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IMPROVING POSITIONING ACCURACY OF AIRCRAFT USING SPP METHOD IN GLONASS SYSTEM

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Abstract:

The paper presents the results of a study showing the accuracy of the determination of aircraft position coordinates based on the SPP (Single Point Positioning) solution in the GLONASS (Globalnaja Navigatsionnaya Sputnikovaya Sistema) system. For this purpose, the paper develops and implements an algorithm for the correction of position errors as parameters describing positioning accuracy. The proposed algorithm uses position error values determined for a single GNSS (Global Navigation Satellite Systems) receiver, which are joined in a linear combination to determine the positioning accuracy of the aircraft. The algorithm uses linear coefficients as an inverse function of the number of GLONASS satellites being tracked by the GNSS receiver. The developed algorithm was tested for GLONASS satellite data recorded by Topcon HiPer Pro and Javad Alpha geodetic receivers, during a flight test carried out with a Cessna 172 aircraft around the military airport in Dęblin. Navigation calculations were carried out using RTKLIB v.2.4.3 and Scilab v.6.0.0 software. On the basis of the tests carried out, it was found that for single Topcon HiPer Pro and Javad Alpha receivers, position errors were up to ±11.4 m. However, by using the position error correction algorithm for both receivers, GLONASS positioning accuracy is up to ±3.6 m. The developed algorithm reduces position errors by 60-80% for all BLh (B- Latitude, L- Longitude, h- ellipsoidal height) coordinates. The paper shows the possibility of testing and implementing the proposed mathematical algorithm for the SPP solution in a GPS (Global Positioning System) navigation system. In this case the position errors from the GPS SPP solution range from -0.9 m to +0.9 m for all BLh coordinates. The obtained results showed that application the GLONASS and GPS system in air transport is important. The algorithm used in this work can also be applied to other global GNSS navigation systems (e.g. Galileo (European Navigation Satellite system) or BeiDou (Chinese Navigation Satellite System)) in air transport and navigation.

Keywords: *GLONASS, GPS, accuracy, position errors, SPP code method*

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1. Introduction

Currently, in air navigation, we can fully use two global GNSS satellite navigation systems to determine the position of an aircraft. Namely, the American NAVSTAR GPS system and the Russian GLONASS system. While the NAVSTAR GPS system is used quite widely for the precise positioning of aircraft, the GLONASS system represents a rather interesting alternative for the aviation community. According to ICAO requirements in Annex 10 of the Chicago Convention, the GLONASS system provides 3 quality parameters for aviation GNSS positioning, i.e., continuity, availability and accuracy (ICAO, 2006). The continuity of GLONASS positioning in air navigation should not exceed the value of 2x10-4/h. On the other hand, the availability of the GLONASS constellation in aircraft positioning should be higher than 90% during the measurement. The positioning accuracy of the aircraft with GLONASS solution should be better than 12 m for horizontal coordinates and better than 25 m for the vertical component (ICAO, 2006). Other navigational and technical parameters defined by ICAO for the use of GLONASS in aviation include (ICAO, 2006):

- the time error shall not exceed 700 ns,
- pseudorange measurement error shall not exceed 18 m,
- GLONASS satellite velocity measurement error shall not exceed 0,02 m/s,
- GLONASS satellite acceleration measurement error shall not exceed 0,007 m/s2,
- resultant 3D position error of GLONASS satellite should not exceed 6 m,
- primary signal carrier frequency: L1~1.6 GHz,
- minimum elevation angle of GLONASS observations used in kinematic measurement: 5□,
- system time: UTC(SU),
- coordinate system: PZ-90.

It should be noted that the GLONASS positioning quality parameters refer to the Single Point Positioning (SPP) positioning method using C/A code measurement at L1 frequency (ICAO, 2006).

In the context of the implementation of flight experiments using the GLONASS system, the determination of the positioning accuracy parameter appears to be crucial for flight safety. Therefore, this paper presents a mathematical algorithm to improve the GLONASS positioning accuracy using the SPP code method in aeronautical navigation. The paper proposes to determine the GLONASS positioning accuracy for a system of two GNSS receivers mounted on board an aircraft during a flight test. The developed GLONASS positioning accuracy algorithm is based on a linear combination of single position error values obtained for each on-board GNSS receiver. The linear combination uses linear coefficients calculated as a function of the number of GLONASS satellites tracked by each on-board GNSS receiver.

Our most important author contributions during the development of the publication are:

- development of a mathematical algorithm to improve GLONASS positioning accuracy in air navigation,
- testing the developed mathematical algorithm on GLONASS kinematic data during a flight test with a Cessna aircraft,
- comparison of the obtained GLONASS accuracy results with the GPS solution,
- comparison of GLONASS accuracy results obtained with reference to the state-of-the-art analysis.

The paper is divided into 7 parts: 1) Introduction, 2) State-of-the-art analysis of the research topic, 3) Research problem, 4) Research method, 5) Research test, 6) Research results, 7) Discussion, 8) Conclusions. The whole publication ends with a rather extensive literature list.

2. Scientific knowledgde analysis

The research subject of the publication concerns SPP code positioning using the GLONASS navigation system. Accordingly, Chapter 2 presents an analysis of the state of the art regarding the research topic discussed in the article. Scientific research on the application of the SPP code method using the GLONASS system in aviation began actively in the 1990s. Scientific research was mainly conducted with a view to the precise positioning of GLONASS or GPS/GLONASS in air navigation. In addition, the number of GLONASS and GPS satellites tracked was identified in the research, and the geometric coefficients of the DOP were also determined. An important aspect of the research experiments was the accuracy of GLONASS positioning within the different flight operations required by ICAO. In the 1990s, the first attempts were made to use GLONASS and GPS systems in aircraft positioning together with augmentation systems, such as Inmarsat placed in geostationary orbit. Subsequent research demonstrates the feasibility of using GLONASS and GPS with WAAS corrections in the precise positioning of aircraft as part of a PA Cat I approach and landing procedure. The research was based on the development of algorithms for the RAIM module for the onboard GNSS receiver. Further research into the use of GLONASS in the approach and landing procedure is given in paper. The paper showed the use of GLONASS for different types of GNSS receivers as part of a LAAS support system for the US area. Another research topic that emerged at the time was combined GPS/GLONASS positioning and the problems of unifying the time scale and coordinate system for kinematic measurements in air navigation. This in turn had an impact on the accuracy of aircraft positioning using the GPS/GLONASS solution. The 1990s also saw the first aircraft traffic monitoring and management systems based on GPS/GLONASS positioning in navigation.

The first years of the 21st century saw problems with the GLONASS navigation system concerning the overall operation and future of the GLONASS satellite programme itself, which in turn translated directly into the poor quality of GLONASS positioning in air navigation (Grzegorzewski, 2005). With the GLONASS satellite system modernisation programme announced in 2007 (Sarkar & Bose, 2017), the effective use of GLONASS receivers (Kokorin et al., 2005; Marathe et al., 2012) in aviation, especially civil aviation, became possible (Hegarty & Chatre, 2008; Ilcev, 2011; Blanch et al., 2012). Recent years have seen a great number of research works on SPP code positioning using the GLONASS system. Noteworthy is the research on the development of algorithms for the RAIM module of a GNSS receiver using GLONASS observations (Walter et al., 2013; Bang et al., 2018). Furthermore, for the quality of GLONASS positioning in civil aviation, the use of observations from GLONASS-K generation satellites is an interesting issue, as shown in the paper (Dumas, 2011). In paper (Jin et al., 2009), the ambiguity solution was estimated using GPS/GLONASS data for purpose of aircraft attitude orientation. More recently, a scientific issue of interest using GLONASS is also the continuous monitoring and optimisation of flight trajectory with control functions during approach and landing (Skrypnik & Arefyeveva, 2017; Baburov et al., 2017; Skrypnik et al., 2018; Gorskiy et al., 2019; Skrypnik & Arefyev, 2020). Furthermore, the GLONASS system is being used together with other telemetry sensors at airports within the Russian Federation as part of the 'Digital Smart Airport' programme (Khadonova et al., 2020). Moreover, the monitoring of HPL/VPL protection level based on GPS/GLONASS solution is still very important subject in aviation area (Sayim, 2018; Pereira et al., 2021). In addition the paper (Baburov et al., 2018) analyses the composition and information characteristics of working satellite constellations in integrated GLONASS and pseudolites positioning in the Arctic region of Russia. The another paper (Ivan et al., 2023) shows the analysis of the impact of GNSS disruptions on aircraft operations at Romanian airports. A final research topic of importance related to this article is the development of algorithms for integrating GLONASS observations with GPS/Galileo/BeiDou satellite observations in air navigation (Ilyin et al., 2022).

3. Research problem

The following conclusions can be drawn from the state-of-the-art analysis presented in Chapter 2:

– in the case of the implementation of flight experiments using the GLONASS system, the leading research topic was the accuracy of aircraft positioning,
– the GLONASS system has ICAO certification and can be used widely in aviation as an alternative to GPS,
– in the last few years, there has been an increase in scientific research on the application of GLONASS code-based measurements to aircraft positioning in air navigation,
– an important element of scientific research in the aspect of GLONASS use in aviation is air traffic management and monitoring,
– in research on the application of GLONASS in aviation, the compatibility and interoperability with other GNSS navigation systems has emerged.

The literature review shows that it is necessary to develop a research method to improve the accuracy of GLONASS positioning by using appropriate mathematical algorithms. Accuracy as one of the quality parameters of GNSS satellite positioning is crucial for conducting and executing flight operations. This is obviously related to the on-board avionics of aircraft equipped with GNSS satellite receivers, including GLONASS. Hence, research is needed to develop new mathematical algorithms to ensure improvement of the accuracy parameter for the GLONASS navigation system. This is all the more important because GLONASS like GPS has official ICAO certification for aviation applications. Thus, the development of new algorithms to improve the accuracy of GLONASS positioning can also be used in GPS positioning and, in the future, Galileo and BeiDou.

4. Research method

Chapter 4 describes the test methodology with the developed mathematical algorithms for improving GLONASS positioning accuracy. The first stage of the methodology is concerned with the recording and collection of GLONASS kinematic data by GNSS receivers during test flights for preliminary processing in preset GNSS software. This stage mainly concerns the unification of GLONASS navigation and observation data in RINEX format. The second stage of the methodology concerns the processing of GLONASS data in GNSS software for the determination of aircraft position coordinates, followed by the calculation of GLONASS positioning accuracy for a single GNSS receiver. The GLONASS positioning accuracy for a single GNSS receiver was expressed in terms of position errors as a $(dB_{Rx1}, dL_{Rx1}, dh_{Rx1})$ and $(dB_{Rx2}, dL_{Rx2}, dh_{Rx2})$ parameters. In turn, the error values of position $(dB_{Rx1}, dL_{Rx1}, dh_{Rx1})$ and $(dB_{Rx2}, dL_{Rx2}, dh_{Rx2})$ are determined as follows:

$$\begin{cases} dB_{Rx1} = Bspp_{Rx1} - Brtk_{Rx1} \\ dL_{Rx1} = Lspp_{Rx1} - Lrtk_{Rx1} \\ dh_{Rx1} = hspp_{Rx1} - hrtk_{Rx1} \end{cases} \quad (1)$$

$$\begin{cases} dB_{Rx2} = Bspp_{Rx2} - Brtk_{Rx2} \\ dL_{Rx2} = Lspp_{Rx2} - Lrtk_{Rx2}\,, \\ dh_{Rx2} = hspp_{Rx2} - hrtk_{Rx2} \end{cases} \quad (2)$$

where:
$(Bspp_{Rx1}, Lspp_{Rx1}, hspp_{Rx1})$ - position of the aircraft, given in ellipsoidal coordinates BLh, derived from the SPP code method in the GLONASS system for the receiver $Rx1$ (Krasuski, et al., 2022),
$(Brtk_{Rx1}, Lrtk_{Rx1}, hrtk_{Rx1})$ - reference trajectory of the aircraft given in ellipsoidal coordinates BLh, derived from the GLONASS differential RTK-OTF technique for the receiver $Rx1$ (Grzegorzewski, 2005),
$(Bspp_{Rx2}, Lspp_{Rx2}, hspp_{Rx2})$ - position of the aircraft, given in ellipsoidal coordinates BLh, derived from the SPP code method in the GLONASS system for the receiver $Rx2$ (Krasuski, al., 2022),
$(Brtk_{Rx2}, Lrtk_{Rx2}, hrtk_{Rx2})$ - reference trajectory of the aircraft given in ellipsoidal coordinates BLh, derived from the GLONASS differential RTK-OTF technique for the receiver $Rx2$ (Grzegorzewski, 2005).

The third stage concerns the development of an algorithm to improve GLONASS positioning accuracy for two GNSS receivers. The mathematical model describing the determination of GLONASS positioning accuracy can be written as follows:

$$\begin{cases} dB = A \cdot dB_{Rx1} + B \cdot dB_{Rx2} \\ dL = A \cdot dL_{Rx1} + B \cdot dL_{Rx2} \\ dh = A \cdot dh_{Rx1} + B \cdot dh_{Rx2} \end{cases} \quad (3)$$

where:
(dB, dL, dh) - final position errors for ellipsoidal coordinates BLh, accuracy of the GLONASS positioning accuracy for a system of two on-board GNSS receivers,
$Rx1$ - GNSS satellite receiver No. 1,
$Rx2$ - GNSS satellite receiver No. 2,
A - linear coefficient for the receiver $Rx1$,
$A = \frac{1}{NS_{Rx1}}$,
NS_{Rx1} - number of GLONASS satellites being tracked by the receiver $Rx1$ (Krasuski et al., 2022),
B - linear coefficient for the receiver $Rx2$,
$B = \frac{1}{NS_{Rx2}}$,
NS_{Rx2} - number of GLONASS satellites being tracked by the receiver $Rx2$ (Krasuski et al., 2022).

By substituting the values of the linear coefficients in the form of $A = \frac{1}{NS_{Rx1}}$ and $B = \frac{1}{NS_{Rx2}}$ the final formula of equation (3) will take the following form:

$$\begin{cases} dB = \frac{1}{NS_{Rx1}} \cdot dB_{Rx1} + \frac{1}{NS_{Rx2}} \cdot dB_{Rx2} \\ dL = \frac{1}{NS_{Rx1}} \cdot dL_{Rx1} + \frac{1}{NS_{Rx2}} \cdot dL_{Rx2} \\ dh = \frac{1}{NS_{Rx1}} \cdot dh_{Rx1} + \frac{1}{NS_{Rx2}} \cdot dh_{Rx2} \end{cases} \quad (4)$$

In the example analysed, the number of GNSS receivers is 2, hence equation (4) is based on the position error values obtained for both GNSS receivers and the number of linear coefficients is also 2. In the case of a greater number of GNSS receivers used in the aerial experiment, equation (4) can be further developed with further linear coefficients and position error values for individual GNSS receivers. The algorithm described by equation (3) is only valid in practice for the number of GNSS receivers greater than or equal to 2. Finally, the described research method is shown in the flowchart in Figure 1.

5. Research test

In this Chapter the flight test and adjustment strategy was described in details. Firstly, the flight test was carried out using a Cessna 172 aircraft. Two GNSS geodetic receivers were mounted on board the aircraft: Javad Alpha (designation Rx1) and TPS Topcon HiperPro (designation Rx2), which recorded raw GLONASS observations with an interval of 1 second. The test flight took place around the EPDE military airfield in Dęblin. The duration of the flight experiment was from 09:40:19 to 10:35:03 according to GPS Time. Fig. 2 shows the horizontal trajectory of the Cessna 172 aircraft around the airfield in Dęblin. In turn, Fig. 3 shows the vertical trajectory of the flight of the Cessna 172 aircraft as a function of time. The change in flight altitude during the experiment ranged from about 145 m to about 710 m.

The GLONASS satellite data collected by both GNSS receivers were used in the stage 2) to determine the coordinates of the aircraft using the GLONASS SPP solution and to calculate the positioning accuracy for a single GNSS receiver. At this stage of the research, the calculations were performed in the RTKLIB v.2.4.3 program in the RTKPOST module (RTKLIB Website, 2022; Takasu, 2013). GLONASS observations in RINEX 2.12 format and the GLONASS navigation message acquired from both GNSS receivers were used as input data for the positioning calculations. The configuration of the calculations in the RTKPOST module was set as follows:

- positioning method: Single Point Positioning (SPP),
- elevation mask: 5°,
- ionospheric data source: broadcast navigation message,
- tropospheric correction source: Saastamoinen model,
- ephemeris and clock data source: broadcast navigation message,
- navigation system: GLONASS,
- GLONASS observations: code C/A on L1 frequency,
- calculation interval: 1 s,
- coordinate frame: ellipsoidal BLh (B - geodetic latitude, L - geodetic longitude, h - ellipsoidal height).

Step 1: recording and collection of GLONASS kinematic data by GNSS receivers during test, unification of GLONASS navigation and observation data in RINEX format

Step 2: GLONASS data adjustment processing, computation the aircraft position from single GNSS receiver, estimation the position accuracy from single GNSS receiver

Step 3: prepare the algorithm for improved the position accuracy, computation the resultant accuracy of GLONASS positioning for both GNSS receivers

Fig. 1. The flowchart of proposed research method

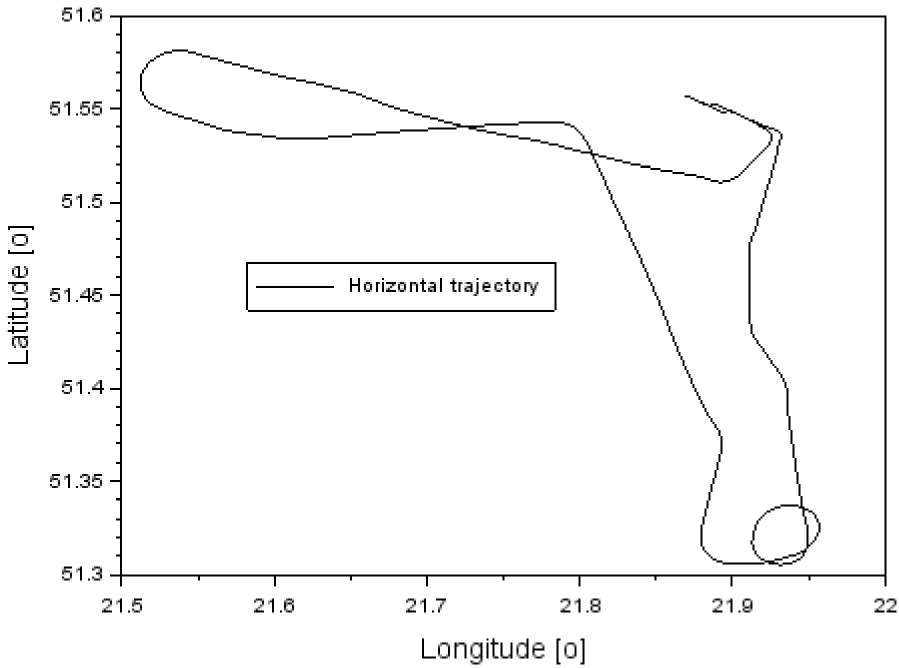

Fig. 2. Horizontal trajectory of Cessna 172 airplane

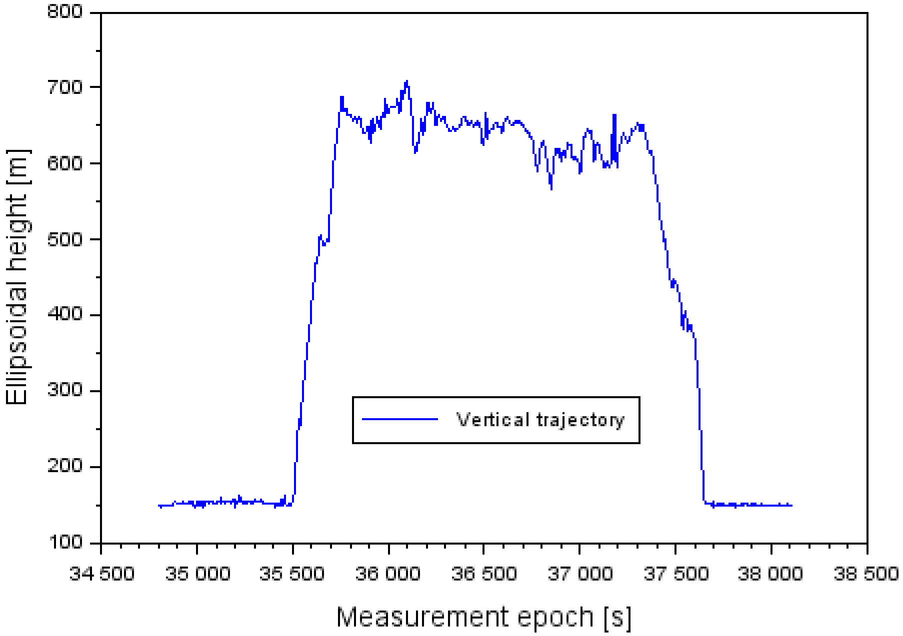

Fig. 3. Vertical trajectory of Cessna 172 airplane

For each GNSS receiver, the position of the aircraft based on GLONASS SPP solution was determined, expressed in BLh ellipsoidal coordinates. Then, for the determined position coordinates of the Cessna 172 aircraft, their accuracy was determined with respect to the differential RTK solution in OTF mode. The flight reference position derived from the RTK solution was also calculated in the RTKPOST module using the 'MOVING BASE' function. In addition, the RTK calculation used GLONASS phase observations from the GNSS reference station installed at the Polish Air Force University in Dęblin. Having the coordinates of the Cessna 172 aircraft determined using the SPP code method and the differential RTK technique, it was possible to determine the GLONASS positioning accuracy for each GNSS receiver and thus determine position errors according to formula (1-2).
The last stage 3) of the research concerned the development and implementation of the proposed mathematical algorithm (3-4) for determining the GLONASS positioning accuracy for the SPP code method for an array of two GNSS satellite receivers. For this purpose, a numerical script was developed in the Scilab v.6.0.0 programming environment (Scilab Website, 2022). In the script, the mathematical algorithm (3-4) was implemented, and the numerical and graphical analysis of the obtained test results was carried out. The test results will be presented in Chapter 6 of this paper.

6. Research results

The presentation of the test results began by showing the number of GLONASS satellites tracked by the Topcon HiPer Pro and Javad Alpha receivers, as shown in Figure 4. The number of GLONASS satellites tracked by the Javad Alpha receiver ranged from 4 to 8 during the test flight. In the same way, for the Topcon Hiper Pro receiver, the number of tracked GLONASS satellites also ranged from 4 to 8. A low number of GLONASS satellites being tracked (equal to 4) by the Javad Alpha receiver (receiver Rx1) can be seen in the initial as well as the final phase of the flight. For the Topcon HiPer Pro receiver, in the middle phase of the flight, for very short moment, only 4 tracked satellites of the GLONASS constellation can be observed. For both GNSS receivers, the average number of tracked GLONASS satellites during the experiment was above 6.

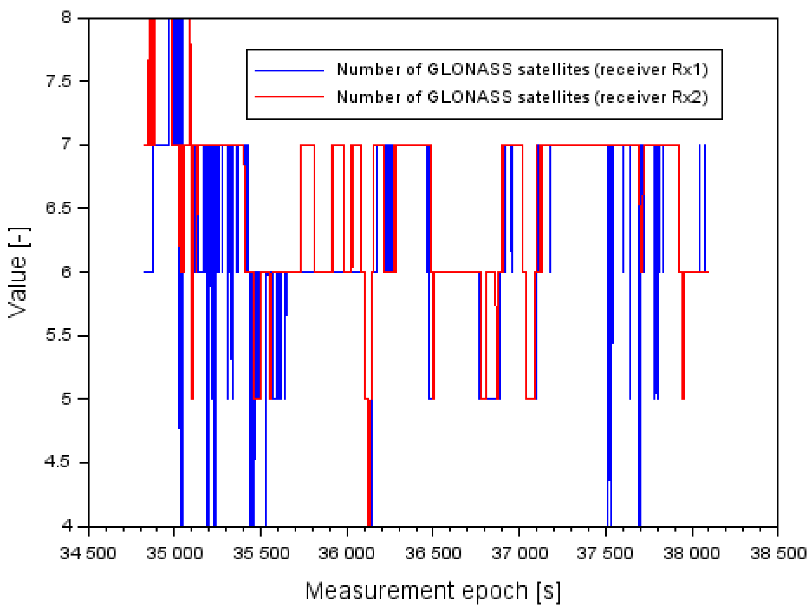

Fig. 4. Number of tracked GLONASS satellites

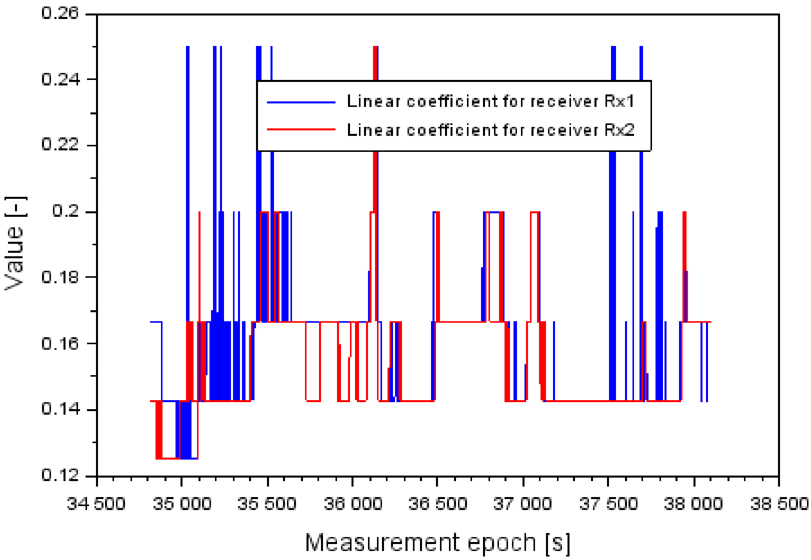

Fig. 5. Values of linear coefficients

Following this, Fig. 5 shows the results of the linear coefficients calculated according to equation (3). The values of coefficient A for receiver $Rx1$ ranged from 0.125 to 0.250. Similar values can be observed for linear coefficient B for receiver $Rx2$.

Figure 6 shows the results of GLONASS positioning accuracy in the form of position errors for the B component, determined according to equations (1-4). The GLONASS positioning accuracy for the $Rx1$ receiver was between -5.8 m and +5.3 m. In turn, the GLONASS positioning accuracy for the $Rx2$ receiver ranged from -7.9 m to +4.5 m. In contrast, the GLONASS positioning accuracy for the B coordinate, calculated according to equation (4), ranged from -1.6 m to +0.7 m. As the obtained position error results show, the use of the proposed mathematical algorithm (4) reduces position errors quite significantly. This is clearly visible in the initial phase of the flight, when the GLONASS positioning accuracy for both GNSS receivers is relatively low, and the applied algorithm (4) reduces the outlier position errors. The apparent low GLONASS positioning accuracy in the initial phase of the flight is due to the change in the number of GLONASS satellites tracked by both GNSS receivers. Also noteworthy are the large position error values in the middle phase of the flight and the improvement in position accuracy using the mathematical algorithm (4).

Figure 7 shows the results of GLONASS positioning accuracy in the form of position errors for the horizontal coordinate L, determined according to equations (1-4). The GLONASS positioning accuracy for the $Rx1$ receiver was between -3.5 m and +6.1 m. In turn, the GLONASS positioning accuracy for the $Rx2$ receiver ranged from -4.2 m to +11.4 m. The accuracy of GLONASS positioning along the L-axis, on the other hand,, calculated according to equation (4), ranged from -0.9 m to +2.8 m. In a similar way to the B-component, the application of algorithm (4) improved the positioning accuracy along the L-axis and enabled a reduction in dL position errors.

Figure 8 shows the results of GLONASS positioning accuracy in terms of position errors for the vertical coordinate h, determined according to equations (1-4). The GLONASS positioning accuracy for the $Rx1$ receiver ranged from -9.3 m to +11.4 m. Whereas the GLONASS positioning accuracy for the $Rx2$ receiver ranged from -10.9 m to +7.3 m. In contrast, the GLONASS positioning accuracy along the h-axis, calculated according to equation (4), ranged from -3.6 m to +2.6 m. Similarly to the horizontal components B and L, the application of algorithm (4) improved the positioning accuracy for the vertical component h and enabled a reduction in position errors dh.

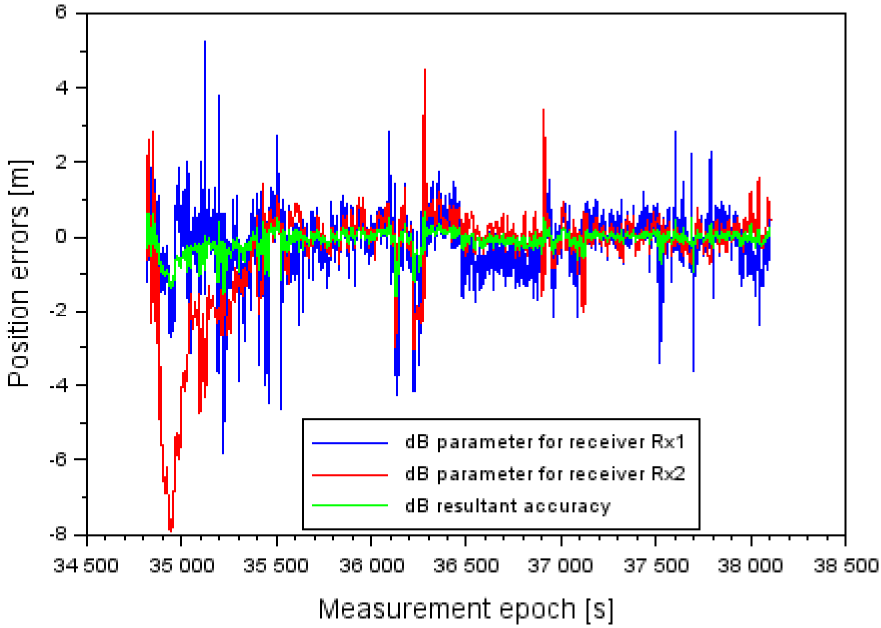

Fig. 6. Values of position errors for Latitude coordinates

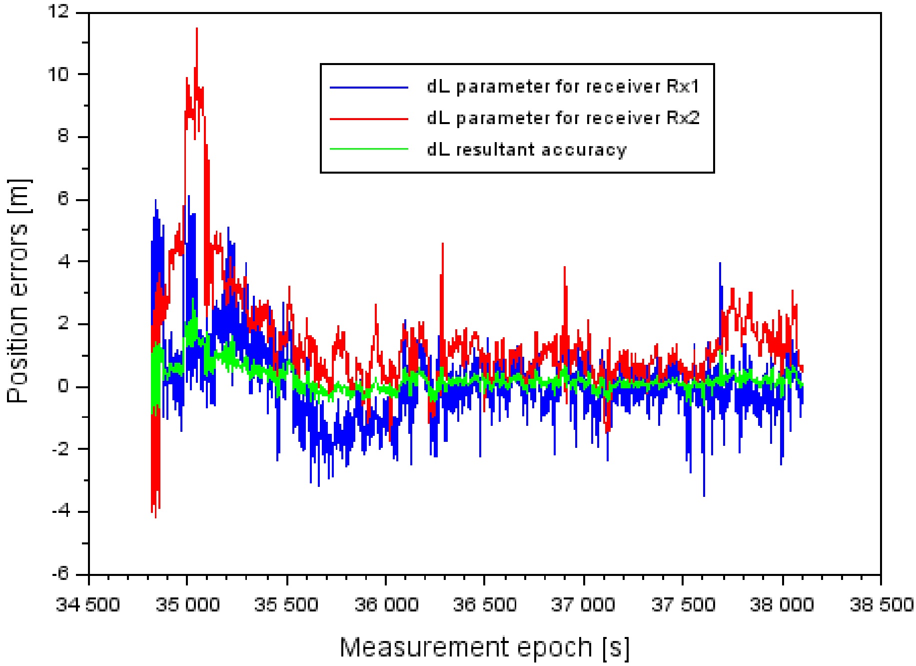

Fig. 7. Values of position errors for Longitude coordinates

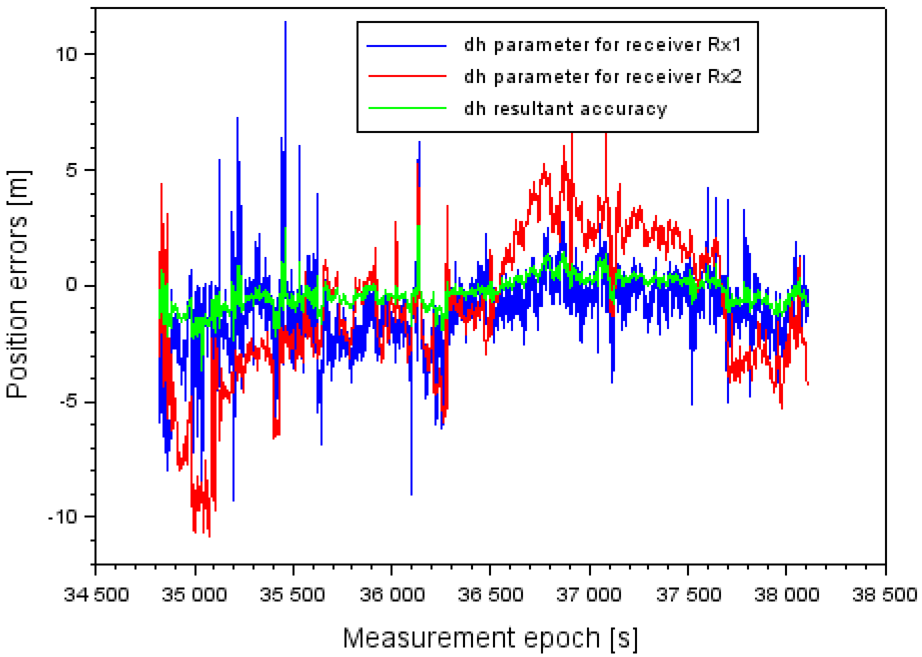

Fig. 8. Values of position errors for ellipsoidal height coordinates

The obtained position error results are summarized in Table 1 for a collective summary. From Table 1, it is apparent that the proposed algorithm (3-4) effectively reduces position errors and thus increases the accuracy of GLONASS dual-receiver positioning in air navigation. In addition, it was determined in percentage terms how the proposed algorithm (3-4) increases the accuracy of GLONASS dual-receiver positioning relative to single-receiver positioning. For the B component, position errors were reduced by 68% and 76%, respectively, compared to the $Rx1$ and $Rx2$ results. For the L component, position errors were reduced by 64% for the $Rx1$ receiver and 80% for the $Rx2$ receiver. On the other hand, for the vertical component h, position errors were reduced by 60% relative to the results obtained for the $Rx1$ receiver and 78% relative to the results obtained for the $Rx2$ receiver. On this basis, it can be concluded that the proposed algorithm (4) for determining GLONASS positioning accuracy for the GNSS receiver array is efficient in operation and effective in navigation calculations.

7. Discussion

The discussion of the results obtained is divided into 3 parts. The first part of the discussion concerns the development of the proposed mathematical model with other linear coefficients. The second part of the discussion shows the possibility of testing and implementing the proposed algorithm (4) for the SPP solution in a GPS navigation system. The third discussion strand will deal with the analysis of the obtained test results in the context of the existing state of knowledge of the research problem undertaken.

Table 1. The obtained results of position errors

Parameter	**Receiver no. 1**	**Receiver no. 2**	**Proposed algorithm (3-4)**
dB [m]	-5.8 m to +5.3 m	-7.9 m to +4.5 m	-1.6 m to +0.7 m
dL [m]	-3.5 m to +6.1 m	-4.2 m to +11.4 m	-0.9 m to +2.8 m
dh [m]	-9.3 m to +11.4 m	-10.9 m to +7.3 m	-3.6 m to +2.6 m
Parameter	**Receiver no. 1**	**Receiver no. 2**	**Proposed algorithm (3-4)**

7.1. Implementation of new values for linear coefficients in proposed mathematical algorithm

As part of the first section of the discussion, it was shown how important and crucial it is to select appropriate values of linear coefficients for the mathematical algorithm (3). Namely, in this part of the discussion, different values of the linear coefficients to the mathematical algorithm (3) were tested and then the resulting positioning accuracy was evaluated against the nominal values shown in Fig. 6-8. In this step, the linear coefficients were applied in the following form:

$$\begin{cases} A = \frac{1}{NS_{Rx1}^{0.5}} = \frac{1}{\sqrt{NS_{Rx1}}} \\ B = \frac{1}{NS_{Rx2}^{0.5}} = \frac{1}{\sqrt{NS_{Rx2}}} \end{cases} \tag{5}$$

Then the equation describing the determination of GLONASS positioning accuracy can be written as follows:

$$\begin{cases} dB = \frac{1}{\sqrt{NS_{Rx1}}} \cdot dB_{Rx1} + \frac{1}{\sqrt{NS_{Rx2}}} \cdot dB_{Rx2} \\ dL = \frac{1}{\sqrt{NS_{Rx1}}} \cdot dL_{Rx1} + \frac{1}{\sqrt{NS_{Rx2}}} \cdot dL_{Rx2} \\ dh = \frac{1}{\sqrt{NS_{Rx1}}} \cdot dh_{Rx1} + \frac{1}{\sqrt{NS_{Rx2}}} \cdot dh_{Rx2} \end{cases} \tag{6}$$

Based on equation (6), new position error values (dB, dL, dh) were calculated and are shown in Fig. 9-11. The position error values dB based on equation (6) range from -3.8 m to +1.8 m. In contrast, the dL position error values range from -2.4 m to +6.2 m. Furthermore, the position errors dh range from -8.0 m to +5.3 m. The respective position error values (dB, dL, dh) based on equation (4) were previously shown in Figure 6-8 and described in the text. Now comparing the position errors (dB, dL, dh) determined from equation (4) and equation (6), it can be seen that by using equation (4), the position errors (dB, dL, dh) have been reduced by more than 60% relative to the results obtained from equation (6). Based on this, it can be stated that the selection of appropriate values for the coefficients (A, B) is crucial for the optimal performance of the algorithm (3) to improve the accuracy of GLONASS positioning. Moreover, the proposed computational strategy with the selection of the coefficients (A, B) determined from equation (3) proved to be valid for the research method presented in this paper.

7.2. Implementation of the proposed algorithm in GPS system

The second part of the discussion shows the possibility of testing and implementing the proposed algorithm (4) for the SPP solution in the GPS navigation system. Furthermore, within this part of the discussion, the obtained position error results from the GLONASS SPP solution are compared to the position errors obtained from the GPS SPP solution. The results of the position errors calculated according to the mathematical algorithm (4) for the GLONASS and GPS navigation systems are presented in the comparative analysis.

Fig. 12 displays the results of the comparative position error analysis for the B component for the proposed mathematical algorithm (4). The results of the position errors from the GLONASS SPP solution are shown in Fig. 6 and described in the text. In contrast, the position errors from the GPS SPP solution range from -0.2 m to +0.9 m. The average value of the position errors for the B component from the GPS SPP solution was +0.1 m. Similarly, in the GLONASS SPP solution, this value for this coordinate was -0.1 m. Disregarding the sign of the position error values, it can be seen that the proposed mathematical algorithm (4) is also effective for the GPS navigation system.

Fig. 13 shows the results of the comparative position error analysis for the L component for the proposed mathematical algorithm (4). The results of the position errors from the GLONASS SPP solution are shown in Fig. 7 and described in the text. In contrast, the position errors from the GPS SPP solution range from -0.2 m to +0.3 m. The average value of the position errors for the B component from the GPS SPP solution was -0.1 m. Similarly, in the GLONASS SPP solution, this value for L coordinate was +0.2 m. As with the B component, the positioning accuracy obtained from the GPS SPP solution along the L axis is satisfactory.

Fig. 14 presents the results of the comparative position error analysis for the h component for the proposed mathematical algorithm (4). The results of the position errors from the GLONASS SPP solution are shown in Fig. 8 and described in the text. In contrast, the position errors from the GPS SPP solution range from -0.9 m to +0.5 m. The average value of the position errors for the h component from the GPS SPP solution was -0.2 m. Similarly, in the GLONASS SPP solution, this value for this coordinate was -0.3

m. As for the horizontal components B and L, the obtained positioning accuracy from the GPS SPP solution along the h axis is high.

To summarise this discussion thread, the validity of the developed mathematical algorithm (4) has also been shown for the GPS navigation system. This is particularly relevant from the point of view of independent control of navigation calculations for GLONASS and GPS systems. The verification of the performance of the mathematical algorithm (4) has therefore been demonstrated in the paper, as shown by the obtained position error results (dB, dL, dh) for the GLONASS and GPS solution.

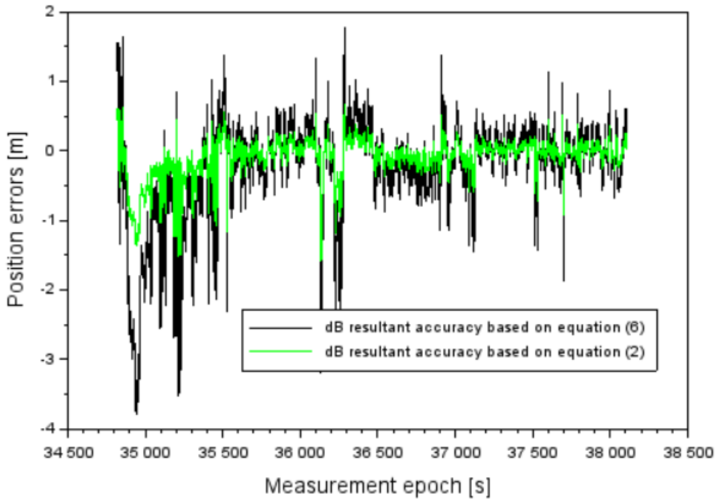

Fig. 9. Comparison of position errors for Latitude coordinates

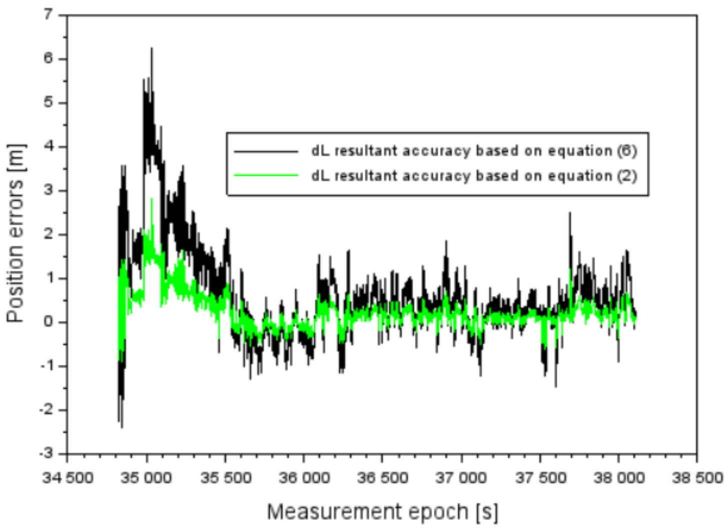

Fig. 10. Comparison of position errors for Longitude coordinates

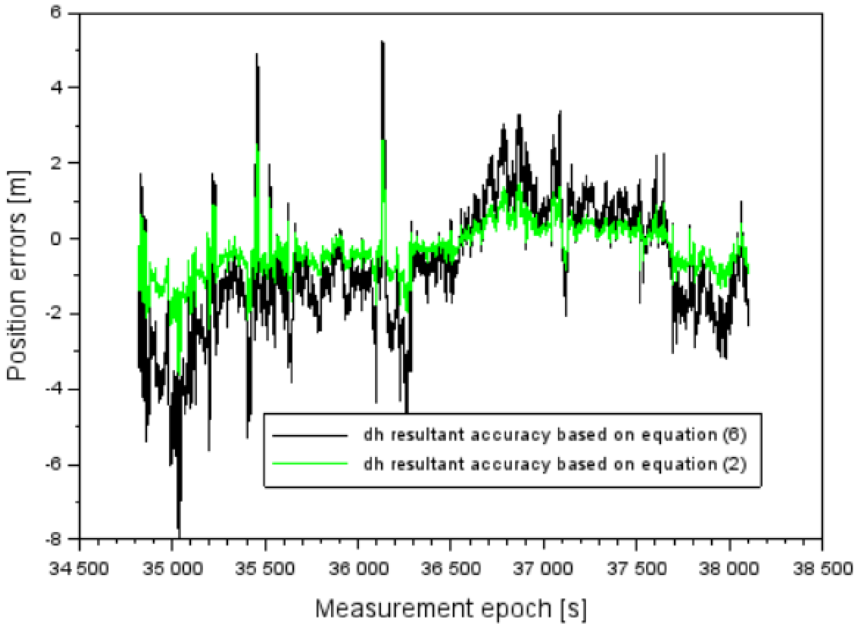

Fig. 11. Comparison of position errors for ellipsoidal height coordinates

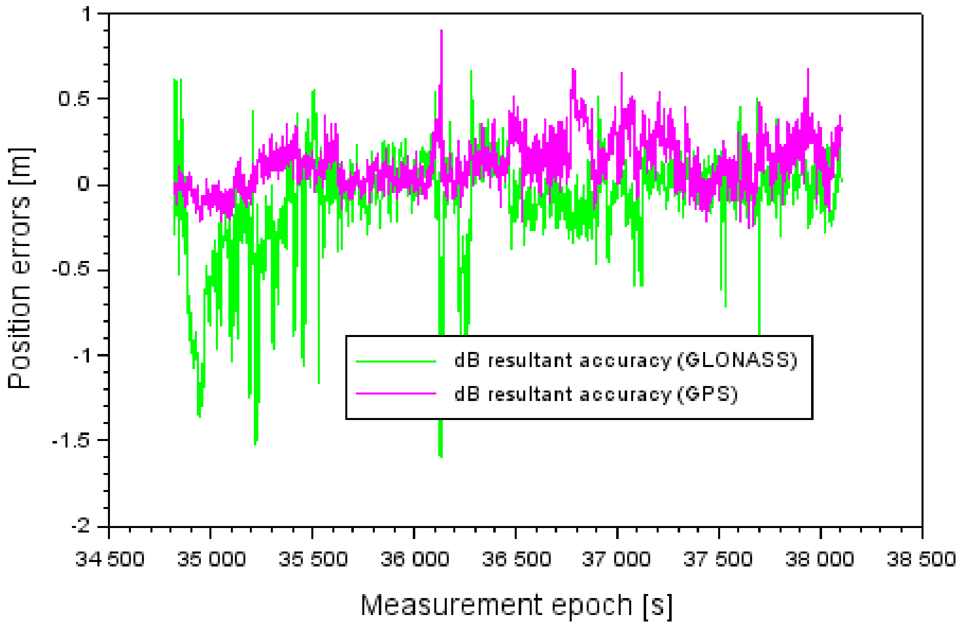

Fig. 12. Comparison of position errors along B axis based on GLONASS and GPS solution

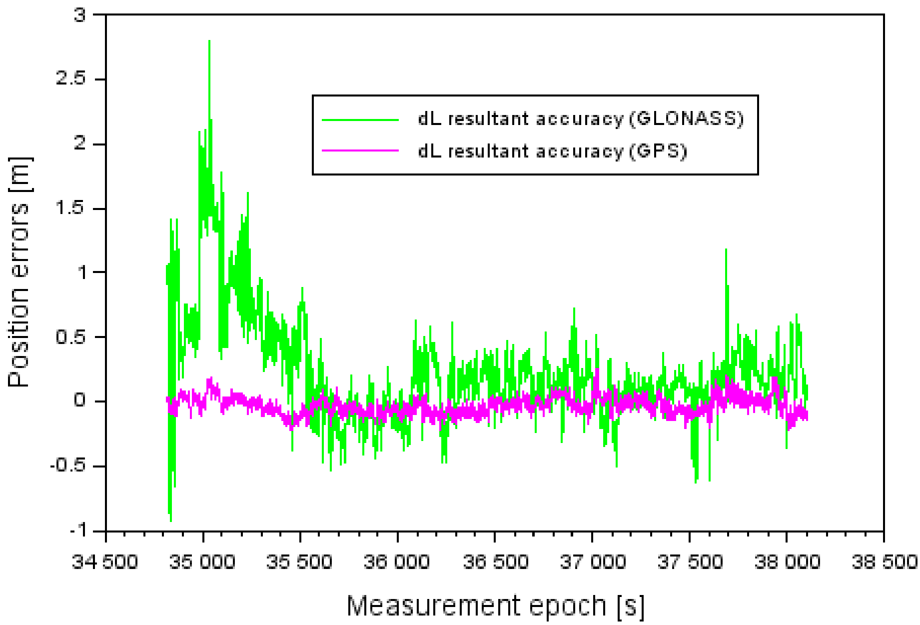

Fig. 13. Comparison of position errors along L axis based on GLONASS and GPS solution

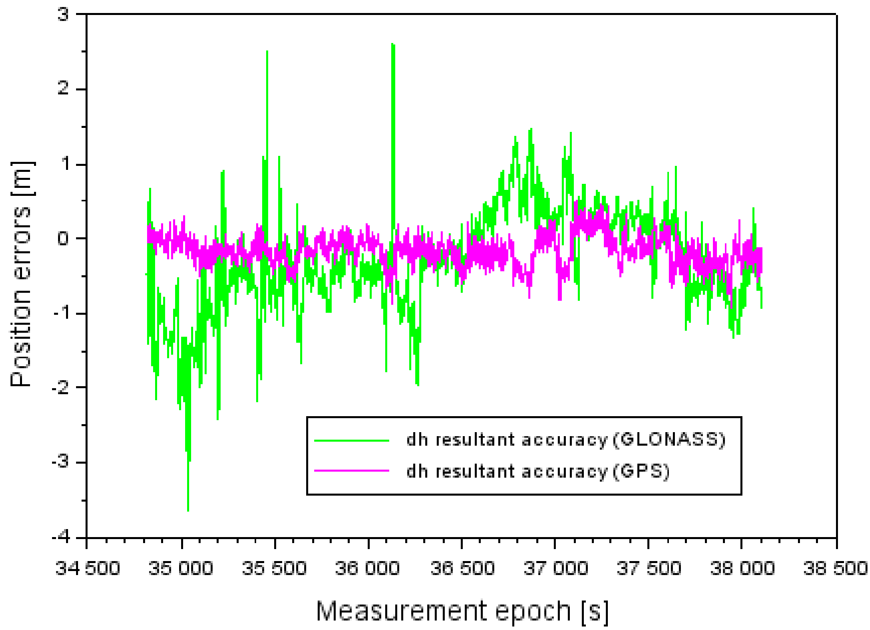

Fig. 14. Comparison of position errors along h axis based on GLONASS and GPS solution

7.3. Comparison between Research Method and Analysis of Scientific Knowledge

The final stage of the discussion concerns the comparison of the obtained test results in the context of the existing state of knowledge. By comparing the obtained GLONASS positioning accuracy results for the applied test method in relation to the analysis of the state of the art, it can be concluded that:

- in the case of the application of the GLONASS SPP solution in air navigation, it is possible to determine the accuracy of the aircraft coordinate calculation, similarly to what was done in the works (Grzegorzewski, 2005; Marathe, et al., 2012; Walter, et al., 2013),
- the GLONASS positioning accuracy obtained in this work is higher than the results published in works (Grzegorzewski, 2005; Marathe, et al., 2012; Walter, et al., 2013),
- the mathematical algorithm published in this work is effective and efficient for use in the SPP code method, which has already been shown by the results presented in the scientific publication (Krasuski et al., 2022),
- the mathematical algorithm published in the paper can also be applied to other GNSS positioning methods in aerial navigation, as presented in papers (Krasuski et al., 2022).

8. Conclusions

This paper shows the results of a study on determining the accuracy of GLONASS positioning in air navigation. The paper proposes the use of a GLONASS positioning accuracy correction model within the SPP code method. The developed algorithm is based on the position error values calculated for a single GNSS receiver and the linear coefficients used for the combination of position errors. The proposed algorithm can be used in a GLONASS SPP solution for a system of at least two on-board GNSS receivers. The linear coefficients used in the mathematical algorithm were calculated as an inverse function of the number of GLONASS satellites being tracked. The developed algorithm was tested for GLONASS data recorded by Topcon HiPer Pro and Javad Alpha geodetic receivers, during a flight test carried out with a Cessna 172 aircraft around the military airport in Dęblin. Navigation calculations were carried out using RTKLIB v.2.4.3 and Scilab v.6.0.0 software. GLONASS observation and navigation data from two onboard GNSS receivers were used in the calculations. Based on the tests performed, it was found that for the Javad Alpha receiver, the positioning accuracy for the aircraft's BLh ellipsoidal coordinates was up to ±11.4 m. For the Topcon HiPer Pro receiver, on the other hand, the positioning accuracy for ellipsoidal coordinates BLh of the aircraft is also up to ±11.4 m. In contrast, the GLONASS positioning accuracy is up to ±3.6 m thanks to the position error correction algorithm. The developed algorithm reduces position errors by 60-80% for both GNSS receivers. In addition, the paper shows the results of the developed algorithm for the SPP solution using GPS. In this case, the positioning accuracy results coincide with those obtained from the GLONASS solution. The mathematical algorithm shown in the paper can also be applied in the future for other GNSS navigation systems (e.g. Galileo or BeiDou) in air navigation.

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