

Prediction of the Discharge Coefficient of a Labyrinth Weir Type D by an Artificial Neural Network Method

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Abstract. This study presents the use, and its advantages, of artificial intelligence methods to predict the discharge coefficient (C_w), considering the approach conditions of the labyrinth weir type D. The study suggests modifying the training and validation rates in AI tools, which are often fixed without proper justification in previous studies. Unlike most studies that use geometric dimensions as inputs, this work focuses on the approach conditions (the emplacement of the labyrinth weir and filling the alveoli upstream and downstream) of the labyrinth weir type D. The results, based on laboratory experiments, show that these modified inputs significantly impact the efficiency and cost of constructing the weir. Moreover, the C_w predictions based on these inputs are highly satisfactory compared to laboratory test results. In terms of training and validation ratios, the study confirms that the optimal ratio is 70/30 for accurate and highly satisfactory predictions.

Key words: ANN model, approach conditions, discharge coefficient, data splitting, labyrinth weir

1. Introduction

Throughout the 20th century, the global construction of numerous dams reached a remarkable peak, resulting in approximately 60,000 large dams being in operation. Currently, over 50% of these dams have surpassed 50 years of service, prompting a reevaluation of dam safety criteria (Biener 1985). Recent studies indicate an intensification of maximum flood flows compared to initial predictions, necessitating the rehabilitation of existing dam weirs (Belaabed 2019). Research has shown that non-linear weirs, such as piano key weirs (PK-Weir), classical mazes, and fuse gates, offer effective solutions (Belaabed et al 2021, Ben Said and Ouamane 2022).

Labyrinth weirs, with their non-linear configurations featuring trapezoidal, triangular, or rectangular geometric repetitions, offer cost-effective advantages (Crookston and Tullis 2010). The capacity of a non-linear weir depends primarily on its height,

crest shape, and length (Falvey and Treille 1995). However, the mathematical resolution of this problem is complex due to the dependence on various geometric parameters and approach conditions (Lux III 1987). Model experiments have been conducted to assess the impact of approach conditions on labyrinth weir performance.

Previous studies have explored different orientations and placements of labyrinth weirs relative to flood discharge channels (Houston 1983), revealing that normal orientation yields higher flow rates compared to reverse orientation. Additionally, partially submerging the weir in the reservoir further increases discharge (Houston 1983). However, these results are limited, emphasizing the need for additional research to provide more precise design recommendations. Another study evaluated the effect of different upstream entry shapes on labyrinth weir flow (Ouamane and Lempérière 2006). Models without guide walls exhibited higher flow coefficients than those with curved guide walls, even for high relative loads. The study also demonstrated that labyrinth weirs with curved guide walls displayed better hydraulic performance than those with straight guide walls, even under high relative loads (Ouamane and Lempérière 2006).

Recent research has explored the use of artificial intelligence (AI) to calculate labyrinth weir discharge coefficients (Filo 2023). Despite significant growth in research on application of AI algorithms in hydraulic science, it is surprising how few studies are dedicated to investigating the weir performance assessment under a combination of factors during the model development phase (Idrees and Al-Ameri 2022). These factors could include the choice of data splitting or the selection of sampling techniques. Regarding data splitting, the data sample is often divided into two datasets: a training set for model training and a testing set for model validation. Many researchers have proposed a ratio of 70/30 or 80/20 (training/testing set) for creating datasets in discharge coefficient calculations (Hekmat et al 2023, Majedi-Asl et al 2022, Salmasi et al 2021). Additionally, in the selection of sampling techniques, the geometry of labyrinth weirs in these studies was used as input for ML models. Recently, Majedi-Asl et al (2024) conducted a study on estimating the discharge coefficient of labyrinth weirs by varying the model's input parameters, including the total water head ratio, magnification (L_c/W), and cycle wall angle (α). The study revealed that each of these parameters demonstrated satisfactory performance in predicting the discharge coefficient. Moreover, Seyedian et al (2023) found that the most effective parameters in predicting C_w are the ratio (L/h) and the Froude number. However, additional studies are required to evaluate the impact of various input parameters of the labyrinth weir on machine learning (ML) models.

The main objective of the present study is to evaluate the performance of an ANN model considering different ratios of labyrinth weir data splitting for the prediction of the discharge coefficient. In this research, two labyrinth weirs approach conditions were adopted to estimate the discharge coefficient of labyrinth weir type D, based on different splitting ratios of input data for the training and testing phases. The first approach condition involved the emplacement of the labyrinth weir, and the second

involved filling the alveoli upstream and downstream. The main difference between this study and previously published works is that this is the first time when the influence of labyrinth weir approach conditions and the splitting strategy of training and testing datasets used in the ANN model has been investigated to predict the value of C_w .

2. Experimental Study

2.1. Description of the Test Facility

An experimental study was conducted at the Hydraulic Planning and Environment Laboratory (LAHE) of the University of Biskra in Algeria to investigate the effects of upstream and downstream approach conditions on the performance of a Type D labyrinth weir. Tests were carried out in a testing facility comprising a supply channel with a 0.95 m × 0.95 m cross-section and a 4.30 m length. This channel is connected to the simulation basin, with a rectangular shape 4.0 m × 5.0 m and a 1.1 m height. The labyrinth model is inserted at the outlet of the simulation basin. The so-called return channel is 2.0 m long and 1.0 m wide, connected to outlet (Fig. 1). The setup included a pumping unit with two pumps capable of discharging 0.180 m³/s and two conduits equipped with valves to test a wide range of flow rates (from 0.030 m³/s to 0.180 m³/s).

The study focused on a type D labyrinth weir model (Fig. 2) constructed by using metal sheets with a thickness (T_s) of 0.002 m. The geometric characteristics of the model are outlined in Table 1.

Table 1. Geometrical characteristics of the experimented model

Model	n	L	Wt	P	B	Wu	a	b	r	L/Wt	W/P	a/b
	–	m	m	m	m	m	m	m	m		–	–
RL with rounded entrance – type D	6	3.55	0.908	0.15	0.25	0.598	0.9	0.6	0.3	3.91	3.97	1.5
a : Inlet width (alveoli the upstream);		Wt: Total width of the labyrinth weir										
b: Outlet width (alveoli the downstream);		Wu: Width of a labyrinth weir unit										
B: Length of lateral wall;		Ts: Sidewall thickness										
P: Height of labyrinth weir;		L: Length of crest development labyrinth weir										
r: Raduit of round shape;		n: Developed length ratio of the labyrinth weir ($n=L/Wt$)										

2.2. Labyrinth Weir Emplacement

Labyrinth weirs are versatile flow control and discharge structures used in rivers, streams, canal entrances for earth dam spillways, and even on concrete dam crests. This diversity necessitates different placement strategies. In the first three cases, the weir is installed directly on the channel bed. However, for dam spillways, the weir is positioned on the dam’s crest, which acts as a concrete foundation.

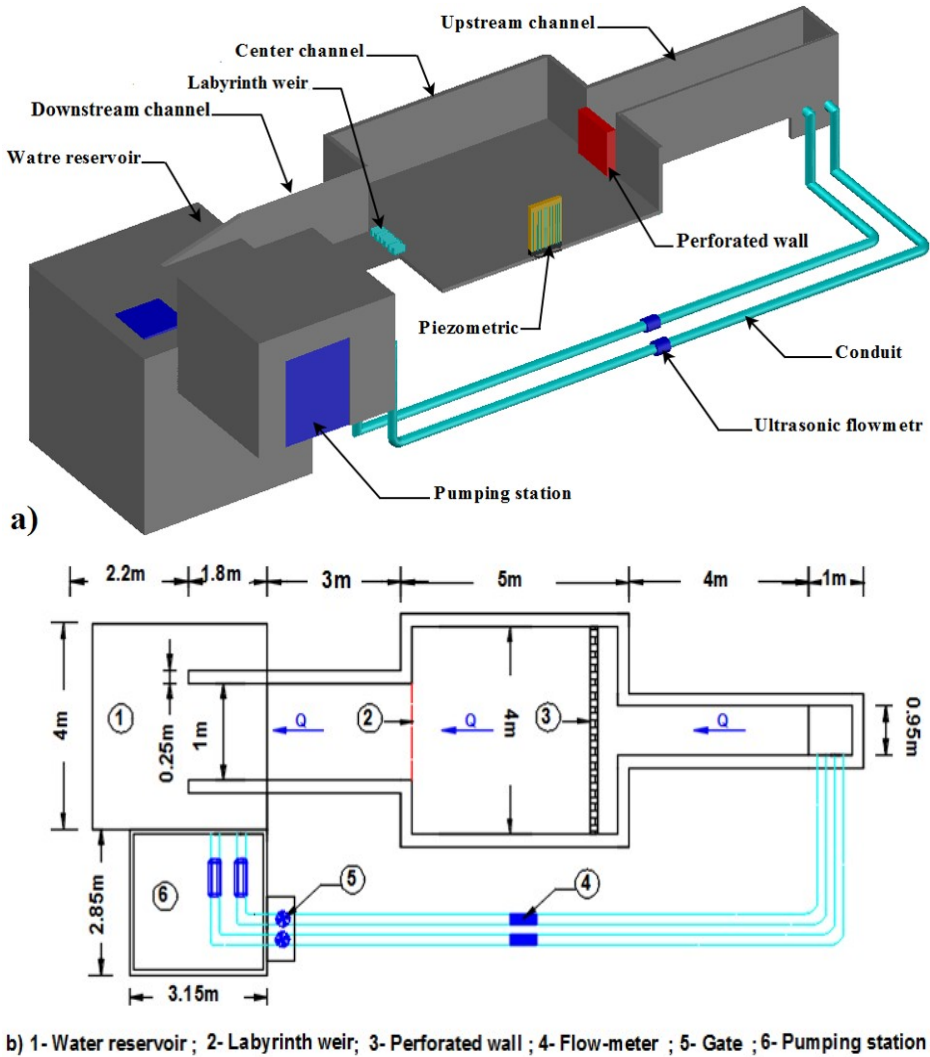


Fig. 1. Experimental station: (a) 3D view; (b) planar view

To investigate the effects of these placements, two labyrinth weir configurations were studied. The first, designated (Model A', Fig. 3b) involves placing the weir on the raft of the experimental channel. The second, designated (Model A, Fig. 3a) involves mounting the weir on a concrete base to simulate chute flow conditions. Both placement types were implemented in the experimental model of the labyrinth weir.

2.3. Filling Alveoli of the Upstream and Downstream

Labyrinth weirs typically have wall heights ranging from 3.0 to 15.0 meters, with varying thickness. Taller walls require reinforced concrete to withstand water pres-

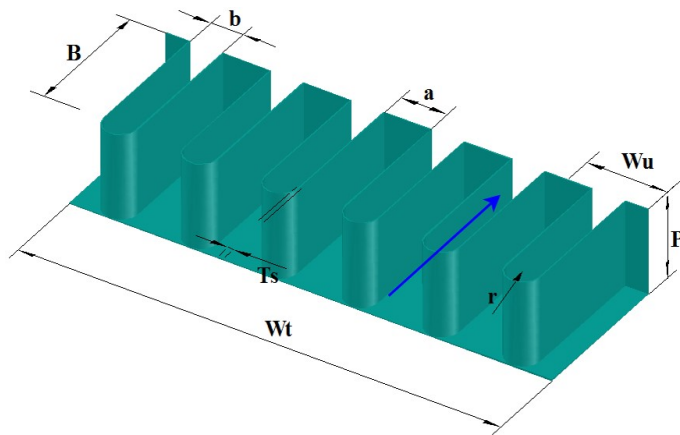


Fig. 2. Rectangular Labyrinth with rounded entrance-type D – a 3D view

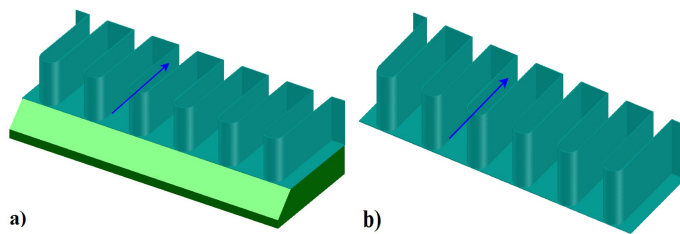


Fig. 3. Labyrinth weir placement: (a) placement on base “Model A”;
(b) placement on the raft channel “Model A’”

sure, significantly increasing costs. To reduce costs, we can lower the height of the free-standing parts of the walls while keeping the overall height of the weir. This is achieved by filling the inlet, outlet, or both with concrete, forming an inclined foundation or stepped arrangement. This reduces the need for reinforcing steel to only the shorter sections of the walls. We studied three cases to explore this approach:

- Effect of filling alveoli downstream: Two scenarios were tested for the labyrinth weir with a rounded upstream: Model B1 and Model B3 (Fig. 4a and c);
- Effect of filling alveoli upstream: The flow in the upstream cells can be stable or disturbed based on the bed design. Two designs were considered: Model B1 and Model B2 (Fig. 4a and b);
- Effect of filling alveoli upstream bed inclination: To study the influence of bed inclination, two scenarios were considered: Model B3 and Model B4 (Fig. 4c and d).

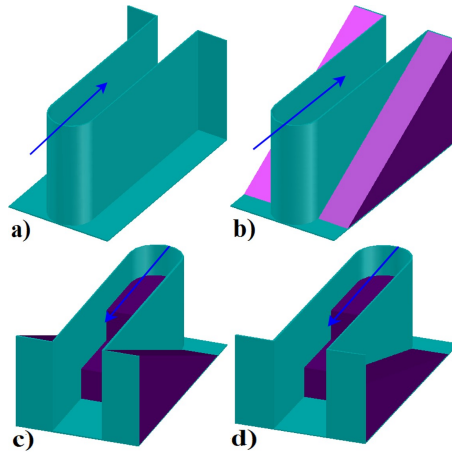


Fig. 4. Labyrinth weir: (a) With a raft horizontally positioned upstream of the cells “Model B₁”; (b) Inclined bed covering the entire height of the upstream cell “Model B₂”; (c) Inclined bed covering the entire height of the upstream cell and with two steps, each 5 cm high “Model B₃”; (d) Inclined bed covering 2/3 of the height of the upstream cell and with two steps, each 5 cm high “Model B₄”

3. Neural Network Architecture

ML models are recognized as sophisticated methodologies for rapid and accurate forecasting of real-world issues. These models, relying on objective computational techniques, can handle complex interactions between input and output variables. ML models show notable sensitivity both to data quality and the methodology used throughout the modeling process, especially to the ratio employed to partition datasets for ML model training and testing. In this study, we examined the impact of the training/validation ratio on the performance of the widely used ML model in predicting the discharge coefficient C_w .

Artificial Neural Networks (ANNs) are widely recognized as a powerful ML technique, modeled after the structure and function of biological neural networks, specifically the human brain’s nervous system. This method has been successfully applied to a variety of hydraulic engineering problems (Belaabed et al 2021). ANNs are used to identify relationships between input and output neurons in both linear and nonlinear patterns, enabling them to make decisions by analyzing patterns and relationships within the data. In this study, a multilayer perceptron neural network, a well-known type of ANN, was utilized as a regression technique to estimate the discharge coefficient.

We conducted numerous simulations to determine the best configuration of our ANN, testing different activation functions, numbers of hidden layers, and numbers of neurons in the hidden layer. The Levenberg-Marquardt back-propagation algorithm,

recognized as the most commonly used algorithm for supervised learning, allowed us to adjust the weights and biases of the neural network, thereby enhancing its predictive ability.

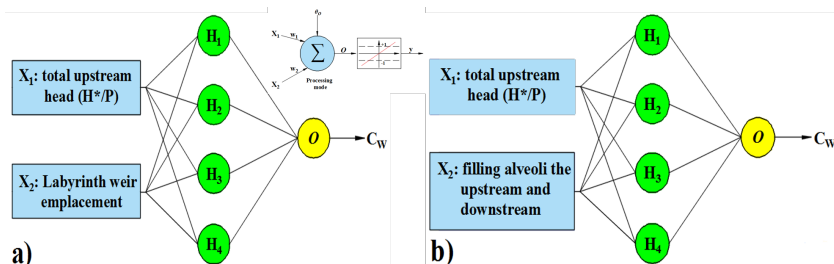


Fig. 5. Architecture of the ANN model: (a) Model 1, (b) Model 2

In Figure 5, we present two distinct models for selecting inputs. Model 1 uses factors like the total relative upstream height ratio (H^*/P) and weir placement, as shown in Models A and A' of Figure 3. In contrast, Model 2 incorporates the total relative upstream height ratio (H^*/P) and the filling of alveoli in the upstream and downstream cells, as illustrated in Models B₁, B₂, B₃, and B₄ of Figure 4. Both models predict the discharge coefficient C_w as indicated in Figure 5.

For the static evaluation of ML models, we divided the dataset into two parts using varying training/validation ratios (10/90, 20/80, 30/70, up to 90/10). The training set was employed to build the model, and the validation set was utilized to assess the predictive capability of the model. Ultimately, we gauged the model's performance using several criteria outlined in Equations 1 through 3.

$$R^2 = 1 - \left[\frac{\sum_1^n (y_{pr} - y_{(target)})^2}{\sum_1^n (y_{target} - \text{mean}(y_{(target)}))^2} \right], \quad (1)$$

$$MAE = \frac{1}{2} - \sum_{i=1}^n |y_{pr} - y_{(target)}|, \quad (2)$$

$$RMSE = \sqrt{\frac{1}{2} - \sum_1^n (y_{pr} - y_{(target)})}, \quad (3)$$

where:

- R^2 : the coefficient of determination,
- MAE : the Mean absolute error,
- $RMSE$: the Root Mean Squared Error,
- y_{pr} and $y_{(target)}$ are predicted and target values of the discharge coefficient C_w of labyrinth weir, respectively.

4. Results and Discussions

4.1. The Influence of the Weir Emplacement

The experimental results are represented by the relationship between the discharge coefficient and the upstream relative head (H^*/P) (Fig. 6a). The impact of the labyrinth weir's emplacement was studied by comparing two installation types: Model A and A' (Fig. 3). The results indicate that placing the weir on a pedestal results in a performance loss of 9.0% for upstream relative heads (H^*/P) below 0.40 and 3.0% for H^*/P values above 0.40.

4.2. Filling Alveoli of the Upstream and Downstream

The graphical representation (Fig. 6b) of the performance of the rounded upstream labyrinth weir, as a function of the downstream alveolus clutter (with 2 steps and without steps), shows that Model B3 does not affect performance. Furthermore, it was observed that for low and medium relative heads, there is a slight performance improvement of about 4.0%. These results also indicate that the stepped design of the downstream alveoli raft does not affect performance as long as the step height does not exceed 2/3 of the weir height.

Additionally, the discharge coefficients indicate that designing a more optimal hydraulic shape (Model B₁) would increase the efficiency of the labyrinth weir by approximately 8.0% for relative heads $H^*/P < 0.7$, compared to Model B₂.

Finally, the difference between the values of the model B₃ and B₄ decreases as the relative head H^*/P increases. For a relative head of $H^*/P = 0.3$, the difference is 11%; for $H^*/P = 0.5$, the difference decreases to 5%; and for $H^*/P = 0.7$, the difference between the two models is 3%. This supports the previous result obtained with the model B₁. However, the difference between the last two cases (model B₃ and B₄) is not significant enough to determine the better choice definitively, so the final decision will depend on economic considerations.

4.3. Influence of Training/Validation Ratio

Table 2 displays various simulations conducted using the ANN method for both types of labyrinth models. We observed that the optimal number of neurons varies depending on the type of transfer function, the number of hidden layers, and the approach conditions employed.

Figure 7 illustrates the fluctuation of performance criteria based on the percentage ratio between the training set and validation set for both Model 1 and Model 2. To assess the impact of changing this ratio on the performance of the ANN model, this analysis was conducted on the training and validation datasets. A close examination of the figure reveals a notable trend: as the training ratio increases, the RMSE and MAE performance criteria show significant instability, reaching high values until the

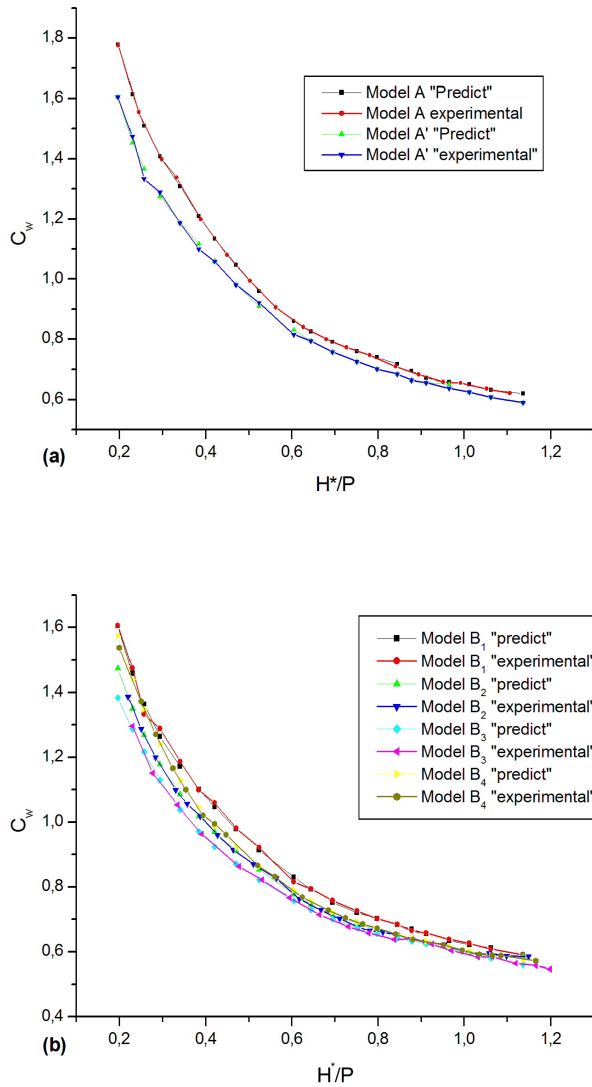


Fig. 6. C_w results obtained by our ANN model and our experimental tests: (a) Model 1, (b) Model 2

ratio reaches the 70/30 configuration (Figures 7a and 7b). An intriguing observation emerges at this crucial 70/30 ratio point: both RMSE and MAE values reach a minimum, suggesting a significant improvement in the accuracy of the ANN model. This indicates that the model effectively generalizes across the training and validation sets, achieving optimal performance at this specific point.

Table 2. Various simulations executed using the ANN model

Nr of H.L.	Activation function	Type of Model ANN	Training_ Testing (%)	Nr of the best neurons
1	T-T (1 to 30 neurons)	Model 1	(10/90); (20/80); (30/70); (40/60); (50/50); (60/40); (70/30); (80/20) and (90/10)	(19); (15); (12); (13); (24); (26); (29); (28) and (26)
		Model 2	(10/90); (20/80); (30/70); (40/60); (50/50); (60/40); (70/30); (80/20) and (90/10)	(2); (2); (2); (2); (2); (2); (2); (2) and (2)
1	L-L (1 to 30 neurons)	Model 1	(10/90); (20/80); (30/70); (40/60); (50/50); (40/60); (30/70); (20/80) and (90/10)	(13); (7); (16); (16); (16); (19); (28); (19) and (19)
		Model 2	(10/90); (20/80); (30/70); (40/60); (50/50); (60/40); (70/30); (80/20) and (90/10)	(6-22); (4-16); (13-14); (12-2); (14-28); (8-12); (5-28); (10-29) and (8-5)
1	P-P (1 to 30 neurons)	Model 1	(10/90); (20/80); (30/70); (40/60); (50/50); (60/40); (70/30); (80/10) and (90/10)	(9); (22); (9); (30); (25); (21); (27); (30) and (10)
		Model 2	(10/90); (20/80); (30/70); (40/60); (50/50); (60/40); (70/30); (80/20) and (90/10)	(16); (27); (11); (17); (2); (2); (17); (30) and (10)
2	T-T-P (1 to 30 neurons in each) H. L.	Model 1	(10/90); (20/80); (30/70); (40/60); (50/50); (60/40); (70/30); (80/20) and (90/10)	(19); (1); (15); (30); (21); (28); (30); (23) and (21)
		Model 2	(10/90); (20/80); (30/70); (40/60); (50/50); (60/40); (70/30); (80/20) and (90/10)	(6-4); (8-17); (14-20); (13-20); (7-11); (5-15); (6-15); (8-18) and (13-29)
H. L.: Hidden Layer T: Tangent Hyperbolic			L: Sigmoid P: Linear	

Furthermore, the examination of the R^2 criterion reveals a specific dynamics. It can be generally seen that the ANN model with a 70/30 split (training/testing) achieved the highest R^2 values compared to other training/testing percentage models. However, the ANN model with the lowest training percentage and highest testing percentage exhibited the most unstable values R^2 (i.e., the lowest R^2 values). As depicted in sub-figure 7c, the results compellingly demonstrate that the ANN model performs better on both training and validation sets when set at 70/30. A thorough analysis of average, standard, and quantitative performance levels supports this assertion, underscoring the crucial importance of this ratio in achieving optimal model accuracy.

4.4. Prediction of C_w by Optimal Ratio

Adhering to the previously identified optimal proportion, Table 3 presents a detailed study using our ANN model (70/30). For each input type, we crafted four models by adjusting the activation function and the number of hidden layers. Subsequently, we identified the optimal number of neurons for each input type. Finally, based on the RMSE, MAE, and R^2 values, we selected the most effective solution.

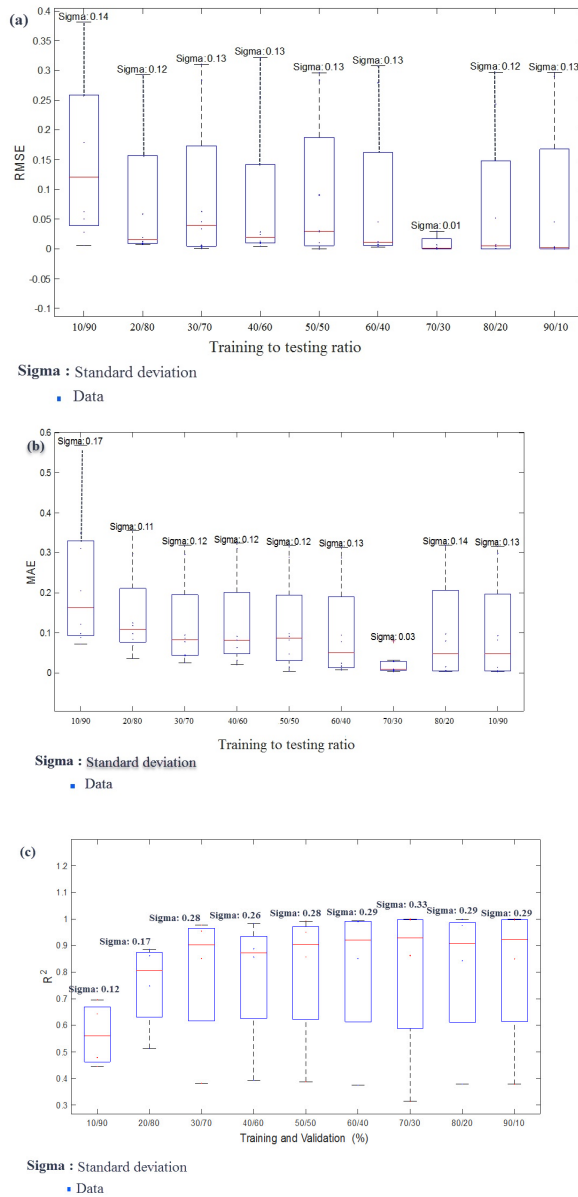


Fig. 7. Testing of the ANN model performance under different percentage values of the training/testing split ratio for the all datasets: (a) RMSE; (b) MAE; (c) R²

The results in the table indicate that the “Sigmoid” activation function exhibits extremely poor performance, while the “Linear” activation function shows average performance. However, the “Tangent Hyperbolic” activation function demonstrates

Table 3. Different simulations executed for the 70/30 rati

ANN Model	Nr of H.L.	Nr of best neurons	Activation function	Training		Validation		Global prediction		
				RMSE (10^{-2})	MAE (10^{-2})	RMSE (10^{-2})	MAE (10^{-2})	RMSE (10^{-3})	MAE (10^{-3})	R ²
M1_a	1	29	T-T	7.652	4.423	1.369	1.843	2.236	8.734	0.997
M1_b	1	2	L-L	144.4	29.58	117.7	35.76	29.70	31.48	0.314
M1_c	1	28	P-P	0.29e-4	7.715	4.946	12.84	0.439	9.292	0.862
M1_d	2	5–28	T-T-P	0.027	6.33e-3	1.035	1.234	0.883	3.843	0.999
M1_a	1	27	T-T	0.567	0.620	0.715	7.21	0.899	8.15	0.997
M1_b	1	17	L-L	214	27.53	150	29.91	28.06	28.24	0.637
M1_c	1	30	P-P	7.3e-13	7.807	11.91	8.37	6.982	79.75	0.856
M1_d	2	6–15	T-T-P	0.255	4.561	2.643	6.059	0.131	5.009	0.999

M1: Model 1 and M2: Model 2

highly satisfactory performance, becoming exceptional in both models when increasing the number of hidden layers to two.

Overall, the results for both input models revealed significant variations in model performance based on the transfer function and the number of hidden layers. The findings demonstrated that the “Tangent Hyperbolic” transfer function and two hidden layers together were optimal for training and validating ANN models with labyrinth approach conditions and h^*/P as inputs. This discovery contradicts other published works, such as the study by (Ayaz and Mansoor 2021), which investigated C_w predictions using the ANN for a triangular labyrinth model with different geometric aspects as inputs. They showed that the “Sigmoid” transfer function yields better performance. Additionally, Ahmad et al (2023) observed that errors (RMSE and MAE) decrease when using the “Linear” transfer function in their ANN learning model for an arc labyrinth weir and its geometric conditions as inputs.

We have presented Figures 7a and b to better understand the discrepancies between the results obtained by our developed model and the results from experimental trials. According to these figures, we observed a very good agreement between the results of the developed model and the experimental outcomes in both studied types.

5. Conclusions

Our investigation into the labyrinth weir discharge coefficient computation method highlights the difficulty involved and the rising interest in using ANNs to facilitate this computation. The ANN techniques need a thorough understanding of the components that influence the outcome, which makes creating accurate and significant equations for forecasting labyrinth weir discharge coefficients a challenging issue. In contrast to conventional methods, our work avoids predetermined ratios for training and validation and concentrates on the weir approach circumstances as inputs. The variance in these ratios provides insight into how they affect the efficacy and precision of ANN models, providing a unique viewpoint to improve the state-of-the-art ANN applications in this domain.

The Hydraulic Planning and Environment Laboratory (LAHE) in Algeria conducted an experimental study that sheds light on how to approach circumstances that affect the functionality of a Type D labyrinth weir. The results show the relevance of weir emplacement and filling upstream and downstream compartment designs. The results suggest that filling the upstream cells as an inclined weir and the downstream cells as stair steps does not impact the weir performance, as long as the height of these inclined weirs and steps remains below $2/3$ of the cell height. The research underscores the need to consider these aspects throughout the design and operation process by highlighting the performance losses that occur when the weir is positioned on a pedestal.

An interesting pattern emerges when the performance criteria for the ANN model about the training-validation data ratio are examined. The crucial point is the 70–30 ratio, which significantly increases the model accuracy. The research highlights how well the model generalizes the training and validation sets and the importance of the 70–30 ratio to attaining optimum performance.

Moreover, the assessment of activation functions in ANN models reveals the significance of choosing an appropriate transfer function and the number of hidden layers. Consistently outperforming other functions, “Tangent Hyperbolic” with two hidden layers offers important insights for model building. This result highlights the need to consider context-specific factors when selecting activation functions, even if they contradict certain previous studies.

Our research adds a nuanced knowledge of the labyrinth weir discharge coefficient prediction, highlighting the relevance of certain model configurations, the influence of approach circumstances, and the possibility of ANNs. The results of this study have significance for the development of ANNs in hydraulic engineering to enhance the prediction accuracy and guidelines for designing decision making.

Declarations

Conflict of interest. The researchers claim no conflict of interest.

References

- Ahmad F., Hussain A., Ansari M. A. (2023) Development of ANN model for the prediction of discharge coefficient of an arced labyrinth side weir, *Modeling Earth Systems and Environment*, **9** (2), 1835–1842.
- Ayaz M., Mansoor T. (2021) Development of ANN model for discharge prediction and optimal design of sharp-crested triangular plan form weir for maximum discharge using linked ANN–optimization model, *Water Supply*, **21** (6), 3027–3041.
- Belaabed F. (2019) *Etude des déversoirs non rectilignes noyés par l’aval (Study of non-rectilinear weirs submerged downstream)*, Université Mohamed Khider–Biskra, Biskra University [in French].
- Belaabed F., Goudjil K., Arabet L., Ouamane A. (2021) Utilization of computational intelligence approaches to estimate the relative head of PK-Weir for submerged flow, *Neural Computing and Applications*, **33** (19), 13001–13013.

- Ben Said M., Ouamane A. (2022) Performance of rectangular labyrinth weir – an experimental and numerical study, *Water Supply*, **22** (4), 3628–3644.
- Biener E. (1985) Rehabilitation of old gravity dams, Paper presented at the *International Congress of Large Dams*, France.
- Crookston B. M., Tullis B. (2010), Labyrinth weirs, *Hydraulic Structures*, **59**.
- Falvey H. T., Treille P. (1995) Hydraulics and design of fusegates, *Journal of Hydraulic Engineering*, **121** (7), 512–518.
- Filo G. (2023) Artificial Intelligence Methods in Hydraulic System Design, *Energies*, **16** (8), 3320.
- Hekmat M., Sarkardeh H., Jabbari E., Samadi M. (2023) Application of a hybrid ANFIS with meta-heuristic algorithms to estimate the aeration design parameters, *Water Supply*, **23** (6), 2249–2266.
- Houston K. L. (1983) *Hydraulic Model Study of Hyrum Dam Auxiliary Labyrinth Spillway*, U.S. Department of the Interior, Bureau of Reclamation, Division of Research, Hydraulics Branch, All U.S. Government Documents (Utah Regional Depository). Paper 159.
- Idrees A. K., Al-Ameri R. (2022) A review of hydraulic performance and design methods of labyrinth weirs, *Water Supply*, **22** (11), 8120–8138. Lux III F. (1987) Discussion of “Boardman Labyrinth-Crest Spillway” by John J. Cassidy, Christopher A. Gardner and Robert T. Peacock (March, 1985, Vol. 111, No. 3), *Journal of Hydraulic Engineering*, **113** (6), 808–811.
- Majedi-Asl M., Fuladipناه M., Arun V., Tripathi R. P. (2022) Using data mining methods to improve discharge coefficient prediction in Piano Key and Labyrinth weirs, *Water Supply*, **22** (2), 1964–1982.
- Majedi-Asl M., Ghaderi A., Kouhdaragh M., Alavian T. O. (2024) A performance comparison of the meta model methods for discharge coefficient prediction of labyrinth weirs, *Flow Measurement and Instrumentation*, 102563.
- Ouamane, A., Lempérière F. (2006) Nouvelle conception de déversoir pour l’accroissement de la capacité des retenues des barrages (New spillway design to increase the capacity of dam reservoirs), Paper presented at the *Colloque international sur la protection et la préservation des ressources en eau*, Bilda, Algérie [in French].
- Salmasi F., Nouri M., Sihag P., Abraham J. (2021) Application of SVM, ANN, GRNN, RF, GP and RT models for predicting discharge coefficients of oblique sluice gates using experimental data, *Water Supply*, **21** (1), 232–248.
- Seyedian S. M., Haghiabi A., Parsaie A. (2023) Reliable prediction of the discharge coefficient of triangular labyrinth weir based on soft computing techniques, *Flow Measurement and Instrumentation*, **92**, 102403.