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Mathematical model and characteristics of dynamic modes for managing the asynchronous motors at voltage asymmetry

Abstract: The development of a comprehensive mathematical model depicting an asynchronous motor in generalised (resultant) vectors, accounting for voltage asymmetry, stands as a significant stride in this research endavour. This model serves as a powerful tool for conducting an exhaustive analysis of the motor's characteristics in dynamic operational modes. Its unique capability lies in enabling a granular examination of the impact of both amplitude and phase asymmetry on the motor's dynamic performance, thereby providing a nuanced understanding crucial for the effective management of asynchronous motors operating under the conditions of voltage asymmetry. In this pursuit, distinct models for an asynchronous motor are delineated separately for the direct and reverse sequence, conceptualised as two distinct machines

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integrated on a single shaft. Additionally, a unified model is presented, encapsulating the voltage equations along the α and β axes within the fixed coordinate system of the generalised machine. The empirical findings derived from this research present compelling insights into the behaviour of asynchronous motors in transient modes during direct starts under varying degrees of voltage asymmetry. These findings, summarised in a tabulated format, illustrate a clear correlation between the degree of voltage asymmetry and the subsequent reduction in motor performance indices. These findings not only enrich the theoretical understanding of asynchronous motor behaviour in the presence of voltage asymmetry but also lay a solid foundation for devising practical approaches to optimize their performance, further enhancing their operational efficiency and reliability.

KEYWORDS: asynchronous motor, management of the voltage asymmetry, generalised vectors, mathematical model, dynamic modes

Introduction

Managing the characteristics of dynamic modes of asynchronous motors at voltage asymmetry is essential for ensuring the reliable and efficient operation of these motors in various industrial applications. Asynchronous motors, also known as induction motors, are commonly used in industrial settings due to their robustness and simplicity. One of the main indicators of power quality is voltage asymmetry caused by the inequality of phase voltages and the angles of shift between them. In addition, voltage asymmetry has the highest probability (Malyar 2020). Voltage asymmetry refers to an imbalance in the supply voltage to the motor phases, which can occur for various reasons, such as unbalanced loads, voltage fluctuations, or issues with the electrical distribution system (Seheda et al. 2024). Voltage asymmetry can also result in unequal torque generation in the motor phases, leading to decreased efficiency, increased heat generation, and potential damage to the motor (Shen 2009). Managing the characteristics of dynamic modes under voltage asymmetry involves the following elements:

- Reasons and understanding of the voltage asymmetry. Voltage asymmetry occurs when the magnitudes of the voltages in the three phases of the motor are not equal. This can lead to uneven performance and potential motor issues (Kolber and Piechocki 2008);
- Voltage monitoring and management. Implementation of the voltage monitoring system to continuously measure and monitor the voltage levels in all three phases of the motor operation helps to detect voltage asymmetry early and provide the measures for its prevention (Kolber and Piechocki 2008; Bosma 2003; Polyanska et al. 2024);
- Voltage balancing. Corrective measures to balance the voltages across the motor phases can involve adjusting the connections, using voltage-balancing transformers, and addressing issues in the electrical distribution system (Barić et al. 2019; Kononenko et al. 2023; Kononenko et al. 2024);

- Asymmetry prevention and motor protection. Installing protective devices such as voltage relays and phase imbalance protection gives the possibility to disconnect the motor in case of severe voltage asymmetry to prevent its damage (Stewart 1997);
- Controller usage and adjustments. It is necessary to use and modify the motor controller settings to compensate for voltage asymmetry. Today, advanced motor control systems are sophisticated technologies and techniques used to control electric motors. These systems aim to optimize motor performance, energy efficiency, and overall functionality (Polyanska et al. 2023). They can adjust the motor's operating parameters to maintain optimal performance. For this purpose, it is necessary to modify the motor controller settings to compensate for voltage asymmetry. Some advanced motor control systems can adjust the motor's operating parameters to maintain optimal performance (Kostic 2012; Escobar et al. 2005);
- Application of the variable frequency drives (VFDs). Using VFDs with built-in voltage regulation capabilities can adjust the motor's operating frequency and voltage to maintain stable performance under varying voltage conditions. VFDs can stabilize the output voltage to the motor, ensuring that it receives a balanced and controlled supply even in the presence of voltage asymmetry. This helps maintain consistent motor performance and prevents damage due to excessively high or low voltages (Bondarenko et al. 2009; Jahns 1996);
- Maintenance and regular inspection. Routine maintenance and inspection can help prolong the motor's lifespan (Robery 2018). It is necessary to inspect the motor and its electrical connections for signs of damage or wear that may be exacerbated by voltage asymmetry;
- Necessity of the equipment voltage correction. Correcting the voltage imbalance helps ensure that the motor operates improving performance and energy efficiency (High-voltage TT. 2016). In cases of severe and chronic voltage asymmetry, voltage correction equipment such as static compensators and voltage stabilizers are installed to regulate and stabilize the supply voltage;
- Providing the load balancing. Load balancing in asynchronous motors helps prevent phase imbalances, reduces adverse effects, mitigates the effects of voltage asymmetry, and ensures that the motor operates more efficiently and reliably. The loads connected to the motor have to be balanced across the three phases to minimize voltage imbalances (Lusk et al. 2005; Dychkovskyi et al. 2018).

So, the voltage unbalance has a particularly negative effect on the operation and service life of asynchronous motors (AM). Even a small negative sequence voltage component will generate a large negative sequence current (Kuznetsov et al. 2021). This causes additional losses and heating of the motor, reduced efficiency and power factor, service life, and load capacity (Asalomia 2020; Yin et al. 2005). In addition, the ripple of the motor torque in case of voltage asymmetry can disrupt the normal course of the technological process. Managing through the records of voltage measurements and any corrective actions taken to address voltage asymmetry proves to be valuable for troubleshooting and improving motor reliability.

The main aim of the paper is to develop a mathematical model of an asynchronous motor in generalised (resulting) vectors with voltage asymmetry and to analyze the characteristics of the motor in dynamic modes.

1. Materials and methods

The most common method for AM analysis with voltage asymmetry is the method of symmetric components (Ponce et al. 2017; Bently et al. 2002; Sobolev et al. 2020). This method is often used in power systems analysis, but it can also be applied to analyze voltage asymmetry in asynchronous motors. This method is based on the representation of instantaneous values of variables (currents, voltages, etc.) for systems without a neutral wire as the sum of two symmetrical components (direct and reverse), differing in the order of phase rotation.

This method breaks down the voltage or current into its positive-sequence, negative-sequence, and zero-sequence components, making it a powerful tool for understanding the effects of voltage asymmetry. Here's a methodology for how to use the method of symmetric components for voltage asymmetry analysis in asynchronous motors (Lobry et al. 2008; Vladyko et al. 2008; Shamir et al. 2021):

- Providing the symmetrical components analysis, which is based on the concept that any unbalanced set of phasors, can be represented as a combination of three symmetrical sets of phasors: positive-sequence, negative-sequence, and zero-sequence components (Kolb et al. 2020);
- Measuring the three-phase voltages at the terminals of the asynchronous motor by recording both the magnitudes and phase angles of these voltages (Cabana et al. 2018; Beshta et al. 2019);
- Converting the measured voltages into phasor form. Phasors are complex numbers that represent the magnitude and phase angle of each voltage in the three phases;
- Calculating the positive-sequence component, using the well-known positive-sequence operator to extract the positive-sequence component of the voltage. This represents the balanced part of the voltage.

$$V_1 = (V_a + aV_\beta + a^2V_\gamma) \tag{1}$$

where:

V_1 – the positive-sequence voltage.
V_a, V_β, and V_γ – the phasor voltages in phases A, B, and C, respectively,
a – the complex operator representing a 120-degree phase shift ($a = e(j2\pi/3)$.

- Calculating the negative-sequence component by using the negative-sequence operator (a^2) to extract the negative-sequence component of the voltage. This represents the unbalanced voltage with a 180-degree phase shift:

$$V_2 = (V_a + a^2V_\beta + aV_\gamma)/3 \tag{2}$$

where:

V_2 – the negative-sequence voltage.

- Calculate the zero-sequence component using the zero-sequence operator (0) to extract the zero-sequence component of the voltage. This represents the common-mode voltage or the voltage with no phase shift:

$$V_0 = (V_a + V_\beta + V_\gamma)/3 \tag{3}$$

where:

V_0 – the zero-sequence voltage.

- Results interpreting thought analysing the positive-sequence, negative-sequence, and zero-sequence components to understand the nature and extent of voltage asymmetry. The presence of a significant negative-sequence or zero-sequence voltage indicates voltage imbalance or asymmetry (Kolb 2013).

Based on the analysis, it is necessary to provide corrective actions to address voltage asymmetry issues. This may involve adjusting the electrical connections, implementing voltage correction equipment, or improving the balance of the electrical system.

The method of symmetric components provides a clear and systematic way to analyze voltage asymmetry in asynchronous motors and identify the underlying causes. By isolating the positive-sequence, negative-sequence, and zero-sequence components, one can gain valuable insights into the nature of the asymmetry and make informed decisions on how to mitigate its effects and improve motor performance.

In this case, the AM is considered as a set of two machines, for each of which an equivalent circuit is drawn up for both steady-state and transient modes (Moschynskyi and Petrov 2002; Kolb 2011). The analysis uses traditional assumptions: uniform air gap; the magnetic conductivity of the machine is the same in all directions; linearity of the magnetization curve; the spatial higher harmonics, eddy current losses, and hysteresis are neglected.

Noteworthy is the method for AM analysing with voltage asymmetry, proposed in (Kolb 2011; Dychkovskyi et al. 2019), which is based on the expressions of the instantaneous power consumed in each phase.

Under the accepted assumptions, the mathematical description of the dynamic processes of an asynchronous motor is most easily implemented using generalised (spatial) vectors of three-phase variables (voltage, current, flux linkage, etc.) (Moschynskyi and Petrov 2002; Kolb 2011; Sytchev and Chernyi 2002). In this case, the action totality of the variables of the three phases is expressed by one vector.

Differential equations for direct sequence voltages (index "1"), written in spatial vectors of variables and a fixed coordinate system (α, β), whose α axis is aligned with the magnetic phase A axis of the stator, has the form (Moschynskyi and Petrov 2002; Kolb 2011; Sytchev and Chernyi 2002):

$$\overline{U}_{s1} = R_s \overline{I}_{s1} + \frac{d\overline{\Psi}_{s1}}{dt} \tag{4}$$

$$\overline{U}_0 = R_r \overline{I}_{r1} + \frac{d\overline{\Psi}_{r1}}{dt} - jZ_n \omega \overline{\Psi}_{r1} \quad (5)$$

$$\overline{\Psi}_{s1} = L_s \overline{I}_{s1} + L_m \overline{I}_{r1} \quad (6)$$

$$\overline{\Psi}_{r1} = L_r \overline{I}_{r1} + L_m \overline{I}_{s1} \quad (7)$$

where:

$\overline{I}_{s1}, \overline{\Psi}_{s1}, \overline{I}_{r1}, \overline{\Psi}_{r1}$	–	stand for generalised (spatial) vectors of currents and flux linkages of the positive sequence of the stator and rotor,
U_{s1}	–	the positive sequence space vector of voltage,
$L_s = L_m + L_{s\sigma}$; $L_r = L_m + L_{r\sigma}$	–	stand for equivalent total inductance of the stator and rotor windings;
$Ls\sigma$, $Lr\sigma$	–	leakage inductances of stator and rotor windings,
L_m = 1,5M	–	equivalent mutual inductance of the stator and rotor, taking into account the action of other phases, which is one and a half times greater than the mutual inductance of a single phase,
Z_n	–	stands for the number of pole pairs; ω stands for rotor angular velocity (Dychkovskyi et al. 2019; Robyns et al. 2000).

Similar expressions can be written for the negative sequence voltages by replacing the index "1" with the index "2", corresponding to the variables of the negative sequence.

It should be noted that the space vector of variables (voltage, current, flux linkage) of the positive sequence coincides in magnitude and phase with the complex time vector representing the corresponding phase variable A, and the space vector of the negative sequence variables is equal to the conjugate complex time vector representing the corresponding phase variables A inverse sequences (Kolb 2011). Therefore, the transition to scalar differential equations for the direct sequence is realised by the expansion of all vectors in the equations (Moschynskyi and Petrov 2002; Kolb 2011) into the components along the real (α) and imaginary (β) axes (real and imaginary components), writing them in the form:

$$\overline{U}_{s1} = U_{s1\alpha} + jU_{s1\beta}; \overline{I}_{s1} = I_{s1\alpha} + jI_{s1\beta} \quad (8)$$

$$\overline{\Psi}_{s1} = \Psi_{s1\alpha} + j\Psi_{s1\beta}; \overline{\Psi}_{r1} = \Psi_{r1\alpha} + j\Psi_{r1\beta} \quad (9)$$

Based on (1–7) it is possible to obtain various options for the structural diagrams of an asynchronous motor. A structural diagram, in which only the currents of the stator and rotor windings appear, is the most convenient for analysis. This makes it possible to investigate the main energy indicators of an asymmetric power supply to an asynchronous motor.

Based on (3) and (4), considering (5–7), the differential equations for the direct sequence, expanded along the real (α) and imaginary (β) axes, are written in the form:

$$U_{s1\alpha} = R_s I_{s1\alpha} + L_s p I_{s1\alpha} + L_m p I_{r1\alpha} \tag{10}$$

$$U_{s1\beta} = R_s I_{s1\beta} + L_s p I_{s1\beta} + L_m p I_{r1\beta} \tag{11}$$

$$0 = R_r I_{r1\alpha} + L_r p I_{r1\alpha} + L_m p I_{s1\alpha} + Z_n \omega L_r I_{r1\beta} + Z_n \omega L_m I_{s1\beta} \tag{12}$$

$$0 = R_r I_{r1\beta} + L_r p I_{r1\beta} + L_m p I_{s1\beta} - Z_n \omega L_r I_{r1\alpha} - Z_n \omega L_m I_{s1\alpha} \tag{13}$$

where:

p – the differentiation operator.

In a similar way, scalar differential equations for the negative sequence voltages can be obtained using, as was already noted, the correspondence of the generalised negative sequence voltage vector to the conjugate complex time vector of the negative sequence variables. So, for example, for the negative sequence voltage, we have:

$$\bar{U}_{s2} = U_{s2\alpha} - jU_{s2\beta} \tag{14}$$

Electromagnetic torque of the motor, expressed in terms of symmetrical components of positive sequence currents (Moschynskyi and Petrov 2002; Kolb 2011; Sytchev and Chernyi 2002):

$$M_1 = \frac{3}{2} Z_n L_m \left(I_{r1\alpha} I_{s1\beta} - I_{s1\alpha} I_{r1\beta} \right) \tag{15}$$

Braking torque for negative sequence currents:

$$M_2 = \frac{3}{2} Z_n L_m \left(I_{r2\alpha} I_{s2\beta} - I_{s2\alpha} I_{r2\beta} \right) \tag{16}$$

Supplementing the system of equations (7–12) with the equation of motion:

$$M_1 + M_2 - M_l = J \frac{d\omega}{dt} \tag{17}$$

where:

J – the moment of inertia of the rotor and associated rotating masses, reduced to the speed of the motor,

M_l – stands for the moment of load resistance; we obtain a complete system of differential equations describing the electromechanical transients of an asynchronous motor with asymmetric power supply.

2. Results

Theoretical studies were carried out to analyze the influence of various types of supply voltage asymmetry on the dynamic processes and characteristics of an asynchronous motor.

To describe the mathematical model of an asynchronous motor in a fixed coordinate system (α, β) using generalised vectors for the stator and rotor currents, we typically use complex phasor notation. This approach simplifies the representation of AC quantities and makes it easier to analyze the motor's behaviour. The mathematical model in the (α, β) reference frame with complex phasor notation involves the following key components (Kolb 2011):

1. Stator Current Phasor (I_s): The stator current phasor represents the current in the (α, β) reference frame. It combines the α and β components of the stator current and is typically represented as a complex number:

$$I_s = I_\alpha + jI_\beta \tag{18}$$

where:

I_α – the α-component of the stator current,

I_β – the β-component of the stator current,

j – the imaginary unit.

2. Rotor Current Phasor (I_r): Similarly, the rotor current phasor represents the rotor current in the (α, β) reference frame as a complex number:

$$I_r = I_{\alpha r} + jI_{\beta r} \tag{19}$$

where:

$I_{\alpha r}$ – the α-component of the rotor current.

$I_{\beta r}$ – the β-component of the rotor current.

3. Stator Voltage Phasor (U_s): The stator voltage phasor represents the voltage applied to the stator windings and is typically given as a complex number:

$$U_s = U_\alpha + jU_\beta \tag{20}$$

where:

U_α – the α-component of the stator voltage.
U_β – the β-component of the stator voltage.

4. Rotor Voltage Phasor (U_r): The rotor voltage phasor represents the voltage applied to the rotor windings (in the case of a wound rotor motor). For a squirrel cage rotor, this can be assumed to be zero in many cases.

$$U_r = 0$$

5. Equations in the (α, β) Reference Frame:

Using complex phasor notation, the equations for the asynchronous motor in the (α, β) reference frame can be expressed as follows:

- stator equation:

$$U_s = R_s I_s + j\omega_s L_s I_s + j\Psi_m \tag{21}$$

where:

R_s – the stator resistance,
L_s – the stator inductance,
ω_s – the electrical angular velocity,
Ψ_m – the magnetising flux;

- rotor equation:

$$U_r = R_r I_r + j\omega_r L_r I_r \tag{22}$$

where:

R_r – the rotor resistance,
L_r – the rotor inductance,
ω_r – the electrical angular velocity;

- torque equation:

$$T_e = 3/2\ P_s\ (\Psi_m I_\beta - \Psi_r I_{\alpha r}) \tag{23}$$

where:

P_s – the number of pole pairs,
Ψ_r – the rotor flux;

✦ mechanical equation:

$$T_e - T_l = Jd\omega_m/dt + B\omega_m \tag{24}$$

where:

T_l – the mechanical load torque.

ω_m – the mechanical angular velocity.

J – the moment of inertia.

B – the friction and damping coefficient.

In this mathematical model, complex phasor notation is used to represent AC quantities, and the equations are written in terms of these phasors. This representation simplifies the analysis and control of the asynchronous motor in the (α, β) reference frame, making it a valuable tool for motor control and performance analysis.

The above equations correspond to the structural diagram of an asynchronous motor in a fixed coordinate system (α, β) for positive sequence voltages, shown in Figure 1.

It should be emphasised that for the negative sequence voltages, the structural diagram of Fig. 1 is valid, with the only difference being that the input voltage is taken according to expression (11), and the index "2" will appear in the designation of the variables.

It should be noted that under the assumptions made above, for the analysis of the dynamic characteristics of the AM with voltage asymmetry, it is possible to use only one structural diagram of Figure 1, having determined only the components $U_{s\alpha}$ and $U_{s\beta}$ preliminary for a given level of voltage asymmetry. In this case, the index "1" will disappear in the circuit in Figure 1, denoting the variables of the direct sequence.

In the case of symmetrical voltage, a symmetrical two-phase voltage system is applied to the stator windings of a generalised machine (Moschynskyi and Petrov 2002; Kolb 2011)

$$U_{s\alpha} = U_{smax} \cos \omega_{1s\beta} \sin \omega_{1smax} \tag{25}$$

where:

U_{smax} – the peak value of phase voltage, and ω1 is the angular frequency of supply voltage.

With amplitude asymmetry of the supply voltage, the generalised positive-sequence $\overline{U}_1$ voltage vector rotates with an angular velocity ω_1 in the positive direction and the generalised $\overline{U}_2 = \dot{U}_2^*$ voltage vector rotates at the same speed but in the opposite direction ($\dot{U}_2^*$ is a conjugated vector of negative sequence voltage). In this case, the resulting generalised vector $\overline{U}_s = \overline{U}_1 + \overline{U}_2$ describes an ellipse in space, forming an elliptical MMF.

Considering the above, the major axis of the ellipse, which coincides with the axis α in the fixed coordinate system α, β, is equal to the sum of the moduli of the generalised voltage vectors of the direct and negative sequence:

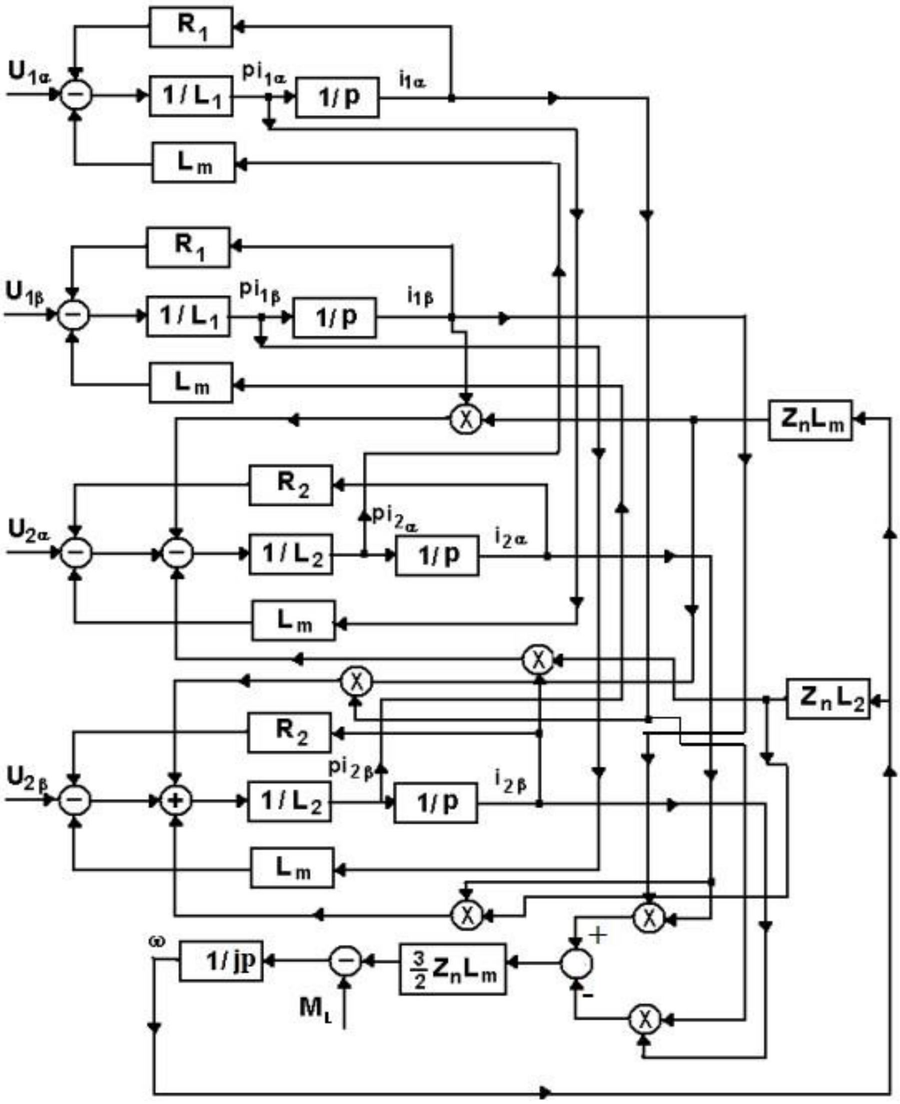

Fig. 1. Structural diagram of an asynchronous motor in a fixed coordinate system (α, β) in the generalised vectors of the stator and rotor currents for the direct sequence of the supply voltage

Rys. 1. Schemat strukturalny silnika asynchronicznego w ustalonym układzie współrzędnych (α, β) w uogólnionych wektorach prądów stojana i wirnika dla ciągu prostego napięcia zasilania

$$U\left|\bar{U}_1\right|\left|\bar{U}_2\right|_{s\alpha\max} \tag{26}$$

and the minor axis coinciding with the β axis is determined by the difference between the absolute values of the indicated vectors

$$U\left|\bar{U}_1\right|\left|\bar{U}_2\right|_{s\beta\min} \tag{27}$$

When analysing the dynamic characteristics of the AM with voltage asymmetry in operation, the supply voltage asymmetry coefficient was determined as:

$$k_a = \frac{\left|\dot{U}_2\right|}{\left|\dot{U}_1\right|} = \frac{\left|\bar{U}_2\right|}{\left|\bar{U}_1\right|} = \frac{U_{s\alpha\max} - U_{s\beta\min}}{U_{s\alpha\max} + U_{s\beta\min}} \tag{28}$$

For a given asymmetry factor, the positive sequence voltage must not exceed the rated motor voltage. Voltages are applied to the inputs of the structural diagram of the generalised machine in Figure 1

$$U_\alpha = Ua\cos\omega_{1r\max} \tag{29}$$

$$U_\beta = Ua\sin\omega_{1r\max} \tag{30}$$

where:

$U_{r\max}$ – the peak value of the rated phase voltage of the motor.

To assess the effect of phase asymmetry between the positive and negative sequence voltages, the voltage components U_α and U_β supplied to the input of the block diagram in Figure 1 are determined as:

$$U_\alpha = \left|\bar{U}_1\right|\cos\omega_1 t + \left|\bar{U}_2\right|\cos(\omega_1 t \pm \varphi) \tag{31}$$

$$U_\beta = \left|\bar{U}_1\right|\sin\omega_1 t + \left|\bar{U}_2\right|\sin(\omega_1 t \pm \varphi) \tag{32}$$

For a detailed study of the influence of various types of asymmetries of the supply voltage on the dynamic processes and characteristics of the AM it is advisable to consider it as a set of two symmetrical machines located on the same shaft.

In this case, the voltage components along the α and β axes are found in the ratios

$$\begin{aligned}
&\bar{U}_1 = U_{1\alpha} + jU_{1\beta}; \quad \bar{U}_2 = U_{2\alpha} - jU_{2\beta};\\
&U_{1\alpha} = U_{r\max}\cos\omega_1 t; \quad U_{1\beta} = U_{r\max}\sin\omega_1 t;\\
&U_{2\alpha} = k_a U_{r\max}\cos(\omega_1 t \pm \varphi; \quad U_{2\beta} = k_a U_{r\max}\sin(\omega_1 t \pm \varphi)
\end{aligned} \tag{33}$$

where the angle φ is measured from the vector $\bar{U}_1$.

The efficiency factor of an asynchronous motor with voltage asymmetry can be determined as:

$$\eta = \frac{P_2}{P_1} = \frac{M_s \omega}{M_c \omega + \Delta P_c + \Delta P_{var}} \tag{34}$$

where:

P_1, P_2	–	stand for consumed and useful power on the motor shaft respectively,
M_s, ω	–	the static moment and angular speed of the motor,
$\Delta P_c = \Delta P_{mech} + \Delta P_{st}$	–	permanent losses,
$\Delta P_{mech}, \Delta P_{st}$	–	mechanical losses in steel.

Variable loss:

$$\Delta P \sum \left(I_{si}^2 R_s + I_{ri}^2 R_r \right)_{var} \tag{35}$$

In three-phase symmetric systems in the absence of zero components of currents and voltages, the instantaneous power value is defined as the scalar product of the generalised (resulting) voltage vector $\bar{U}$ and conjugated current vector $\bar{I}^*$ (Polyanska et al. 2022; Kovach and Rats 1963; Akagi et al. 1983)

$$p = \frac{3}{2} \mathrm{Re}\left[\bar{U} \cdot \bar{I}^* \right] \tag{36}$$

If, in a fixed coordinate system α, β, the angles between the α axis and the resulting voltage and current vectors are denoted by φ_u and φ_i, then the expressions for the voltage and conjugate current vectors are written in the form:

$$\bar{U}_{\alpha\beta} = U_\alpha + jU_\beta = U_m \cos\varphi_u \sin\varphi_{u m_m} \tag{37}$$

$$\bar{I}^* = I_\alpha - jI_\beta = I \cos\varphi_i \sin\varphi_{i m_m} \tag{38}$$

where:

U_m – modules of the corresponding vectors equal to the amplitude values of the phase voltages and currents.

Considering (37) and (38), expressions for instantaneous power (36) in a fixed coordinate system take the form:

$$p_{\alpha,\beta} = \frac{3}{2} Re\left[\bar{U}_{\alpha,\beta} \cdot \bar{I}^*_{\alpha,\beta} \right] == \frac{3}{2} \mathrm{Re}\left[\left(U_m \cos\varphi_u \sin\varphi_{u m_m} \left(I \cos\varphi_i \sin\varphi_{i m_m} \right) \right) \right] \tag{39}$$

In the above expression, the real component is equal to the average value of the instantaneous power, i.e. active power:

$$P = \frac{3}{2} U \left(\cos\varphi_u \cos\varphi_i + \sin\varphi_u \sin\varphi_i \right) \cos\varphi_{ui} \cos\varphi_{m_m} \tag{40}$$

and the imaginary component corresponds to the reactive shift power:

$$Q = \frac{3}{2} U \left(\sin\varphi_u \cos\varphi_i - \cos\varphi_u \sin\varphi_i \right) \sin\varphi_{m_m} \tag{41}$$

Taking into account (37) and (38), the above expressions for active and reactive power can be represented as:

$$\begin{gathered} P = \frac{3}{2}\left(U_\alpha I_\alpha + U_\beta I_\beta \right); \\ Q = \frac{3}{2}\left(U_\beta I_\alpha - U_\alpha I_\beta \right) \end{gathered} \tag{42}$$

By analogy with (36), the instantaneous value of the power of an asymmetric system is determined as the scalar product of the generalised voltage vector $\overline{U} = \dot{U}_1 + \dot{U}_2^*$ and conjugate current vector $\overline{I}^* = \dot{I}_1^* + \dot{I}_2$ (Nabae and Tanaka 1996; Kolb 2006)

$$p = \frac{3}{2}\mathrm{Re}\left[\overline{U} \cdot \overline{I}^* \right] = \frac{3}{2}\mathrm{Re}\left[\left(\dot{U}_1 + \dot{U}_2^* \right)\left(\dot{I}_1^* + \dot{I}_2 \right) \right] == \frac{3}{2}\mathrm{Re}\left[\dot{U}_1 \dot{I}_1^* + \dot{U}_1 \dot{I}_2 + \dot{U}_2^* \dot{I}_1^* + \dot{U}_2^* \dot{I}_2 \right] \tag{43}$$

As (Kovach and Rats 1963; Kolb 2005)

$$\begin{gathered} \mathrm{Re}\left[\dot{U}_2^* \dot{I}_2 \right] \quad \mathrm{Re}\left[\dot{U}_2 \dot{I}_2^* \right]; \\ \mathrm{Re}\left[\dot{U}_2^* \dot{I}_1^* \right] \quad \mathrm{Re}\left[\dot{U}_2 \dot{I}_1 \right] \end{gathered} \tag{44}$$

then the expression (35) takes the form:

$$p = \frac{3}{2}\mathrm{Re}\left[\dot{U}_1 \dot{I}_1^* + \dot{U}_2 \dot{I}_2^* + \dot{U}_2 \dot{I}_1 + \dot{U}_1 \dot{I}_2 \right] \tag{45}$$

In the above expression, by analogy with (36), the components

$$\frac{3}{2}\mathrm{Re}\left[\dot{U}_1 \dot{I}_1^* \right] = P_1; \quad \frac{3}{2}\mathrm{Re}\left[\dot{U}_2 \dot{I}_2^* \right] = P_2 \tag{46}$$

correspond to the instantaneous power value of the direct and reverse sequences, and the component

$$\frac{3}{2}\mathrm{Re}\left[\dot{U}_1 \dot{I}_2 + \dot{U}_2 \dot{I}_1 \right] \tag{47}$$

there is a variable power pulsating with a double frequency near the zero-mean value (Kovach and Rats 1963; Kolb 2005). The peak value of this power is called the asymmetry power.

Table 1 shows some results of the authors' study of the decrease in indicators in transient modes of an asynchronous motor of the type 4A200M6Y3 (P_r = 22 kW, efficiency factor = 90.5%; cosφ = 0.9; I_r= 55.8 A; S_r = 0.023; S_{cr} = 0.135; ω_0 =104.7 1/c)

Table 1. Results of the study of the decrease in indicators in transient modes of an asynchronous motor of the type 4A200M6Y3

Tabela 1. Wyniki badania spadku wskaźników w stanach przejściowych silnika asynchronicznego typu 4A200M6Y3

Indicators of AM	Voltage asymmetry [%]				Mode of operation
Asymmetry coefficient [%]	1	2	3	4	
Increase in stator losses	1.74	2.4	3.66	4.35	$M_1 = 0$
	1.96	3.0	4.24	5.45	$M_1 = M_r$
Increase in rotor losses	1.43	1.6	1.87	2.17	$M_1 = 0$
	1.52	2.38	3.49	3.56	$M_1 = M_r$
Active energy consumption growth [%]	1.1	1.3	1.8	2.6	$M_1 = 0$
	15.1	15.95	16.1	16.5	$M_1 = M_r$
Growth in reactive energy consumption [%]	1.3	1.15	1.68	1.83	$M_1 = 0$
	22.5	23.2	23.7	24.2	$M_1 = M_r$
Efficiency factor decrease [%]	1.1	4.11	5.01	6.36	$M_1 = M_r$

It should be added that the integral value of the power factor during direct start remains approximately the same at the level of 0.39.

To confirm the obtained results, laboratory studies were carried out on the decrease in the performance of an asynchronous motor at various values of voltage asymmetry.

Voltage asymmetry was created using laboratory autotransformers with a variable transformation ratio.

The research results are shown in Table 2.

As is evident from Table 2, the research results confirmed the sufficient reliability of the developed mathematical model of an induction motor in generalised vectors.

Also, practical confirmation of the results was performed in the conditions of engines located at various enterprises in the city of Dnipro, at two electrical repair enterprises that have specialised equipment for measuring the necessary parameters of an asynchronous motor.

The results obtained at the enterprises confirmed the high reliability of the performed theoretical research.

TABLE 2. Results of the laboratory studies of the decrease in indicators in transient modes of an asynchronous motor

TABELA 2. Wyniki badań laboratoryjnych spadku wskaźników w stanach przejściowych silnika asynchronicznego

Indicators of AM	Voltage asymmetry [%]					Mode of operation
Asymmetry coefficient [%]	0	1	2	3	4	
Active energy consumption [kW]	2.810	2.842	2.848	2.863	2.888	$M_l = 0$
	24.27	28.12	28.32	28.38	28.48	$M_l = M_r$
Active energy consumption growth [%]	0	1.155	1.365	1.872	2.78	$M_l = 0$
	0	15.86	16.7	16.93	17.35	$M_l = M_r$
Reactive energy consumption [kVAr]	12.38	12.55	12.57	12.8	12.62	$M_l = 0$
	12.43	15.37	15.46	15.53	15.6	$M_l = M_r$
Reactive energy consumption growth [%]	0	1.38	1.52	1.79	1.94	$M_l = 0$
	0	23.63	24.36	24.96	25.48	$M_l = M_r$

Conclusions

Managing the characteristics of dynamic modes of asynchronous motors under voltage asymmetry requires a combination of proactive measures, monitoring, and appropriate corrective actions to ensure the motor's safe and efficient operation. Depending on the severity and frequency of voltage asymmetry and the specific strategies and equipment used, we have adopted concrete conditions for its usage. The following conclusions were established regarding the conducted research.

1. The mathematical model of an induction motor with asymmetry of the supply voltage is proposed based on generalised (resulting) vectors of variables and the method of symmetric components.

2. The mathematical model of an asynchronous motor for the negative sequence voltage has the same form as for the positive sequence voltage, with the only difference that a generalised voltage vector corresponding to the conjugate negative sequence voltage vector is fed to the input of the structural circuit. This vector rotates at the same speed as the generalised positive sequence vector but in the opposite direction.

3. Models of an asynchronous motor are presented separately for direct and reverse sequences in the form of a set of two separate machines located on one shaft. The general model is also presented and the voltage equations along the axes α, β of the fixed coordinate system of the

generalised machine are presented. This model allows you to study the influence of amplitude and phase asymmetry on the dynamic performance of the motor in detail.

4. The results of the study of the decrease in the indices of an asynchronous motor in transient modes with direct start and various values of voltage asymmetry are given in the form of a table.

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References

Akagi et al. 1983 – Akagi, H., Kanazawa, Y. and Nabae, A. 1983. Generalized theory of the instantaneous reactive power in three-phase circuits. *IPEC'83 – Int. Power Electronics Conf.*, pp. 1375–1386.

Asalomia, B. 2020. Analysis of the quality indicators of electricity and of the equipment used in the navigation boat. *Scientific Bulletin of Naval Academy* XXIII, 2, pp. 251–259, DOI: 10.21279/1454-864x-20-i2-102.

Barić et al. 2019 – Barić, T., Glavaš, H. and Barukčić, M. 2019. Impact of balance resistor uncertainty on voltages across supercapacitors. *Journal of Energy Storage* 22, pp. 131–136, DOI: 10.1016/j.est.2019.02.005.

Bently et al. 2002 – Bently, E., Hatch, Ch. and Grissom, B. 2002. High Vibration in an Electric Motor. *Fundamentals of Rotating Machinery Diagnostics*, pp. 569–578, DOI: 10.1115/1.801frm_ch28.

Beshta et al. 2019 – Beshta, A., Beshta, A., Balakhontsev, A. and Khudolii, S. 2019. Performances of Asynchronous Motor within Variable Frequency Drive with Additional Power Source Plugged via Combined Converter. *IEEE 6th International Conference on Energy Smart Systems* (*ESS*), pp. 11–20, DOI: 10.1109/ess.2019.8764192.

Bondarenko et al. 2009 – Bondarenko, V., Dychkovskiy, R. and Falshtynskiy, V. 2009. Synthetic Stowing of Rockmass at Borehole Underground Coal Gasification (BUCG). *Deep Mining Challenges*, pp. 169–177, DOI: 10.1201/noe0415804288.ch18.

Bosma, A. 2003. Condition monitoring and maintenance strategies for high-voltage circuit-breakers. 6th International Conference on Advances in Power System Control, Operation and Management. *Proceedings* 11, pp. 191–196, DOI: 10.1049/cp:20030585.

Cabana et al. 2018 – Cabana, E., Falshtynskyi, V., Saik, P., Lozynskyi, V. and Dychkovskyi, R. 2018. A concept to use energy of air flows of technogenic area of mining enterprises. *E3S Web of Conferences* 60, DOI: 10.1051/e3sconf/20186000004.

Dychkovskyi et al. 2018 – Dychkovskyi, R., Falshtynskyi, V., Ruskykh, V., Cabana, E. and Kosobokov, O. 2018. A modern vision of simulation modelling in mining and near mining activity. *E3S Web of Conferences* 60, DOI: 10.1051/e3sconf/20186000014.

Dychkovskyi et al. 2019 – Dychkovskyi, R., Tabachenko, M., Zhadiaieva, K. & Cabana, E. 2019. Some aspects of modern vision for geo-energy usage. *E3S Web of Conferences* 123, DOI: 10.1051/e3sconf/201912301010.

Escobar et al. 2005 – Escobar, G., Martinez, P.R. and Valdez, A.A. 2005. A Repetitive-based Controller to Compensate for Harmonic Distortion in the Output Voltage of a Boost Converter. *IEEE 36th Conference on Power Electronics Specialists*, pp. 12–31, DOI: 10.1109/pesc.2005.1582016

High-voltage TT. 2016. High-voltage test techniques for low-voltage equipment. *Definitions, test and procedure requirements, test equipment.* DOI: 10.3403/30265514.

Jahns, T.M. 1996. Variable Frequency Permanent Magnet AC Machine Drives. Portico. *Power Electronics and Variable Frequency Drives*, pp. 277–331, DOI: 10.1002/9780470547113.ch6.

KOLB, A. 2005. Power quality control in electric drive systems with common direct current supply buses. *Naukovyi visnyk NGU* 7, pp. 85–91.

KOLB, A. 2006. Method I_x, I_y theories of instantaneous power in problems of power quality management in electric drive systems with capacitive storage. *Vistnyk Kremenchutskoho derzavnoho politehnichnoho universytetu* 3(38), pp. 65–67.

KOLB, A. 2011. Theory of electric drive. 2-nd edition. *D.: National Mining University*, 565 p.

KOLB, A. 2013. Energy saving in electrified transport by capacity storages. *Energy Efficiency Improvement of Geotechnical Systems*, pp. 131–134, DOI: 10.1201/b16355-17.

KOLB et al. 2020 – KOLB, A., PAZYNICH, Y., MIREK, A. and PETINOVA, O. 2020. Influence of voltage reserve on the parameters of parallel power active compensators in mining. *E3S Web of Conferences* 201, DOI: 10.1051/e3sconf/202020101024.

KOLBER, P. and PIECHOCKI, J. 2008. Simulation Model to Evaluate Voltage Asymmetry in a Rural Low Voltage. *Technical Sciences* 11(1), pp. 77–86, DOI: 10.2478/v10022-008-0005-0.

KONONENKO et al. 2023 – KONONENKO, M., KHOMENKO, O., SADOVENKO, I., SOBOLEV, V., PAZYNICH, Y. and SMOLIŃSKI, A. 2023. Managing the rock mass destruction under the explosion. *Journal of Sustainable Mining* 22(3), p. 240, DOI: 10.46873/2300-3960.1391.

KONONENKO et al. 2024 – KONONENKO, M., KHOMENKO, O., KOSENKO, A., MYRONOVA, I., BASH, V. and PAZYNICH, Y. 2024. Raises advance using emulsion explosives. *E3S Web of Conferences* 526, DOI: 10.1051/e3sconf/202452601010.

KOSTIC, M. 2012. Effects of Voltage Quality on Induction Motors' Efficient Energy Usage. *Induction Motors – Modelling and Control*, DOI: 10.5772/51223.

KOVACH, K.P. and RATS, I.L. 1963. Processes in alternating current machines. *Gosenergoizdat*, 744 p.

KUZNETSOV et al. 2021 – KUZNETSOV, V., TRYPUTEN, M., TYTIUK, V., ROZHNENKO, Z., LEVCHENKO, S. and KUZNETSOV, V. 2021. Modeling of thermal process in the energy system "Electrical network – asynchronous motor". *E3S Web of Conferences* 280, DOI: 10.1051/e3sconf/202128005003.

LOBRY et al. 2008 – LOBRY, J., NENS, E. and BROCHE, C. 2008. Symmetric Components and Numerical Modeling. [In:] *The Finite Element Method for Electromagnetic Modeling*. *Portico,* pp. 369–404, DOI: 10.1002/9780470611173.ch9.

LUSK et al. 2005 – LUSK, E., BUTLER, R. and PIEPER, S.C. 2015. Asynchronous Dynamic Load Balancing. *Programming Models for Parallel Computing*, pp. 187–204, DOI: 10.7551/mitpress/9486.003.0010.

MALYAR, V. 2020. Mathematical modeling start-up and steady state modes of asynchronous motors operation with capacitive compensation of reactive power. *Przegląd elektrotechniczny* 1(11), pp. 111–116, DOI: 10.15199/48.2020.11.22.

MOSCHYNSKYI, Ju.A. and PETROV, A.P. 2002. Mathematical model of an asymmetrical asynchronous motor based on substitution schemes for transient modes. *Elektromekhanika* 3, pp. 24–30.

NABAE, A. and TANAKA, T.A. 1996. new definition of instantaneous active-reactive current and power based on instantaneous space vectors on polar coordinates in three-phase circuits. *IEEE Transactions on Power Delivery* 11(3), pp. 1238–1243, DOI: 10.1109/61.517477.

POLYANSKA et al. 2022 – POLYANSKA, A., SAVCHUK, S., DUDEK, M., SALA, D., PAZYNICH, Y. and CICHO, D. 2022. Impact of digital maturity on sustainable development effects in energy sector in the condition of Industry 4.0. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu* 6, pp. 97–103, DOI: 10.33271/nvngu/2022-6/097.

POLYANSKA et al. 2023 – POLYANSKA, A., PAZYNICH, Y., SABYROVA, M. and VERBOVSKA, L. 2023. Directions and prospects of the development of educational services in conditions of energy transformation: the aspect of the coal industry. *Polityka Energetyczna – Energy Policy Journal* 26(2), pp. 195–216, DOI: 10.33223/epj/162054.

POLYANSKA et al. 2024 – POLYANSKA, A., PAZYNICH, Y., MYKHAILYSHYN, K., BABETS, D. and TOŚ, P. 2024. Aspects of energy efficiency management for rational energy resource utilization. *Rudarsko-Geološko -Naftni Zbornik* 39(3), pp. 13–26, DOI: 10.17794/rgn.2024.3.2.

PONCE et al. 2017 – PONCE, P., MOLINA, A., MATA, O., IBARRA, L. and MACCLEERY, B. 2017. Power Flow and Electric Machinery Basics. *Power System Fundamentals*, pp. 55–121, DOI: 10.1201/9781315148991-2.

ROBERY, P. 2018. Daily surveillance and routine inspection. *Recommendations for the Inspection, Maintenance and Management of Car Park Structures*, pp. 47–49, DOI: 10.1680/rimmcps.58392.047.

ROBYNS et al. 2000 – ROBYNS, B., BERTHEREAU, F., COSSART, G., CHEVALIER, L., LABRIQUE, F. and BUYSE, H. 2000. A methodology to determine gains of induction motor flux observers based on a theoretical parameter sensitivity analysis. *IEEE Transactions on Power Electronics* 15(6), pp. 983–995, DOI: 10.1109/63.892703.

SEHEDA et al. 2024 – SEHEDA, M.S., BESHTA, O.S., GOGOLYUK, P.F., BLYZNAK, YU.V., DYCHKOVSKYI, R.D. and SMOLIŃSKI, A. 2024. A mathematical model for the management of the wave processes in three -winding transformers with consideration of the main magnetic flux in the mining industry. *Journal of Sustainable Mining* 23(1), pp. 20–39, DOI: 10.46873/2300-3960.1402.

SHAMIR et al. 2021 – SHAMIR, O., SCHWARTZ, C., GARFINKEL, C. and PALDOR, N. 2021. The power distribution between the symmetric and anti-symmetric components of the tropical wavenumber-frequency spectrum. *EGU General Assembly 2021*, DOI: 10.5194/egusphere-egu21-6246.

SHEN, P. 2009. Dynamic Characteristics of the Intelligent Compactor Model with Adjustable Vibration Modes. *International Conference on Transportation Engineering* 23, pp. 1–45, DOI: 10.1061/41039(345)384.

SOBOLEV et al. 2020 – SOBOLEV, V., BILAN, N., DYCHKOVSKYI, R., CASERES CABANA, E. and SMOLINSKI, A. 2020. Reasons for breaking of chemical bonds of gas molecules during movement of explosion products in cracks formed in rock mass. *International Journal of Mining Science and Technology* 30(2), pp. 265–269, DOI: 10.1016/j.ijmst.2020.01.002.

STEWART, D. 1997. Motor vehicle occupant protection for children. *Injury Prevention* 3(4), pp. 312–312, DOI: 10.1136/ip.3.4.312.

SYTCHEV, S.D. and CHERNYI, A.P. 2002. Modes of operation of an asynchronous motor with asymmetry of the supply voltage. *Vistnyk Kremenchutskoho derzavnoho politehnichnoho universytetu* 1(12), pp. 266–269.

VLADYKO et al. 2008 – VLADYKO, O., MALTSEV, D., SALA, D., CICHOŃ, D., BUKETOV, V. and DYCHKOVSKYI, R. 2022. Simulation of leaching processes of polymetallic ores using the similatory theorem. *Rudarsko-Geološko-Naftni Zbornik* 37(5), pp. 169–180, DOI: 10.17794/rgn.2022.5.14.

YIN et al. 2005 – YIN, M., LI, G., ZHOU, M. AND LIU, Y. 2005. Analysis and Control of Wind Farm Incorporated VSC-HVDC in Unbalanced Conditions. *IEEE/PES Transmission; Distribution Conference; Exposition: Asia and Pacific*, pp. 14–31, DOI: 10.1109/TDC.2005.1547105.

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Model matematyczny i charakterystyki trybów dynamicznych do zarządzania silnikami asynchronicznymi przy asymetrii napięcia

Streszczenie

Opracowanie kompleksowego modelu matematycznego przedstawiającego silnik asynchroniczny w uogólnionych (wypadkowych) wektorach, uwzględniających asymetrię napięcia, stanowi znaczący krok w tym przedsięwzięciu badawczym. Model ten służy jako potężne narzędzie do przeprowadzania wyczerpującej analizy charakterystyk silnika w dynamicznych trybach pracy. Jego unikalna zdolność polega na umożliwieniu szczegółowego badania wpływu zarówno asymetrii amplitudy, jak i fazy na dynamiczną wydajność silnika, zapewniając w ten sposób niuansowe zrozumienie kluczowe dla skutecznego zarządzania silnikami asynchronicznymi pracującymi w warunkach asymetrii napięcia. W tym dążeniu odrębne modele silnika asynchronicznego są określane oddzielnie dla sekwencji bezpośredniej i odwrotnej, pojmowanej jako dwie odrębne maszyny zintegrowane na jednym wale. Dodatkowo przedstawiono ujednolicony model, obejmujący równania napięciowe wzdłuż osi α i β w stałym układzie współrzędnych uogólnionej maszyny. Wyniki empiryczne uzyskane w ramach tych badań przedstawiają przekonujące spostrzeżenia na temat zachowania silników asynchronicznych w trybach przejściowych podczas rozruchów bezpośrednich przy różnym stopniu asymetrii napięcia. Wyniki te, podsumowane w formie tabeli, ilustrują wyraźną korelację między stopniem asymetrii napięcia a późniejszym zmniejszeniem wskaźników wydajności silnika. Wyniki te nie tylko wzbogacają teoretyczne zrozumienie zachowania silnika asynchronicznego w obecności asymetrii napięcia, ale także stanowią solidną podstawę do opracowywania praktycznych podejść w celu optymalizacji ich wydajności, co dodatkowo zwiększa ich wydajność operacyjną i niezawodność.

Słowa kluczowe: silnik asynchroniczny, zarządzanie asymetrią napięcia, wektory uogólnione, model matematyczny, tryby dynamiczne