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Research paper

Statistical Approach in Determining Energy Ballistic Parameters of Gunpowder Contained in the 7.62×39 mm Ammunition Based on Measuring the Development of Pressure in a Manometric Bomb

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Abstract: Currently, most design processes are carried out with the use of specialized tools in the form of advanced computer programs. Assuming that mathematical models are formulated without significant simplifications and their numerical solution is performed with high accuracy, the quality of the problem solution is determined each time by the input data. In the case of the main problem of internal ballistics, these will be the reliably obtained energy-ballistic values of the gunpowder. In the present work, classical methodology for determining the energy-ballistic parameters of gunpowder based on the recorded pressure curve in a manometric bomb was implemented using mathematical statistics. For the assumed distribution of the probability function, which is the Student's t-distribution, confidence intervals have been established for the assumed confidence level for the values of the parameters of the distribution of ballistic features, such as the loading density power, gunpowder force, covolume and proper burn rate. These parameters were determined for three different gunpowder charges contained in the cal. 7.62x39 mm ammunition: SM FMJ 8 g, Barnaul FMJ 123gr 8.1 g ZN FMJ and STV FMJ-CIP 8gr/123grs.

Keywords: internal ballistic, pyrostatic test, manometric bomb, 7.62x39mm ammunition, mathematical statistics

1 Introduction

The main laboratory tool for testing the pyrostatic energy-ballistic properties of gunpowder is the manometric bomb [1-8]. In the listed publications, the authors point out various aspects related to pyrostatic research. As it is presented in these publications, the idea of determining the energy ballistic quantities is very similar for powders with plasma ignition or black powder ignition. Experimental tests were executed in a closed chamber with conventional ignition, as shown below in Figure 1.

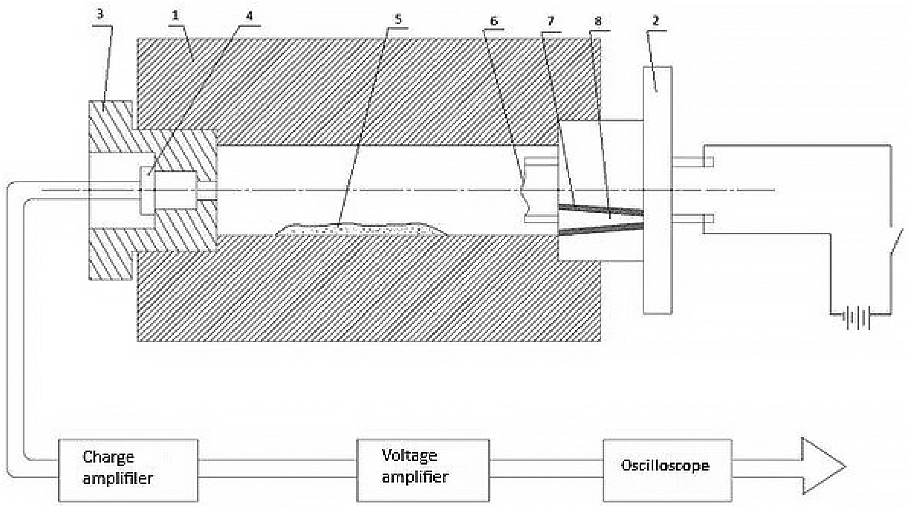


Figure 1. Manometric bomb scheme [8]: 1 – bomb body, 2 – ignition plug, 3 – measuring plug, 4 – pressure transducer, 5 – powder charge, 6 – bridge for black powder ballast ignition, 7 – insulating insert, 8 – insulated electrode

This is a tightly closed vessel with a constant construction volume W_0, enabling the combustion of materials under the so-called pyrostatic conditions, *i.e.* in the isochoric transformation for the gas-powder mixture in the substantial system [8].

The main element of the manometric bomb is its body (1), which is made of high-strength thick-walled steel pipe. Two threaded plugs with copper seals close the two internally threaded pipe ends. In the ignition plug (2) there is an insulating insert (7), insulating the electrode (8) from the mass of the bomb. The insulated electrode (8) supplies current to the glow filament connected to the second non-insulated electrode, which is integral with the mass of the bomb, thus enabling closure of the electric circuit powered by the battery. The helix of the glow wire (6) heated by the flowing current ignites the ignition charge located around it, usually made of a small amount of fine-grained black powder placed in bags.

The mass of the ignition charge is selected so that the pressure of the gaseous combustion products (GCP), called the ignition pressure (p_z), reaches a value of several MPa and ensures practically instantaneous ignition of the tested (basic) powder charge, and at the same time minimally affects the combustion of the tested powder (5).

The gaseous combustion products GCP of the black powder priming, acting thermally on the surface of the grains of the tested charge (5) of the gunpowder placed in the bomb, cause its ignition, thus initiating the spontaneous combustion process. As a result of the combustion of the tested powder inside the manometric bomb, the GCP mass resource and the amount of emitted heat increase. The GCP pressure increases from the value of the ignition pressure p_z, to the value of the maximum pressure p_m at the end of combustion. The value of the maximum pressure depends on the properties of the powder as well as on the loading density. For typical powders, the loading density is of the order of several hundred kg/m^3, and the maximum pressure of several hundred MPa. Generally, powder tests are carried out at pressures ranging from 100 to 400 MPa, *i.e.* at pressures within the range of typical barrel weapon systems. The typical shape of pressure development is shown in Figure 2.

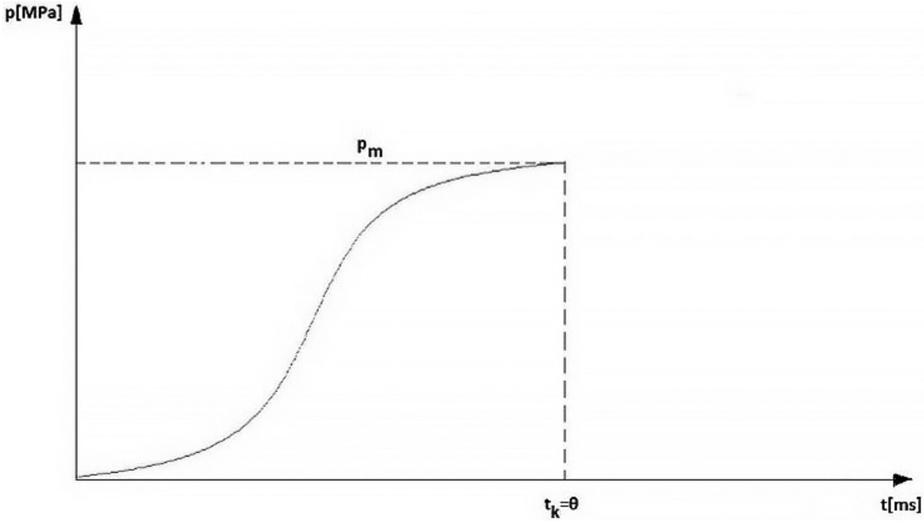


Figure 2. Illustration of GCP pressure development as a function of time in a manometric bomb [8]

For pressure measurement, piezo quartz transducers ((4) in Figure 1) are used, characterized by a linear characteristic of accumulating electric charge with increasing pressure in the manometric bomb. The electric charge from the piezo quartz converter (4) is fed to the charge amplifier and then to the voltage amplifier, from which it is directed to the oscilloscope, with memory. The change in pressure in the manometric bomb as a function of time is stored on the oscilloscope or computer. The time of the end of burning of the powder, $t_k = \theta$, is determined from the condition of zeroing of the GCP pressure derivative after time: $dp(t_k)/dt = 0$ for $t = t_k = \theta$.

Knowledge of the energy-ballistic parameters [9-17] is important when conducting digital simulations for solving the main problem of internal ballistics. The properties of the gunpowders determined in this work were input parameters for the digital tools that allowed the pressure development curve along the barrel to be determined. By accurately determining these parameters, we were able to obtain a solution to the main problem of internal ballistics with such an accuracy that will satisfactorily reflect the results obtained experimentally.

The aim of the work was to determine the energetic ballistic properties of the gunpowders contained in the 7.62x39mm ammunition and to demonstrate the statistical approach in determining these quantities.

2 Energy-Ballistic Properties of Gunpowder

The ballistic properties of the gunpowder include [8]:
- powder force (f) [J/kg],
- proper burn rate (u_1) [m/sPa],
- minimum burn rate (u_0) [m/s],
- pressure sensitivity coefficient (v),
- the liveliness of the gunpowder (θ) [1/s].

The results of the measurement of the maximum pressure (p_m) in the manometric bomb are the basis for determining the value of the powder force f and the GCP covolume (α). Due to the insignificant impact of the igniter on the energy ballistic parameters, it was omitted in the calculations. The fact that the entire experimental process is very fast, heat loss associated with heating of the chamber has also been omitted. In order to determine these values, two combustion tests of the same type of gunpowder with two different loading densities should be carried out. For the charging density $\Delta = \Delta_1$, the pressure $p_{m1} = p_m(\Delta_1)$ is obtained, and for $\Delta = \Delta_2$, the pressure $p_{m2} = p_m(\Delta_2)$. From the Noble-Abel equation it follows:

$$\begin{cases} p_{m_1} = \frac{f\Delta_1}{1-\alpha\Delta_1} \\ p_{m_2} = \frac{f\Delta_2}{1-\alpha\Delta_2} \end{cases} \tag{1}$$

From the above system of two equations, the constants of the characteristics α and f are determined for the tested powder.

The covolume α is the volume of atoms/molecules in a gaseous mass unit, which is interpreted as the excluded volume (gas volume cannot be lower than α). This is one of the terms of the van der Waals equation and is shown in Equation 2.

$$\alpha = \frac{\frac{p_{m_2}}{\Delta_2} - \frac{p_{m_1}}{\Delta_1}}{p_{m_2} - p_{m_1}} \tag{2}$$

The force of the powder is a ballistic property of the material and describes the mass density of the GCP potential energy resource, which was created during the conversion of the material into GCP in an adiabatic transformation in a space with a constant structural volume W_0. The powder force is determined according to the literature as follows:

$$f = RT_{\vartheta} = \frac{p_m}{\omega}(W_0 - \alpha\omega) \tag{3}$$

where R is the individual gas constant of GCP, T_{ϑ} is the combustion temperature, p_m is the GCP maximum pressure after combustion of material in a space with a constant structural volume W_0, *i.e.* W_0 is the construction volume of the combustion space, ω is the GCP mass resource equal to mass resource of the material.

However, using the values determined during the tests of gunpowder in a manometric bomb, it can be written as:

$$f = \frac{p_{m_1} p_{m_2}}{p_{m_2} - p_{m_1}} * \frac{\Delta_2 - \Delta_1}{\Delta_1 \Delta_2} \tag{4}$$

The proper burn rate is a component of the relationship determining the burn rate of a material in accordance with the geometric law of combustion and depends on the chemical composition of the material. The minimum burn rate u_0 is also a component of the same relationship determining the burn rate of materials and is the burn rate in a vacuum (at zero pressure). The pressure sensitivity coefficient v is the power exponent at the GCP pressure in the relationship determining the burn rate of a material and depends on the type of material. The liveness of the powder θ is the reciprocal of the end-of-burning time of the powder grains.

In the study of classical barreled weapons generally a linear law of combustion is used. Knowing that the gunpowder is from ammunition for that kind of weapon, the linear law of combustion, $u = u_1 \cdot p$, is used for the calculations. In using them, it is necessary to define the notion of post-explosion gas impulse ($I(t)$) as a function of the upper limit of the integral of the GCP pressure resulting from the combustion of the material in the manometric bomb. By writing the equation of the linear combustion law in the form:

$$\frac{de_s}{dt} = u_1 p \tag{5}$$

and then separating the variables and integrating within the limits with the variable upper limit of integration (Equation 6), Equation 7 was obtained:

$$\int_0^{e_s} de_s = u_1 \int_0^t p(t)dt \tag{6}$$

$$e_s = u_1 \int_0^t p(t)dt = u_1 I(t) \tag{7}$$

wherein:

$$I(t) = \int_0^t p(t)dt \tag{8}$$

which is the definition of the post-explosion gas impulse function. At the moment of reaching the state of complete combustion of the powder charge, *i.e.* for $t = t_k = \theta$, the post-explosion gas impulse function defines the concept of the total post-explosion gas impulse (I_k), (Equation 9), Equation 7 takes the form of Equation 10, and the total impulse of post-explosion gases reaches the value according to Equation 11.

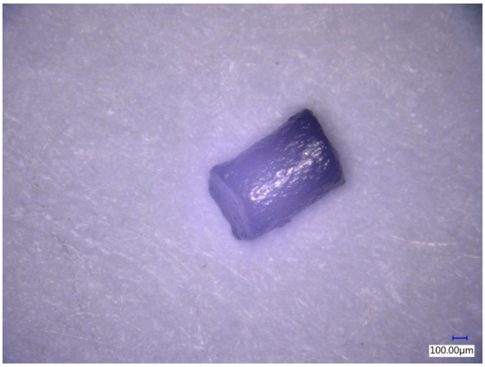
$$I_k = I(t_k) = \int_0^{t_k} p(t)dt \tag{9}$$

$$e_1 = u_1 \int_0^{t_k} p(t)dt = u_1 I_k \tag{10}$$

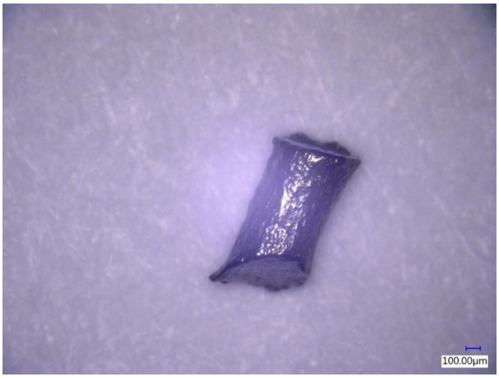
$$I_k = \frac{e_1}{u_1} \tag{11}$$

3 Materials and Test Method

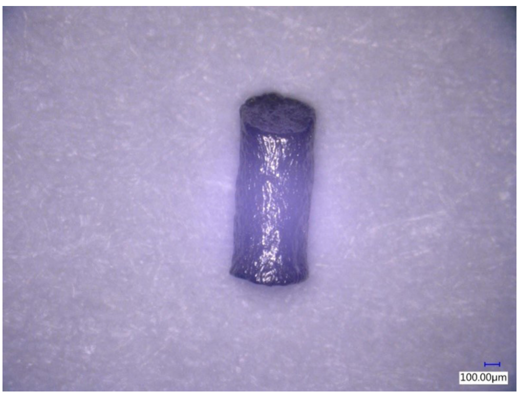
The required parameters were determined for three different gunpowder charges contained in the cal. 7.62x39 mm ammunition: SM FMJ 8 g, Barnaul FMJ 123gr 8.1 g ZN FMJ and STV FMJ-CIP 8gr/123grs. The grain shape of each type of gunpowder was cylindrical, as is shown in Figure 3.



(a)



(b)



(c)

Figure 3. Microscope pictures of grains of SM FMJ (a), Barnaul (b) and STV (c) gupowders

During the experimental tests a manometric bomb located in the Department of Mechanics and Weaponry Technology, which is part of the Institute of Mechanics and Printing at the Faculty of Mechanical and Industrial Engineering, Warsaw University of Technology, was used. The experimental tests took place at ambient temperature, 20 °C. The diagram in Figure 4 shows an example of the pressure waveform recorded by the apparatus during experimental tests.

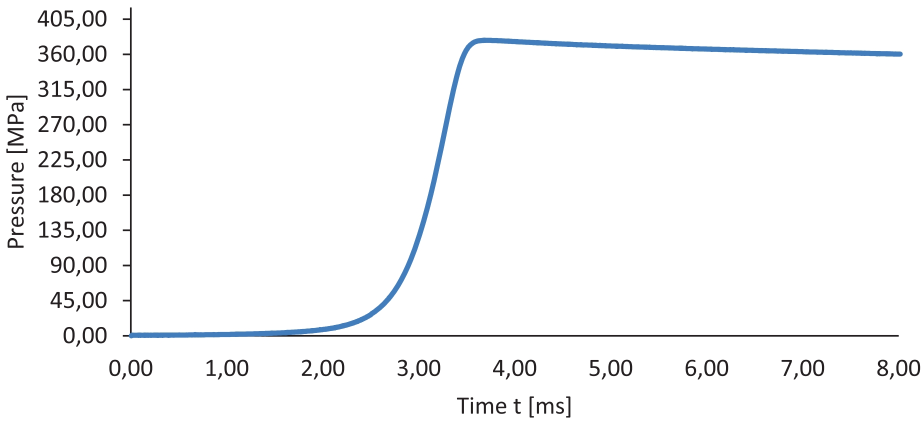


Figure 4. Pressure chart for 9 g gunpowder from SM ammunition

4 Analysis of Simple Quantities Obtained from the Measurement of the Ballistic Properties of Powders from SM, Barnaul and STV Cartridges

Taking into account the random nature of the average values of the ballistic properties of the tested powders, the probability of uncertainty in the measured result was determined. Statistical analysis of the measured simple quantity was carried out according to the following scheme [18].

The elements of the set $\mu = \{\mu_1, \mu_2, ..., \mu_{16}\}$ were, respectively, the values of the distribution parameters of the measured quantities (see Table 1).

Table 1. Description of the distribution parameters of the analysed simple quantities

Element of the set	Distribution parameter value of:
$\mu_1=W_0$	construction volume of a manometric bomb
$\mu_2=\omega_n$	mass of powder charge for low pressure
$\mu_3=\omega_w$	mass of powder charge for high pressure
$\mu_4=d$	powder grain thickness
$\mu_5=t_{zn}$	ignition timing of the powder charge for a set value of ignition pressure p_z = *const* (low pressure)
$\mu_6=t_{zw}$	ignition timing of the powder charge for a set value of ignition pressure p_z = *const* (high pressure)
$\mu_7=t_{mn}$	time to reach maximum pressure (low pressure)
$\mu_8=p_{mn}$	maximum pressure (low pressure)
$\mu_9=t_{mw}$	time to reach maximum pressure (high pressure)
$\mu_{10}=p_{mw}$	maximum pressure (high pressure)
$\mu_{11}=t_{pn}$	inflection time on the pressure curve (low pressure)
$\mu_{12}=p_{pn}$	inflection pressure on the pressure curve (low pressure)
$\mu_{13}=t_{pw}$	inflection time on the pressure curve (high pressure)
$\mu_{14}=p_{pw}$	inflection pressure on the pressure curve (high pressure)
$\mu_{15}=I_n$	total impulse of post-explosion gases (low pressure)
$\mu_{16}=I_w$	total impulse of post-explosion gases (high pressure)

The description low and high pressure means that the parameter is determined for samples containing a small and a large mass, respectively, and applies to the entire paper. The values of the distribution parameters are not random and usually their point value is unknown. The elements of the set $\overline{x} = \{\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_{16}\}$ are respectively the arithmetic means of $\overline{x}_j$ samples with *n* elements.

$$\bar{x}_j = \frac{1}{n}\sum_{i=1}^{n} x_{j,i} \qquad (12)$$

where j = 1, 2,…, 16 and i =1, 2,…, n.

The arithmetic means of the samples for the *j*-th feature may differ in their values, so they are random quantities (see Table 2). In other words, $\overline{x}_1 = \overline{W}_0$, $\overline{x}_2 = \overline{\omega}_n$, $\overline{x}_3 = \overline{\omega}_w$, *etc.*

Table 2. Relationships between elements $\bar{x}_1$, $\bar{x}_2$, …, $\bar{x}_{16}$ and the distribution parameters of the analysed simple quantities

Element of the set	Mean value	Distribution parameter
$\bar{x}_1$	$\bar{W}_0$	W_0
$\bar{x}_2$	$\bar{\omega}_n$	ω_n
$\bar{x}_3$	$\bar{\omega}_w$	ω_w
$\bar{x}_4$	$\bar{d}$	d
$\bar{x}_5$	$\bar{t}_{z_n}$	t_{z_n}
$\bar{x}_6$	$\bar{t}_{z_w}$	t_{z_w}
$\bar{x}_7$	$\bar{t}_{m_n}$	t_{m_n}
$\bar{x}_8$	$\bar{p}_{m_n}$	p_{m_n}
$\bar{x}_9$	$\bar{t}_{m_w}$	t_{m_w}
$\bar{x}_{10}$	$\bar{p}_{m_w}$	p_{m_w}
$\bar{x}_{11}$	$\bar{t}_{p_n}$	t_{p_n}
$\bar{x}_{12}$	$\bar{p}_{p_n}$	p_{p_n}
$\bar{x}_{13}$	$\bar{t}_{p_w}$	t_{p_w}
$\bar{x}_{14}$	$\bar{p}_{p_w}$	p_{p_w}
$\bar{x}_{15}$	$\bar{I}_n$	I_n
$\bar{x}_{16}$	$\bar{I}_w$	I_w

Element $x_{j,i}$ of the set $x = \{x_{1,I}, x_{2,I}, ..., x_{16,I}\}$ where $j = 1, 2, …,16$ and $i = 1, 2,…, n$ define the i-th value of the j-th feature of the measured quantity. Thus, the elements of the set x take the following values:

$$\left.\begin{array}{c} x_{1,i} = W_{0,i}\ ,\ x_{2,i} = \omega_{n,i}\ ,\ x_{3,i} = \omega_{w,i}\ ,\ x_{4,i} = d_{1,i} \\ x_{5,i} = t_{z_{n,i}},\ x_{6,i} = t_{z_{w,i}},\ x_{7,i} = t_{m_{n,i}},\ x_{8,i} = p_{m_{n,i}} \\ x_{9,i} = t_{m_{w,i}},\ x_{10,i} = p_{m_{w,i}},\ x_{11,i} = t_{p_{n,i}},\ x_{12,i} = p_{p_{n,i}} \\ x_{13,i} = t_{p_{w,i}},\ x_{14,i} = p_{p_{w,i}},\ x_{15,i} = I_{n,i},\ x_{16,i} = I_{w,i} \end{array}\right\} \quad (13)$$

The unbiased, consistent and effective statistic of the expected standard deviation for the mean value of the j-th sample of the measured quantity is determined by Equation 14.

$$S'_{\bar{x}_j} = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}\left(x_{j,i} - \bar{x}_j\right)^2} \quad (14)$$

The standard deviation of the sample mean value $\bar{x}_j$ of the j-th measured quantity $S_{\bar{x}_j}$ with a small number of measurements, $n \leq 30$, is determined by using the t-Student probability distribution and is equal to the product of the quantile $t_{\alpha,n-1}$ of the t-Student distribution and the expected value of the standard deviation of the values of the sample means of the j-th measured quantity (:), according to Equation 15.

$$S_{\bar{x}_j} = t_{\alpha,n-1} S'_{\bar{x}_j} \tag{15}$$

where $t_{\alpha,n-1}$ is the quantile value from the Student's t-distribution tables for $(n-1)$ degrees of freedom fulfilling the condition presented in Equation 16.

$$p(\bar{x}_j - t_{\alpha,n-1} S'_{\bar{x}_j} < \mu_j < \bar{x}_j + t_{\alpha,n-1} S'_{\bar{x}_j}) = 1 - \alpha \tag{16}$$

The quantity $(1 - \alpha)$ is called the confidence level, in other words the probability that the value of the distribution parameter μ_j of the j-th feature of the measured value is within the confidence interval of the measurement limited by the deviation from the sample mean value , the positive and negative values of the standard deviation of the Student's t-distribution for $(n-1)$ degrees of freedom.

$$\bar{x}_j - t_{\alpha,n-1} S'_{\bar{x}_j} < \mu_j < \bar{x}_j + t_{\alpha,n-1} S'_{\bar{x}_j} \tag{17}$$

or

$$\mu_j = \bar{x}_j \pm t_{\alpha,n-1} S'_{\bar{x}_j}$$

On the other hand, for a confidence interval equal to three standard deviations $\sigma(\bar{x}_j) = S_{\bar{x}_j}$, the value of the distribution parameter μ_j, j-th feature of the measured value with probability $(1-\alpha) = 0.997$ will be included in the range $\mu_j = \bar{x}_j \pm 3\sigma(\bar{x})$.

According to the results of the measurements of the ballistic properties of the tested powders in the manometric bomb measuring set for simple quantities and Equations 12-15 quoted above, the results of the calculations of their mean values, standard deviations and uncertainty of the measurement for all simple values, the values obtained for the distribution parameters of the individual characteristics of the measured quantities for the tested powders from the 7.62x39 mm cartridges are presented in Table 3.

Table 3. Calculation results of simple quantities for the tested SM, Barnaul and STV powders in 7.62x39 mm ammunition

No. of distribution parameter	Units	SM FMJ	Barnaul ZN FMJ	STV FMJ
$\mu_1 = W_0$	[m³]	$(31.8 \pm 0.72) \cdot 10^6$		
$\mu_2 = \omega_n$	[g]	4.0003 ±0.0006	4.0004 ±0.0003	
$\mu_3 = \omega_w$	[g]	9.0003 ±0.0006	9.0005 ±0.0003	
$\mu_4 = d$	[μm]	607.09 ±17.43	556.19 ±28.08	511.45 ±23.82
$\mu_5 = t_{zn}$	[ms]	2.285 ±0.378	1.175 ±0.267	0.574 ±0.279
$\mu_6 = t_{zw}$	[ms]	0.660 ±0.279	2.161 ±0.183	1.813 ±0.090
$\mu_7 = t_{mn}$	[ms]	8.022 ±0.144	8.022 ±0.144	6.674 ±0.342
$\mu_8 = p_{mn}$	[MPa]	135.376 ±4.578	134.351 ±0.699	133.008 ±0.762
$\mu_9 = t_{mw}$	[ms]	3.714 ±0.099	4.772 ±0.054	4.832 ±0.105
$\mu_{10} = p_{mw}$	[MPa]	379.334 ±4.365	381.201 ±2.019	378.613 ±1.731
$\mu_{11} = t_{pn}$	[ms]	6.790 ±0.105	5.300 ±0.000	5.450 ±0.114
$\mu_{12} = p_{pn}$	[MPa]	89.111 ±2.568	88.843 ±2.340	88.037 ±1.683
$\mu_{13} = t_{pw}$	[ms]	3.200 ±0.075	4.340 ±0.036	4.390 ±0.036
$\mu_{14} = p_{pw}$	[MPa]	221.985 ±14.169	223.779 ±14.853	230.664 ±17.904
$\mu_{15} = I_n$	[MPa·ms]	236.763 ±16.317	212.016 ±31.104	230.578 ±37.098
$\mu_{16} = I_w$	[MPa·ms]	250.504 ±18.675	216.078 ±1.992	223.429 ±34.083

As can be seen in Table 3, the relative error in determining the volume of the chamber and the weight of the powder is very small, while the relative error from recording the maximum pressure exceeds 3% in some cases. This may be caused by the character of the measurement. Determination of the gunpowder's weight or chamber's volume is a static measurement, whereas recording pressure is a very dynamic process.

5 Analysis of Complex Quantities as a Function of the Mean Values of Simple Quantities

5.1 General approach in the determination of complex quantities

Taking into account the statistical nature of the average values of the ballistic properties of the tested powders, the probability of uncertainty in the measured result was determined. Statistical analysis of the measurement of the examined complex quantities was carried out according to the following scheme [18].

The set of examined complex quantities is defined by the symbol $\overline{F}_z = \{\overline{F}_{z1}, \overline{F}_{z2}, \overline{F}_{z3}, \overline{F}_{z4}, \overline{F}_{z5}\}$, where $\overline{F}_{zj}$ is *j-th* element of the set $\overline{F}_z$ and $j = 1, 2, \ldots, 5$. The elements of the set $\overline{F}_z$ are complex quantities, which are functions of the mean values of simple quantities and take the following forms:

$$\left.\begin{aligned} \overline{F}_{z_1} &= \overline{\Delta_w} = \frac{\overline{\omega}_w}{\overline{W_0}} \\ \overline{F}_{z_2} &= \overline{\Delta_n} = \frac{\overline{\omega}_n}{\overline{W_0}} \\ \overline{F}_{z_3} &= \bar{f} = \frac{\overline{p_{m_W}}\,\overline{p_{m_n}}}{(\overline{p_{m_W}} - \overline{p_{m_n}})} \frac{(\bar{\Delta}_w - \bar{\Delta}_n)}{\bar{\Delta}_w \bar{\Delta}_n} \\ \overline{F}_{z_4} &= \bar{\alpha} = \frac{\frac{\overline{p_{m_W}}}{\bar{\Delta}_w} - \frac{\overline{p_{m_n}}}{\bar{\Delta}_n}}{\overline{p_{m_W}} - \overline{p_{m_n}}} \\ \overline{F}_{z_5} &= \overline{u_1} = \frac{\overline{e_1}}{\bar{I}_{m_{sr}}} \end{aligned}\right\} \tag{18}$$

The standard deviation of the *j*-th complex quantity is determined by the relationship:

$$S_{\overline{F}_{z_j}} = \sqrt{\sum_{i=1}^{k} S_{\bar{x}_i}^2 \left(\frac{\partial \overline{F}_{z_j}}{\partial \bar{x}_i}\right)^2} \tag{19}$$

The set of arguments of complex quantities is marked with $\bar{x}$ and contains nine elements *(note that the set of arguments is different from that in the previous section)*:

$$\bar{x} = \bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4, \bar{x}_5, \bar{x}_6, \bar{x}_7, \bar{x}_8, \bar{x}_9 \tag{20}$$

which are described by the relations:

$$\left.\begin{aligned} \bar{x}_1 = \bar{\omega}_w,\ \bar{x}_2 = \overline{W_0},\ \bar{x}_3 = \bar{\omega}_n,\ \bar{x}_4 = \overline{p_{m_n}},\ \bar{x}_5 = \overline{p_{m_w}} \\ \bar{x}_6 = \overline{\Delta_n},\ \bar{x}_7 = \overline{\Delta_w},\ \bar{x}_8 = \bar{d},\ \bar{x}_9 = \bar{I}_{m_{sr}} \end{aligned}\right\} \tag{20a}$$

$\bar{x}_1 = \bar{\omega}_w$ mean value of mass of powder charge for high pressure
$\bar{x}_2 = \overline{W}_0$ mean value of construction volume of the manometric bomb
$\bar{x}_3 = \bar{\omega}_n$ mean value of mass of powder charge for low pressure
$\bar{x}_4 = \bar{p}_{m_n}$ mean value of maximum pressure (low pressure)
$\bar{x}_5 = \bar{p}_{m_w}$ mean value of maximum pressure (high pressure)

$\bar{x}_6 = \bar{\Delta}_n$ mean value of loading density for low pressure
$\bar{x}_7 = \bar{\Delta}_w$ mean value of loading density for high pressure
$\bar{x}_8 = \bar{d}$ mean value of powder grain thickness
$\bar{x}_9 = \bar{I}_{msr}$ mean value of total GCP impulse
j-th subsets $(\bar{x})_j$ of the set $\bar{x}$ contains selected independent variables of the functions of complex quantities $\bar{F}_{zj}$:

$$\left.\begin{array}{c}(\bar{x})_1 = \{\bar{x}_1, \bar{x}_2\},\ (\bar{x})_2 = \{\bar{x}_2, \bar{x}_3\}, \\ (\bar{x})_3 = (\bar{x})_4 = \{\bar{x}_4, \bar{x}_5, \bar{x}_6, \bar{x}_7\}, (\bar{x})_5 = \{\bar{x}_8, \bar{x}_9\}\end{array}\right\} \tag{21}$$

the sum of which is equal to:

$$\cup_{j=1}^{5} (\bar{x})_j = \bar{x} \tag{22}$$

All calculations presented in Section 5 were made using the example of SM gunpowder. Calculations for the remaining gunpowders were made in the same way and the results are presented in the appropriate tables.

5.2 Determination of the measurement uncertainty for the values of the loading density Δ_n for low pressure and Δ_w for high pressure

For the powder charge taken from the 7.62x39 mm ammunition, the average values of the loading density (high $\bar{\Delta}_w$ and low $\bar{\Delta}_n$ pressure) are equal to the quotient of the mean values of powder charge masses equal to $\bar{\omega}_w$, $\bar{\omega}_n$, respectively, and the mean value of the construction volume of the manometric bomb $\bar{W}_0$:

$$\bar{F}_{z_1} = \overline{\Delta_w} = \frac{\bar{\omega}_w}{\bar{W}_0} = \frac{0.0090003}{0.0000318} = 283.028 \text{ kg/m}^3 \tag{23}$$

$$\bar{F}_{z_2} = \overline{\Delta_n} = \frac{\bar{\omega}_n}{\bar{W}_0} = \frac{0.0040003}{0.0000318} = 125.796 \text{ kg/m}^3 \tag{24}$$

Taking into account the independent variables being elements of the subsets $(\bar{x})_1 = \{\bar{\omega}_w, \bar{W}_0\}$ and $(\bar{x})_2 = \{\bar{W}_0, \bar{\omega}_n\}$ according to Equation 21, and their mean values according to Equation 20, the partial derivatives of the complex quantities (Equations 23 and 24) after the independent variables will take the following formulations, respectively:

i) for high pressure:

$$\frac{\partial \overline{F}_{z_1}}{\partial \bar{x}_1} = \frac{\partial \overline{\Delta_w}}{\partial \overline{\omega}_w} = \frac{\partial \frac{\overline{\omega}_w}{\overline{W_0}}}{\partial \overline{\omega}_w} = \frac{1}{\overline{W_0}} \tag{25}$$

$$\frac{\partial \overline{F}_{z_1}}{\partial \bar{x}_2} = \frac{\partial \overline{\Delta_w}}{\partial \overline{W_0}} = \frac{\partial \frac{\overline{\omega}_w}{\overline{W_0}}}{\partial \overline{W_0}} = -\frac{\overline{\omega_w}}{\overline{W_0}^2} \tag{26}$$

ii) for low pressure:

$$\frac{\partial \overline{F}_{z_2}}{\partial \bar{x}_1} = \frac{\partial \overline{\Delta_n}}{\partial \overline{\omega}_n} = \frac{\partial \frac{\overline{\omega}_n}{\overline{W_0}}}{\partial \overline{\omega}_n} = \frac{1}{\overline{W_0}} \tag{27}$$

$$\frac{\partial \overline{F}_{z_2}}{\partial \bar{x}_2} = \frac{\partial \overline{\Delta_n}}{\partial \overline{W_0}} = \frac{\partial \frac{\overline{\omega}_n}{\overline{W_0}}}{\partial \overline{W_0}} = -\frac{\overline{\omega_n}}{\overline{W_0}^2} \tag{28}$$

For the powder charge from the SM cartridge cal. 7.62x39 mm FMJ for high and low pressure, the mean values and the values of the standard deviations of the mean values of the powder charge masses, and the volume of the manometric bomb, for the t-Student distribution were taken from Table 3, which allowed the standard deviation for the complex quantity to be determined. For high and low pressure these will take the following values:

$$S_{\overline{\Delta_w}} = \sqrt{(S_{\overline{\omega}_w})^2 \left(\frac{1}{\overline{W_0}}\right)^2 + (S_{\overline{W_0}})^2 \left(-\frac{\overline{\omega}_w}{\overline{W_0}^2}\right)^2} = 2.118 \text{ kg/m}^3 \tag{29}$$

$$S_{\overline{\Delta_n}} = \sqrt{(S_{\overline{\omega}_n})^2 \left(\frac{1}{\overline{W_0}}\right)^2 + (S_{\overline{W_0}})^2 \left(-\frac{\overline{\omega}_n}{\overline{W_0}^2}\right)^2} = 0.942 \text{ kg/m}^3 \tag{30}$$

The measurement uncertainty of the high-pressure loading density distribution parameter Δ_w for a confidence interval equal to the standard deviation $S_{\overline{\Delta_w}}$ in the Student's t-probability distribution with $(n-1)$ degrees of freedom and confidence level $(1-\alpha) = 0.683$ is defined by Equation 31.

$$\Delta_w = \overline{\Delta_w} \pm S_{\overline{\Delta_w}} = 283.028 \pm 2.118 \text{ kg/m}^3 \tag{31}$$

The measurement uncertainty of the loading density distribution parameter Δ_n for low pressure with probability $(1 - \alpha) = 0.683$ is included in the confidence interval of the measurement limited by the standard deviation $S_{\overline{\Delta_n}}$ from the value of the mean loading density Δ_n for low pressure.

$$\Delta_n = \overline{\Delta_n} \pm S_{\overline{\Delta_n}} = 125.796 \pm 0.942 \text{ kg/m}^3 \tag{32}$$

On the other hand, for a confidence interval equal to three standard deviations $S_{\overline{\Delta_w}}$ and $S_{\overline{\Delta_n}}$ the values of the loading density distribution parameters Δ_w and Δ_n with probability $(1 - \alpha) = 0.997$ will be included in the ranges:

$$\Delta_w = \overline{\Delta_w} \pm 3S_{\overline{\Delta_w}} = 283.028 \pm 6.354 \text{ kg/m}^3 \tag{33}$$

$$\Delta_n = \overline{\Delta_n} \pm 3S_{\overline{\Delta_n}} = 125.796 \pm 2.826 \text{ kg/m}^3 \tag{34}$$

5.3 Determination of the measurement uncertainty for the value of the powder force *f*

In order to calculate the value of the mean powder force $\bar{f}$, it is necessary to know the mean values of the pressures: maximum $\bar{p}_{m_n}$ for low pressures and maximum $\bar{p}_{m_w}$ for high pressures and the loading density for low pressure $\overline{\Delta}_n$ and for high pressures $\overline{\Delta}_w$, which were taken from Table 3.

Having the values mentioned above, it was possible to calculate the mean value of the powder force $\bar{f}$ (for the powder charge from the SM cartridge cal.7.62x39 mm FMJ), which was carried out according to the following relationship:

$$\bar{F}_{z_3} = \bar{f} = \frac{\overline{p_{m_w}}\,\overline{p_{m_n}}}{(\overline{p_{m_w}} - \overline{p_{m_n}})} \frac{(\bar{\Delta}_w - \bar{\Delta}_n)}{\bar{\Delta}_w \bar{\Delta}_n} = \frac{379.334 * 135.376}{(379.334 - 135.376)} \frac{(283.028 - 125.796)}{283.028 * 125.796} = 0.9296 \text{ MJ/kg} \tag{35}$$

The standard deviation of the complex quantity $\bar{F}_{z_3}$ (powder force) was calculated from the relationship:

$$S_{\bar{F}_{z_3}} = \sqrt{\sum_{i=1}^{k} S_{\bar{x}_i}^2 \left(\frac{\partial \bar{F}_{z_3}}{\partial \bar{x}_i}\right)^2} \tag{36}$$

$(\bar{x})_3 = \{\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4\}$

For $\bar{x}_1 = \overline{p_{m_w}}$, $\bar{x}_2 = \overline{p_{m_n}}$ and $\bar{x}_3 = \overline{\Delta}_w$, $\bar{x}_4 = \overline{\Delta}_n$, this leads to:

$$\frac{\partial \bar{F}_{z3}}{\partial \bar{x}_1} = \frac{\partial \bar{f}}{\partial \overline{p_{m_w}}} = \frac{\partial\left(\frac{\overline{p_{m_w}}\,\overline{p_{m_n}}}{(\overline{p_{m_w}}-\overline{p_{m_n}})}\right)}{\partial \overline{p_{m_w}}} \frac{(\bar{\Delta}_w-\bar{\Delta}_n)}{\bar{\Delta}_w\bar{\Delta}_n} = \frac{-\left(\overline{p_{m_n}}\right)^2}{\left(\overline{p_{m_w}}-\overline{p_{m_n}}\right)^2} \frac{(\bar{\Delta}_w-\bar{\Delta}_n)}{\bar{\Delta}_w\bar{\Delta}_n} \tag{37}$$

$$\frac{\partial \bar{F}_{z3}}{\partial \bar{x}_2} = \frac{\partial \bar{f}}{\partial \overline{p_{m_n}}} = \frac{\overline{p_{m_w}}^2}{\left(\overline{p_{m_w}}-\overline{p_{m_n}}\right)^2} \frac{(\bar{\Delta}_w-\bar{\Delta}_n)}{\bar{\Delta}_w\bar{\Delta}_n} \tag{38}$$

$$\frac{\partial \bar{F}_{z3}}{\partial \bar{x}_3} = \frac{\partial \bar{f}}{\partial \bar{\Delta}_w} = \frac{\bar{\Delta}_n^{\,2}}{(\bar{\Delta}_w\bar{\Delta}_n)^2} \frac{\overline{p_{m_w}}\,\overline{p_{m_n}}}{(\overline{p_{m_w}}-\overline{p_{m_n}})} \tag{39}$$

$$\frac{\partial \bar{F}_{z3}}{\partial \bar{x}_4} = \frac{\partial \bar{f}}{\partial \bar{\Delta}_n} = \frac{-\bar{\Delta}_w^{\,2}}{(\bar{\Delta}_w\bar{\Delta}_n)^2} \frac{\overline{p_{m_w}}\,\overline{p_{m_n}}}{(\overline{p_{m_w}}-\overline{p_{m_n}})} \tag{40}$$

The standard deviation of the mean powder force $\bar{f}$ will have the form:

$$S_{\bar{f}} = \sqrt{\begin{array}{c}(S_{\overline{p_{m_w}}})^2\left(\frac{\partial \bar{f}}{\partial \overline{p_{m_w}}}\right)^2 + (S_{\overline{p_{m_n}}})^2\left(\frac{\partial \bar{f}}{\partial \overline{p_{m_n}}}\right)^2 + \\ +\left(S_{\bar{\Delta}_w}\right)^2\left(\frac{\partial \bar{f}}{\partial \bar{\Delta}_w}\right)^2 + \left(S_{\bar{\Delta}_n}\right)^2\left(\frac{\partial \bar{f}}{\partial \bar{\Delta}_n}\right)^2\end{array}} = 13713.32532 \text{ J/kg} \tag{41}$$

The uncertainty of measurement of the powder force distribution parameter f for a confidence interval equal to the standard deviation $S_{\bar{f}}$ in the Student's t-distribution with $(n-1)$ degrees of freedom and confidence level $(1-\alpha) = 0.683$, is determined by the relationship:

$$f = \bar{f} \pm S_{\bar{f}} = 0.9296 \pm 0.0137 \text{ MJ/kg} \tag{42}$$

For a confidence interval equal to three standard deviations $S_{\bar{f}}$, the value of the powder force distribution parameter f with probability $(1-\alpha) = 0.997$ will be in the range:

$$f = \bar{f} \pm 3S_{\bar{f}} = 0.9296 \pm 0.0411 \text{ MJ/kg} \tag{43}$$

5.4 Determination of the measurement uncertainty for the value of the covolume α

In order to calculate the measurement uncertainty of the expected value of the covolume $\overline{\alpha}$ (see Table 4) it is necessary to know the values of the mean pressures: maximum $\overline{p}_{m_n}$ for low pressure and maximum $\overline{p}_{m_w}$ for high pressure and the loading density for low pressure $\overline{\Delta}_n$ and for high pressure $\overline{\Delta}_w$, which was taken from Table 3.

For the powder charge taken from the SM cartridge cal.7.62x39 mm FMJ, calculation of the mean value of the covolume $\overline{\alpha}$ was made according to Equation 44.

$$\overline{F}_{z_4} = \overline{\alpha} = \frac{\frac{\overline{p_{m_w}}}{\overline{\Delta}_w} - \frac{\overline{p_{m_n}}}{\overline{\Delta}_n}}{\overline{p_{m_w}} - \overline{p_{m_n}}} = \frac{\frac{379.334}{283.028} - \frac{135.376}{125.796}}{379.334 - 135.376} \approx 1.0826 \cdot 10^{-3}\ \mathrm{m^3/kg} \tag{44}$$

The standard deviation of the complex quantity (covolume) was calculated from:

$$S_{\overline{F}_{z_4}} = \sqrt{\sum_{i=1}^{k} S_{\bar{x}_i}^2 \left(\frac{\partial \overline{F}_{z_4}}{\partial \bar{x}_i}\right)^2} \tag{45}$$

$$(\overline{x})_4 = \{\overline{x}_1, \overline{x}_2, \overline{x}_3, \overline{x}_4\}$$

For $\overline{x}_1 = \overline{p_{m_w}}$, $\overline{x}_2 = \overline{p_{m_n}}$ and $\overline{x}_3 = \overline{\Delta}_w$, $\overline{x}_4 = \overline{\Delta}_n$ received:

$$\frac{\partial \overline{F}_{z_4}}{\partial \bar{x}_1} = \frac{\partial \overline{\alpha}}{\partial \overline{p_{m_w}}} = \frac{\overline{p_{m_n}}\left(\frac{1}{\overline{\Delta}_n} - \frac{1}{\overline{\Delta}_w}\right)}{\left(\overline{p_{m_w}} - \overline{p_{m_n}}\right)^2} \tag{46}$$

$$\frac{\partial \overline{F}_{z_4}}{\partial \bar{x}_2} = \frac{\partial \overline{\alpha}}{\partial \overline{p_{m_n}}} = \frac{\overline{p_{m_w}}\left(\frac{1}{\overline{\Delta}_w} - \frac{1}{\overline{\Delta}_n}\right)}{\left(\overline{p_{m_w}} - \overline{p_{m_n}}\right)^2} \tag{47}$$

$$\frac{\partial \overline{F}_{z_4}}{\partial \bar{x}_3} = \frac{\partial \overline{\alpha}}{\partial \overline{\Delta}_w} = -\frac{\overline{p_{m_w}}}{\left(\overline{p_{m_w}} - \overline{p_{m_n}}\right)\overline{\Delta}_w^{\,2}} \tag{48}$$

$$\frac{\partial F_{z_4}}{\partial \bar{x}_4} = \frac{\partial \overline{\alpha}}{\partial \overline{\Delta}_n} = \frac{\overline{p_{m_n}}}{\left(\overline{p_{m_w}} - \overline{p_{m_n}}\right)\overline{\Delta}_n^{\,2}} \tag{49}$$

The standard deviation of the mean covolume value $\bar{\alpha}$ is:

$$S_{\bar{\alpha}} = \sqrt{(S_{\overline{p_{mw}}})^2\left(\frac{\partial\bar{\alpha}}{\partial\overline{p_{mw}}}\right)^2 + (S_{\overline{p_{mn}}})^2\left(\frac{\partial\bar{\alpha}}{\partial\overline{p_{mn}}}\right)^2 + \left(S_{\bar{\Delta}_w}\right)^2\left(\frac{\partial\bar{\alpha}}{\partial\bar{\Delta}_w}\right)^2 + \left(S_{\bar{\Delta}_n}\right)^2\left(\frac{\partial\bar{\alpha}}{\partial\bar{\Delta}_n}\right)^2} = 0.0696 \cdot 10^{-3}\ \mathrm{m^3/kg} \quad (50)$$

The measurement uncertainty of the covolume distribution parameter α for a confidence interval equal to the standard deviation $S_{\bar{\alpha}}$ in the Student's t-probability distribution with $(n - 1)$ degrees of freedom and confidence level $(1 - \alpha) = 0.683$, is defined by Equation 51.

$$\alpha = \bar{\alpha} \pm S_{\bar{\alpha}} = 1.0826 \pm 0.0696 \cdot 10^{-3}\ \mathrm{m^3/kg} \quad (51)$$

For a confidence interval equal to three standard deviations $\pm(\sigma(\bar{\alpha}) = S_{\bar{\alpha}})$, the value of the covolume distribution parameter α with probability $(1 - \alpha) = 0.997$ will be in the range:

$$\alpha = \bar{\alpha} \pm 3S_{\bar{\alpha}} = 1.0826 \pm 0.2088 \cdot 10^{-3}\ \mathrm{m^3/kg} \quad (52)$$

5.5 Determination of the measurement uncertainty for the value of the proper burn rate u_1

In order to calculate the measurement uncertainty of the expected value of the proper burn rate $\overline{u_1}$ (see Table 4), it is necessary to know: the shape of the grain, the values of the mean thickness of the powder grains $\bar{d}$ and the mean value of the averaged total impulse $\bar{I}_{msr}$, which was taken from Table 3.

The shape of the grain of SM cartridge cal.7.62x39 mm FMJ was cylindrical, so the thickness of the combustible layer was:

$$\overline{e_1} = \frac{\bar{d}}{2} = \frac{607.09}{2} = 303.54\ \mu\mathrm{m}$$

$$\bar{I}_{msr} = 243634\ \mathrm{Pa{\cdot}s}$$

The mean value of the proper burn rate $\overline{u_1}$ is equal to the quotient of the mean value of the powder grain thickness $\overline{e_1}$ and the mean value of the averaged total impulse $\overline{I(t_m)}$.

$$\bar{F}_{z_5} = \overline{u_1} = \frac{\overline{e_1}}{\overline{I(t_m)}} = \frac{303.54 * 10^{-6}}{243634} = 1.246 \cdot 10^{-9}\ \mathrm{m/(Pa{\cdot}s)} \quad (53)$$

The standard deviation of the complex quantity $\overline{F}_{z5}$ (proper burn rate) was calculated from the relations:

$$S_{\overline{F}_{z_5}} = \sqrt{\sum_{i=1}^{k} S_{\bar{x}_i}^2 \left(\frac{\overline{F}_{z_5}}{\partial \bar{x}_i}\right)^2} \tag{54}$$

$k = 2$

$(\overline{x})_5 = \{\overline{x}_1, \overline{x}_2\}$

For $\overline{x}_1 = \overline{e_1}$ and $\overline{x}_1 = \overline{I(t_m)}$, this is:

$$\frac{\partial \overline{F}_{z_5}}{\partial \bar{x}_1} = \frac{\partial \overline{u_1}}{\partial \overline{e_1}} = \frac{1}{\overline{I(t_m)}} \tag{55}$$

$$\frac{\partial \overline{F}_{z_5}}{\partial \bar{x}_2} = \frac{\partial \overline{u_1}}{\partial \overline{I(t_m)}} = -\frac{\overline{e_1}}{(\overline{I(t_m)})^2} \tag{56}$$

The standard deviations of the mean grain thickness $\overline{e_1}$ and of the average total impulse are:

$$S_{\overline{e_1}} = \frac{S_{\bar{d}}}{2} = \frac{5.28}{2} = 2.64\ \mu\text{m}$$

$$S_{\bar{I}_{m_{sr}}} = 5832\ \text{Pa·s}$$

The standard deviation of the complex mean value of the proper burn rate $\overline{u_1}$ is equal to:

$$S_{\overline{u_1}} = \sqrt{(S_{\overline{e_1}})^2 \left(\frac{1}{\overline{I(t_m)}}\right)^2 + \left(S_{\overline{I(t_m)}}\right)^2 \left(-\frac{\overline{e_1}}{(\overline{I(t_m)})^2}\right)^2} = 3.17 \cdot 10^{-11}\ \text{m/(Pa·s)} \tag{57}$$

The measurement uncertainty of the proper burn rate distribution parameter u_1 for a confidence interval equal to the standard deviation $S_{\overline{u_1}}$ in the Student's t-probability distribution with (n – 1) degrees of freedom and confidence level (1 – α) = 0.683, is defined by Equation 58.

$$u_1 = \overline{u_1} \pm S_{\overline{u_1}} = (1.246 \pm 0.032) \cdot 10^{-9}\ \text{m/(Pa·s)} \tag{58}$$

For a confidence interval equal to three standard deviations $\pm(\sigma(\overline{\alpha}) = S_{\overline{u_1}})$, the value of the proper burn rate distribution parameter u_1 with probability$(1 - \alpha) = 0.997$ will be in the range:

$$u_1 = \overline{u_1} \pm 3S_{\overline{u_1}} = (1.246 \pm 0.096) \cdot 10^{-9} \text{ m/(Pa·s)} \quad (59)$$

Table 4. Calculation results of complex quantities for the tested powders from cal. 7.62x39 mm ammunition

Parameter	Units	SM FMJ	Barnaul, ZN FMJ	STV FMJ
Δw	[kg/m^3]	283.028 ±6.354	283.035 ±6.354	
Δn	[kg m^3]	125.796 ±2.826	125.799 ±2.826	
f	[MJ/kg]	0.9296 ±0.0411	0.9162 ±0.0834	0.9055 ±0.0825
α	[m^3/kg]	(1.0826 ±0.2088)·10^{-3}	(1.1296 ±0.2268)·10^{-3}	(1.1416 ±0.2256)·10^{-3}
u_1	[m/(Pa·s)]	(1.246 ±0.096)·10^{-9}	(1.299 ±0.117)·10^{-9}	(1.127 ±0.099)·10^{-9}

6 Conclusions

- As part of this work, the energy-ballistic properties of gunpowders, such as loading density, powder force, covolume, proper burn rate, were experimentally determined using a manometric bomb. The paper also shows the classical methodology for determining these quantities, but elements of the mathematical statistics were implemented by the classical method. The results obtained using the estimator (statistics) of the variance of the random sample mean, in which the parameter of the distribution of the population mean value has been replaced with the value of the sample mean, are presented. The applied estimator of the variance of the mean value of the random sample is a consistent and unbiased estimator, therefore, for the assumed distribution of the probability function, which is the Student's t-distribution, confidence intervals have been established for the assumed confidence level for the values of the parameters of the distribution of ballistic features, such as the loading density power, gunpowder force, covolume and proper burning rate. This may be important in some cases because very often the properties provided by manufacturers are not fully reflected in reality.
- The differences between the energy-ballistic parameters for individual powders from the 7.62x39 mm ammunitions are small. The largest difference is between the proper combustion rate for SM cal.7.62x39 mm FMJ powders and STV cal.7.62x39 mm FMJ powders, being less than 10%. Differences between the values of covolume and powder force are smaller and extend up to 5%.

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