Wilemski Szymon 0000-0002-7988-0834
Gdynia Maritime University, Gdynia, Poland, szymonwilemski (at) o2.pl

Reliability analysis and optimization of multistate parallel-serial fuel transportation system in variable operating conditions

Keywords

reliability, multi-state technical system, operation process, optimization, fuel transportation

Abstract

The chapter presents reliability analysis of the piping fuel transportation system with a multistate approach and with parallel-series structure including process operation impact. A model of operation process is presented based on evaluated expert data and estimated parameters. The most important reliability characteristics are analysed for the fuel transportation system and its components for example lifetimes, damage intensities, reliability functions or risk functions. The influence of process operation on the system aging was shown by comparing the fuel transportation system without the operation process and including operating conditions. A method of optimizing the reliability of such system is also presented by changing the initial parameters of the operation process.

1. Introduction

Today's technical systems belong to the class of complex systems due to the structural diversity and technical advancement. The complexity of technical systems is manifested mainly in the possession of a large number of subsystems and individual elements, as well as in difficulty of estimating operational processes. Such a trend causes that the determination of reliability and modeling of the operation processes of complex technical systems becomes more and more difficult and complicated. Moreover the theory of systems reliability analysis has expanded recently [2]–[3], [7]–[9]. Increasingly, a multi-state approach is introduced to the reliability testing of complex systems. Such an assumption allows for a more detailed analysis, distinguishing, for example, the critical state of the system, the exceeding of which may result in wrong operational efficiency [14]–[16]. An example of such systems could be a fuel transportation system having a complex structure and many components [4]–[5]. The chapter is devoted to introducing the method of analyzing the reliability of the multistate parallel-series fuel transportation system and presenting the possibility of its real application in practice. The paper is comprised of 7 components, this Introduction as Section 1, Sections 2-6 and Summary as Section 7. Section 2 is devoted to introducing the piping fuel transportation system, its subsystems and the structure of this system. In Section 3, the analysis of the operation process is shown as well as estimated basic parameters and characteristics of operation process based on expert data and their opinions. In Section 4 the reliability characteristics of fuel transportation system excluding operation process impact are determined, for example, reliability functions, lifetimes or damage intensities. The risk function of the fuel transportation system is also presented in this section. Section 5 is devoted to showing the operation process impact on reliability of the fuel transportation system. The system reliability with aging components and with operated components is compared. In Section 6, the exemplary optimization of fuel transportation system parameters is presented using linear programming method. In Summary, the evaluation of the results and the chance for their real practical applications are discussed.

2. Description of fuel transportation system

We consider a certain fuel base intended to receive petroleum products from vessels (e.g. gasoline or diesel oil), store them and send them using trucks.

Safety and Reliability of Systems and Processes, Summer Safety and Reliability Seminar 2020

DOI: 10.26408/srsp-2020-17

This base also operates in reverse way, starting with receiving the cargo from trucks and ending with loading the cargo onto the vessel. The considered fuel transportation terminal is composed of three parts *A*, *B* and *C*. These points are linked by the piping transportation systems [9]. The scheme of the fuel transportation base is presented in *Figure 1*.

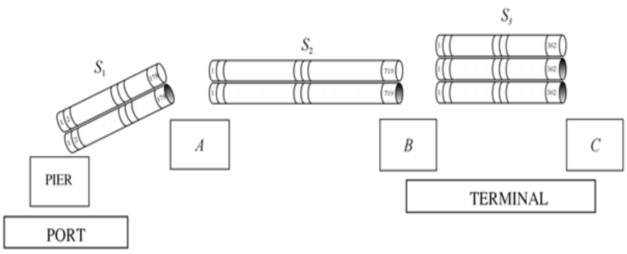

Figure 1. The scheme of the fuel transportation base

The considered piping fuel transportation system consists of three subsystems:

- the subsystem S_1 composed of two pipelines connecting the Pier and part *A*;
- the subsystem S_2 composed of two pipelines connecting part *A* and part *B*;
- the subsystem S_2 composed of three pipelines connecting part *B* and part *C*.

The piping fuel transportation system is a parallel-series system consisting of three serially connected subsystems that contain parallel components (each component includes pipelines, pumps and valves) (*Figure 2*). The subsystem S_1 contains two elements (E_{11}, E_{12}), the subsystem S_2 contains two elements (E_{21}, E_{22}) and the subsystem S_3 contains three elements (E_{31}, E_{32}, E_{33}). The considered subsystems together form a complete piping fuel transportation system at the terminal [13].

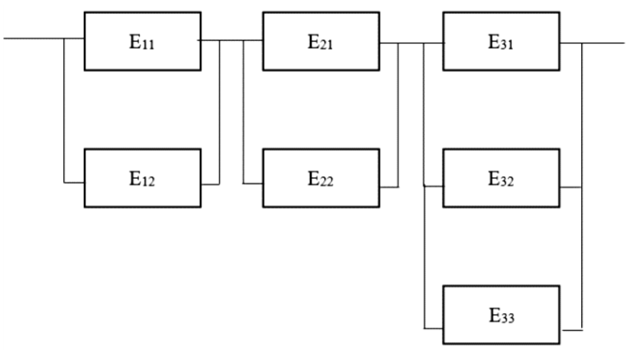

Figure 2. Parallel-series structure of the piping fuel transportation system

3. Operation process of fuel transportation system

Subsystems create a reliability structure of the fuel transportation system. The structure and reliability of subsystems depend on operation process. On the basis of the expert's opinions and statistical data (period 3 years), it is possible to evaluate the basic unknown parameters of the operation process of the fuel transportation system [13]:

- the number of operation states $\nu = 8$ with:
  - the operation state z_1 – transport of one kind of medium from part *B* to part *C* using three elements (E_{31}, E_{32}, E_{33}) of the subsystem S_3;
  - the operation state z_2 – transport of one kind of medium from part *C* to part *B* using two elements (E_{31}, E_{32}) of the subsystem S_3;
  - the operation state z_3 – transport of one kind of medium from part *B* to Pier using two elements (E_{21}, E_{22}) of the subsystem S_2 and two elements (E_{11}, E_{12}) of the subsystem S_1;
  - the operation state z_4 – transport of two kinds of medium from part *C* to Pier using two elements (E_{31}, E_{32}) of the subsystem S_3, two elements (E_{21}, E_{22}) of the subsystem S_2 and two elements (E_{11}, E_{12}) of the subsystem S_1;
  - the operation state z_5 – transport of one kind of medium from Pier to part *C* using one element (E_{11}) of the subsystem S_1, one element (E_{21}) of the subsystem S_2 and two elements (E_{31}, E_{32}) of the subsystem S_3;
  - the operation state z_6 – transport of one kind of medium from Pier to part *C* using one pipeline (E_{11}) of the subsystem S_1, two pipelines (E_{21}, E_{22}) of the subsystem S_2 and three pipelines (E_{31}, E_{32}, E_{33}) of the subsystem S_3;
  - the operation state z_7 – transport of two kinds of medium from Pier to part *C* using two elements (E_{11}, E_{12}) of the subsystem S_1, two elements (E_{21}, E_{22}) of the subsystem S_2 and three elements (E_{31}, E_{32}, E_{33}) of the subsystem S_3;
  - the operation state z_8 – no fuel transport, system doesn't work;
- the matrix of probabilities p_{bl}, $b, l = 1,2,\ldots,8$, $b \neq l$, of the system operation process *Z*(*t*) transitions between the operation states z_b and z_l:

$$[p_{bl}]_{8x8} = \begin{bmatrix} 0 & 0.213 & 0.053 & 0.156 & 0.165 & 0.082 & 0.051 & 0.280 \\ 0.183 & 0 & 0.090 & 0.180 & 0.176 & 0.073 & 0.086 & 0.212 \\ 0.100 & 0.142 & 0 & 0.150 & 0.178 & 0.053 & 0.086 & 0.310 \\ 0.144 & 0.131 & 0.125 & 0 & 0.160 & 0.099 & 0.098 & 0.243 \\ 0.096 & 0.156 & 0.096 & 0.149 & 0 & 0.103 & 0.124 & 0.276 \\ 0.150 & 0.134 & 0.054 & 0.120 & 0.136 & 0 & 0.100 & 0.306 \\ 0.123 & 0.151 & 0.076 & 0.119 & 0.110 & 0.076 & 0 & 0.345 \\ 0.275 & 0.110 & 0.121 & 0.182 & 0.180 & 0.080 & 0.052 & 0 \end{bmatrix} \quad (1)$$

- the matrix of mean values of the system operation process $Z(t)$ conditional sojourn times θ_{bl} $b,l = 1,2,\ldots,8,$ $b \neq l,$ on condition of transition from operation states z_b to z_l:

$$[M_{bl}]_{8x8} = \begin{bmatrix} 0 & 454 & 654 & 638 & 543 & 432 & 689 & 888 \\ 643 & 0 & 812 & 442 & 758 & 543 & 498 & 674 \\ 213 & 457 & 0 & 432 & 325 & 511 & 140 & 625 \\ 456 & 442 & 550 & 0 & 689 & 613 & 371 & 525 \\ 312 & 256 & 425 & 513 & 0 & 697 & 732 & 725 \\ 342 & 450 & 250 & 415 & 198 & 0 & 423 & 628 \\ 400 & 423 & 450 & 546 & 206 & 253 & 0 & 513 \\ 723 & 817 & 540 & 467 & 902 & 725 & 1024 & 0 \end{bmatrix}. \quad (2)$$

Thanks to the above parameters (1), (2) and the theory of determining the characteristics of the operation process contained in [7]–[8] we can fix the most important operation process values needed for further analysis [13]:

- the mean values of the system operation process $Z(t)$ conditional sojourn times θ_{bl}, $b,l = 1,2,\ldots,8,$ $b \neq l$ at the particular operation states (in hours):

$$M_1 \cong 639.69,\ M_2 \cong 629.072,$$
$$M_3 \cong 439.057,\ M_4 \cong 527.053,$$
$$M_5 \cong 548.754,\ M_6 \cong 436.296,$$
$$M_7 \cong 431.120,\ M_8 \cong 712.637; \quad (3)$$

- the limit values of transient probabilities of the operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,8$,:

$$p_1 \cong 0.160,\ p_2 \cong 0.139,\ p_3 \cong 0.065,$$
$$p_4 \cong 0.124,\ p_5 \cong 0.134,\ p_6 \cong 0.058,$$
$$p_7 \cong 0.055, p_8 \cong 0.265. \quad (4)$$

Moreover, the parameters of the impact of the operation process on the considered fuel transportation system are given. The coefficients of the operation process impact on the fuel transportation system intensities of ageing at the operation states z_b, $b = 1,2,\ldots,8$, are as follows [4]:

$$[\rho(1)]^{(b)} = 1, [\rho(2)]^{(b)} = 1,\ b = 8,$$
$$[\rho(1)]^{(b)} = 1.1, [\rho(2)]^{(b)} = 1.1,\ b = 1,2,$$
$$[\rho(1)]^{(b)} = 1.2, [\rho(2)]^{(b)} = 1.2,\ b = 4,5,$$
$$[\rho(1)]^{(b)} = 1.3, [\rho(2)]^{(b)} = 1.3,\ b = 3,6,7. \quad (5)$$

4. Reliability characteristics of fuel transportation system without of operation process impact

After considering expert's opinions and comments, taking account of the effectiveness and reliability aspects of the piping fuel transportation system operation, we can fix for them the following parameters [13]:

- the number of reliability states $z = 2$ (excluding state 0);
- the reliability states:
  - reliability state 2 – the fuel transportation system is fully safe;
  - reliability state 1 – the fuel transportation system is less safe and more dangerous because of the environmental pollution;
  - reliability state 0 – the fuel transportation system is destroyed.

After considering above parameters we assume that:

- there is possible the transition between reliability states only from better to worse ones;
- the critical reliability state of fuel transportation system is $r = 1$;
- the fuel transportation system risk function permitted level is $\delta = 0.05$.

Another assumption for the fuel transportation system is that the elements of the system do not change their reliability characteristics during the transitions between the operation states. The considered system is not homogenous which means that the individual elements have different reliability functions.

Now, we will perform detailed reliability analysis of the fuel piping transportation system. The mean values of the elements lifetimes in the reliability state subsets {1,2}, {2} approximately evaluated on the basis of reliability data of its components coming from experts, are (in years) [4]:

- for the elements E_{11}, E_{12}:

$$\mu_{11}^0(1) = \mu_{12}^0(1) = 276,$$
$$\mu_{11}^0(2) = \mu_{12}^0(2) = 185; \quad (6)$$

- for the elements E_{21}, E_{22}:

$$\mu_{21}^0(1) = \mu_{22}^0(1) = 69,$$
$$\mu_{21}^0(2) = \mu_{22}^0(2) = 46; \quad (7)$$

- for the elements E_{31}, E_{32}:

$$\mu_{31}^0(1) = \mu_{32}^0(1) = 137,$$
$$\mu_{31}^0(2) = \mu_{32}^0(2) = 110; \quad (8)$$

- for the element E_{33}:

$$\mu_{33}^0(1) = 114,\ \mu_{33}^0(2) = 102. \quad (9)$$

Using the above data, we get the intensities of ageing the elements of the fuel transportation system without operation process impact [10]–[12]:

- for the elements E_{11}, E_{12}:

$$\lambda_{11}^{0}(1) = \lambda_{12}^{0}(1) = 0.00362, \tag{10}$$

$$\lambda_{11}^{0}(2) = \lambda_{12}^{0}(2) = 0.0054; \tag{11}$$

- for the elements E_{21}, E_{22}:

$$\lambda_{21}^{0}(1) = \lambda_{22}^{0}(1) = 0.01444, \tag{12}$$

$$\lambda_{21}^{0}(2) = \lambda_{22}^{0}(2) = 0.02163; \tag{13}$$

- for the elements E_{31}, E_{32}:

$$\lambda_{31}^{0}(1) = \lambda_{32}^{0}(1) = 0.0073, \tag{14}$$

$$\lambda_{31}^{0}(2) = \lambda_{32}^{0}(2) = 0.00912; \tag{15}$$

- for the element E_{33}:

$$\lambda_{33}^{0}(1) = 0.00874, \lambda_{33}^{0}(2) = 0.00984. \tag{16}$$

And then we get reliability functions of each elements without operation process impact [13]:

- for the elements E_{11}, E_{12}:

$$[R_{11}(t,\cdot)]^{(0)} = [R_{12}(t,\cdot)]^{(0)} = [1, e^{-0.00362t}, e^{-0.0054t}]; \tag{17}$$

- for the elements E_{21}, E_{22}:

$$[R_{21}(t,\cdot)]^{(0)} = [R_{22}(t,\cdot)]^{(0)} = [1, e^{-0.01444t}, e^{-0.02163t}]; \tag{18}$$

- for the elements E_{31}, E_{32}:

$$[R_{31}(t,\cdot)]^{(0)} = [R_{32}(t,\cdot)]^{(0)} = [1, e^{-0.0073t}, e^{-0.00912t}]; \tag{19}$$

- for the element E_{33}:

$$[R_{33}(t,\cdot)]^{(0)} = [1, e^{-0.00874t}, e^{-0.00984t}], \quad t \geq 0. \tag{20}$$

Using the theory of reliability structures and methods of determining the elements lifetimes [1], we state that the considered fuel transportation system is a parallel-series multistate system with its lifetimes *T*(*u*):

- in the reliability state subsets {1, 2}:

$$[\boldsymbol{T}(1)]^{(0)} = \min\left\{\begin{array}{l} \max\{[T_{11}(1)]^{(0)}, [T_{12}(1)]^{(0)}\}, \\ \max\{[T_{21}(1)]^{(0)}, [T_{22}(1)]^{(0)}\}, \\ \max\{[T_{31}(2)]^{(0)}, [T_{32}(2)]^{(0)}, [T_{33}(2)]^{(0)}\} \end{array}\right\}; \tag{21}$$

- in the reliability state subsets {2}:

$$[\boldsymbol{T}(2)]^{(0)} = \min\left\{\begin{array}{l} \max\{[T_{11}(2)]^{(0)}, [T_{12}(2)]^{(0)}\}, \\ \max\{[T_{21}(2)]^{(0)}, [T_{22}(2)]^{(0)}\}, \\ \max\{[T_{31}(2)]^{(0)}, [T_{32}(2)]^{(0)}, [T_{33}(2)]^{(0)}\} \end{array}\right\}. \tag{22}$$

After applying formulae for the reliability function of parallel-series systems given in [1]–[3] (exponential case), we obtain the reliability function of the considered fuel transportation system without operation process impact [13]:

$$[\boldsymbol{R}(t,\cdot)]^{(0)} = [1, [\boldsymbol{R}(t,1)]^{(0)}, [\boldsymbol{R}(t,2)]^{(0)}], \tag{22}$$

where:

$$\begin{aligned}[\boldsymbol{R}(t,1)]^{(0)} &= [1-(1-e^{-0.00362t})^2] \\ &\cdot [1-(1-e^{-0.01444t})^2] \\ &\cdot [1-(1-e^{-0.0073t})^2(1-e^{-0.00874t})] \\ &= e^{-0.05946t} - 2e^{-0.05584t} - 2e^{-0.05216t} \\ &- e^{-0.05072t} + 4e^{-0.04854t} + 2e^{-0.0471t} \\ &- 2e^{-0.04502t} + e^{-0.04486t} + 2e^{-0.04342t} \\ &+ 4e^{-0.0414t} - 2e^{-0.04124t} - 4e^{-0.0398t} \\ &+ 4e^{-0.03772t} + 2e^{-0.03628t} - 8e^{-0.0341t} \\ &- 4e^{-0.03266t} - 2e^{-0.03042t} - 4e^{-0.02898t} \\ &+ 4e^{-0.0268t} + 8e^{-0.02536t}, \quad t \geq 0, \end{aligned} \tag{23}$$

$$\begin{aligned}[\boldsymbol{R}(t,1)]^{(0)} &= [1-(1-e^{-0.0054t})^2] \\ &\cdot [1-(1-e^{-0.02163t})^2] \cdot \\ &\cdot [1-(1-e^{-0.00912t})^2(1-e^{-0.00984t})] \\ &= e^{-0.08214t} - 2e^{-0.07674t} - 2e^{-0.07302t} \\ &- e^{-0.0723t} + 4e^{-0.06762t} + 2e^{-0.0669t} \\ &+ e^{-0.0639t} + 2e^{-0.06318t} - 2e^{-0.06051t} \\ &- 2e^{-0.0585t} - 4e^{-0.05778t} + 4e^{-0.05511t} \\ &+ 4e^{-0.05139t} + 2e^{-0.05067t} - 8e^{-0.04599t} \\ &- 4e^{-0.04527t} - 2e^{-0.04227t} - 4e^{-0.04155t} \\ &+ 4e^{-0.03687t} + 8e^{-0.03615t}, \quad t \geq 0. \end{aligned} \tag{24}$$

Hence, the mean values of the fuel transportation system lifetimes are [13]:

- in the reliability state subsets {1, 2}:

$$\mu(1)^{(0)} = \int_0^\infty [\boldsymbol{R}(t,1)]^{(0)} dt \cong 84.39 \text{ years}; \tag{25}$$

- in the reliability state subsets {2}:

$$\mu(2)^{(0)} = \int_0^\infty [\boldsymbol{R}(t,2)]^{(0)} dt \cong 58.56 \text{ years}; \tag{26}$$

with intensities of ageing of considered system:

$$\lambda(1)^{(0)} = 0.01185,\ \lambda(2)^{(0)} = 0.01708. \qquad (27)$$

Another reliability characteristics are:

- standard deviations of the fuel transportation system lifetimes in the reliability state subsets {1,2}, {2}:

$$[\sigma(1)]^{(0)} \cong 55.59,\ [\sigma(2)]^{(0)} \cong 39.26; \qquad (28)$$

- the mean values of the fuel transportation system lifetimes in the particular reliability states:

$$[\bar{\mu}(1)]^{(0)} = 25.83,\ [\bar{\mu}(2)]^{(0)} = 58.56. \qquad (29)$$

As the critical reliability state is $r = 1$, then by (23) and (27), the fuel transportation system risk function without process operation impact is given by:

$$r^0(t) = 1 - [\boldsymbol{R}(t,1)]^{(0)} = 1 - e^{-0.01185t}$$
$$t \geq 0. \qquad (30)$$

The moment when the fuel transportation system risk function exceeds a permitted level $\delta = 0.05$ is:

$$\tau^0 = -\frac{1}{0.01185} \cdot \ln(1 - 0.05) \cong 4.33. \qquad (31)$$

The graph of the risk function of the fuel transportation system with the moment τ^0 is presented in *Figure 3*.

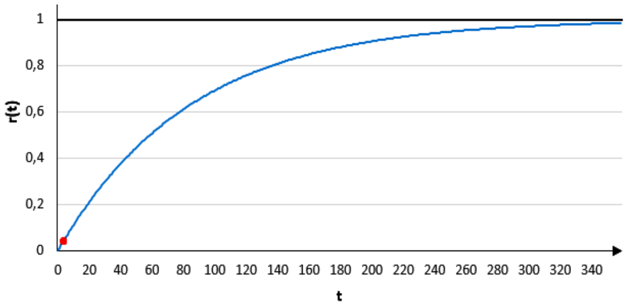

Figure 3. Risk function of the piping fuel transportation system impacted by operation process

5. Reliability characteristics of the fuel transportation system

As previously mentioned in Section 3, the fuel transportation system is subject to 8 operation states. The conditional reliability functions in each operation state are different and depend on operated components (individual intensities multiplied by appropriate coefficients $\rho(u)$ according to (5)). The conditional functions of the fuel transportation system including operation process impact are as follows [13]:

- in the operation state z_1:

$$[\boldsymbol{R}(t,\cdot)]^{(1)} = [1, [\boldsymbol{R}(t,1)]^{(1)}, [\boldsymbol{R}(t,2)]^{(1)}], \qquad (32)$$

where:

$$[\boldsymbol{R}(t,1)]^{(1)} = [1 - (1 - e^{-0.00362t})^2]$$
$$\cdot [1 - (1 - e^{-0.01444t})^2]$$
$$\cdot \begin{bmatrix} 1 - \left(1 - e^{-0.0073 \cdot [\rho(1)]^{(1)} t}\right)^2 \cdot \\ \cdot \left(1 - e^{-0.00874 \cdot [\rho(1)]^{(1)} t}\right) \end{bmatrix}$$
$$= e^{-0.061814t} - 2e^{-0.058194t} - 2e^{-0.053774t}$$
$$- e^{-0.0522t} + 4e^{-0.050154t} + 2e^{-0.04858t}$$
$$- 2e^{-0.047374t} + e^{-0.045734} + 2e^{-0.04416t}$$
$$+ 4e^{-0.04374t} - 2e^{-0.042114t} - 4e^{-0.04054t}$$
$$+ 4e^{-0.039334t} + 2e^{-0.03776t} - 8e^{-0.035714t}$$
$$- 4e^{-0.03414t} - 2e^{-0.031294t} - 4e^{-0.02972t}$$
$$+ 4e^{-0.027674t} + 8e^{-0.0261t}, t \geq 0, \qquad (33)$$

$$[\boldsymbol{R}(t,2)]^{(1)} = [1 - (1 - e^{-0.0054t})^2]$$
$$\cdot [1 - (1 - e^{-0.02163t})^2]$$
$$\cdot \begin{bmatrix} 1 - \left(1 - e^{-0.00912 \cdot [\rho(2)]^{(1)} t}\right)^2 \\ \left(1 - e^{-0.00984 \cdot [\rho(2)]^{(1)} t}\right) \end{bmatrix}$$
$$= e^{-0.084944t} - 2e^{-0.079544t} - 2e^{-0.074914t}$$
$$- e^{-0.07412t} + 4e^{-0.069514t} + 2e^{-0.06872t}$$
$$+ e^{-0.064884t} + 2e^{-0.06409} - 2e^{-0.063314t}$$
$$- 2e^{-0.059484t} - 4e^{-0.05869t} + 4e^{-0.057914t}$$
$$+ 4e^{-0.053284t} + 2e^{-0.05249t} - 8e^{-0.047884t}$$
$$- 4e^{-0.04709t} - 2e^{-0.043254t} - 4e^{-0.04246t}$$
$$+ 4e^{-0.037854t} + 8e^{-0.03706t}, t \geq 0; \qquad (34)$$

- in the operation state z_2:

$$[\boldsymbol{R}(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}(t,1)]^{(2)}, [\boldsymbol{R}(t,2)]^{(21)}], \qquad (35)$$

where:

$$[\boldsymbol{R}(t,1)]^{(2)} = [1 - (1 - e^{-0.00362t})^2]$$
$$\cdot [1 - (1 - e^{-0.01444t})^2]$$
$$\cdot \left[1 - \left(1 - e^{-0.0073 \cdot [\rho(1)]^{(2)} t}\right)^2 (1 - e^{-0.00874t})\right]$$
$$= e^{-0.06086t} - 2e^{-0.05724t} - 2e^{-0.05286t}$$
$$- e^{-0.05212t} + 4e^{-0.04924t} + 2e^{-0.0485t}$$
$$- 2e^{-0.04642t} + e^{-0.04486t} + 2e^{-0.04412t}$$
$$+ 4e^{-0.0428t} - 2e^{-0.04124t} - 4e^{-0.0405t}$$
$$+ 4e^{-0.03842t} + 2e^{-0.03768t} - 8e^{-0.0348t}$$
$$- 4e^{-0.03406t} - 2e^{-0.03042t} - 4e^{-0.02968t}$$
$$+ 4e^{-0.0268t} + 8e^{-0.02606t}, t \geq 0, \qquad (36)$$

$$[\boldsymbol{R}(t,2)]^{(2)} = [1 - (1 - e^{-0.0054t})^2]$$
$$\cdot [1 - (1 - e^{-0.02163t})^2]$$
$$\cdot \left[1 - \left(1 - e^{-0.00912 \cdot [\rho(2)]^{(2)} t}\right)^2 (1 - e^{-0.00984t})\right]$$
$$= e^{-0.083964t} - 2e^{-0.078564t} - e^{-0.074124t}$$

$$- 2e^{-0.073932t} + 2e^{-0.068724t} + 4e^{-0.068532t} + 2e^{-0.064092t} + e^{-0.0639t} - 2e^{-0.062334t} - 4e^{-0.058692t} - 2e^{-0.0585t} + 4e^{-0.056934t} + 2e^{-0.052494t} + 4e^{-0.052302t} - 4e^{-0.047094t} - 8e^{-0.046902t} - 4e^{-0.042462t} - 2e^{-0.04227t} + 8e^{-0.037062t} + 4e^{-0.03687t}, t \geq 0; \quad (37)$$

- in the operation state z_3:

$$[\boldsymbol{R}(t,\cdot)]^{(3)} = [1, [\boldsymbol{R}(t,1)]^{(3)}, [\boldsymbol{R}(t,2)]^{(3)}], \quad (38)$$

where:

$$[\boldsymbol{R}(t,1)]^{(3)} = \left[1 - \left(1 - e^{-0.00362\cdot[\rho(1)]^{(3)}t}\right)^2\right] \cdot \left[1 - \left(1 - e^{-0.01444\cdot[\rho(1)]^{(3)}t}\right)^2\right] \cdot [1 - (1 - e^{-0.0073t})^2(1 - e^{-0.00874t})] = e^{-0.066688t} - 2e^{-0.062344t} - 2e^{-0.059388t} - e^{-0.057948t} + 4e^{-0.055044t} + 2e^{-0.053604t} + e^{-0.052088t} + 2e^{-0.050648t} - 2e^{-0.049358t} - 2e^{-0.047744t} - 4e^{-0.046304t} + 4e^{-0.045014t} + 4e^{-0.042058t} + 2e^{-0.040618t} - 8e^{-0.037714t} - 4e^{-0.036274t} - 2e^{-0.034758t} - 4e^{-0.033318t} + 4e^{-0.030414t} + 8e^{-0.028974t}, t \geq 0, \quad (39)$$

$$[\boldsymbol{R}(t,2)]^{(3)} = \left[1 - \left(1 - e^{-0.0054\cdot[\rho(2)]^{(3)}t}\right)^2\right] \cdot \left[1 - \left(1 - e^{-0.02163\cdot[\rho(2)]^{(3)}t}\right)^2\right] \cdot [1 - (1 - e^{-0.00912t})^2(1 - e^{-0.00984t})] = e^{-0.09295t} - 2e^{-0.08647t} - 2e^{-0.08383t} - e^{-0.08311t} + 4e^{-0.07735t} + 2e^{-0.07663t} + e^{-0.07471t} + 2e^{-0.07399t} - 2e^{-0.06823t} - 4e^{-0.06751t} - 2e^{-0.066995t} + 4e^{-0.060515t} + 4e^{-0.057875t} + 2e^{-0.057155t} - 8e^{-0.051395t} - 4e^{-0.050675t} - 2e^{-0.048755t} - 4e^{-0.048035t} + 4e^{-0.042275t} + 8e^{-0.041555t}, t \geq 0; \quad (40)$$

- in the operation state z_4:

$$[\boldsymbol{R}(t,\cdot)]^{(4)} = [1, [\boldsymbol{R}(t,1)]^{(4)}, [\boldsymbol{R}(t,2)]^{(4)}], \quad (41)$$

where:

$$[\boldsymbol{R}(t,1)]^{(4)} = \left[1 - \left(1 - e^{-0.00362\cdot[\rho(1)]^{(4)}t}\right)^2\right] \cdot \left[1 - \left(1 - e^{-0.01444\cdot[\rho(1)]^{(4)}t}\right)^2\right] \cdot \left[1 - \left(1 - e^{-0.0073\cdot[\rho(1)]^{(4)}t}\right)^2 (1 - e^{-0.00874t})\right] = e^{-0.074672t} - 2e^{-0.069966t} - e^{-0.065932t} - 2e^{-0.065182t} + 2e^{-0.061226t} + 4e^{-0.060476t} + 2e^{-0.056442t} - 2e^{-0.055902t} + e^{-0.055692t} - 4e^{-0.051736t} + 4e^{-0.051196t} - 2e^{-0.050986t} + 2e^{-0.047162t} + 4e^{-0.046412t} - 4e^{-0.042456t} - 8e^{-0.041706t} - 4e^{-0.037672t} - 2e^{-0.036922t} + 8e^{-0.032966t} + 4e^{-0.032216t}, t \geq 0, \quad (42)$$

$$[\boldsymbol{R}(t,2)]^{(4)} = \left[1 - \left(1 - e^{-0.0054\cdot[\rho(2)]^{(4)}t}\right)^2\right] \cdot \left[1 - \left(1 - e^{-0.02163\cdot[\rho(2)]^{(4)}t}\right)^2\right] \cdot \left[1 - \left(1 - e^{-0.00912\cdot[\rho(2)]^{(4)}t}\right)^2 (1 - e^{-0.00984t})\right] = e^{-0.10383t} - 2e^{-0.09681t} - e^{-0.09399t} - 2e^{-0.091974t} + 2e^{-0.08697t} + 4e^{-0.084954t} + 2e^{-0.082134t} + e^{-0.080118t} - 2e^{-0.075711t} - 4e^{-0.075114t} - 2e^{-0.073098t} + 4e^{-0.068691t} + 2e^{-0.065871t} + 4e^{-0.063855t} - 4e^{-0.058851t} - 8e^{-0.056835t} - 4e^{-0.054015t} - 2e^{-0.051999t} + 8e^{-0.046995t} + 4e^{-0.044979t}, t \geq 0; \quad (43)$$

- in the operation state z_5:

$$[\boldsymbol{R}(t,\cdot)]^{(5)} = [1, [\boldsymbol{R}(t,1)]^{(5)}, [\boldsymbol{R}(t,2)]^{(5)}], \quad (44)$$

where:

$$[\boldsymbol{R}(t,1)]^{(5)} = \left[1 - \left(1 - e^{-0.00362\cdot[\rho(1)]^{(5)}t}\right)(1 - e^{-0.00362t})\right] \cdot \left[1 - \left(1 - e^{-0.01444\cdot[\rho(1)]^{(5)}t}\right)(1 - e^{-0.01444t})\right] \cdot \left[1 - \left(1 - e^{-0.0073\cdot[\rho(1)]^{(5)}t}\right)^2 (1 - e^{-0.00874t})\right] = e^{-0.065984t} - e^{-0.062364t} - e^{-0.06164t} - e^{-0.057244t} - 2e^{-0.057224t} + e^{-0.053624t} + 2e^{-0.053604t} + e^{-0.0529t} + 2e^{-0.05288t} - e^{-0.051544t} - e^{-0.048664t} + 2e^{-0.048484t} + e^{-0.048464t} + e^{-0.047924t} + e^{-0.0472t} + e^{-0.045044t} - 2e^{-0.044864t} - e^{-0.044844t} + e^{-0.04432t} - 2e^{-0.04414t} - e^{-0.04412t} + e^{-0.042804t} + 2e^{-0.042784t} + e^{-0.039924t} + 2e^{-0.039904t} - e^{-0.039184t} - 2e^{-0.039164t} - e^{-0.03846t} - 2e^{-0.03844t} - e^{-0.036304t} - 2e^{-0.036284t} - e^{-0.03558t} - 2e^{-0.03556t} - 2e^{-0.034044t} - e^{-0.034024t} - 2e^{-0.031164t} - e^{-0.031144t} + 2e^{-0.030424t} + e^{-0.030404t} + 2e^{-0.0297t} + e^{-0.02968t} + 2e^{-0.027544t} + e^{-0.027524t} + 2e^{-0.02682t} + e^{-0.0268t}, t \geq 0, \quad (45)$$

$$[\boldsymbol{R}(t,2)]^{(5)} = \left[1 - \left(1 - e^{-0.0054\cdot[\rho(2)]^{(5)}t}\right)(1 - e^{-0.0054t})\right] \cdot \left[1 - \left(1 - e^{-0.02163\cdot[\rho(2)]^{(5)}t}\right)(1 - e^{-0.02163t})\right] \cdot \left[1 - \left(1 - e^{-0.00912\cdot[\rho(2)]^{(5)}t}\right)^2 (1 - e^{-0.00984t})\right] = e^{-0.091194t} - e^{-0.085794t} - e^{-0.084714t} - e^{-0.081354t} - 2e^{-0.08025t} + e^{-0.075954t}$$

$$
\begin{aligned}
&+e^{-0.074874t}+2e^{-0.07485t}+2e^{-0.07377t}\\
&+2e^{-0.07041t}-e^{-0.069564t}+e^{-0.069306t}\\
&-e^{-0.065238t}-2e^{-0.06501t}+e^{-0.064164t}\\
&-2e^{-0.06393t}-e^{-0.063906t}+e^{-0.063084t}\\
&-e^{-0.062826t}+e^{-0.059838t}+e^{-0.059724t}\\
&+e^{-0.058758t}+2e^{-0.05862t}+e^{-0.055398t}\\
&-e^{-0.054324t}+2e^{-0.054294t}-e^{-0.053244t}\\
&-2e^{-0.05322t}-2e^{-0.05214t}-e^{-0.049998t}\\
&-e^{-0.048918t}-2e^{-0.048894t}-2e^{-0.04878t}\\
&-2e^{-0.047814t}-e^{-0.047676t}-2e^{-0.044454t}\\
&+2e^{-0.04338t}-e^{-0.04335t}+2e^{-0.0423t}\\
&+e^{-0.042276t}+e^{-0.041196t}+2e^{-0.039054t}\\
&+2e^{-0.037974t}+e^{-0.03795t}+e^{-0.03687t},\\
&t\ge 0; \qquad (46)
\end{aligned}
$$

- in the operation state z_6:

$$[\boldsymbol{R}(t,\cdot)]^{(6)}=[1,[\boldsymbol{R}(t,1)]^{(6)},[\boldsymbol{R}(t,2)]^{(6)}], \qquad (47)$$

where:

$$
\begin{aligned}
&[\boldsymbol{R}(t,1)]^{(6)}=\\
&\left[1-\left(1-e^{-0.00362\cdot[\rho(1)]^{(6)}t}\right)(1-e^{-0.00362t})\right]\\
&\cdot\left[1-\left(1-e^{-0.01444\cdot[\rho(1)]^{(6)}t}\right)^2\right]\\
&\cdot\left[\begin{matrix}1-\left(1-e^{-0.0073\cdot[\rho(1)]^{(6)}t}\right)^2\\ \left(1-e^{-0.00874\cdot[\rho(1)]^{(6)}t}\right)\end{matrix}\right]\\
&=e^{-0.076212t}-e^{-0.072592t}-e^{-0.071506t}\\
&-2e^{-0.066722t}-e^{-0.06485t}+2e^{-0.063102t}\\
&+2e^{-0.062016t}+e^{-0.06123t}+e^{-0.060144t}\\
&-2e^{-0.05744t}+e^{-0.057232t}+2e^{-0.05536t}\\
&+2e^{-0.05382t}-e^{-0.053612t}+2e^{-0.052734t}\\
&-e^{-0.052526t}-2e^{-0.05174t}-2e^{-0.050654t}\\
&+4e^{-0.04795t}+2e^{-0.046078t}-4e^{-0.04433t}\\
&-4e^{-0.043244t}-2e^{-0.042458t}-2e^{-0.041372t}\\
&-2e^{-0.03846t}-4e^{-0.036588t}+2e^{-0.03484t}\\
&+2e^{-0.033754t}+4e^{-0.032968t}+4e^{-0.031882t},\\
&t\ge 0, \qquad (48)
\end{aligned}
$$

$$
\begin{aligned}
&[\boldsymbol{R}(t,2)]^{(6)}=\\
&\left[1-\left(1-e^{-0.0054\cdot[\rho(2)]^{(6)}t}\right)(1-e^{-0.0054t})\right]\\
&\cdot\left[1-\left(1-e^{-0.02163\cdot[\rho(2)]^{(6)}t}\right)^2\right]\\
&\cdot\left[\begin{matrix}1-\left(1-e^{-0.00912\cdot[\rho(2)]^{(6)}t}\right)^2\\ \left(1-e^{-0.00984\cdot[\rho(2)]^{(6)}t}\right)\end{matrix}\right]=\\
&=e^{-0.105162t}-e^{-0.099762t}-e^{-0.098142t}\\
&-2e^{-0.093306t}-e^{-0.09237t}+2e^{-0.087906t}\\
&+e^{-0.08697t}+2e^{-0.086286t}+e^{-0.08535t}\\
&+e^{-0.08145t}+2e^{-0.080514t}-2e^{-0.077043t}\\
&-e^{-0.07605t}-2e^{-0.075114t}-e^{-0.07443t}\\
&-2e^{-0.073494t}+2e^{-0.071643t}+2e^{-0.070023t}\\
&+4e^{-0.065187t}+2e^{-0.064251t}-4e^{-0.059787t}\\
&-2e^{-0.058851t}-4e^{-0.058167t}-2e^{-0.057231t}\\
&-2e^{-0.053331t}-4e^{-0.052395t}+2e^{-0.047931t}\\
&+4e^{-0.046995t}+2e^{-0.046311t}+4e^{-0.045375t},\\
&t\ge 0; \qquad (49)
\end{aligned}
$$

- in the operation state z_7:

$$[\boldsymbol{R}(t,\cdot)]^{(7)}=[1,[\boldsymbol{R}(t,1)]^{(7)},[\boldsymbol{R}(t,2)]^{(7)}], \qquad (50)$$

where:

$$
\begin{aligned}
&[\boldsymbol{R}(t,1)]^{(7)}=\left[1-\left(1-e^{-0.0362\cdot[\rho(1)]^{(7)}t}\right)^2\right]\\
&\cdot\left[1-\left(1-e^{-0.01444\cdot[\rho(1)]^{(7)}t}\right)^2\right]\\
&\cdot\left[\begin{matrix}1-\left(1-e^{-0.0073\cdot[\rho(1)]^{(7)}t}\right)^2\\ \left(1-e^{-0.00874\cdot[\rho(1)]^{(7)}t}\right)\end{matrix}\right]\\
&=e^{-0.077298t}-2e^{-0.072592t}-2e^{-0.067808t}\\
&-e^{-0.065936t}+4e^{-0.063102t}+2e^{-0.06123t}\\
&-2e^{-0.058526t}+e^{-0.058318t}+2e^{-0.056446t}\\
&+4e^{-0.05382t}-2e^{-0.053612t}-4e^{-0.05174t}\\
&+4e^{-0.049036t}+2e^{-0.047164t}-8e^{-0.04433t}\\
&-4e^{-0.042458t}-2e^{-0.039546t}-4e^{-0.037674t}\\
&+4e^{-0.03484t}+8e^{-0.032968t}, t\ge 0, \qquad (51)
\end{aligned}
$$

$$
\begin{aligned}
&[\boldsymbol{R}(t,2)]^{(7)}=\left[1-\left(1-e^{-0.0054\cdot[\rho(2)]^{(7)}t}\right)^2\right]\\
&\cdot\left[1-\left(1-e^{-0.02163\cdot[\rho(2)]^{(7)}t}\right)^2\right]\\
&\cdot\left[\begin{matrix}1-\left(1-e^{-0.00912\cdot[\rho(2)]^{(7)}t}\right)^2\\ \left(1-e^{-0.00984\cdot[\rho(2)]^{(7)}t}\right)\end{matrix}\right]\\
&=e^{-0.106782t}-2e^{-0.099762t}-2e^{-0.094926t}\\
&-e^{-0.09399t}+4e^{-0.087906t}+2e^{-0.08697t}\\
&+e^{-0.08307t}+2e^{-0.082134t}-2e^{-0.078663t}\\
&-2e^{-0.07605t}-4e^{-0.075114t}+4e^{-0.071643t}\\
&+4e^{-0.066807t}+2e^{-0.065871t}-8e^{-0.059787t}\\
&-4e^{-0.058851t}-2e^{-0.054951t}-4e^{-0.054015t}\\
&+4e^{-0.047931t}+8e^{-0.046995t}, t\ge 0; \qquad (52)
\end{aligned}
$$

- in the operation state z_8:

$$[\boldsymbol{R}(t,\cdot)]^{(8)}=[1,[\boldsymbol{R}(t,1)]^{(8)},[\boldsymbol{R}(t,2)]^{(8)}], \qquad (53)$$

where:

$$
\begin{aligned}
&[\boldsymbol{R}(t,1)]^{(8)}=[1-(1-e^{-0.00362t})^2]\\
&\cdot[1-(1-e^{-0.01444t})^2]\\
&\cdot[1-(1-e^{-0.0073t})^2(1-e^{-0.00874t})]\\
&=e^{-0.05946t}-2e^{-0.05584t}-2e^{-0.05216t}\\
&-e^{-0.05072t}+4e^{-0.04854t}+2e^{-0.0471t}
\end{aligned}
$$

$$- 2e^{-0.04502t} + e^{-0.04486t} + 2e^{-0.04342t} + 4e^{-0.0414t} - 2e^{-0.04124t} - 4e^{-0.0398t} + 4e^{-0.03772t} + 2e^{-0.03628t} - 8e^{-0.0341t} - 4e^{-0.03266t} - 2e^{-0.03042t} - 4e^{-0.02898t} + 4e^{-0.0268t} + 8e^{-0.02536t}, t \geq 0, \quad (54)$$

$$[\boldsymbol{R}(t,2)]^{(8)} = [1-(1-e^{-0.0054t})^2] \cdot [1-(1-e^{-0.02163t})^2] \cdot \cdot [1-(1-e^{-0.00912t})^2(1-e^{-0.00984t})] = e^{-0.08214t} - 2e^{-0.07674t} - 2e^{-0.07302t} - e^{-0.0723t} + 4e^{-0.06762t} + 2e^{-0.0669t} + e^{-0.0639t} + 2e^{-0.06318t} - 2e^{-0.06051t} - 2e^{-0.0585t} - 4e^{-0.05778t} + 4e^{-0.05511t} + 4e^{-0.05139t} + 2e^{-0.05067t} - 8e^{-0.04599t} - 4e^{-0.04527t} - 2e^{-0.04227t} - 4e^{-0.04155t} + 4e^{-0.03687t} + 8e^{-0.03615t}, t \geq 0. \quad (55)$$

Hence, the expected values of the fuel transportation system lifetimes at the operation states z_b, $b = 1,2,\ldots,8$ respectively are [13]:

- in the reliability state subset {1, 2}:

$$\mu(1)^{(1)} = \int_0^\infty [\boldsymbol{R}(t,1)]^{(1)} dt \cong 82.63 \text{ years},$$
$$\mu(1)^{(2)} = \int_0^\infty [\boldsymbol{R}(t,1)]^{(2)} dt \cong 83.26 \text{ years},$$
$$\mu(1)^{(3)} = \int_0^\infty [\boldsymbol{R}(t,1)]^{(3)} dt \cong 72.70 \text{ years},$$
$$\mu(1)^{(4)} = \int_0^\infty [\boldsymbol{R}(t,1)]^{(4)} dt \cong 66.01 \text{ years},$$
$$\mu(1)^{(5)} = \int_0^\infty [\boldsymbol{R}(t,1)]^{(5)} dt \cong 76.77 \text{ years},$$
$$\mu(1)^{(6)} = \int_0^\infty [\boldsymbol{R}(t,1)]^{(6)} dt \cong 65.68 \text{ years},$$
$$\mu(1)^{(7)} = \int_0^\infty [\boldsymbol{R}(t,1)]^{(7)} dt \cong 64.92 \text{ years},$$
$$\mu(1)^{(8)} = \int_0^\infty [\boldsymbol{R}(t,1)]^{(8)} dt \cong 84.39 \text{ years}; \quad (56)$$

- in the reliability state subset {2}:

$$\mu(2)^{(1)} = \int_0^\infty [\boldsymbol{R}(t,2)]^{(1)} dt \cong 57.6 \text{ years},$$
$$\mu(2)^{(2)} = \int_0^\infty [\boldsymbol{R}(t,2)]^{(2)} dt \cong 57.9 \text{ years},$$
$$\mu(2)^{(3)} = \int_0^\infty [\boldsymbol{R}(t,2)]^{(3)} dt \cong 50.0 \text{ years},$$
$$\mu(2)^{(4)} = \int_0^\infty [\boldsymbol{R}(t,2)]^{(4)} dt \cong 45.6 \text{ years},$$
$$\mu(2)^{(5)} = \int_0^\infty [\boldsymbol{R}(t,2)]^{(5)} dt \cong 53.4 \text{ years},$$
$$\mu(2)^{(6)} = \int_0^\infty [\boldsymbol{R}(t,2)]^{(6)} dt \cong 45.6 \text{ years},$$
$$\mu(2)^{(7)} = \int_0^\infty [\boldsymbol{R}(t,2)]^{(7)} dt \cong 45.05 \text{ years},$$
$$\mu(2)^{(8)} = \int_0^\infty [\boldsymbol{R}(t,2)]^{(8)} dt \cong 58.56 \text{ years}; \quad (57)$$

- with intensities of ageing of considered system in the reliability state subsets {1, 2},{2}:

$$\lambda(1)^{(1)} = 0.01210,\ \lambda(2)^{(1)} = 0.01735,$$
$$\lambda(1)^{(2)} = 0.01201,\ \lambda(2)^{(2)} = 0.01725,$$
$$\lambda(1)^{(3)} = 0.01376,\ \lambda(2)^{(3)} = 0.01999,$$
$$\lambda(1)^{(4)} = 0.01515,\ \lambda(2)^{(4)} = 0.02192,$$
$$\lambda(1)^{(5)} = 0.01303,\ \lambda(2)^{(5)} = 0.01872,$$
$$\lambda(1)^{(6)} = 0.01523,\ \lambda(2)^{(6)} = 0.02192,$$
$$\lambda(1)^{(7)} = 0.01540,\ \lambda(2)^{(7)} = 0.02220,$$
$$\lambda(1)^{(8)} = 0.01185,\ \lambda(2)^{(8)} = 0.01708. \quad (58)$$

From results (4) and (58), the fuel transportation system unconditional reliability function is given by [13]:

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), \boldsymbol{R}(t,2)], t \geq 0, \quad (59)$$

where:

$$\boldsymbol{R}(t,1) = 0.16 \cdot e^{-0.0121t} + 0.139 \cdot e^{-0.01201t} + 0.065 \cdot e^{-0.01376t} + 0.124 \cdot e^{-0.01515t} + 0.134 \cdot e^{-0.01303t} + 0.058 \cdot e^{-0.01523t} + 0.055 \cdot e^{-0.01540t} + 0.265 \cdot e^{-0.01185t}, t \geq 0, \quad (60)$$

$$\boldsymbol{R}(t,2) = 0.16 \cdot e^{-0.01735t} + 0.139 \cdot e^{-0.01725t} + 0.065 \cdot e^{-0.01999t} + 0.124 \cdot e^{-0.02192t} + 0.134 \cdot e^{-0.01872t} + 0.058 \cdot e^{-0.02192t} + 0.055 \cdot e^{-0.02220t} + 0.265 \cdot e^{-0.01708t}, t \geq 0. \quad (61)$$

Hence, from (4) and (56)–(57), the expected values of the fuel transportation system lifetimes are (in years) [13]:

- in the reliability state subset {1, 2}:

$$\mu(1) = 0.16 \cdot 82.63 + 0.139 \cdot 83.26 + 0.065 \cdot 72.7 + 0.124 \cdot 66.01 + 0.134 \cdot 76.77 + 0.058 \cdot 65.68 + 0.055 \cdot 64.92 + 0.265 \cdot 84.39 \cong 77.74; \quad (62)$$

- in the reliability state subset {2}:

$$\mu(2) = 0.16 \cdot 57.65 + 0.139 \cdot 57.97 + 0.065 \cdot 50.01 + 0.124 \cdot 45.626 + 0.134 \cdot 53.41 + 0.058 \cdot 45.618 + 0.055 \cdot 45.05 + 0.265 \cdot 58.56 \cong 53.99; \quad (63)$$

- with intensities of ageing of considered system:

$$\lambda(1) = 0.01286,\ \lambda(2) = 0.01852. \quad (64)$$

Another reliability characteristics are [13]:

- standard deviations of the fuel transportation system lifetimes in the reliability state subsets {1, 2}, {2}:

$$\sigma(1) \cong 78.44 \text{ years}, \sigma(2) \cong 54.5 \text{ years}; \quad (65)$$

- the expected values of the fuel transportation system lifetimes in the particular reliability states:

$$\bar{\mu}(1) = 23.75, \bar{\mu}(2) = 53.99. \quad (66)$$

As the critical reliability state is $r = 1$, then by (60) and (64), the fuel transportation system risk function including process operation impact is respectively given by [13]:

$$r^1(t) = 1 - \boldsymbol{R}(t,1) = 1 - e^{-0.01286t}, t \geq 0. \quad (67)$$

The moment when the fuel transportation system risk function exceeds a permitted level $\delta = 0.05$ is:

$$\tau^1 = -\frac{1}{0.01286} \cdot \ln(1 - 0.05) \cong 3.99. \quad (68)$$

The graph of the risk function of the fuel transportation system with the moment τ^1 is presented below in *Figure 4* [13].

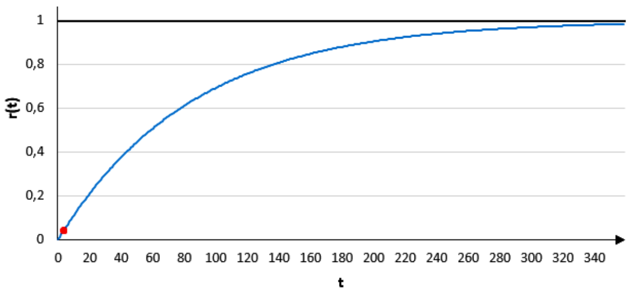

Figure 4. The graph of fuel transportation system risk function impacted by operation process

Considering (27) and (64), the coefficients of the operation process impact on the fuel transportation system intensities of ageing, respectively are [13]:

$$\rho(t,1) = \frac{\lambda(t,1)}{\lambda(t,1)^{(0)}} = \frac{0.01286}{0.01185} \cong 1.085,$$
$$\rho(t,2) = \frac{\lambda(t,2)}{\lambda(t,2)^{(0)}} = \frac{0.01852}{0.01708} \cong 1.085. \quad (69)$$

Finally, by (69), the fuel transportation system resilience indicator (the coefficient of the fuel transportation system resilience to operation process impact, is [13]:

$$RI\ (t) = \frac{1}{\rho(t,1)} \cong 0,922 = 92,2\%. \quad (70)$$

The comparison of reliability indicators (22)–(31) and (59)–(68) proves a noticeable influence of the operation process on the fuel transportation system reliability what is clearly presented in the resilience indicators (69)–(70).

6. Reliability optimization of fuel transportation system

As we can notice in Section 5, the operation process has a significant impact on the fuel transportation system reliability. In order to improve the reliability of system, it is proposed to optimize it based on linear programming [6]–[8]. This approach consists in finding appropriate optimal $\dot{p}_b$ values in order to maximise the mean values of the fuel transportation system lifetimes in the reliability state subsets. As mentioned in Section 4, it is assumed that critical state is $r = 1$. The first step is to define the objective function using the mean value of the fuel transportation system lifetime in critical state as follows:

$$\mu(1) = p_1 \cdot 82.63 + p_2 \cdot 83.26 + p_3 \cdot 72.7 + p_4 \cdot 66.01 + p_5 \cdot 76.77 + p_6 \cdot 65.68 + p_7 \cdot 64.92 + p_8 \cdot 84.39. \quad (71)$$

Moreover, it is necessary to arbitrarily assume the boundary limits for the limit probabilities p_b, $b = 1,2 \ldots 8$:

$$0.14 \leq p_1 \leq 0.17, \quad 0.11 \leq p_5 \leq 0.15,$$
$$0.12 \leq p_2 \leq 0.15, \quad 0.04 \leq p_6 \leq 0.07,$$
$$0.05 \leq p_3 \leq 0.07, \quad 0.04 \leq p_7 \leq 0.06,$$
$$0.11 \leq p_4 \leq 0.14, \quad 0.23 \leq p_8 \leq 0.28. \quad (72)$$

The mean values $[\mu(1)]^{(b)}, b = 1,2, \ldots ,8,$ are set in non-increasing order as follows:

$$\mu_8(1) \geq \mu_2(1) \geq \mu_1(1) \geq \mu_5(1) \geq \mu_3(1) \geq \mu_4(1) \geq \mu_6(1) \geq \mu_7(1). \quad (73)$$

Then, using (73), we take some substitutions as follows:

$$x_1 = p_8,\ x_2 = p_2,\ x_3 = p_1,\ x_4 = p_5,$$
$$x_5 = p_3,\ x_6 = p_4,\ x_7 = p_6,\ x_8 = p_7 \quad (74)$$

and using (60), we create the new objective function:

$$\mu(1) = x_1 \cdot 84.39 + x_2 \cdot 83.26 + x_3 \cdot 82.63 + x_4 \cdot 76.77 + x_5 \cdot 72.7 + x_6 \cdot 66.01 + x_7 \cdot 65.68 + p_8 \cdot 64.92, \quad (75)$$

with the new boundary limits ($\sum_{i=1}^{8} x_i = 1$):

$$\breve{x}_1 = 0.23 \le x_1 \le 0.28 = \hat{x}_1,$$
$$\breve{x}_2 = 0.12 \le x_2 \le 0.15 = \hat{x}_2,$$
$$\breve{x}_3 = 0.14 \le x_3 \le 0.17 = \hat{x}_3,$$
$$\breve{x}_4 = 0.11 \le x_4 \le 0.15 = \hat{x}_4,$$
$$\breve{x}_5 = 0.05 \le x_5 \le 0.07 = \hat{x}_5,$$
$$\breve{x}_6 = 0.11 \le x_6 \le 0.14 = \hat{x}_6,$$
$$\breve{x}_7 = 0.04 \le x_7 \le 0.07 = \hat{x}_7,$$
$$\breve{x}_8 = 0.04 \le x_8 \le 0.06 = \hat{x}_8. \quad (76)$$

In order to find optimal values $x_i, i = 1,2,\ldots,8$, the following characteristics are defined:

$$\breve{x} = \sum_{i=1}^{8} \breve{x}_i = 0.84, \quad (77)$$

$$\hat{y} = 1 - \breve{x} = 1 - 0.84 = 0.16, \quad (78)$$

$$\breve{x}^1 = \sum_{i=1}^{1} \breve{x}_i = 0.23, \hat{x}^1 = \sum_{i=1}^{1} \hat{x}_i = 0.28,$$
$$\breve{x}^2 = \sum_{i=1}^{2} \breve{x}_i = 0.35, \hat{x}^2 = \sum_{i=1}^{2} \hat{x}_i = 0.43,$$
$$\breve{x}^3 = \sum_{i=1}^{3} \breve{x}_i = 0.49, \hat{x}^3 = \sum_{i=1}^{3} \hat{x}_i = 0.60,$$
$$\breve{x}^4 = \sum_{i=1}^{4} \breve{x}_i = 0.60, \hat{x}^4 = \sum_{i=1}^{4} \hat{x}_i = 0.75,$$
$$\breve{x}^5 = \sum_{i=1}^{5} \breve{x}_i = 0.65, \hat{x}^5 = \sum_{i=1}^{5} \hat{x}_i = 0.82,$$
$$\breve{x}^6 = \sum_{i=1}^{6} \breve{x}_i = 0.76, \hat{x}^6 = \sum_{i=1}^{6} \hat{x}_i = 0.96,$$
$$\breve{x}^7 = \sum_{i=1}^{7} \breve{x}_i = 0.80, \hat{x}^7 = \sum_{i=1}^{7} \hat{x}_i = 1.03,$$
$$\breve{x}^8 = \sum_{i=1}^{8} \breve{x}_i = 0.84, \hat{x}^8 = \sum_{i=1}^{8} \hat{x}_i = 1.09. \quad (79)$$

Then we find the highest value of I for $I \in \{0,1,\ldots,8\}$ that satisfies the equation $\hat{x}^I - \breve{x}^I < \hat{y}$:

$$\hat{x}^4 - \breve{x}^4 = 0.15 < \hat{y} = 0.16. \quad (80)$$

Considering (78)–(80), we determine optimal solutions $\dot{x}_i$, $i = 1,2 \ldots 8$, as follows:

- for $i = 1,2 \ldots I$:

$$\dot{x}_1 = \hat{x}_1 = 0.28,$$
$$\dot{x}_2 = \hat{x}_2 = 0.15,$$
$$\dot{x}_3 = \hat{x}_3 = 0.17,$$
$$\dot{x}_4 = \hat{x}_4 = 0.15; \quad (81)$$

- for $i = I + 1$:

$$\dot{x}_5 = \hat{y} - \hat{x}^4 + \breve{x}^4 + \breve{x}_5 = 0.06; \quad (82)$$

- for $i = I + 2, \ldots, 8$:

$$\dot{x}_6 = \breve{x}_6 = 0.11,$$
$$\dot{x}_7 = \breve{x}_7 = 0.04,$$
$$\dot{x}_8 = \breve{x}_8 = 0.04. \quad (83)$$

Then we define the optimal boundary limit probabilities $\dot{p}_i, i = 1,2,\ldots,8$, the same as in (74):

$$\dot{p}_1 = \dot{x}_3 = 0.17,$$
$$\dot{p}_2 = \dot{x}_2 = 0.15,$$
$$\dot{p}_3 = \dot{x}_5 = 0.06,$$
$$\dot{p}_4 = \dot{x}_6 = 0.11,$$
$$\dot{p}_5 = \dot{x}_4 = 0.15,$$
$$\dot{p}_6 = \dot{x}_7 = 0.04,$$
$$\dot{p}_7 = \dot{x}_8 = 0.04,$$
$$\dot{p}_8 = \dot{x}_1 = 0.28. \quad (84)$$

Finally, using (71) and (84), we get the maximized mean value of the fuel transportation system lifetime in critical state:

$$\dot{\mu}(1) = \dot{p}_1 \cdot 82.63 + \dot{p}_2 \cdot 83.26 + \dot{p}_3 \cdot 72.7 + \dot{p}_4 \cdot 66.01 + \dot{p}_5 \cdot 76.77 + \dot{p}_6 \cdot 65.68 + \dot{p}_7 \cdot 64.92 + \dot{p}_8 \cdot 84.39 = 78.58. \quad (85)$$

Thanks to the above parameters, we can optimize the reliability function and its reliability characteristics of the fuel transportation system. The optimized unconditional reliability function is presented as follows:

$$\dot{\boldsymbol{R}}(t,\cdot) = [1, \dot{\boldsymbol{R}}(t,1), \dot{\boldsymbol{R}}(t,2)], t \ge 0, \quad (86)$$

where:

$$\dot{\boldsymbol{R}}(t,1) = 0.17 \cdot e^{-0.0121t} + 0.15 \cdot e^{-0.01201t} + 0.06 \cdot e^{-0.01376t} + 0.11 \cdot e^{-0.01515t} + 0.15 \cdot e^{-0.01303t} + 0.04 \cdot e^{-0.01523t} + 0.04 \cdot e^{-0.01540t} + 0.28 \cdot e^{-0.01185t}, t \ge 0, \quad (87)$$

$$\dot{\boldsymbol{R}}(t,2) = 0.17 \cdot e^{-0.01735t} + 0.15 \cdot e^{-0.01725t} + 0.06 \cdot e^{-0.01999t} + 0.11 \cdot e^{-0.02192t} + 0.15 \cdot e^{-0.01872t} + 0.04 \cdot e^{-0.02192t} + 0.04 \cdot e^{-0.02220t} + 0.28 \cdot e^{-0.01708t}, t \ge 0. \quad (88)$$

Hence, the optimized the expected values of the fuel transportation system lifetimes are (in years):

- in the reliability state subsets {1, 2}:

$$\dot{\mu}(1) = 78.58; \quad (89)$$

- in the reliability state subsets {2}:

$$\dot{\mu}(2) = 54.55; \quad (90)$$

- with optimized intensities of ageing of the fuel transportation system:

$$\dot{\lambda}(1) = 0.01273; \quad \dot{\lambda}(2) = 0.01833. \quad (91)$$

The optimized expected values of the fuel transportation system lifetimes in the particular reliability states respectively are:

$$\dot{\bar{\mu}}(1) = 24.03 \text{ years}, \quad \dot{\bar{\mu}}(2) = 54.55 \text{ years}. \quad (92)$$

As the critical reliability state is $r = 1$, the optimized fuel transportation system risk function including process operation impact is given by:

$$\dot{r}^1(t) = 1 - \dot{\boldsymbol{R}}(t,1) = 1 - e^{-0.01273t}. \quad (93)$$

The moment when the optimized fuel transportation system risk function exceeds a permitted level $\delta = 0.05$ is:

$$\dot{\tau}^1 = -\frac{1}{0.01273} \cdot \ln(1 - 0.05) \cong 4.03. \quad (94)$$

The graph of the optimized risk function of the fuel transportation system with the moment $\dot{\tau}^1$ is presented in *Figure 5*.

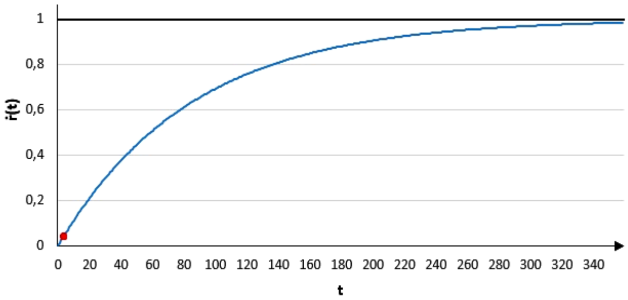

Figure 5. The graph of optimized risk function of fuel transportation system impacted by operation process

Considering (27) and (91), the coefficients of the operation process impact on the optimized fuel transportation system intensities of ageing, respectively are:

$$\dot{\rho}(t,1) = \frac{\lambda(t,1)}{\lambda(t,1)^{(0)}} = \frac{0.01273}{0.01185} \cong 1.074,$$
$$\dot{\rho}(t,2) = \frac{\dot{\lambda}(t,2)}{\lambda(t,2)^{(0)}} = \frac{0.01833}{0.01708} \cong 1.073. \quad (95)$$

Finally, the optimized fuel transportation system resilience indicator (the coefficient of the fuel transportation system resilience to operation process impact), is:

$$\dot{RI}\ (t) = \frac{1}{\dot{\rho}(t,1)} \cong 0{,}931 = 93.1\%. \quad (96)$$

Comparing (59)–(70) with (86)–(96), we conclude that the performed reliability optimization has a good effect on the operation of the fuel transportation system. The obtained characteristics after optimization are more favourable for the fuel transportation system and increase its reliability.

7. Conclusion

The approach to the reliability analysis and optimization of the multistate parallel-serial fuel transportation system in variable operating conditions is presented in the paper. The comparison of the fuel transportation system reliability without operation process impact and including operation process is introduced as well. The obtained results prove a noticeable influence of the operation process on the fuel transportation system reliability what is clearly presented in the resilience indicators (69)–(70). The reliability of the fuel transportation system with the participation of operation process is also optimized using the method of linear programming. The performed optimization shows the possibility of increasing the fuel transportation system reliability by changing the initial operating parameters of the system, which are the limit values of transient probabilities of the operation process at the particular operating states.

The proposed model of the fuel transportation system reliability without operation process impact and including operation process can be applied to the reliability and resilience analysis of various technical systems with large structures. Moreover the presented optimization shows the opportunities for increasing the reliability of such systems.

The reliability analysis presented in this paper may turn out to be a useful tool in estimating the reliability characteristics of large technical systems. The obtained results may contribute to the improvement of the operational safety of the systems, the increase its operational efficiency and the further research in the field of reliability and optimization of the large technical systems.

Acknowledgement

The chapter presents results developed in the scope of the author's MSc Thesis "Analysis of the reliability of multistate parallel-series system in variable operating conditions" at Department of Mathematics of Gdynia Maritime University in 2020.

References

[1] Kołowrocki, K. 1993. *On a Class of Limit Reliability Functions for Series-Parallel and Parallel-Series Systems*. Monograph. Maritime University Press, Gdynia.

[2] Kołowrocki, K. 2004. *Reliability of Large Systems*. Elsevier, Amsterdam – Boston – Heidelberg – London – New York – Oxford – Paris – San Diego – San Francisco – Singapore – Sydney – Tokyo.

[3] Kołowrocki, K. 2014. *Reliability of Large and Complex Systems*. 2nd, ed., Elsevier, Amsterdam – Boston – Heidelberd – London – New York – Oxford – Paris – San Diego – San Francisco – Singapore – Sydney – Tokyo.
[4] Kołowrocki, K. 2019. Examination of the safety of a port oil terminal, *Scientific Journals of the Maritime University of Szczecin*, 61(133), 143–151.
[5] Kołowrocki, K., Kuligowska, E., Soszyńska-Budny J. & Torbicki M. 2018. Oil transport in port. Part 0. Port oil piping transportation system safety without outside impacts. *Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars* 9(2), 1–10.
[6] Kołowrocki, K. & Magryta, B. 2020. Port oil terminal reliability optimization, *Scientific Journals of the Maritime University of Szczecin* 62(134), 161–167.
[7] Kołowrocki, K. & Soszyńska-Budny, J. 2011. 2011/2015. *Reliability and Safety of Complex Systems and Processes: Modeling – Identification – Prediction – Optimization*, Springer English / Chinese Edition.
[8] Kołowrocki, K. & Soszyńska-Budny, J. 2018. Critical infrastructure impacted by operation safety and resilience indicators, *International Conference on Industrial Engineering and Engineering Management – IEEM*.
[9] Kołowrocki, K., Soszyńska-Budny, J. 2019. Safety indicators of critical infrastructure application to port terminal examination. *Proceedings of 29th ISOPE Conference*, Honolulu.
[10] Szymkowiak, M. 2018. Characterizations of distributions through aging intensity. *IEEE Transactions on Reliability* 67.
[11] Szymkowiak, M. 2018. Generalized aging intensity functions. *Reliability Engineering and System Safety* 178.
[12] Szymkowiak, M. 2019. *Lifetime Analysis by Aging Intensity Functions*. Monograph in series: Studies in Systems, Decision and Control 196, Springer International Publishing.
[13] Wilemski, S. 2020. *Analysis of the Reliability of Multistate Parallel-Series System in Variable Operating Conditions*. MSc Thesis. Department of Mathematics. Gdynia Maritime University.
[14] Xue, J. 1985. On multi-state system analysis. *IEEE Transactions on Reliability* 34, 329–337.
[15] Xue, J & Yang, K. 1985. Dynamic reliability analysis of coherent multi-state systems. *IEEE Transactions on Reliability* 4(44), 683–688.
[16] Xue, J. & Yang, K. 1995. Symmetric relations in multi-state systems. *IEEE Transactions on Reliability* 4(44), 689–693.