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EXPLOITATION PARAMETERS OF THE TRANSVERSE SLIDER BEARINGS, LUBRICATED WITH USED OIL OF NON-NEWTONIAN PROPERTIES

Key words: slide journal bearing, non-Newtonian fluid, Cross model, load carrying capacity, apparent viscosity.

Abstract: In this paper authors present result of the experimental research and numerical calculations of the exploitation parameters of transverse slider bearings, lubricated with the used oil of non-Newtonian properties. The results of the experimental research are based on rheological examinations of the used oil in its exploitation time. These results leaded to the preparation of the dependence models of the dynamic viscosity on diverse influences, such as temperature and pressure and also shear rate and oil ageing. The effect of temperature on the dynamic viscosity was described by an exponential function. The pressure effect is modelled with a combined function, which takes into account the initial viscosity and the high pressure viscosity. For modelling of the shear rate influence, the modified Cross model was used, while oil ageing process has been related with exponential function to the exploitation period between oil changes as variable. Apparent viscosity is a product of dynamic viscosity of diverse influences. In this way, the modelled viscosity was substituted for the fundamental equations, which are momentum conservation, energy conservation, and the continuity equation. After that, received equations were non-dimensionalized, and a value of each part was estimated. Parts in an order of the same magnitude as radial clearance were neglected, as they have insignificant influence on the further numerical calculations. Using Mathcad 15 software, our own calculation procedures, the finite element method, and iterative procedure, the influence of each dependence on exploitation parameters were calculated.

Influences which are commonly taken into account in the calculations of friction nodes are the influence of temperature and pressure. Such influences as non-Newtonian properties and oil ageing are much more rarely in the literature and scientific publications. The authors have not met such a complex research yet. In this way, the prepared research conduction, which binds correlations used in the literature and the results of the experimental research, made it possible to indicate the influences of non-Newtonian properties and oil ageing on exploitation parameters of journal slider bearings. From the numerical calculations results that drop of the dynamic viscosity caused by the increase of the shear rate can cause a drop of the carrying capacity of the bearing even in 21%. A similar drop of the carrying capacity can be observed after taking into account oil ageing on the last stage of its exploitation time. When taking all of the above mentioned influences into account in the numerical calculations of the flow and exploitation parameters of the slider bearings, it appears that at the last stage of exploitation time, the drop of the carrying capacity may exceed 30%.

Parametry eksploatacyjne poprzecznych łożysk ślizgowych smarowanych przepracowanym olejem o właściwościach nienewtonowskich

Słowa kluczowe: łożyska ślizgowe, olej nienewtonowski, model Crossa, siła nośna, lepkość pozorna.

Streszczenie: W artykule przedstawiono wyniki badań doświadczalnych oraz obliczenia numeryczne parametrów przepływowych i eksploatacyjnych poprzecznych łożysk ślizgowych, smarowanych przepracowanym olejem. Wyniki badań doświadczalnych opierają się na pomiarach właściwości reologicznych oleju silnikowego w trakcie jego eksploatacji. Posłużyły one do opracowania modeli zależności lepkości dynamicznej oleju od poszczególnych wpływów, tj. temperatury i ciśnienia, a także szybkości ścinania i starzenia oleju. Wpływ temperatury na lepkość dynamiczną został opisany funkcją wykładniczą. Wpływ ciśnienia zamodelowano funkcją mieszaną, uwzględniającą lepkość początkową oraz lepkość w wysokich ciśnieniach. Do zamodelowania wpływu prędkości ścinania użyto zmodyfikowanego modelu Crossa. Natomiast wpływ starzenia uzależniono funkcją wykładniczą, uwzględniając okres pomiędzy planowanymi wymianami oleju jako zmienną. Lepkość pozorna jest iloczynem poszczególnych zmian lepkości od odpowiednich wpływów. Tak zamodelowaną lepkość podstawiono do równań podstawowych, tj. równania zachowania pędu, energii i ciągłości strugi, a następnie ubezwymiarowiono i oszacowano wielkości

poszczególnych członów. Człony rzędu wielkości luzu promieniowego pominięto jako mające nieznaczny wpływ na wyniki obliczeń numerycznych. Przy pomocy oprogramowania komputerowego Mathcad 15, własnych procedur obliczeniowych, metody różnic skończonych oraz metody kolejnych przybliżeń obliczono wpływy poszczególnych zależności na parametry eksploatacyjne poprzecznego łożyska ślizgowego.
Wpływy powszechnie uwzględniane w obliczeniach węzłów tarcia to wpływ temperatury oraz ciśnienia. Wpływy nienewtonowskie oraz starzenia o wiele rzadziej pojawiają się w literaturze fachowej oraz artykułach naukowych. Autorzy nie spotkali się dotychczas z tak kompleksowymi badaniami. Tak opracowany proces badawczy, wiążący zależności stosowane w literaturze oraz wyniki badań doświadczalnych, pozwolił na ustalenie wpływu właściwości nienewtonowskich oraz starzenia oleju na parametry eksploatacyjne poprzecznych łożysk ślizgowych. Z obliczeń numerycznych wynika, iż spadek lepkości oleju spowodowany zwiększeniem szybkości ścinania może powodować spadek siły nośnej łożyska ślizgowego nawet o 21%. Podobny spadek siły nośnej łożyska następuje w wyniku uwzględnienia starzenia oleju, na koniec jego eksploatacji w silniku. Uwzględniając wszystkie z wymienionych wpływów w obliczeniach numerycznych parametrów eksploatacyjnych i przepływowych łożysk ślizgowych, okazuje się, że w końcowym okresie eksploatacji oleju spadek siły nośnej łożyska może przekraczać 30%.

Introduction

The issue of hydrodynamic lubrication of journal bearings is a well examined and described phenomenon [1–9, 15, 16]. Most of the researchers investigate this issue in a much simpler way, i.e. using rectangular coordinate system, omitting oil changes in exploitation time, or treating engine oil as a Newtonian fluid. However, considering the effect of oil viscosity changes and its non-Newtonian properties, the influence of these changes on exploitation parameters of a slider bearing can be determined [9–14, 16]. The authors of this paper decided to model the hydrodynamic lubrication of the slider bearing as accurately as possible; therefore, based on the theoretical basis and the results of experimental research, they determined the influence of individual changes on carrying capacity and the friction force in a transverse journal bearing. In order to model hydrodynamic lubrication in a cylindrical coordinate system, the equations of conservation of momentum, energy, and continuity equation were designated from the fundamentals. A steady, non-isothermal laminar flow was assumed. These equations were non-dimensionalized and estimated in the order of magnitude. The velocity vector components and the Reynolds equation were determined from the momentum and continuity equations. The obtained equations were used for numerical calculations of hydrodynamic pressure, friction force, carrying capacity, and the friction coefficient.

1. Theoretical basics

Considering the hydrodynamic lubrication of the journal bearings, it is necessary to start from the following basic equations: momentum conservation, continuity and conservation of energy [8, 9, 15]:

$$\rho \frac{d\boldsymbol{v}}{dt} = Div\,\boldsymbol{S} \tag{1}$$

$$\frac{\partial \rho}{\partial t} + div(\rho \boldsymbol{v}) = 0 \tag{2}$$

$$div(\kappa\; grad\; T) + div\,(\boldsymbol{vS}) - \boldsymbol{v}Div\boldsymbol{S} + \Omega = \rho \frac{d(c_v T)}{dt} \tag{3}$$

where
$\boldsymbol{v}$ – oil velocity vector [$m \cdot s^{-1}$],
ρ – oil density [$kg \cdot m^{-3}$],
$\boldsymbol{S}$ – oil stress tensor with coordinates τ_{ij} for i,j=Φ,r,z [Pa],
κ – oil thermal conductivity coefficient [$W \cdot m^{-1} \cdot K^{-1}$],
T – oil temperature [K],
Ω – heat per volume unit applied to the oil from other sources [$W \cdot m^{-3}$],
c_v – specific heat of the oil at constant volume [$J \cdot kg^{-1} \cdot K^{-1}$].

In next step, the rheological model of the oil was chosen. The authors in their earlier research investigated different rheological models [1, 10, 11, 13]. One of them was Rivlin-Ericksen viscoelastic fluid model [13]. With the experimental results, in which the dynamic viscosity of the oil is dependent on shear rate, the Newtonian model and the Rivlin-Ericksen model (4), which has α and β coefficients, are rejected. These coefficients show a strong interdependence [13, 14] which makes it difficult to estimate:

$$\boldsymbol{S} = -p\boldsymbol{I} + \eta \boldsymbol{A}_1 + \alpha \boldsymbol{A}_1 \boldsymbol{A}_1 + \beta \boldsymbol{A}_2 \tag{4}$$

where
I – unity tensor,
$\mathbf{A}_1$ – first tensor of the shear rate [s^{-1}],
$\mathbf{A}_2$ – second tensor of the shear rate [s^{-2}],
α,β – experimental coefficients, which describe viscoelastic properties of the oil [$Pa \cdot s^2$],
η – dynamic viscosity coefficient [Pa·s].

The viscosity changes from shear rate for Rivlin-Ericksen model based oils differ from the nature of the viscosity changes of tested marine and automotive engine oils. For this reason, it was decided to use a first order model [5, 16] with apparent viscosity (5).

$$\boldsymbol{S} = -p\boldsymbol{I} + \eta_p \boldsymbol{A}_1 \tag{5}$$

The apparent viscosity of the oil can be described as a function of temperature, pressure, exploitation time, and shear rate $\eta_p = \eta_p(p, T, \tau, \theta)$. The authors propose to describe apparent viscosity function η_p as a product of the dimensional value h_o and dimensionless dependences of individual influences:

$$\begin{gathered}\eta_p = \eta_0 \cdot \eta_1, \eta_1 = \eta_{1p} \cdot \eta_{1T} \cdot \eta_{1t} \cdot \eta_{1\theta} \\ \eta_{1p}(\varphi, z) = \frac{\eta_{hp}}{\eta_0} - \frac{\eta_{hp}-\eta_0}{\eta_0} \cdot e^{\delta p \cdot p_0 \cdot p_1} \\ \eta_{1T}(\varphi, r, z) = a \cdot e^{-\delta_T(T-T_0)} = a \cdot e^{-Q_{Br}T_1} \\ \eta_{1\tau}(\tau) \equiv b \cdot e^{\delta_\tau \cdot \tau} = b \cdot e^{\delta_{\tau 1} \cdot \tau_1} \\ \eta_{1\theta}(\varphi, r, z) = \frac{\eta_{inf}}{\eta_0} - \frac{1-\frac{\eta_{inf}}{\eta_0}}{1-c\theta^d}\end{gathered} \tag{6}$$

where:
- η_0 – dimensional value of the dynamic viscosity in reference conditions [Pa·s];
- η_1 – dimensionless function of the viscosity changes, dependent on pressure, temperature, shear rate and exploitation time;
- η_{1p} – dimensionless function of the viscosity changes, dependent on pressure;
- $\eta_{1\tau}$ – dimensionless function of the viscosity changes, dependent on exploitation time;
- η_{1T} – dimensionless function of the viscosity changes, dependent on temperature;
- $\eta_{1\theta}$ – dimensionless function of the viscosity changes, dependent on shear rate;
- η_{hp} – dimensional value of the dynamic viscosity, after reaching maximal concentration of the molecules;
- η_{inf} – dimensional value of the minimal dynamic viscosity, by the high shear rates;
- δ_T, δ_t, δ_p – dimensionless values of the material coefficients, which include viscosity changes from temperature, pressure and exploitation time;
- $\delta_{\tau 1}$ – dimensionless value of the material coefficient, which includes viscosity changes in exploitation time;
- Q_{Br} – dimensionless coefficient of the viscosity changes in temperature T;
- T_1 – dimensionless function of the oil temperature;
- a,b – dimensionless coefficients, which include different values of the dimensional value of the characteristic viscosity η_o designated from the experimental research, by the several influences (temperature, pressure, exploitation time);
- c,d – dimensionless coefficients of the fitting curve of viscosity changes in shear rate.

The following relationship was assumed for the shear rate:

$$\theta = \frac{1}{2}\sqrt{\sum_i \boldsymbol{A}_1 \cdot \sum_j \boldsymbol{A}_1} \tag{7}$$

In order to introduce in dimensionless form and to estimate the order of magnitude of the individual parts of the equations, i.e. conservation of momentum, energy, and continuity, the following dimensional and non-dimensional symbols and criteria have been assumed [8, 9, 15]:

$$\begin{gathered}t = t_o \cdot t_1,\ r = R(1+\psi r_1),\ z = bz_1,\ h_p = \varepsilon \cdot h_{p1},\ p = p_o p_1,\ \kappa = \kappa_o \kappa_1, \\ \rho = \rho_o \cdot \rho_1,\ v_\phi = Uv_1,\ v_r = U\psi v_2,\ \varepsilon = R' - R, \\ v_z = \frac{U}{L_1} v_3,\ \psi \equiv \frac{\varepsilon}{R} \cong 10^{-3},\ L_1 \equiv \frac{b}{R},\ Re \equiv \frac{U\varepsilon\rho_o}{\eta_o}, \\ p_o \equiv \frac{RU\eta_o}{\varepsilon^2},\ Str \equiv \frac{R}{Ut_o},\ T = T_o + T_o Br T_1, \\ Gz = \frac{\varepsilon^2 \rho_o \omega \cdot c_v}{\kappa_o},\ Br \equiv \frac{U^2 \eta_o}{\kappa_o T_o},\ 0 < Q_{Br} \equiv Br T_o \delta_T < 1\end{gathered} \tag{8}$$

where:
- Br – dimensionless Brinkman number,
- Gz – Graetz number which describes forced convection,
- L_1 – dimensionless bearing length,
- R – journal radius [m],
- R' – sleeve radius [m],
- Re – Reynolds number, which describes type of the flow,
- Str – Strouhal number, which describes unsteady flow,
- U – dimensional value of the perimeter velocity [$m \cdot s^{-1}$],
- 2b – bearing length [m],
- h_p – dimensional height of the lubrication gap, which depends on relative eccentricity and axes skew [m],
- h_{p1} – dimensionless height of the lubrication gap, which depends on relative eccentricity and axes skew,
- p_o – dimensional value of the characteristic pressure [Pa],
- p_1 – dimensionless value of the hydrodynamic pressure,
- r – radial coordinate in the lubrication gap [m],
- r_1 – dimensionless radial coordinate,
- t_o – dimensional time [s],
- t_1 – dimensionless time,
- z – lengthwise coordinate [m],
- z_1 – dimensionless lengthwise coordinate,
- ε – radial clearance [m],
- k_o – dimensional value of the lubricants heat conduction coefficient [$W \cdot m^{-1} \cdot K^{-1}$],
- k_1 – dimensionless value of the lubricants heat conduction coefficient,
- λ – relative eccentricity,
- υ_o – dimensional value of the lubricants convective heat transfer coefficient [$W \cdot m^{-2} \cdot K^{-1}$],
- υ_1 – dimensionless value of the lubricants convective heat transfer coefficient,
- ρ_o – dimensional value of the lubricants density [$kg \cdot m^{-3}$],
- ρ_1 – dimensionless value of the lubricants density,
- φ – perimeter coordinate,
- ψ – dimensionless value of the relative radial clearance,
- ω – angular velocity of the bearings journal [s^{-1}].

It was assumed for the further analysis of the basic equations (1)-(3) that dimensionless heat transfer coefficient $k_1=1$, dimensionless convective heat transfer coefficient $\upsilon_1=1$, and dimensionless density $r_1=1$ are constant and independent of the temperature and pressure [3, 5]. Neglected are the inertia forces in the momentum equations – elements multiplied by $Re\cdot\psi$. The elements, which are multiplied by the Graetz number Gz, concern forced convection and are also neglected. This neglect is reasonable in the low – and medium speed bearings [6, 7]. Another assumption is steady and stationary flow, so the elements which include derivatives relative to time were neglected. Neglected were also parts in the same order as relative radial clearance $y \approx 0.001$.

2. Experimental research

Experimental tests were carried out for engine oil Castrol Professional 5W30. This oil was exploited in a car engine, and the samples were taken according to a research plan. The dynamic viscosity test was performed on a rheometer Haake Mars III, using for this purpose different modules of the rheometer, according to the investigated influences. The results of the experimental research have been presented in the following papers [2, 12]. The influence of individual physical changes on the dynamic viscosity was approximated according to the respective models (6) using Matlab software. The determined parameters for the characteristics for this oil are presented in Tab.1 and have been used in the numerical calculation.

Table 1. Oil parameters, designated from the experimental research

Investigated influence	Parameter	Designated value	Unit
Pressure	η_{hp}	0.02106	[Pa·s]
	α_p	$-5.384\cdot10^{-6}$	[-]
Temperature	δ_T	0.04818	[-]
Shear rate	η_{inf}	0.01200	[Pa·s]
	c	0.3013	[-]
	d	0.3598	[-]
Oil ageing	τ	30000*	[km]**
	δ_τ	$-7.668\cdot10^{-6}$	[-]

*considered exploitation time;

** different unit, used in exploitation time: km, mth, h.

3. Numerical calculations

Numerical calculations of hydrodynamic pressure followed by load and friction force were made for relative eccentricity from $\lambda=0.1$ to $\lambda=0.9$, dimensionless bearing length $L_1=1$, and the angle between journal and sleeve axis $\gamma=0$. For the numerical calculations, Mathcad 15 software, the successive approximations method, and our own calculation procedures were used. In order to solve Reynolds differential equation, the finite difference method was used. In the first calculation step, the constant viscosity, which is independent on individual influences, was assumed. Hydrodynamic pressure distribution and temperature in dimensionless form were designated. Results from the first calculation step were used for the designation of the viscosity changes from individual influences in the second calculation step. In the second calculation step, the hydrodynamic pressure distribution and temperature were designated with taking viscosity changes dependence on pressure, temperature, shear rate, and oil ageing into account. In the further steps, the calculation procedure has been repeated (see Fig. 1). Depending on the influence and complexity of the equation describing the apparent viscosity, the convergence of the result at the level of 0.5% has been achieved after 3–6 calculation steps.

As a result of such numerical calculations, the percentage change caused by individual influences has been determined. Changes in carrying capacity are shown in Fig. 2, while changes in friction force are shown in Fig. 3.

Another important parameter at the design stage of slider friction nodes is the end of the oil film. Its changes, depending on the investigated influence, are shown in Fig 4.

The tabular summary the values of carrying capacity and friction force, depending on the individual influences and relative eccentricities, is shown in Tab. 2.

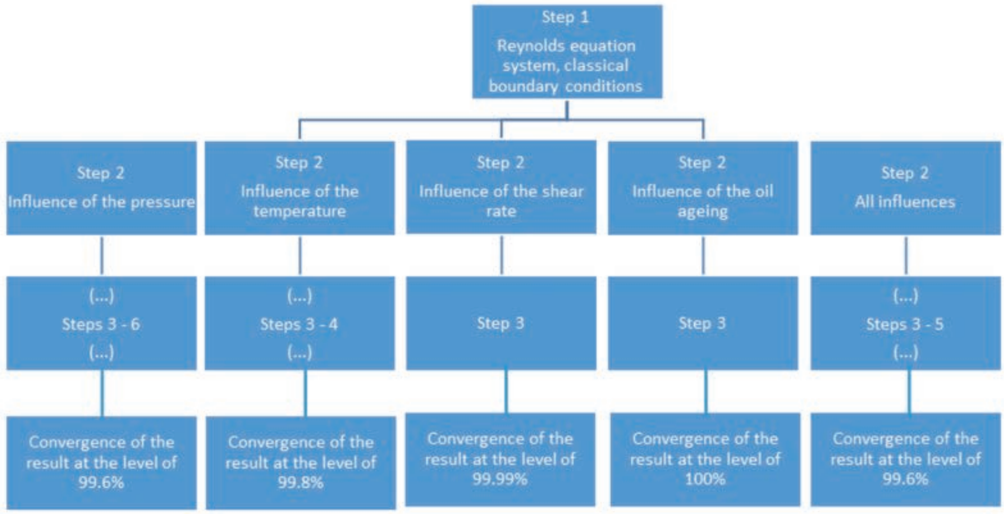

Fig. 1. Diagram of the calculation steps for the individual influences

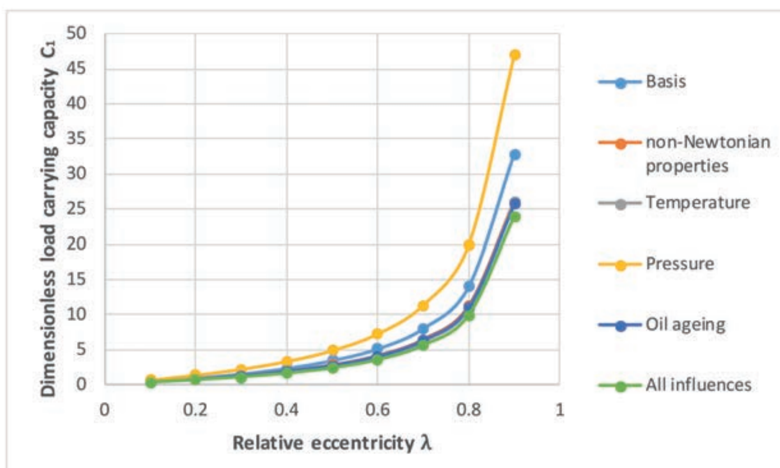

Fig. 2. Load carrying capacity changes

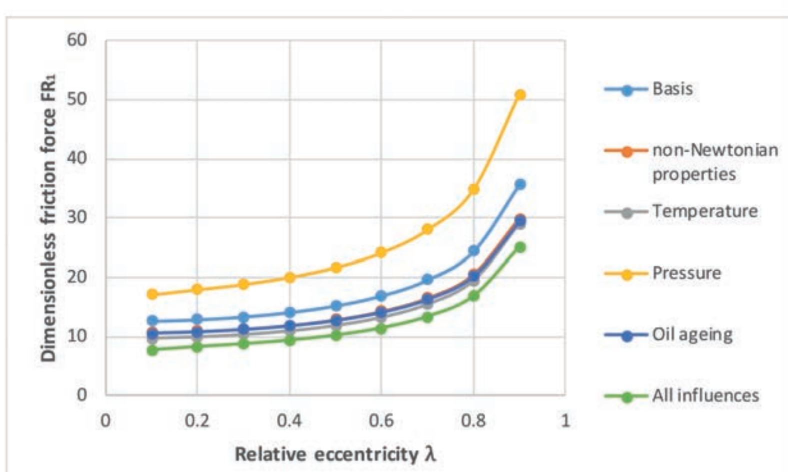

Fig. 3. Friction force changes

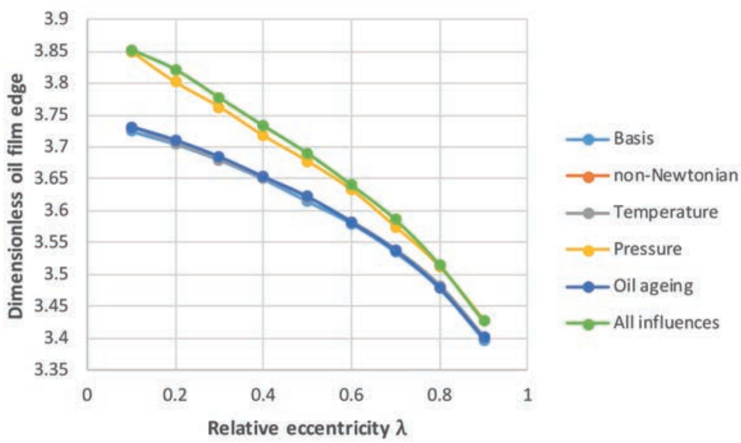

Fig. 4. End of the oil film

Table 2. Numerical values and percentage changes of the dimensionless carrying capacity and friction force as a function of relative eccentricity and type of influence on the change of viscosity of the lubricating oil

	Dimensionless load carrying capacity C_1								
Relative eccentricity	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	0.460	0.969	1.578	2.366	3.471	5.144	8.024	14.001	32.768
	0.336	0.711	1.163	1.756	2.592	3.891	6.147	10.883	25.902
	0.676	1.390	2.241	3.347	4.898	7.283	11.382	19.957	46.902
	0.371	0.781	1.271	1.905	2.786	4.138	6.437	11.223	26.017
	0.365	0.769	1.252	1.877	2.746	4.080	6.351	11.083	25.722
	0.319	0.669	1.077	1.608	2.360	3.537	5.590	9.947	23.946
	Dimensionless friction forces F_{R1}								
	12.65	12.91	13.38	14.13	15.24	16.93	19.59	24.35	35.59
	9.72	9.94	10.33	10.94	11.85	13.23	15.45	19.44	29.01
	17.16	18.02	18.89	20.07	21.72	24.15	28.00	34.79	50.86
	10.72	10.94	11.33	11.96	12.89	14.29	16.51	20.47	29.83
	10.55	10.77	11.16	11.78	12.69	14.08	16.27	20.19	29.43
	7.65	8.20	8.69	9.30	10.15	11.39	13.34	16.83	25.15
	Percent change in the dimensionless load carrying capacity C_1								
$\Delta C_1(T_1)$ [%]	-27	-27	-26	-26	-25	-24	-23	-22	-21
$\Delta C_1(p_1)$ [%]	47	43	42	41	41	42	42	43	43
$\Delta C_1(\theta)$ [%]	-19	-19	-19	-19	-20	-20	-20	-20	-21
$\Delta C_1()$ [%]	-21	-21	-21	-21	-21	-21	-21	-21	-22
$\Delta C_1(T_1, p_1, \theta)$ [%]	-31	-31	-32	-32	-32	-31	-30	-29	-27
	Percent change in the dimensionless friction forces F_{R1}								
$\Delta F_{R1}(T_1)$ [%]	-23	-23	-23	-23	-22	-22	-21	-20	-18
$\Delta F_{R1}(p_1)$ [%]	36	40	41	42	42	43	43	43	43
$\Delta F_{R1}(\theta)$ [%]	-15	-15	-15	-15	-15	-16	-16	-16	-16
$\Delta F_{R1}(\tau)$ [%]	-17	-17	-17	-17	-17	-17	-17	-17	-17
$\Delta F_{R1}(T_1, p_1, \theta)$ [%]	-39	-37	-35	-34	-33	-33	-32	-31	-29

Conclusions

As an effect of the presented studies, the results of calculations of slider friction nodes, based on real rheological models for the given type of oil were obtained. Research also includes influences that have been neglected in the past, such as the effects of oil aging and shear rates. Carrying out the above analysis at the design stage of slider friction nodes would allow engine manufacturers to take a completely new approach to engine oil requirements, optimize production costs, and monitor engine performance parameters. As presented in the example, the pressure has a significant influence on the change of carrying capacity. However, if we take into account the influences that are reducing carrying capacity, such as temperature, aging, or non-Newtonian properties, the carrying capacity reduction may exceed 30% and cause mixed friction in the friction node. This research also has a significant exploitation meaning, showing at the concrete values, what can be caused by exceeding the oil change interval, exceeding the rotational speed of the journal, or even small increasing of the temperature.

In this paper, dynamic viscosity changes in exploitation time and shear rate have been taken into account. In this case, oil ageing reduces viscosity. In real objects, an increase in viscosity with exploitation time can also be observed.

References

1. Awais M.: Applications of the Numerical Inversion of the Laplace transform to unsteady problems of the third grade fluid, Applied Mathematics and Computation, 2015, 250, 228–234.
2. Czaban A.: The Influence of temperature and shear rate on the viscosity of selected motor oils. Solid State Phenomena, 2013, 199, 188–193.
3. Dunn J. E., Rajagopal K. R.: Fluids of differential type: critical review and thermodynamic analysis, International Journal of Engineering Science, 1995, 33 (5), 689–729.
4. Ellahi R., Arshad Riaz: Analytical solutions for MHD flow in a third-grade fluid with variable viscosity, Mathematical and Computer Modelling, 2010, 52, 1783–1793.
5. Galindo-Rosales F.J., Rubio-Hernández F.J., Sevilla A.: An apparent viscosity function for shear thickening fluids. Journal of Non-Newtonian Fluid Mechanics, 2011, 166, 321–325.
6. Khan M., Nadeem S., Hayat T., Siddiqui A. M.: Unsteady Motions of a Generalized Second-Grade Fluid, Mathematical and Computer Modelling, 2005, 41, 629–637.
7. Hayat T., Hina S., Hendi A.A., Asghar S.: Effect of wall properties on the peristaltic flow of a third grade fluid in a curved channel with heat and mass transfer, International Journal of Heat and Mass Transfer, 2011, 54, 5126–5136.
8. Lang O.R., Steinhilper W., Gleitlager, Springer Verlag, Berlin-Heidelberg-New York 1978.
9. Miszczak, A., Analiza hydrodynamicznego smarowania ferrocieczą poprzecznych łożysk ślizgowych. Monografia. Fundacja Rozwoju Akademii Morskiej, Gdynia 2006.
10. Miszczak A., Sikora G.: Carrying capacity and friction forces in a transverse journal bearing, lubricated with non-Newtonian oil. Journal of Kones, 2017, Vol. 24, No. 3, pp. 203–210.
11. Miszczak A., Sikora G.: Analysis of hydrodynamic lubrication of journal bearings with oil of non-Newtonian properties. Tribologia No. 2/2017, pp. 127–138.
12. Sikora G., Miszczak A.: Viscosity in Exploitation Time Analysis of the Lubricating Oil Used in the Combustion Engine of the Personal Car . Solid State Phenomena, 2015 No. 220–221, pp. 271–276.
13. Sikora G., Miszczak A.: Równania hydrodynamicznego smarowania poprzecznych łożysk ślizgowych z uwzględnieniem starzenia się oleju. Logistyka, 2015, 4, 5649–5656.
14. Sikora G., Miszczak A.: Determination of the material coefficients of the selected lubrication oils including ageing process, Logistyka, 2015, 4, 5641–5648.
15. Wierzcholski K., Mathematical Methods of Hydrodynamic Theory of Lubrication. Politechnika Szczecińska, Monografia, nr 511, Szczecin 1993.
16. Wierzcholski K.: Teoria niekonwencjonalnego smarowania łożysk ślizgowych. Prace Naukowe Politechniki Szczecińskiej, No. 527, Szczecin 1995.