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Determination of the linear correlation coefficient between Young's modulus and the compressive strength in fibre-reinforced concrete based on experimental studies

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Abstract
The test procedures for determining Young's modulus in concrete are complicated and time-consuming. Therefore, attempts to search for alternative methods of its determination are not surprising. The relationship between the value of compressive strength and Young's modulus in concrete is known. However, the strength of this relationship in fibre-reinforced concrete has not been exactly described. The article attempts to investigate the strength of the correlation between Young's modulus and the compressive strength of fibre-reinforced concrete. The influence of the amount of fibres on this relationship was also checked. Two types of specimen were used for the tests. The specimens differed in the content of steel fibres, 0.25% and 0.50%, respectively. In order to determine the correlation relationship, the method of linear regression and the coefficient of linear correlation were used. The use of the determination coefficient allowed us to examine the degree of explanation of one variable by another.

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1. Introduction

The continuous development of the construction industry related to the increasing needs of society and industry, the need to build newer and more durable buildings but also bigger and bigger demands of customers have caused that scientists are looking for better and better opportunities to improve the quality of materials used in this field (Boadu et al., 2020). Anamani and Osei-Amponsah(2007) claimed that there are significant relationships between the growth rate of the construction industry and the rate of macroeconomic growth of developing countries. It should also be remembered that other sectors of the economy largely dependent on the construction industry, as their development often requires the expansion or construction of new industrial facilities, infrastructure, water supply and power lines. On the other hand, there is also a growing demand among society for residential buildings and other infrastructure which is useful for every-day living.

Important aspect of the management in the construction industry is the reasonable allocation of resources in order to complete the project in accordance with the approved budget, time allowance and the set quality level (Ulewicz and Ulewicz, 2020). This will allow not only for good construction organization, but also for timely execution of orders and execution of constructions that meet the design assumptions.

Each construction, each building is exposed to different influences during its operation. As a result, various defects and damage to these elements arise, which affect their safe operation (Kopiika et al., 2021). That is why the quality of the materials used in construction is so essential. Many authors emphasize that the quality of the materials used in the production is the factor that can have the greatest impact on the final result (Kraus et al., 2018; Krynke et al., 2021; Kucharikova et al., 2019).

The use of appropriate solutions and materials in this area allows not only to accelerate the construction process, but also improves the durability of the constructed buildings. The development of technology, also in the field of construction, means that also in this area new solutions are sought or the already existing ones are improved. Work on improving the builder system is ongoing. Most of them have led to the creation of modern materials that can even perform several functions. Therefore, it is necessary to emphasize the need to grant innovation and improvement of the materials that are currently used (Pachura, 2021; Szajnaret al., 2013; Fidlerova et al., 2022).

One of the important areas of research in the field of improvement in the construction industry is the search for methods to strengthen concrete. One solution is to use different types of fibres to improve the mechanical properties of concrete. Steel, synthetic (polypropylene, polyester, polyacrylonitrile), glass, carbon, basalt and even organic fibers are used. However, one of the most commonly used fibers, both because of their versatile effectiveness and relatively low cost, is steel fibers.

Fibres added to concrete improve its strength and eliminate the formation of shrinkage cracks. In the past, it was a solution reserved for industry. Currently, reinforcing fibres are used more and more often for the needs of individual construction.

Already in the ancient time there were the first attempts to modify the building material with fibres. The first fibres were of organic origin. However, the turning point came in 1874, when Bernard patented the possibility of strengthening concrete with steel filings (Katzer, 2003). Then Alfsen in 1918 tried to strengthen the concrete with long steel fibres. Another turning point was Zitkiewic's research on the strength and impact toughness of concrete using pieces of mild steel wire (Jamroży, 2008). Steelfibres in concrete were first used by Romuladi and Baston in 1963 to improve its properties (Ghaffar et al., 2014). The most common additive found in fibroconcrete are steel fibres (Thomas and Ramaswamy, 2007), but due to the research carried out in this field, this group also includes fibres made of glass, carbon, basalt and artificial materials.

Improvement of the properties of a material commonly used in construction, i.e. concrete, has been the subject of research for many years. In the literature, it is possible to find many interesting works on steel fibre-reinforced concrete and their properties. For example, the authors of Bencardino and the team(2008; 2010) conducted research aimed at evaluation of fibre reinforced concrete fracture properties. They claimed that fibres contribute immensely to the structural integrity and structural stability of concrete elements and thereby improve their durable service life. While authors Chalioris and Panagiotopoulos (2018) after analyzing their research they concluded that their developed approach also provides rational and more accurate compressive and tensile stress-strain curves along with bending moment versus curvature curves with regards to the predictions of relevant existing models. Additionally, Chalioris with another group of scientists (Kytinou et al., 2020) continued research on steel fibre-reinforced concrete beams. This group of scientists pointed out as the most important conclusion the favorable influence of steel fibres on the flexural behavior, the cracking performance, and the post-cracking residual stress. Galobardes and his colleagues (2014) conducted research on the resilience of the sprayed concrete. Their research indicated the basis to adapt the current formulations, taking into account the specificities of sprayed concrete: porosity and rebound.

Young's modulus (or modulus of linear deformation / modulus of elasticity) is a quantity describing the elasticity of a material. Young's modulus allows the determination of the stiffness of an element (Michałek, 2015). It expresses the specific dependence for a given material of the relative linear strain ε on the stress σ, which occurs in the material in the range of elastic deformations (Fig. 1).

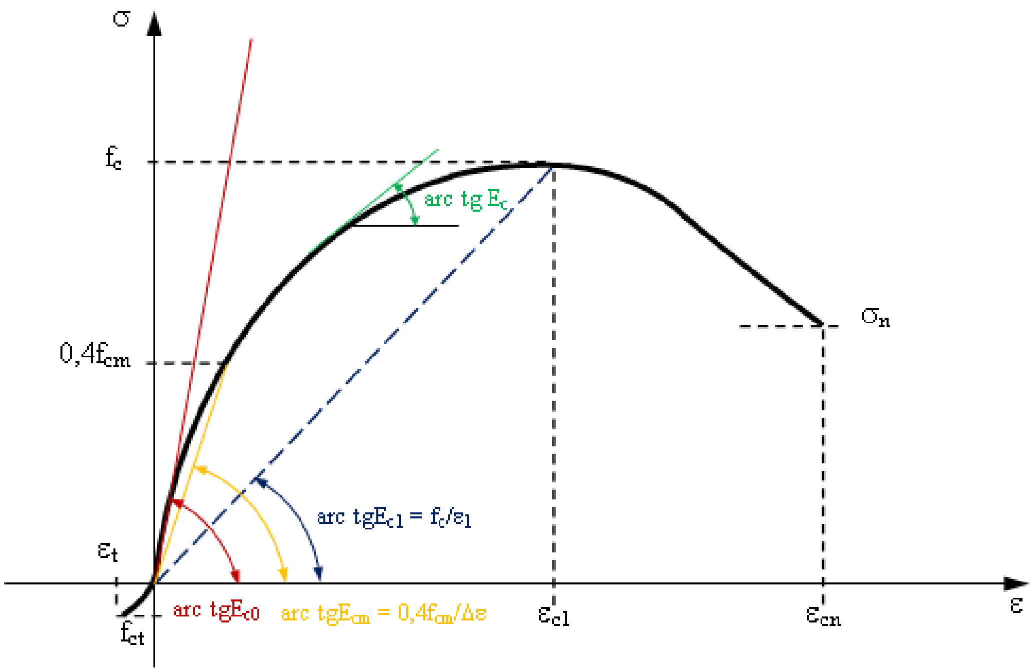
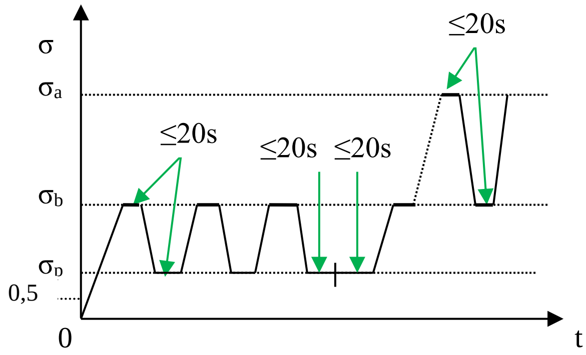


Fig. 1. Diagram of the σ - εrelationship for concrete with marked values of the modulus of elasticity: initial, secant, return (NoguchiandNemati, 1995)

Young's modulus is, apart from compressive strength, the main parameter describing concrete as a construction material. Determining its value for concrete is very important, especially since concrete is a composite material that has a rather heterogeneous structure, and the results obtained from experimental tests are often characterized by a large dispersion. The value of the modulus is influenced by both the qualitative and quantitative composition of the concrete mix (the use of admixtures and additives), as well as the method of its implementation (the order of dosing components, appropriate compaction) and the subsequent care of the hardening concrete. For this reason, despite knowledge of the recipe, production technology and concrete care, appropriate tests should be performed on specimens made of fresh concrete mix or cut from existing structural elements in order to determine the actual parameters of the concrete (Lustosa and Magalhaes, 2019; Steenbergen and Vervuurt, 2012; Jurowski and Grzeszczyk, 2015; Trąmpczyński et al., 2020).

Unfortunately, while the compressive strength tests performed on specimens in the uniaxial stress state do not cause major problems, the determination of Young's modulus is much more time-consuming, complicated, and often involves rejecting the experimental results. According to the PN-EN 12390-13:2014-02 standard (PN-EN 12390-13:2014-02), Young's modulus should be determined according to two methods, A or B. Method A allows the determination of both the initial modulus of concrete elasticity (Ec,0) and the secant modulus (Ec,s) – three preload cycles are initially performed and only then are the actual cycles performed. Method B allows the determination of only the stabilized secant modulus of concrete elasticity (Ec,s) after three load cycles. Regardless of the method adopted, the procedure of determining Young's modulus in concrete is complicated and time-consuming. The test should be performed on specimens whose preparation and parameters are strictly defined in the standard (PN-EN 12390-1:2013-03; PN-EN 12390-2:2001). Specimens should be columnar (cylindrical or rectangular) (PN-EN 12390-2:2001) or cores cut from the structure meeting the requirements of the standard (PN EN 12504-1) with appropriate dimensions and shape proportions. In addition, before the main test of Young's modulus for concrete, it is necessary to prepare an appropriate number of accompanying specimens in order to determine the average compressive strength of the concrete in accordance with the standard (PN-EN 12390-3:2002). This allows the determination of the value of the upper stress level σa = fcm/3, necessary for the correct determination of the elastic modulus by the A or B method. In method A, three preload cycles up to the value of σb = (0.1 ÷ 0.15)fcm should be performed, during which the correctness of the readings of the measuring sensors should be checked (this is the so-called first control) and the correct, centric specimen setting (the so-called second control). If, in the case of the first check, the difference in the readings from the individual sensors, i.e. the strain values on the individual measurement lines (measured at the stress level σb) is greater than εb ± 10%, the test must be stopped, adjusted and restarted. If, after adjustment, the difference is still greater than εb ± 10%, the test shall be terminated without the possibility of determining Young's modulus. If, in the case of the second control, during the third load cycle, the difference in the strain values on individual measurement lines is greater than εb ± 20%, the test must be stopped, the specimen settings corrected and the test restarted. If, after adjusting the settings, the difference is still greater than ± 20%, the test must be terminated and the specimen discarded. Only when both the first and second checks indicate correct sensor readings can Young’s modulus be tested by performing the main cycles (up to stress level σa = fcm/3), as shown in Fig. 2.



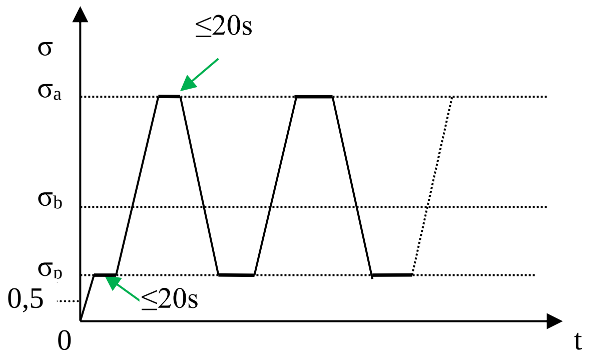
loading cycle
loading cycle for the determination of the initial secant modulus of elasticity
loading cycle for the determination of the stabilized secant modulus of elasticity
σ – applied stress in MPa
σa – upper stress – fc/3
σb –lower stress -0.10xfc≤σb≤0,15xfc
σp– preload stress – 0.5MPa≤ σp≤ σb
t- time in s

Fig. 2. Scheme of the determination of Young's modulus for concrete using the A method PN-EN 12390-13:2014-02 Concrete test. Part 13: Determination of the secant modulus of elasticity in compression (PN-EN 12390-13:2014-02)

Determining Young’s modulus by the B method is slightly less time-consuming and complicated. In this case, no preload cycles are performed, but only main cycles, during which the strain value readings are also checked: after the second load cycle - the first control, in the third load cycle - second control. If the strain values during the controls meet the conditions analogous to those in method A, the test can be completed by finally loading the specimen to failure (Fig. 3), while the compressive strength value should not differ by more than ± 20% of the average value (fcm) obtained from the test with the accompanying specimens.

If the controls performed during the test show greater than the acceptable differences in the strain values on individual sensors (similarly to method A), the test should be stopped, and either the specimen settings or the sensor adjustment should be corrected, and the test repeated. If the strain values still exceed the acceptable differences, the test should be terminated without the possibility of determining a reliable value of Young's modulus.

The above measurement procedures are not only complicated and time-consuming, but often involve the rejection of test specimens, which, with their limited number, may make it impossible to determine Young's modulus.



loading cycle
loading cycle for the determination of the initial secant modulus of elasticity
σ – applied stress in MPa
σ_a – upper stress – fc/3
σ_b –lower stress – 0.10 x fc≤σ_b≤0,15 x fc
σ_p– preload stress – 0,5MPa≤ σ_p≤ σ_b
t- time in s

Fig. 3. Scheme of the determination of Young's modulus for concrete using the B method (PN-EN 12390-13:2014-02)

Therefore, it is worth considering whether in some situations it would be possible to replace the described tests with the analysis of results obtained only from the compressive strength measurements. Perhaps a positive correlation of the results of compressive strength and Young's modulus could be obtained in the case of a material more homogeneous than concrete, e.g. fibre reinforced concrete, i.e. concrete with the addition of randomly dispersed fibres. Fibre reinforced concrete is a type of concrete composite in which various types of fibre are used as an additive.

Research on concrete with dispersed reinforcement, i.e. fibro-concrete, has been conducted for decades. Many properties of this composite have already been recognized and described (Brandt, 2008; Brandt, 2009; Czajkowska and Ingaldi, 2022). Steel, polypropylene, carbon, glass, or even biological fibres are often used (Brandt, 2008; Brandt, 2009; Karwowska and Łapko, 2011; Lee and Barr, 2003; Helbrych, 2021; Šadzevičius et al., 2023). The addition of randomly dispersed micro-reinforcement fibres to the concrete usually allows for obtaining a composite with better properties than ordinary concrete. Concrete with dispersed reinforcement is a "quasi-plastic" and "quasi-homogeneous" material, so it has better adhesion, deformability and tightness, as well as higher early strength (Brandt, 2008; Brandt, 2009; Vavru and Koteš, 2022; Song and Hwang, 2004; Skarżyński and Suchorzewski, 2008), which is important in the case of constructures exposed to aggressive environmental impact, i.e. bridge structures, tunnels, viaducts, parking lots, thin-walled elements (tanks and basins), weirs, retaining walls, elements subjected to dynamic loads, concrete pavements, industrial floors, as well as for repairs of this type of facilities. Randomly dispersed fibres in the concrete mix reduce the stress concentration and thus reduce scratches (Logońet al. 2021). When added to fresh concrete mix, they play the role of micro-reinforcement reducing plastic shrinkage and limiting the formation of shrinkage cracks in hardened concrete (Alsharie, 2015). Adding fibres to the concrete mix additionally influences its aeration, which improves frost resistance (Yan et al., 2018; Chen, 2006).

Steel fibres are among the most commonly used fibres added to concrete. This is due to both their effectiveness and relatively low cost. The short steel fibres added in concrete as mass reinforcement mainly provide crack control due to the tensile stress transfer capability of the fibres across crack surfaces known as crack-bridging, after cracking. This way, fibres provide significant resistance to shear across developing cracks, and therefore, SFRC demonstrate a pseudo-ductile response, increased residual strength (especially in tension) and enhanced energy dissipations capacities, relative to the brittle behaviour of plain concrete mixtures. Steel fibres for concrete reinforcement are supplied in various shapes and sizes. The fibre content of the concrete mix is usually in the range 0.25 – 2% (by volume) per 1m3, although some authors point to a wider range - up to 3.0% (Karwowska and Łapko, 2011), which, according the authors' experience, seems surprisingly a lot. The addition of reinforcement in an amount less than 0.25% is ineffective (studies show that it does not improve the parameters of concrete), while the addition of fibres in an amount more than 2.0% makes the concrete mixture difficult to work (even with the use of special plasticizing admixtures) (Raczkiewicz, 2016; Raczkiewicz and Kossakowski, 2019). The authors' own research indicates that adding 1.5% of fibres is enough to significantly affect the workability of the mix; due to the formation of so-called "nests", i.e. bundles of fibres, the mix loses uniformity and the concrete parameters are worse than those of a mix with a 1.0% fibre content (Raczkiewicz, 2016). The addition of steel fibres primarily increases the homogeneity of the material, improves tensile strength and fatigue strength, and resistance to cracking and abrasion (Brandt, 2008; Brandt, 2009). Concrete with the addition of fibres is more cohesive because the fibres "clamp" the concrete matrix together and prevent the formation of large pores in the concrete mix, and reduce the formation and spread of shrinkage cracks (Raczkiewicz, 2016; Raczkiewicz and Kossakowski, 2019; Czajkowsk aet al., 2020). Due to the steel fibres, the concrete matrix is "sealed".

The article presents the results of experimental tests aimed at determining the relationship between two variables: compressive strength and Young's modulus determined for concrete specimens with the addition of steel fibres. On the basis of the obtained results, it was analysed whether an increase in one variable (compressive strength) was accompanied by an increase in the other variable (Young's modulus), and how strong the relationship between these two variables was. The analysis was performed using the linear regression method. Linear regression analysis is a commonly used statistical technique to measure the relationship between two variables (Marill, 2004). It is a modelling technique in which a dependent variable is predicted from one or more independent variables.

The applied method of linear regression allowed for the determination of such regression coefficients (coefficients in the linear model) for which the adopted model best predicts the value of the dependent variable, i.e. the value of Young's modulus (i.e. with the smallest estimation error).

Moreover, the obtained results and statistical analysis made it possible to estimate the extent to which the addition of steel fibres to concrete influenced the relationship between the compressive strength and the modulus of elasticity of the specimens made. This type of research was carried out for concrete, however, it is difficult to find information in the literature on this type of research on concrete with the addition of fibres.

2. Materials and methods

For the testing of Young's modulus and compressive strength, 16 cylindrical specimens were prepared with standard dimensions: base diameter d = 150 mm and height h = 300 mm. The specimens were made of concrete of C30/37 class, S3 consistency, with a water-cement coefficient w/c = 0.43. In the study a concrete mix formulation used at Sibet Concrete Production Company Kielce was used.

The composition of the concrete mix per 1m^3 is presented in Table 1.

Table 1. The composition of the concrete mix

Components	[kg/m^3]
Cement 42.5 N MSR NA	384
Fine aggregate 0-2	680
Basalt aggregate 2-8	600
Basalt aggregate 8-16	650
Water	166
Plasticizer 0.5%/kg of cement	1.92
Aerator 0.2%/kg of cement	0.768

Steel fibres of the BauMix 60/1 type were used with the following parameters: fibre length lw = 60 mm, fibre diameter ∅ 1.0 mm, shape - straight fibres with hook-shaped ends. The amount of fibres was varied: in 8 specimens (SA) 0.25% fibres were used per 1m3 of the mixture, and in the next 8 specimens (SB) – 0.5% fibres were used. Compressive strength tests in uniaxial stress and determination of the modulus of elasticity were made based on the guidelines contained in (PN-EN 12390-1:2013-03; PN-EN 12390-2:2001; PN-EN 12390-4:2001). Compressive strength tests were carried out in a Zwick/Roell SP-Z6000 testing machine in accordance with the requirements of the standards (PN-EN 12390-2:2001; PN-EN 12390-4:2001). The main components of the testing machine are a hydraulic unit, a load frame and an electronic control and measurement system. The maximum compressive force that can be achieved is 6000 kN (Michałowska-Maziejuk and Teodorczyk, 2018). During the test, the load increases continuously without sudden jumps in force. The measuring system enables the simultaneous electronic measurement and recording of basic parameters. Thanks to testXpert software, it is also possible to generate graphs of dependence of the tested parameters during the test. The program also enables performing statistical calculations of the obtained results. Before starting the measurements, each of the specimens was properly prepared. To avoid non-axial loading of the load, the surfaces of the specimens that directly contact the pressure plates of the testing machine were cleaned (PN-EN 12390-3:2002). Then, three measurement benchmarks - extensometers with a measurement base length of 120 mm were placed at equal intervals on the side surface of each specimen. The concrete specimens prepared in this way were placed between the press plates. The correct positioning of the elements was achieved by using the centering lines located on the lower pressure plate. The articulated connection of the upper pressure plate with the load frame allowed for coaxial alignment with the surface of the test specimen (PN-EN 12390-4:2001). Each time, the specimens were positioned so that the load acted perpendicular to the direction of concreting. Then the specimen was continuously loaded until it was destroyed. The load was generated at a constant speed 0.5 MPa/s (PN-EN 12390-3:2002). The concrete was treated according to the standard (PN-EN 12390-2:2001 Concrete testing, part 2: Execution and maintenance of tests for strength tests). The test took place after 28 days.

The test stand together with the loaded specimen is shown in Fig. 4.

Each time, after the test, the damaged specimen was visually inspected in order to assess the correctness of its destruction. As mentioned above, the values determined during the test (including the value of the destructive force and displacement) were recorded in a computer database.

Fig. 4. The test stand together with the loadeds pecimen (Czajkowska et al., 2020)

Thanks to testXpert software compatible with the testing machine, on the basis of the obtained measurements it was possible to automatically determine the strain, and the compressive strength and Young's modulus were calculated. According to the standard recommendations (PN-EN 12390-13:2014-02), out of 16 tested specimens, 5 had to be rejected as they did not meet the conditions at the time of the controls. Therefore, eleven specimens were accepted for further analysis - six with 0.25% fibres and five with 0.50% fibres.

3. Results and discussion

3.1. Analysis of the results of compressive strength and Young's modulus for fibre-reinforced concrete specimens

According to the standard (PN-EN 12390-13:2014-02), before determining Young's modulus of concrete, the average concrete compressive strength should be determined. The strength should be determined in a uniaxial stress state on the accompanying specimens. It is necessary to determine the upper stress level, σa = fcm/3 during the cyclic loading of the main specimens (Michałek, 2015). Figure 5 shows a plot of the relative strain versus force for one of the specimens containing 0.25% of steel fibres.

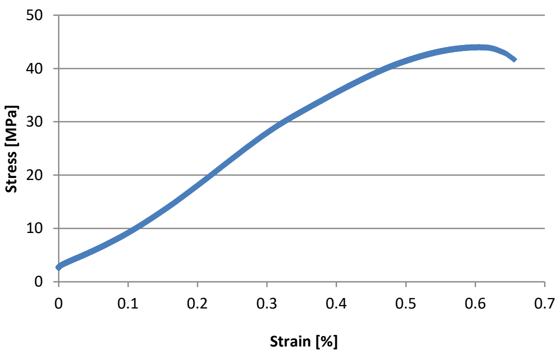


Fig. 5. Relativestrain-force plot generated for one of the testedspecimens (the SA1 specimen with 0.25% of fibres)

Figure 6 shows a plot of the relative strain versus force for one of the specimens with 0.50% of steel fibres.

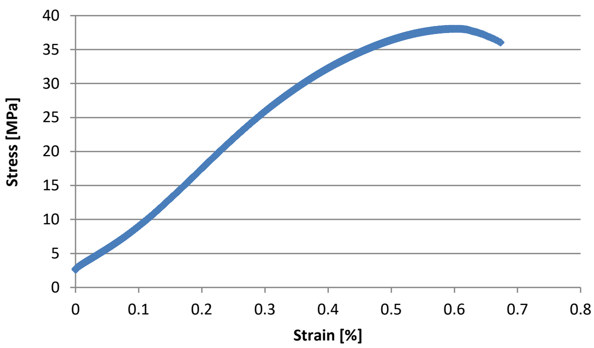


Fig. 6. Relative strain-force plot generated for one of the tested specimens (the SB7 specimen with 0.50% of fibres)

The results of compressive strength and Young's modulus measurements for eleven samples (five were rejected during the process of testing the modulus of elasticity) are presented in Table 2.

Table 2. The results of compressive strength and Young's modulus

fiber content [%]	Samples	Compressive strength [MPa]	Young's modulus [GPa]
0.25 of steel fibres	P_1	33.50	43.35
	P_2	32.89	44.27
	P_3	38.85	43.20
	P_4	36.46	42.90
	P_5	45.46	54.29
	P_6	43.78	54.06
0.50 of steel fibres	P_7	28.63	38.03
	P_8	26.81	32.18
	P_9	30.14	41.15
	P_{10}	38.11	55.56
	P_{11}	36.63	49.92

Compressive strength values expressed in [MPa] for all eleven specimes along with the corresponding values of Young's modulus expressed in [GPa] are presented in Fig. 7.

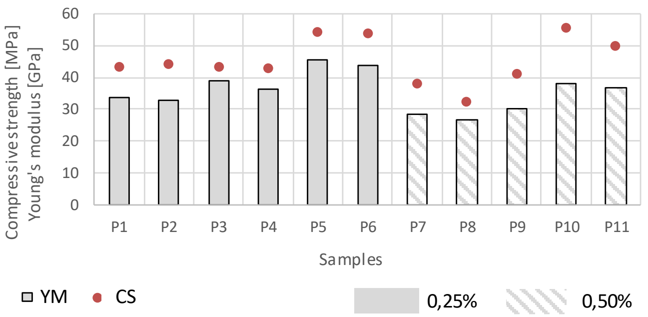


Fig. 7. The values of compressive strength (CS) and Young's Modulus (YM) for all tested specimens

The analysis of the performed tests shows that the compressive strength of SA specimens (containing 0.25% of steel fibres) was higher than the SB specimens (containing 0.50% of steel fibres) by an average of 6 MPa. The analysis shows that for the tested fibre-reinforced concrete specimens, higher values of the compressive strength correspond to the higher value of Young's modulus. To assess the degree of explanation of one variable by the other, the linear regression method was used, and the linear correlation coefficient and the coefficient of determination were calculated.

3.2. Linear regression method, correlation coefficient and coefficient of determination for the relationship between compressive strength and Young's modulus

The situation in which each observation is placed exactly on the regression line (the line of the least squares) proves the best fit of the model to the data (Jóźwiak and Podgórski, 1995). In this case, the regression line will pass through each point delineated by the value pairs of the random variable X and Y. The strength of the linear relationship between two variables is determined by the correlation coefficient from the sample r.

The formula by which the correlation coefficient is calculated has the form (Angelini, 2019):

$$r = \frac{\Sigma_{i=1}^{n}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\Sigma_{i=1}^{n}\left(x_i - \bar{x}\right)^2 \Sigma_{i=1}^{n}\left(y_i - \bar{y}\right)^2}, \tag{1}$$

where x_i and y_i denote the values of the variables x and y, respectively.

The correlation coefficient takes values from the interval <-1;1>(Angelini, 2019; Borkowski and Czajkowska, 2012), which are interpreted as follows (Borkowski and Czajkowska, 2012; Ostasiewiczet al., 1998):
r=-1 the occurrence of a perfect negative correlation,
r= 0 no linear correlation,
r= 1 the occurrence of a perfect positive correlation.

The measure of the usefulness of the regression equation model is the coefficient of determination (r2). The values of the coefficient of determination are in the range from 0 to 1. The value of the coefficient r2 within the limits of zero indicates that the model is not suitable for estimating the dependent variable with the independent variable. If the coefficient values are close to one, they indicate that the regression equation is useful for predicting the value of the dependent variable using the independent variable. Typically, the values of r2 (multiplied by 100%) are interpreted as a percentage of the total variability of the dependent variable Y, which is explained by the variability of the explanatory variable.

The coefficient of determination is a descriptive measure of the fit of the regression model to the data, i.e. it is a measure of the strength of the linear relationship between the data. It measures a part of the variability of the dependent variable Y, which was explained by the linear influence of the explanatory variable X.

In order to check whether there is a similar relationship in the general population to that observed in the sample population, it is necessary to examine the significance of Pearson's linear correlation coefficient.

In order to find the significance of the r coefficient, the values of the t and t(α; f) coefficients should be compared. So we accept the hypotheses. The H0 hypothesis assumes that the investigated quantities X and Y are not correlated. The accepted hypothesis was subjected to the Student's t-test, which is used to test statistical hypotheses. We verify the hypothesis that the variables X and Y are linearly independent.

H_0: r = 0 against H_1: r ≠ 0
If |t| ≥ t(α;f), then the hypothesisH0 is false.

For the previously adopted significance level α=0.01 we read the critical value tn-2,α=3.250. We use Student's t-test. We calculate the significance of the correlation coefficient using the formula:

$$t = \frac{r}{\sqrt{1-r^2}}\sqrt{n-2}, \tag{2}$$

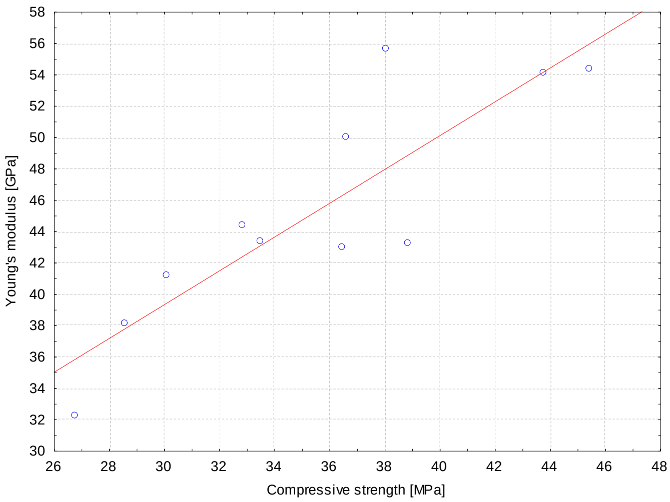
where:
r – coefficient of correlation,
n – the number of degrees of freedom.

$$t = \frac{0.87}{\sqrt{1-0.75}}\sqrt{11-2} = 4.6 \tag{3}$$

As defined, where |t| ≥ tα we reject the hypothesis H_0. This means that the correlation between the compressive strength and Young's modulus is not accidental.

3.3. Analysis of the influence of the number of steel fibres in concrete on the correlation between the compressive strength and Young's modulus

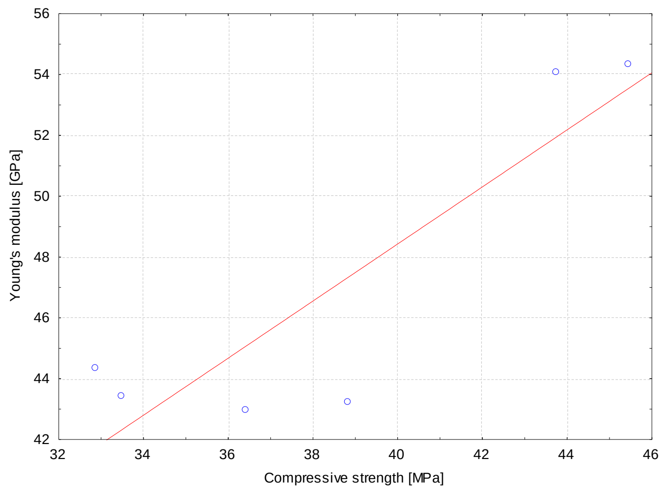
Rejection of the H0 hypothesis provides the basis for further analyses. Using the correlation coefficient and the scatter plot, the relationship between the compressive strength and the value of Young's modulus for eleven specimens was determined, as shown in Figure 8. The correlation coefficient equal to r = 0.8669 shows a very strong positive relationship between these variables (the higher the compressive strength, the higher the Young's modulus). The value of the coefficient of determination (r2 = 0.7516) shows that the compressive strength value explains 75% of Young’s modulus for the tested specimens.



y = 7.0149 + 1.0779*x; r = 0.8669; p = 0.0006; r^2 = 0.7516

Fig. 8. Graph of the relationship between the compressive strength and Young's modulus for eleven fibre-reinforced concrete specimens

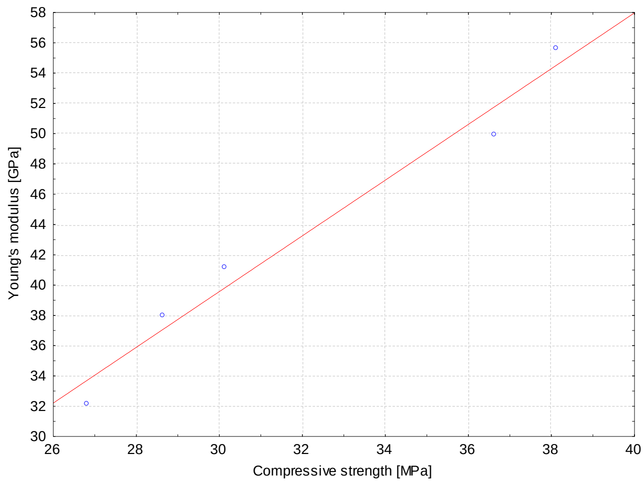
The aim of the performed tests was also to estimate the influence of the amount of steel fibres on the relationship between the compressive strength and Young’s modulus for the tested specimens. Figure 9 shows the scatter plot of the relationship between the compressive strength and Young's modulus for specimens containing 0.25% of steel fibres. The coefficients r and r 2 for specimens containing 0.25% of steel fibres are similar to those for all eleven specimens.



$y = 10.8973 + 0.9383*x;\ r = 0.8823;\ p = 0.0200;\ r^2 = 0.7785$

Fig. 9. The relationship between compressive strength and Young's modulus for specimens containing 0.25% of steel fibres

Figure 10 shows the scatter plot of the relationship between the compressive strength and Young's modulus for specimens containing 0.50% of steel fibres. This chart differs significantly from the previous two. It presents the greatest degree of model fit. Also, the coefficients r and r2 are higher and amount to r = 0.9862 and r2 = 0.9726, respectively. We observe here a strong positive relationship between the compressive strength and the value of Young's modulus.



$y = -15.6575 + 1.8409*x;\ r = 0.9862;\ p = 0.0019;\ r^2 = 0.9726$

Fig. 10. The relationship between compressive strength and Young's modulus for specimens containing 0.50% of steel fibres

4. Summary and conclusion

The article presents an analysis based on experimental studies that allows an assessment of the relationship between compressive strength and Young's modulus in concrete with the addition of steel fibres, and the impact of the amount of microreinforcement on the strength of this compound. The analysis shows that there is a linear positive relationship between compressive strength and the value of Young's modulus, determined on the basis of tests of eleven specimens. For specimens containing 0.50% of steel fibres, the relationship between the modulus of elasticity and the compressive strength turned out to be higher than for specimens with a lower, 0.25% fibre content (as evidenced by the high value of the correlation coefficient r). The above observed effect of the amount of fibre addition on the increase of the correlation between compressive strength and Young's modulus in the tested specimens is probably related to the increase in homogeneity of the concrete mix. The higher the percentage of fibres in the mix, the more homogeneous the concrete obtained. For specimens containing 0.25% of steel fibres, the relationship between the modulus of elasticity and compressive strength is non-linear. Moreover, the following specific conclusions can be made:

- the content of steel fibres affects the strength of the correlation between compressive strength and the value of Young's modulus in fibre-reinforced concrete,
- the higher the compressive strength, the higher the value of Young's modulus,
- in the case of specimens containing 0.25% of steel fibres, 78% compressive strength determines the value of Young's modulus,
- in the case of specimens containing 0.50% of steel fibres, 97% compressive strength determines the value of Young's modulus,
- the addition of fibres increases the homogeneity of concrete, which positively affects the correlation between compressive strength and Young's modulus.

The results obtained and presented in the paper gave good prospects. However, it should be emphasized that these research and presented results are an introduction to wider research in this topic, which will be continued on a larger number of samples with a different recipe of the concrete mix and the addition of fibres. The authors' research is in the form of a preliminary/pilot study, which obviously needs to be extended to a larger number of sample series, with different amounts of fiber addition.

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基于实验研究的纤维混凝土杨氏模量与抗压强度线性相关系数的测定

關鍵詞
杨氏模量
抗压强度
纤维增强混凝土
线性回归法
线性相关系数

摘要
测定混凝土杨氏模量的测试程序复杂且耗时。 因此，尝试寻找替代的测定方法并不令人意外。混凝土的抗压强度值与杨氏模量之间的关系是已知的。 然而，纤维增强混凝土中这种关系的强度尚未得到准确描述。 本文试图研究纤维增强混凝土的杨氏模量与抗压强度之间的相关性强度。 还检查了纤维量对此关系的影响。 测试使用了两种类型的样本。 样品的钢纤维含量不同，分别为 0.25%和 0.50%。 为了确定相关关系，使用了线性回归和线性相关系数的方法。 决定系数的使用使我们能够检查一个变量对另一个变量的解释程度。