$

such that

$$(X^k, Y^k): \langle 0,T\rangle \to R^2,$$

where $X^k(t,\tau)$, $Y^k(t,\tau)$ respectively are an abscissa and an ordinate of the plane Oxy point, in which the oil spill central point is placed at the moment t while the process $A(t)$, $t \in \langle 0,T\rangle$, is at the state k. We set deterministically the central point of oil spill domain in the area in which an accident has happened and an oil release was placed in the water as the origin $O(0,0)$ of the coordinate system Oxy. The value of a parameter t at the moment of accident we assume equal to 0. It means that the process ($X^k(t,\tau)$, $Y^k(t,\tau)$), is a random two-dimensional coordinate (a random position) of the oil spill central point after the time t from the accident moment and that at the accident moment $t = 0$ the oil spill central point is at the point $O(0,0)$, i.e.

$(X^k(0,0), Y^k(0,0)) = (0,0).$

After some time, the central point of the oil spill starts its drift along a curve called a drift curve. In further analysis, we assume that processes

$(X^k(t,\tau), Y^k(t,\tau)),\ t \in (0,T\rangle,\ \tau \in \langle\tau_1,\tau_2\rangle,$

$k \in \{1,2,\ldots,m\},$

are two-dimensional normal processes

$N(m_X^k(t,\tau), m_Y^k(t,\tau), \rho_{XY}^k(t,\tau), \sigma_X^k(t,\tau), \sigma_Y^k(t,\tau)),$

with varying in time expected values

$$m_X^k(t,\tau) = E[X^k(t,\tau)],\ m_Y^k(t,\tau) = E[Y^k(t,\tau)], \quad (16)$$

standard deviations

$$\sigma_X^k(t,\tau),\ \sigma_Y^k(t,\tau) \quad (17)$$

and correlation coefficients

$$\rho_{XY}^k(t,\tau), \quad (18)$$

for $t \in (0,T\rangle$, $\tau \in \langle\tau_1,\tau_2\rangle$, $k \in \{1,2,\ldots,m\}$, i.e. with the joint density functions

$$\varphi_{t,\tau}^k(x,y) = \frac{1}{2\pi\sigma_X^k(t,\tau)\sigma_Y^k(t,\tau)\sqrt{1-(\rho_{XY}^k(t,\tau))^2}}$$

$$\exp\{-\frac{1}{2(1-(\rho_{XY}^k(t,\tau))^2)}[\frac{(x-m_X^k(t,\tau))^2}{(\sigma_X^k(t,\tau))^2}$$

$$-2\rho_{XY}^k(t,\tau)\frac{(x-m_X^k(t,\tau))(y-m_Y^k(t,\tau))}{\sigma_X^k(t,\tau)\sigma_Y^k(t,\tau)}$$

$$+\frac{(y-m_Y^k(t,\tau))^2}{(\sigma_Y^k(t,\tau))^2}]\}, \quad (19)$$

$(x, y) \in R^2,\ t \in (0,T\rangle,\ \tau \in \langle\tau_1,\tau_2\rangle,\ k \in \{1,2,\ldots,m\}.$

Thus, the points

$(m_X^k(t,\tau), m_Y^k(t,\tau)),$

$t \in (0,T\rangle,\ \tau \in \langle\tau_1,\tau_2\rangle,\ k \in \{1,2,\ldots,m\},$

given by (16), create the curve K^k represented parametrically by

$$K^k: \begin{cases} x^k = x^k(t,\tau) = m_X^k(t,\tau) \\ y^k = y^k(t,\tau) = m_Y^k(t,\tau),\ t \in \langle 0,T\rangle,\ \tau \in \langle a,b\rangle. \end{cases} \quad (20)$$

where τ, $\tau \in \langle\tau_1,\tau_2\rangle$ is the initial thickness of oil spill layer at the oil spill central point.

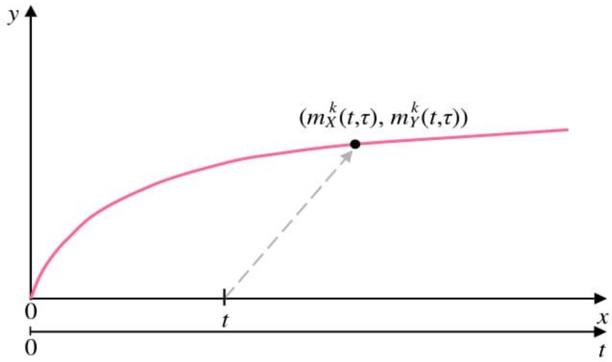

Figure 2. Oil spill central point drift trend.

4. Identification of oil spill drift trend with considering thickness of oil layer

Having in disposal empirical mean positions of the oil spill domain central point

$(m'^k_X(t_1,\tau), m'^k_Y(t_1,\tau)), (m'^k_X(t_2,\tau),$

$$m'^k_Y(t_2,\tau)),\ldots, (m'^k_X(t_{N^k},\tau), m'^k_Y(t_{N^k},\tau)), \quad (21)$$

$k = 1,2,\ldots,m,$

at the moments $t_1,t_2,\ldots,t_{N^k}$, it is possible to fix the equation of the central point of oil spill drift trend curve in the following parametric form

K^k:

$$\begin{cases} x = m_X^k(t,\tau) = \alpha_0^k(\tau) + \alpha_1^k(\tau)t^1 + \ldots + \alpha_a^k(\tau)t^a \\ y = m_Y^k(t,\tau) = \beta_0^k(\tau) + \beta_1^k(\tau)t^1 + \ldots + \beta_b^k(\tau)t^b, \end{cases} \quad (22)$$

$t \in (0,T\rangle,\ \tau \in \langle\tau_1,\tau_2\rangle,\ k = 1,2,\ldots,m,$

where a and b are natural numbers, while

$$\alpha_j^k(\tau), j = 0,1, ...,a, k = 1,2,\ldots,m, \tau \in \langle\tau_1,\tau_2\rangle, \quad (23)$$

and

$$\beta_j^k(\tau), j = 0,1, ...,b, k = 1,2,\ldots,m, \tau \in \langle\tau_1,\tau_2\rangle, \quad (24)$$

are unknown real coefficients of the assumed curve model.

According to the least squares method the unknown coefficients of the central point of oil spill drift trend are determined from the condition of the minimising the sum of squares of the differences between the realisations (21) and values calculated from the curve $\bar{K}^k$ equations (22), i.e. for each fixed k, $k = 1,2,\ldots,m$, the following condition should be satisfied

$$\Delta^k(\tau) = \sum_{\nu=1}^{N^k}[(m'^k_X(t_\nu,\tau) - m^k_X(t_\nu,\tau))^2$$

$$+ (m'^k_Y(t_\nu,\tau) - m^k_Y(t_\nu,\tau))^2]$$

$$= minimum, k = 1,2,\ldots,m, \tau \in \langle\tau_1,\tau_2\rangle. \quad (25)$$

Hence and from the necessary condition for extremes existence for each fixed k, after substituting

$$A^k_{uv} = \sum_{\nu=1}^{N^k} t_\nu^{u+v}, \; u, v = 0,1,\ldots,a,$$

$$E^k_u(\tau) = \sum_{\nu=1}^{N^k} t_\nu^u m'^k_X(t_\nu,\tau), \; u = 0,1,\ldots,a, \quad (26)$$

$k = 1,2,\ldots,m$, $\tau \in \langle\tau_1,\tau_2\rangle$,

$$B^k_{uv} = \sum_{\nu=1}^{N^k} t_\nu^{u+v}, \; u, v = 0,1,\ldots,b,$$

$$F^k_u(\tau) = \sum_{\nu=1}^{N^k} t_\nu^u m'^k_Y(t_\nu,\tau), \; u = 0,1,\ldots,b, \quad (27)$$

$k = 1,2,\ldots,m$, $\tau \in \langle\tau_1,\tau_2\rangle$,

and introducing the matrices

$$A^k = \begin{bmatrix} A^k_{00} & A^k_{01} & . & . & . & A^k_{0a} \\ A^k_{10} & A^k_{11} & . & . & . & A^k_{1a} \\ . & . & . & . & . & . \\ A^k_{a0} & A^k_{a1} & . & . & . & A^k_{aa} \end{bmatrix},$$

$$\alpha^k(\tau) = \begin{bmatrix} \alpha^k_0(\tau) \\ \alpha^k_1(\tau) \\ \ldots \\ \alpha^k_a(\tau) \end{bmatrix}, E^k(\tau) = \begin{bmatrix} E^k_0(\tau) \\ E^k_1(\tau) \\ \ldots \\ E^k_a(\tau) \end{bmatrix}, \quad (28)$$

$k = 1,2,\ldots,m$, $\tau \in \langle\tau_1,\tau_2\rangle$,

$$B^k = \begin{bmatrix} B^k_{00} & B^k_{01} & . & . & . & B^k_{0b} \\ B^k_{10} & B^k_{11} & . & . & . & B^k_{1b} \\ . & . & . & . & . & . \\ B^k_{b0} & B^k_{b1} & . & . & . & B^k_{bb} \end{bmatrix},$$

$$\beta^k(\tau) = \begin{bmatrix} \beta^k_0 \\ \beta^k_1 \\ \ldots \\ \beta^k_b \end{bmatrix}, F^k(\tau) = \begin{bmatrix} F^k_0 \\ F^k_1 \\ \ldots \\ F^k_b \end{bmatrix}, \quad (29)$$

$k = 1,2,\ldots,m$, $\tau \in \langle\tau_1,\tau_2\rangle$,

we get

$$\begin{cases} A^k\alpha^k(\tau) = E^k(\tau) \\ B^k\beta^k(\tau) = F^k(\tau), \; k = 1,2,...,m, \tau \in \langle a,b\rangle. \end{cases} \quad (30)$$

Hence under the assumption that determinants of the matrices A^k and B^k, defined by (28)–(29), are different from zero we get the unknown coefficient of the curve models from the system of equations

$$\begin{cases} \alpha^k(\tau) = (A^k)^{-1}E^k(\tau) \\ \beta^k(\tau) = (B^k)^{-1}F^k(\tau), \; k = 1,2,...,m, \tau \in \langle a,b\rangle, \end{cases} \quad (31)$$

where $(A^k)^{-1}$ and $(B^k)^{-1}$ are the inverse matrices of the matrices A^k and B^k, defined by (28)–(29).

The equations of the remaining dependent on time t functional parameters of the join density functions $\varphi^k_{t,\tau}(x,y)$, $t \in (0,T\rangle$, $\tau \in \langle\tau_1,\tau_2\rangle$,

$k = 1,2,\ldots,m$, may be found in an analogous way. Having in disposal empirical values of standard deviations of the oil spill domain central point positions

$$\sigma'^{k}_{X}(t_1,\tau),\ \sigma'^{k}_{X}(t_2,\tau),\ \ldots,\ \sigma'^{k}_{X}(t_{N^k},\tau), \tag{32}$$

$k = 1,2,\ldots,m,\ \tau \in \langle \tau_1,\tau_2 \rangle,$

at the moments $t_1,t_2,\ldots,t_{N^k}$, it is possible to evaluate the coefficients of functional standard deviations of random variables $X^k(t,\tau)$, $t \in \langle 0,T \rangle$, $\tau \in \langle \tau_1,\tau_2 \rangle$, $k = 1,2,\ldots,m$, given by the following curve in the parametric form

$$\sigma^{k}_{X}(t,\tau) = \gamma^{k}_{0}(\tau) + \gamma^{k}_{1}(\tau)t^1 + \ldots + \gamma^{k}_{c}(\tau)t^c, \tag{33}$$

$t \in \langle 0,T \rangle,\ \tau \in \langle \tau_1,\tau_2 \rangle,\ k = 1,2,\ldots,m,$

determining them from the system of equations

$$\gamma^k(\tau) = (A^k)^{-1}G^k(\tau),\ k = 1,2,\ldots m, \tag{34}$$

where

$$\gamma^k(\tau) = \begin{bmatrix} \gamma_0^k(\tau) \\ \gamma_1^k(\tau) \\ \cdots \\ \gamma_c^k(\tau) \end{bmatrix},\ G^k(\tau) = \begin{bmatrix} G_0^k(\tau) \\ G_1^k(\tau) \\ \cdots \\ G_c^k(\tau) \end{bmatrix},$$

$$G_u^k = \sum_{v=1}^{N^k} t_v^u \sigma'^{k}_{X}(t_v,\tau),\ u = 0,1,\ldots,c, \tag{35}$$

$k = 1,2,\ldots,m,\ \tau \in \langle \tau_1,\tau_2 \rangle.$

Having in disposal empirical values of standard deviations of the oil spill domain central point positions

$$\sigma'^{k}_{Y}(t_1,\tau),\ \sigma'^{k}_{Y}(t_2,\tau),\ \ldots,\ \sigma'^{k}_{Y}(t_{N^k},\tau), \tag{36}$$

$k = 1,2,\ldots,m,\ \tau \in \langle \tau_1,\tau_2 \rangle,$

at the moments $t_1,t_2,\ldots,t_{N^k}$, it is possible to evaluate coefficients of the functional standard deviations of random variables $Y^k(t,\tau)$, $t \in \langle 0,T \rangle$, $\tau \in \langle \tau_1,\tau_2 \rangle$, $k = 1,2,\ldots,m$, given by the following curve in the parametric form

$$\sigma^{k}_{Y}(t,\tau) = \eta^{k}_{0}(\tau) + \eta^{k}_{1}(\tau)t^1 + \ldots + \eta^{k}_{d}(\tau)t^d, \tag{37}$$

$t \in \langle 0,T \rangle,\ \tau \in \langle \tau_1,\tau_2 \rangle,\ k = 1,2,\ldots,m,$

determining them from the system of equations

$$\eta^k(\tau) = (A^k)^{-1}H^k(\tau),\ k = 1,2,\ldots m, \tag{38}$$

where

$$\eta^k(\tau) = \begin{bmatrix} \eta_0^k(\tau) \\ \eta_1^k(\tau) \\ \cdots \\ \eta_d^k(\tau) \end{bmatrix},\ H^k(\tau) = \begin{bmatrix} H_0^k(\tau) \\ H_1^k(\tau) \\ \cdots \\ H_d^k(\tau) \end{bmatrix}, \tag{39}$$

$$H_u^k(u) = \sum_{v=1}^{N^k} t_v^u \sigma'^{k}_{Y}(t_v,\tau),\ u = 0,1,\ldots,d,$$

$k = 1,2,\ldots,m.$

Having in disposal empirical values of correlation coefficients

$$\rho'^{k}_{XY}(t_1,\tau),\ \rho'^{k}_{XY}(t_2,\tau),\ \ldots,\ \rho'^{k}_{XY}(t_{N^k},\tau), \tag{40}$$

$k = 1,2,\ldots,m,\ \tau \in \langle \tau_1,\tau_2 \rangle,$

at the moments $t_1,t_2,\ldots,t_{N^k}$, it is possible to evaluate coefficients of functional correlation coefficients of random variables $X^k(t,\tau)$ and $Y^k(t,\tau)$, $t \in \langle 0,T \rangle$, $\tau \in \langle \tau_1,\tau_2 \rangle$, $k = 1,2,\ldots,m$, given by the following curve in the parametric form

$$\rho^{k}_{XY}(t,\tau) = \varsigma^{k}_{0}(\tau) + \varsigma^{k}_{1}(\tau)t^1 + \ldots + \varsigma^{k}_{e}(\tau)t^e, \tag{41}$$

$t \in \langle 0,T \rangle,\ \tau \in \langle \tau_1,\tau_2 \rangle,$

determining them from the system of equations

$$\varsigma^k(\tau) = (A^k)^{-1}I^k(\tau),\ k = 1,2,\ldots m, \tag{42}$$

where

$$\varsigma^k(\tau) = \begin{bmatrix} \varsigma_0^k(\tau) \\ \varsigma_1^k(\tau) \\ \cdots \\ \varsigma_e^k(\tau) \end{bmatrix},\ I^k(\tau) = \begin{bmatrix} I_0^k(\tau) \\ I_1^k(\tau) \\ \cdots \\ I_e^k(\tau) \end{bmatrix}, \tag{43}$$

$$I_u^k(\tau) = \sum_{\nu=1}^{N^k} t_\nu^u \rho'^k_{XY}(t_\nu, \tau), \; u = 0,1,\ldots,e,$$

$k = 1,2,\ldots,m$, $\tau \in \langle \tau_1,\tau_2 \rangle$.

Having in disposal empirical values of the radius of the oil spill domain

$$r'^k(t_1,\tau),\; r'^k(t_2,\tau),\; \ldots,\; r'^k(t_{N^k},\tau), \tag{44}$$

$k = 1,2,\ldots,m$, $\tau \in \langle \tau_1,\tau_2 \rangle$,

at the moments $t_1,t_2,\ldots,t_{N^k}$, it is possible to evaluate the coefficients of the radius of the oil spill domain $D^k(t,\tau)$, $t \in \langle 0,T \rangle$, $\tau \in \langle \tau_1,\tau_2 \rangle$, $k = 1,2,\ldots,m$, given by the following curve in the parametric form

$$r^k(t,\tau) = \zeta_0^k(\tau) + \zeta_1^k t^1(\tau) + \ldots + \zeta_f^k t^f(\tau), \tag{45}$$

$t \in \langle 0,T \rangle$, $\tau \in \langle \tau_1,\tau_2 \rangle$,

determining them from the system of equations

$$\zeta^k(\tau) = (A^k)^{-1} J^k(\tau),\; k = 1,2,\ldots m,\; \tau \in \langle \tau_1,\tau_2 \rangle, \tag{46}$$

where

$$\zeta^k(\tau) = \begin{bmatrix} \zeta_0^k(\tau) \\ \zeta_1^k(\tau) \\ \ldots \\ \zeta_f^k(\tau) \end{bmatrix},\; J^k(\tau) = \begin{bmatrix} J_0^k(\tau) \\ J_1^k(\tau) \\ \ldots \\ J_f^k(\tau) \end{bmatrix}, \tag{47}$$

$$J_u^k(\tau) = \sum_{\nu=1}^{N^k} t_\nu^u r'^k(t_\nu, \tau),$$

$u = 0,1,\ldots,f$, $k = 1,2,\ldots,m$, $\tau \in \langle \tau_1,\tau_2 \rangle$.

Thus, to determine the evaluations of the central point of oil spill drift trend curves (22) and parameters of joint density functions $\varphi_{t,\tau}^k(x,y)$, $t \in \langle 0,T \rangle$, $\tau \in \langle \tau_1,\tau_2 \rangle$, $k = 1,2,\ldots,m$, given by (19), it is necessary to perform the following steps:

- to fix the numbers

  $$N^k,\; k = 1,2,\ldots,m, \tag{48}$$

  of observations of the central point of oil spill positions,
- to fix the moments of observations

  $t_1,t_2,\ldots,t_{N^k}$, $k = 1,2,\ldots,m$,

  of the central point of oil spill positions,
- to fix the numbers of the process $(X^k(t,\tau), Y^k(t,\tau))$, $t \in \langle 0,T \rangle$, $\tau \in \langle \tau_1,\tau_2 \rangle$, $k = 1,2,\ldots,m$, realisations

  $n^k(1), n^k(2), \ldots, n^k(N^k)$, $k = 1,2,\ldots,m$,

  at the moments $t_1,t_2,\ldots,t_{N^k}$, $k = 1,2,\ldots,m$,
- to fix the central point of oil spill positions

  $(x'^k_1(t_\nu,\tau), y'^k_1(t_\nu,\tau)), (x'^k_2(t_\nu,\tau), y'^k_2(t_\nu,\tau)),\ldots,$

  $(x'^k_{n^k(\nu)}(t_\nu,\tau), y'^k_{n^k(\nu)}(t_\nu,\tau)),$

  $k = 1,2,\ldots,m$, $\tau \in \langle \tau_1,\tau_2 \rangle$,

  at each moment t_ν, $\nu = 1,2,\ldots,N^k$, $k = 1,2,\ldots,m$,
- to fix the most distant points of the oil spill domain

  P_1:

  $(x'^k_{11}(t_\nu,\tau), y'^k_{11}(t_\nu,\tau)), (x'^k_{12}(t_\nu,\tau), y'^k_{12}(t_\nu,\tau)),\ldots,$

  $(x'^k_{1n^k(\nu)}(t_\nu,\tau), y'^k_{1n^k(\nu)}(t_\nu,\tau)),$

  $k = 1,2,\ldots,m$, $\tau \in \langle \tau_1,\tau_2 \rangle$,

  P_2:

  $(x'^k_{21}(t_\nu,\tau), y'^k_{21}(t_\nu,\tau)), (x'^k_{22}(t_\nu,\tau), y'^k_{22}(t_\nu,\tau)),\ldots,$

  $(x'^k_{2n^k(\nu)}(t_\nu,\tau), y'^k_{2n^k(\nu)}(t_\nu,\tau)),$

  $k = 1,2,\ldots,m$, $\tau \in \langle \tau_1,\tau_2 \rangle$,

  at each moment t_ν, $\nu = 1,2,\ldots,N^k$, $k = 1,2,\ldots,m$,
- to evaluate the central point of oil spill mean positions, according to the formulae

$$m'^k_X(t_\nu,\tau) = \frac{1}{n^k(\nu)} \sum_{w=1}^{n^k(\nu)} x'^k_w(t_\nu, \tau), \tag{49}$$

$\nu = 1,2,\ldots,N^k$, $k = 1,2,\ldots,m$, $\tau \in \langle \tau_1,\tau_2 \rangle$,

$$m'^{k}_{Y}(t_\nu,\tau) = \frac{1}{n^k(\nu)} \sum_{w=1}^{n^k(\nu)} y'^{k}_{w}(t_\nu,\tau), \tag{50}$$

$\nu = 1,2,...,N^k$, $k = 1,2,\dots,m$, $\tau \in \langle\tau_1,\tau_2\rangle$,

- to evaluate the central point of oil spill position standard deviations according to the formulae

$$\sigma'^{k}_{X}(t_\nu,\tau)$$

$$= \sqrt{\frac{1}{n^k(\nu)} \sum_{w=1}^{n^k(\nu)} [x'^{k}_{w}(t_\nu,\tau)]^2 - [m^k_X(t_\nu,\tau)]^2}, \tag{51}$$

$\nu = 1,2,...,N^k$, $k = 1,2,\dots,m$, $\tau \in \langle\tau_1,\tau_2\rangle$,

$$\sigma'^{k}_{Y}(t_\nu,\tau)$$

$$= \sqrt{\frac{1}{n^k(\nu)} \sum_{w=1}^{n^k(\nu)} [y'^{k}_{w}(t_\nu,\tau)]^2 - [m^k_Y(t_\nu,\tau)]^2}, \tag{52}$$

$\nu = 1,2,...,N^k$, $k = 1,2,\dots,m$, $\tau \in \langle\tau_1,\tau_2\rangle$,

- to evaluate the central point of oil spill position correlation coefficients according to the formula

$$\rho'^{k}_{XY}(t_\nu,\tau)$$

$$= \frac{\frac{1}{n^k(\nu)} \sum_{w=1}^{n^k(\nu)} x'^{k}_{w}(t_\nu,\tau) y'^{k}_{w}(t_\nu,\tau) - m^k_X(t_\nu,\tau) m^k_Y(t_\nu,\tau)}{\sigma_{X_i}^{k}(t_\nu,\tau)\sigma_{Y_i}^{k}(t_\nu,\tau)},$$

$\nu = 1,2,...,N^k$, $k = 1,2,\dots,m$, $\tau \in \langle\tau_1,\tau_2\rangle$,

- to calculate the radius of the oil spill domain

$$r'^{k}(t_\nu) = \frac{1}{2n^k(\nu)} \cdot$$

$$\cdot \sum_{w=1}^{n^k(\nu)} \sqrt{[x'^{k}_{1w}(t_\nu,\tau) - x'^{k}_{2w}(t_\nu,\tau)]^2 + [y'^{k}_{1w}(t_\nu,\tau) - y'^{k}_{2w}(t_\nu,\tau)]^2},$$

$\nu = 1,2,...,N^k$, $k = 1,2,\dots,m$, $\tau \in \langle\tau_1,\tau_2\rangle$

- to find parametric forms (22) of the central point of oil spill drift trend and remaining parameters (33), (37), (41), (45) of distributions, applying the formulae (31), (34), (38), (42) and (46) respectively.

5. Modelling oil spill domain

Assuming that the experiment takes place in the time interval $(0,T\rangle$, we are interested in finding the oil spill domain $D^k(t,\tau)$, $t \in (0,T\rangle$, $\tau \in \langle\tau_1,\tau_2\rangle$, $k = 1,2,\dots,m$, such that the central point of oil spill domain is placed in it with a fixed probability p. From (Dąbrowska & Kołowrocki, 2019) we have

$$P((X^k(t,\tau), Y^k(t,\tau)) \in D^k(t,\tau))$$

$$= \iint_{D^k(t,\tau)} \varphi^k_{t,\tau}(x,y)dxdy = p, \tag{53}$$

$t \in (0,T\rangle$, $\tau \in \langle\tau_1,\tau_2\rangle$, $k = 1,2,\dots,m$,

where

$$D^k(t,\tau) = \{(x,y): \frac{1}{1-(\rho^k_{XY}(t,\tau))^2}[\frac{(x-m^k_X(t,\tau))^2}{(\sigma^k_X(t,\tau))^2}$$

$$-\ 2\rho^k_{XY}(t,\tau)\frac{(x-m^k_X(t,\tau))(y-m^k_Y(t,\tau))}{\sigma^k_X(t,\tau)\sigma^k_Y(t,\tau)}$$

$$+\ \frac{(y-m^k_Y(t,\tau))^2}{(\sigma^k_Y(t,\tau))^2}] \le c^2\}, \tag{54}$$

$t \in (0,T\rangle$, $\tau \in \langle\tau_1,\tau_2\rangle$, $k = 1,2,\dots,m$,

is the domain bounded by an ellipse being the projection on the plane $0xy$ of the curve resulting from the intersection (Figure 3) of the density function surface

$$\pi^k_1 = \{(x,y,z): z = \varphi^k_{t,\tau}(x,y), (x,y) \in R^2\}, \tag{55}$$

and the plane

$$\pi^k_2 = \{(x,y,z):$$

$$z = \frac{1}{2\pi\sigma^k_X(t,\tau)\sigma^k_Y(t,\tau)\sqrt{1-(\rho^k_{XY}(t,\tau))^2}} \exp[-\frac{1}{2}c^2], \tag{56}$$

$(x,y) \in R^2\}$, $t \in (0,T\rangle$, $\tau \in \langle\tau_1,\tau_2\rangle$, $k = 1,2,\dots,m$.

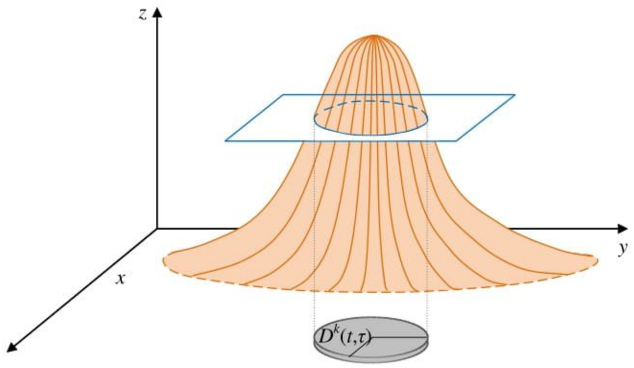

Figure 3. Domain $D^k(t,\tau)$ of integration bounded by an ellipse.

It was shown in (Dąbrowska & Kołowrocki, 2019) that the inequality in (54) holds if $c^2 = -2\ln(1-p)$.
Considering the above and the assumed in Section 3 definition of the central point of oil spill, for each fixed state k, $k \in \{1,2,\ldots,m\}$, of the process $A(t)$ and time $t \in (0,T\rangle$, we define the oil spill domain

$$\bar{D}^k(t,\tau) = \{(x,y): \frac{1}{1-(\rho_{XY}^k(t,\tau))^2}[\frac{(x-m_X^k(t,\tau))^2}{(\bar{\sigma}_X^k(t,\tau))^2} - 2\rho_{XY}^k(t,\tau)\frac{(x-m_X^k(t,\tau))(y-m_Y^k(t,\tau))}{\bar{\sigma}_X^k(t,\tau)\bar{\sigma}_Y^k(t,\tau)} + \frac{(y-m_Y^k(t,\tau))^2}{(\bar{\sigma}_Y^k(t,\tau))^2}] \le -2\cdot\ln(1-p)\}, \quad (57)$$

$t \in (0,T\rangle$, $\tau \in \langle\tau_1,\tau_2\rangle$, $k \in \{1,2,\ldots,m\}$,

where

$$\bar{\sigma}_X^k(t,\tau) = \sigma_X^k(t,\tau) + r^k(t,\tau),$$
$$\bar{\sigma}_Y^k(t,\tau) = \sigma_Y^k(,\tau\ t) + r^k(t), \quad (58)$$

$t \in (0,T\rangle$, $\tau \in \langle\tau_1,\tau_2\rangle$, $k \in \{1,2,\ldots,m\}$,

and

$r^k(t,\tau)$, $t \in (0,T\rangle$, $\tau \in \langle\tau_1,\tau_2\rangle$, $k \in \{1,2,\ldots,m\}$, (59)

is the radius of the oil spill domain $\bar{D}^k(t,\tau)$, $t \in (0,T\rangle$, $\tau \in \langle\tau_1,\tau_2\rangle$, $k \in \{1,2,\ldots,m\}$. The graph of the oil spill domain $\bar{D}^k(t,\tau)$, is given in Figure 4.

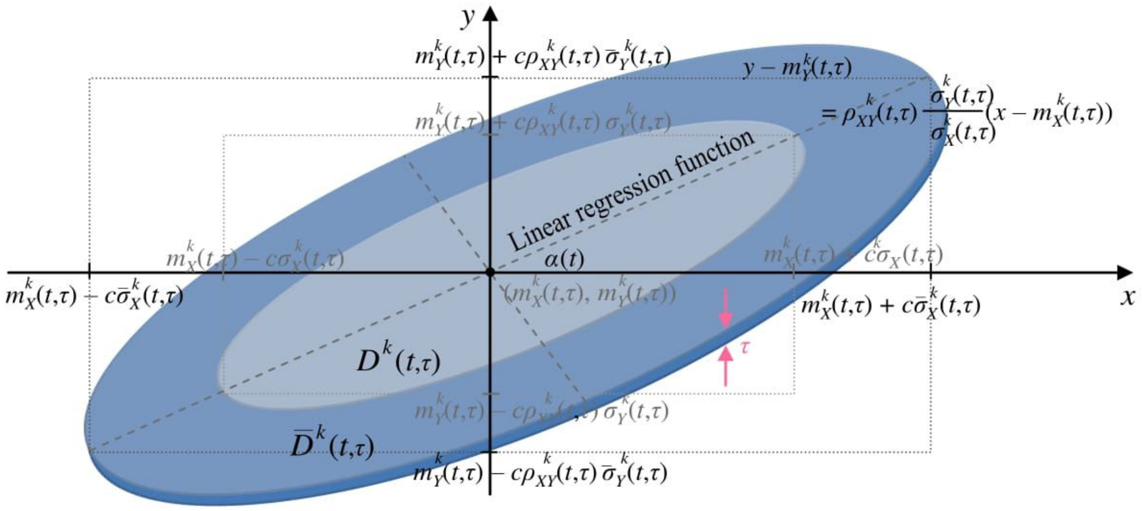

Figure 4. Oil spill domain $\bar{D}^k(t,\tau)$.

5.1. Modelling oil spill domain for fixed hydro-meteorological conditions

We suppose that the process $A(t)$ for all $t \in \langle 0,T\rangle$, is at the fixed state k, $k \in \{1,2,\ldots,m\}$. Assuming a time step Δt and a number of steps, $s \ge 1$, such that

$$(s-1)\Delta t < M_k \le s\Delta t,\ s\Delta t \le T, \quad (60)$$

where

$$M_k = E[\theta_k],\ k \in \{1,2,\ldots,m\}, \quad (61)$$

are the expected value of the process $A(t)$, $t \in \langle 0,T\rangle$, sojourn times θ_k, $k = 1,2...,m$, at the state k determined by (7), after multiple applying sequentially the procedure from Section 5, for

$$t = 1\Delta t,\ 2\Delta t, \ldots,\ s\Delta t, \tag{62}$$

we receive the following sequence of oil spill domains

$$\overline{D}^k(1\Delta t,\tau),\ \overline{D}^k(2\Delta t,\tau), \ldots, \overline{D}^k(s\Delta t,\tau), \tag{63}$$

$\tau \in \langle \tau_1,\tau_2 \rangle$.

Hence, the oil spill domain $\overline{D}^k$, $k \in \{1,2,\ldots,m\}$, is described by the sum of determined domains of the sequence (63)

$$\overline{D}^k = \bigcup_{i=1}^{s} \overline{D}(i\Delta t,\tau) = \overline{D}^k(1\Delta t,\tau) \cup \overline{D}^k(2\Delta t,\tau)$$

$$\cup \ldots \cup \overline{D}^k(s\Delta t,\tau),\ \tau \in \langle \tau_1,\tau_2 \rangle,\ k = 1,2,\ldots,m, \tag{64}$$

and illustrated in Figure 5.

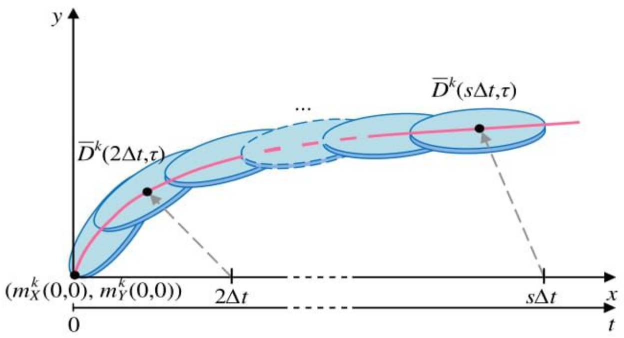

Figure 5. Oil spill domain for fixed hydro-meteorological conditions.

Remark 1. The oil spill domain $\overline{D}^k$ defined by (64) and illustrated in Figure 5 is determined for constant radius $r^k(t,\tau) = r^k$, $t \in \langle 0,T \rangle$, and constant thickness τ, $\tau \in \langle \tau_1,\tau_2 \rangle$, $k \in \{1,2,\ldots,m\}$. If the radius is not constant, we define the sequence of domains (Dąbrowska & Kołowrocki, 2019)

$$\overline{\overline{D}}^k(b\Delta t,\tau) = \bigcup_{a=1}^{b} \overline{\overline{D}}^k(a\Delta t,\tau) = \overline{\overline{D}}^k(1\Delta t,\tau)$$

$$\cup\ \overline{\overline{D}}^k(2\Delta t,\tau) \cup \ldots \cup\ \overline{\overline{D}}^k(b\Delta t,\tau),$$

$b = 1,2,\ldots,s$, $\tau \in \langle \tau_1,\tau_2 \rangle$, $k \in \{1,2,\ldots,m\}$,

where

$\overline{\overline{D}}^k(a\Delta t,\tau) := \overline{D}^k(a\Delta t,\tau)$, $a = 1,2,\ldots,b$, $b = 1,2,\ldots,s$,

$\tau \in \langle \tau_1,\tau_2 \rangle$, $k = 1,2,\ldots,m$,

defined by (57) with the following substitutions:

$$m_X^k(t,\tau) := m_X^k(a\Delta t,\tau),\ m_Y^k(t,\tau) := m_Y^k(a\Delta t,\tau),$$

$$\overline{\sigma}_X^k(t,\tau) := \overline{\overline{\sigma}}_X^k(b\Delta t,\tau) = \sigma_X^k(b\Delta t,\tau) + r^k(b\Delta t,\tau),$$

$$\overline{\sigma}_Y^k(t,\tau) := \overline{\overline{\sigma}}_Y^k(b\Delta t,\tau) = \sigma_Y^k(b\Delta t,\tau) + r^k(b\Delta t,\tau),$$

$a = 1,2,\ldots,b$, $b = 1,2,\ldots,s$, $\tau \in \langle \tau_1,\tau_2 \rangle$,

$k \in \{1,2,\ldots,m\}$.

This oil spill domain movement is illustrated in Figures 6–9.

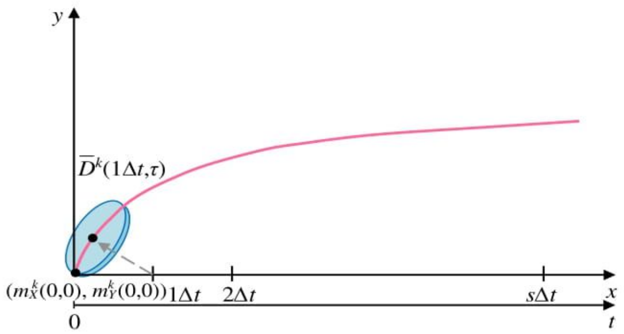

Figure 6. Oil spill domain at the time $1\Delta t$.

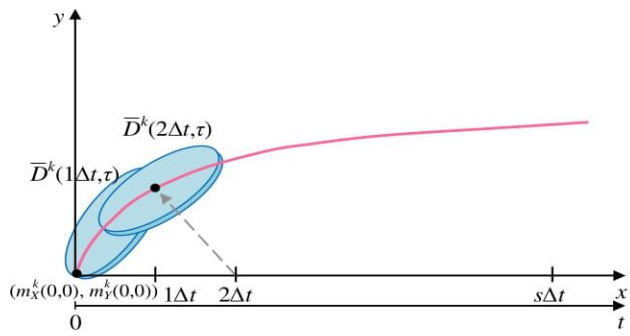

Figure 7. Oil spill domain at the time $2\Delta t$.

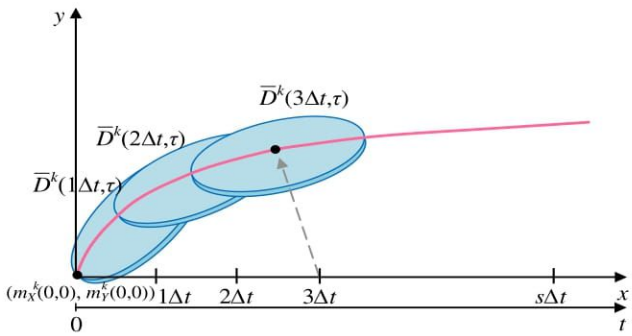

Figure 8. Oil spill domain at the time $3\Delta t$.

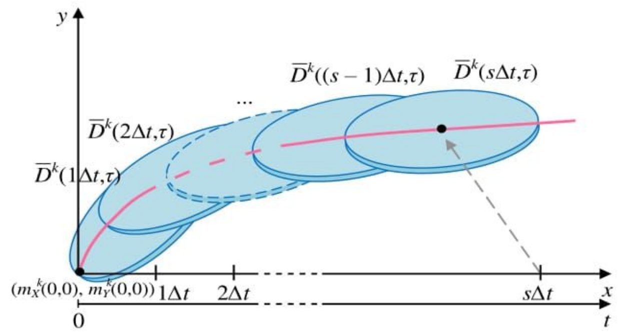

Figure 9. Oil spill domain at the time $s\Delta t$.

Remark 2. The natural generalisation of the presented approach is to assume that the thickness is changing with time, i.e. $\tau = \tau(t)$, $t \in \langle 0,T\rangle$. To consider this generalisation, we can assume the form of the equation $\tau = \tau(t)$ expressing the dependence of thickness tau on the time t from the moment of the accident and to identify it.

In the next Section, a model of oil spill domain movement impacted by changing hydro-meteorological conditions at the sea water area determination, based on a probabilistic approach, is proposed.

5.2. Modelling oil spill domain in varying hydro-meteorological conditions

Considering the varying hydro-meteorological conditions, we assume that the process of changing hydro-meteorological conditions $A(t)$ in succession takes the states

$$k_1, k_2, ..., k_{n+1}, k_i \in \{1,2,...,m\}, i = 1,2,...,n+1.$$

For a fixed step of time Δt, after multiple applying sequentially the procedure from Section 5.1 and assuming that s_i is a number of steps, $i = 1,2,...,n+1$, we have:

- for

$$t = 1\Delta t, 2\Delta t,..., s_1\Delta t, \tag{65}$$

at the process $A(t)$ state k_1,

- for

$$t = (s_1 + 1)\Delta t, (s_1 + 2)\Delta t,..., s_2\Delta t, \tag{66}$$

at the process $A(t)$ state k_2,

...

- for

$$t = (s_{n-1} + 1)\Delta t, (s_{n-1} + 2)\Delta t,..., s_n\Delta t, \tag{67}$$

at the process $A(t)$ state k_n,

we receive the following sequence of oil spill domains:

$$\overline{D}^{k_1}(1\Delta t,\tau), \overline{D}^{k_1}(2\Delta t,\tau),\ldots, \overline{D}^{k_1}(s_1\Delta t,\tau), \tag{68}$$

$$\overline{D}^{k_2}((s_1+1)\Delta t,\tau), \overline{D}^{k_2}((s_1+2)\Delta t,\tau),$$

$$\ldots, \overline{D}^{k_2}(s_2\Delta t,\tau), \tag{69}$$

...

$$\overline{D}^{k_n}((s_{n-1}+1)\Delta t,\tau), \overline{D}^{k_n}((s_{n-1}+2)\Delta t,\tau),\ldots,$$

$$\ldots, \overline{D}^{k_n}(s_n\Delta t,\tau), \tag{70}$$

where s_i, $i = 1,2,...,n$, are such that

$$(s_i - 1)\Delta t < \sum_{j=1}^{i} M_{k_j k_{j+1}} \le s_i\Delta t, \tag{71}$$

$$i = 1,2,...,n,\ s_n\Delta t \le T,$$

and

$$M_{k_j k_{j+1}} = E[\theta_{k_j k_{j+1}}], j = 1,2...,n, \tag{72}$$

are the expected value of the process $A(t)$, $t \in \langle 0,T\rangle$, conditional sojourn times $\theta_{k_j\ k_{j+1}}$, $j = 1,2...,n$ at the states k_j, upon the next state is k_{j+1}, $j = 1,2...,n$, $k_i \in \{1,2,...,m\}$, $i = 1,2...,n$, determined in Section 2.
Therefore, assuming oil spill central point drift trend

$$K^{k_i}: \begin{cases} x^{k_i} = x^{k_i}(t,\tau) \\ y^{k_i} = y^{k_i}(t,\tau), \end{cases} \quad t \in \langle 0,T\rangle, \tau \in \langle \tau_1,\tau_2\rangle,$$

at each state, we obtain the sequences of oil spill domains.
The oil spill domain $\overline{D}^{k_1,k_2,...,k_n}$, where $k_1,k_2,...,k_n \in \{1,2,...,m\}$, in the experiment is described by the sum of determined domains of the sequences, given by

$$\overline{D}^{k_1,k_2,...,k_n} = \bigcup_{i=1}^{n}\bigcup_{j=1}^{s_i-s_{i-1}} \overline{D}^{k_i}((s_{i-1}+j)\Delta t,\tau)$$

$$= [\overline{D}^{k_1}(1\Delta t,\tau) \cup \overline{D}^{k_1}(2\Delta t,\tau) \cup ... \cup \overline{D}^{k_1}(s_2\Delta t,\tau)]$$

$$\cup [\overline{D}^{k_2}\left((s_1+1)\Delta t,\tau\right) \cup \overline{D}^{k_2}\left((s_1+2)\Delta t,\tau\right)$$

$$\cup ... \cup \overline{D}^{k_2}(s_2\Delta t,\tau)]$$

...

$$\cup [\overline{D}^{k_n}\left((s_{n-1}+1)\Delta t,\tau\right) \cup \overline{D}^{k_n}\left((s_{n-1}+2)\Delta t,\tau\right)$$

$$\cup ... \cup \overline{D}^{k_n}(s_n\Delta t,\tau)], \qquad (73)$$

for $k_1, k_2, ..., k_n \in \{1,2,...,m\}$, $s_0 = 0$,

and illustrated in Figure 10.

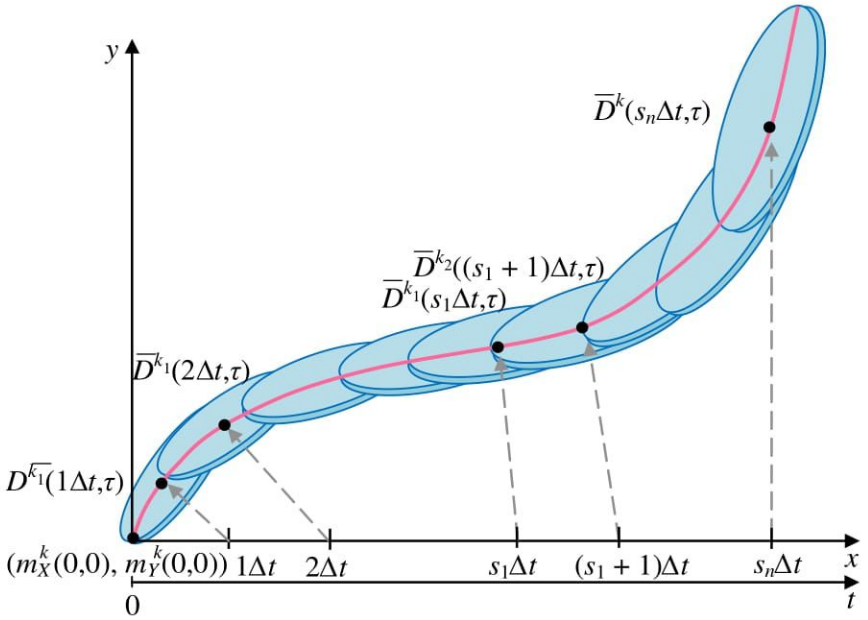

Figure 10. Oil spill domain for changing hydro-meteorological conditions.

The oil spill domain movement determined for varying radiuses is illustrated in Figure 10 in a similar way (a bit more complicated) to that given in Figures 6–9.

Remark 3. The oil spill domain $\overline{D}^{k_1,k_2,...,k_n}$ defined by (73) and illustrated in Figure 10 is determined for constant radiuses

$r^{k_i}(t,\tau) = r^{k_i}$, $t \in (0,T\rangle$, $\tau \in \langle\tau_1,\tau_2\rangle$, $k_i \in \{1,2,...,m\}$,

$i = 1,2,...,n$.

If the radiuses are not constant, we define the sequence of domains for each sate k_i, $k_i \in \{1,2,...,m\}$, $i = 1,2,...,n$, in a way similar to that described in *Remark 1* in Section 5.1, i.e. we define the sequence of domains

$$\overline{\overline{D}}^{k_1,k_2,...,k_n}(b_i\Delta t,\tau) = \bigcup_{i=1}^{n}\bigcup_{a_i=1}^{b_i} \overline{\overline{D}}^{k_i}(s_{i-1}+a_i\Delta t,\tau)$$

$$= [\overline{\overline{D}}^{k_1}(1\Delta t,\tau) \cup \overline{\overline{D}}^{k_1}(2\Delta t,\tau) \cup ... \cup \overline{\overline{D}}^{k_1}(s_1\Delta t,\tau)]$$

$$\cup [\overline{\overline{D}}^{k_2}\left((s_1+1)\Delta t,\tau\right) \cup \overline{\overline{D}}^{k_2}\left((s_1+2)\Delta t,\tau\right)$$

$$\cup ... \cup \overline{\overline{D}}^{k_2}(s_2\Delta t,\tau)]$$

...

$$\cup [\overline{\overline{D}}^{k_n}\left((s_{n-1}+1)\Delta t,\tau\right) \cup \overline{\overline{D}}^{k_n}\left((s_{n-1}+2)\Delta t,\tau\right)$$

$$\cup ... \cup \overline{\overline{D}}^{k_n}(s_n\Delta t,\tau)],$$

for $b_i = 1,2,...,s_i - s_{i-1}$, $\tau \in \langle\tau_1,\tau_2\rangle$, (74)

$k_i \in \{1,2,...,m\}$, $i = 1,2,...,n$,

where the sequences of oil spill domains

$$\overline{\overline{D}}^{k_i}(s_{i-1}+a_i\Delta t,\tau) := \overline{D}^{k}(s_{i-1}+a_i\Delta t,\tau),$$

$a_i = 1,2,...,b_i,\ b_i = 1,2,...,s_i - s_{i-1},\ \tau \in \langle\tau_1,\tau_2\rangle,$ (75)

$k_i \in \{1,2,...,m\},\ i = 1,2,...,n,$

defined by (57) with the following substitutions:

$$m_X^k(t,\tau) := m_X^{k_{i-1}}(s_{i-1}\Delta t,\tau) + m_X^{k_i}(a_i\Delta t,\tau),$$

$$m_Y^k(t,\tau) := m_Y^{k_{i-1}}(s_{i-1}\Delta t,\tau) + m_Y^{k_i}(a_i\Delta t,\tau),$$

$$\bar{\sigma}_X^k(t,\tau) := \bar{\bar{\sigma}}_X^{k_i}(s_{i-1} + a_i\Delta t,\tau)$$

$$= \sigma_X^{k_i}(s_{i-1} + a_i\Delta t,\tau) + \sum_{j=1}^{i} r^{k_j}(b_j\Delta t,\tau),$$

$$\bar{\sigma}_Y^k(t,\tau) := \bar{\bar{\sigma}}_Y^{k_i}(s_{i-1} + a_i\Delta t,\tau)$$

$$= \sigma_Y^{k_i}(s_{i-1} + a_i\Delta t,\tau) + \sum_{j=1}^{i} r^{k_j}(b_j\Delta t,\tau),$$

where

$$m_X^{k_0}(s_0\Delta t,\tau) = 0,\quad m_Y^{k_0}(s_0\Delta t,\tau) = 0,$$

for

$a_i = 1,2,...,b_i,\ b_i = 1,2,...,s_i - s_{i-1},\ \tau \in \langle\tau_1,\tau_2\rangle,$

$k_i \in \{1,2,...,m\},\ i = 1,2,...,n,\ s_0 = 0.$

Hence, according to *Remark 3*, we can obtain the sequences of oil spill domains

$$\bar{D}^{k_i}((s_{i-1}+1)\Delta t,\tau),\ \bar{D}^{k_i}((s_{i-1}+2)\Delta t,\tau),$$

$$\ldots,\ \bar{D}^{k_i}(s_i\Delta t,\tau),\ i = 1,2,...,n, \quad (76)$$

in varying hydro-meteorological conditions, where $\bar{D}^{k_i}(t,\tau)$, $\tau \in \langle\tau_1,\tau_2\rangle$, for t equals to

$(s_{i-1}+ 1)\Delta t,\ (s_{i-1}+ 2)\Delta t,\ ...,\ s_i\Delta t,$

are given by (57) with:

- expected values:

$$m_X^{k_i}(t,\tau) := m_X^{k_{i-1}}(s_{i-1}\Delta t,\tau) + m_X^{k_i}(a_i\Delta t,\tau),$$

$$m_Y^{k_i}(t,\tau) := m_Y^{k_{i-1}}(s_{i-1}\Delta t,\tau) + m_Y^{k_i}(a_i\Delta t,\tau),$$

- standard deviations:

$$\bar{\sigma}_X^{k_i}(t,\tau) := \sigma_X^{k_i}((s_{i-1} + a_i)\Delta t,\tau) + \sum_{j=1}^{i} r^{k_j}(b_j\Delta t,\tau),$$

$$\bar{\sigma}_Y^{k_i}(t,\tau) := \sigma_Y^{k_i}((s_{i-1} + a_i)\Delta t,\tau) + \sum_{j=1}^{i} r^{k_j}(b_j\Delta t,\tau),$$

with radiuses

$$r^{k_j}(t,\tau) := r^{k_j}(b_j\Delta t,\tau),\ j = 1,2,...,i,$$

- correlation coefficients $\rho_{XY}^{k_i}(t,\tau)$,

for

$a_i = 1,2,...,b_i,\ b_i = 1,2,...,s_i - s_{i-1},\ \tau \in \langle\tau_1,\tau_2\rangle,$

$i = 1,2,...,n.$

The oil spill domain in the experiment is described by the sum of domains determined by (76).

6. Conclusion

The chapter presents stochastic method of oil spill domain movement prediction dependent on the thickness of oil spill layer and on changing hydro-meteorological conditions at sea area. The author's further research is intended to be related to the methods of fixing the exact identification of the rescue action area and the ways of their quick reaching. The final effect of the research should be a model for rapid calculation of the situation at sea during a disaster resulting in the oil spills and their consequences mitigation. Practical applications of the developed models and methods, after performing scientific experiments and obtaining suitable statistical data, will be done for the different sea water areas.

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