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MATHEMATICAL MODEL OF MILITARY EQUIPMENT PRODUCTS MAINTENANCE FOR THE CONDITION-BASED OPERATION STRATEGY TAKING INTO ACCOUNT TYPE I ERRORS

Model matematyczny obsługiwania technicznego wyrobów sprzętu wojskowego dla strategii eksploatacji według stanu z uwzględnieniem błędów pierwszego rodzaju

Abstract: *For military equipment products maintained according to the condition-based operation strategy with control of parameters a mathematical model is constructed using semi-Markov random process. The diffusion-monotonic distribution law which is inherent in mechanical type products is taken for the fault model. The model takes into account type I errors. The analytical dependence of the utilization factor on the parameters of the scale and shape of the diffusion-monotonic distribution, the of regulated maintenance periodicity, the duration of complete restoration of a sample of equipment, the reliability of its control, is established. Graphs of the dependence of the utilization factor on the given parameters of the model are shown.*
Keywords: mathematical model, military equipment products, type I errors

Streszczenie: *Model matematyczny wykorzystujący losowy proces semi-Markowa został opracowany dla wyrobów sprzętu wojskowego obsługiwanych zgodnie ze strategią eksploatacji według stanu z kontrolą parametrów. Dla modelu uszkodzenia przyjęto prawo podziału dyfuzyjno-monotonicznego właściwego dla wyrobów mechanicznych. Model uwzględnia błędy pierwszego rodzaju. Ustalono analityczną zależność współczynnika wykorzystania technicznego od parametrów skali i kształtu rozkładu dyfuzyjno-monotonicznego, okresowości przeprowadzenia obsługiwania planowego, czasu trwania pełnej regeneracji próbki sprzętu, niezawodności jego kontroli. Pokazano wykresy zależności współczynnika wykorzystania technicznego od danych parametrów modelu.*
Słowa kluczowe: model matematyczny, wyroby sprzętu wojskowego, błędy pierwszego rodzaju

1. Introduction

A certain part of military equipment is subject to preventive measures after a certain time. Such measures in many cases are called regulated maintenance, which are carried out through a non-random time interval. In the intervals between regulated maintenance are possible equipment faults, which we will call emergency faults. In case of an emergency fault a complete restoration of the sample of equipment is carried out. In addition to emergency faults there may be so-called false faults when the signal from the built-in control system is received about the refusal of serviceable equipment. Such faults are commonly called type I errors, which make up about 5% of the total faults.

The choice of the distribution law of the faultless work time plays an important role in the construction of mathematical models for the military equipment maintenance. Nowadays, diffusion distribution laws are considered to be the most modern, namely: diffusion-monotonic and diffusion-nonmonotonic distributions. The use of these laws was considered in [3-9]. The scientific novelty of the proposed work is the consideration of type I errors. This fact is of considerable practical importance, which determines the relevance of the research.

2. Construction of a mathematical model

The built-in control system constantly monitors the product's general characteristics, for example, battery voltage, fuel availability in tanks, oil level, transmitter power, noise ratio of the receiver, the presence of chips in the engine oil, the temperature of the second stage of the compressor of the aircraft engine, etc.

The external control system is intended for the use of regulated maintenance, current and partly mid-life repair. Such system controls a lot more parameters and has a much higher control reliability than the built-in control system.

For shortening we will indicate the external control system as CS-1, and the built-in control system as CS-2.

A schematic representation of transitions for the proposed model is shown in fig. 1.

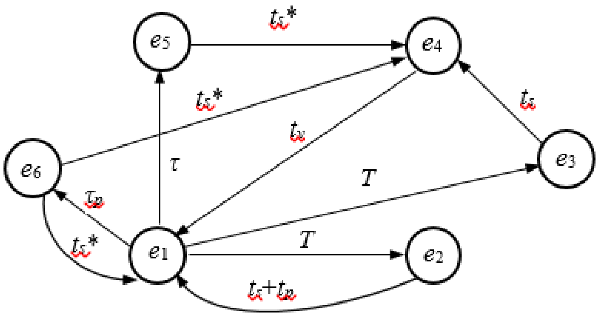

Fig. 1. Transition scheme for the proposed model

For shortening the military equipment sample we will call the object of control (OC). The scheme of transitions (fig. 1) takes into account the following states of the OC and the control systems (CS):

e_1 – OC works according to its intended purpose in perfect state;

e_2 – the technical state of the equipment sample is checked by the ground control system and preventive works are carried out; and there is no fault in OC;

e_3 – the technical state of the equipment sample is checked by the ground control system, and there is a fault in OC which occurred at the transition e_1–e_3 at the time $0<\tau<T$ and wasn't detected by built-in control system;

e_4 – complete restoration of OC is performed;

e_5 – at the moment $0<\tau<T$ the fault of OC was detected by the built-in control system;

e_6 – at the moment $0<\tau_p<T$ the false fault signal of OC was detected by the built-in control system.

In fig. 1. following notations are used:

T – the regulated maintenance periodicity;

t_s – the duration of verification of OC by control system CS-1;

t_s* – the duration of verification of OC by control system CS-2;

t_p – the duration of preventive works carrying out;

t_v – the duration of the emergency restoration;

t_r – the regulated maintenance duration;

τ – the random time of fault signal receiving from the CS-2;

τ_p – the random time of false fault signal receiving from the CS-2.

The the ideal version of the OC operation is shown in fig. 2.

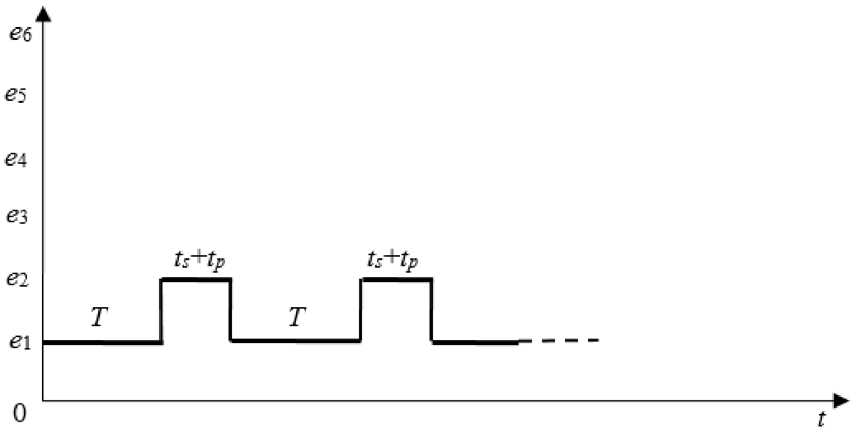

Fig. 2. The ideal version of the OC operation

During the time T OC works in perfect state, then the technical state control (time t_s) is performed and preventive works are carried out within time t_p, whereafter the situation is repeated many times.

Possible variant of real operation is shown in fig.3

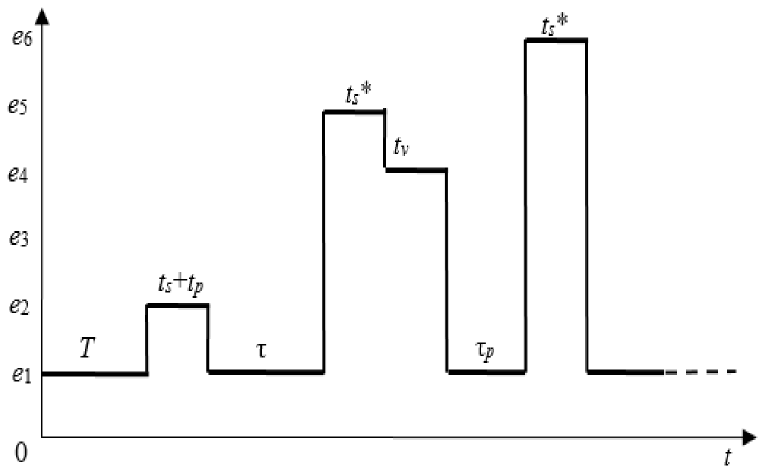

Fig. 3. Possible variant of real OC operation

The states of the OC are laid off along the Y axis in figs. 2 and 3, and the current time is laid off along the X axis.

The process of OC functioning we will describe by a semi-Markov random process. The feature of such a process is that the duration of the presence of an OC in its previous state during the transition to the next state may have an arbitrary distribution law. This circumstance significantly enhances the possibilities of an OC operation process modeling compared to Markov random process for which such a distribution function is exponential. It is proved that the maximum availability factor can be achieved with a deterministic regulated maintenance periodicity. It is not possible to model the nonrandomness of the regulated maintenance periodicity by an exponential distribution function.

3. Choosing a mathematical fault model

The choice of mathematical fault model is important during the development of the mathematical model of OC functioning. For mechanical type products the diffusion--monotonic (DM) law of faultless work time distribution is considered the most suitable [2].

The distribution function and the reliability function for such a distribution law have the form accordingly:

$$F(t, \mu, \nu) = \Phi\left(\frac{t-\mu}{\nu\sqrt{\mu t}}\right) \tag{1}$$

and

$$P(t) = \Phi\left(\frac{\mu - t}{\nu\sqrt{\mu t}}\right) \tag{2}$$

where

t – is the current time;

μ – is the scale parameter;

ν – is the shape parameter;

$$\Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} e^{-\frac{u^2}{2}} du$$ – Laplace function.

The mean operating time between failures T_s for the DM distribution is equal $\mu\left(1+\frac{\nu^2}{2}\right)$ [2]. If ν = 0.5, then the mean operating time between failures almost the same ($T_s = \mu\left(1+\frac{0{,}5^2}{2}\right) = 1{,}125\mu$) with the scale parameter.

The scale parameter has a dimension of h, and the shape parameter is dimensionless.

The initial data taken into account in the calculation task are:

- scale parameter $\mu = 200$ h;
- shape parameter $\nu = 0{,}5$;
- intensity of receiving of false alarm signals $\lambda = 10^{-3}$ 1/h;
- the regulated maintenance periodicity $T = 200$ h;
- the duration of equipment sample control by ground control equipment $t_s = 2{,}5$ h;
- the duration of equipment sample control by built-in control equipment $t_s^* = 1$ h;
- the duration of preventive works $t_p = 4$ h;
- the duration of the OC emergency restoration $t_v = 5$ h;
- the probability of receiving a signal about the equipment sample failure from the built-in control system $\rho = 0{,}7$;
- the reliability of the correct OC good state determination by built-in control equipment $d_r = 0{,}9$.

4. The utilization factor substantiation

The utilization factor of OC can be determined from the equation [1]

$$K_{tv} = \frac{\sum_{i=1}^{6} \pi_i(T)\cdot\omega_i(T)}{\sum_{i=1}^{6} \pi_i(T)\cdot\eta_i(T)}, \tag{3}$$

where

$\pi_i(T)$ – is the frequency of the Markov chain getting to the i state, $i=\overline{1,6}$;

$\omega_i(T)$ – the average time of the OC stayinsg in perfect state;

$\eta_i(T)$ – the average time of the OC staying in any state.

The numerator (3) shows the average time the OC staying of in perfect state, while the denominator indicates the current operation time of the OC.

The frequency of the Markov chain getting to the i state $\pi_i(T)$ can be determined from the formula (3)

$$\left.\begin{aligned} &\overline{\pi_i}(T) = \overline{\pi_i}(T) \cdot P_{ij}(T) \\ &\sum_{i=1}^{6} \pi_i(T) = 1 \end{aligned}\right\} \tag{4}$$

where $\overline{\pi_i}(T)$ – is the vector of getting Markov chain to the i state;

$P_{ij}(T)$ – the transition matrix of OC from state $i = \overline{1,6}$ to state $j = \overline{1,6}$.

The transition probability matrix for the proposed model looks like this

$$P_{ij}(T) = \begin{vmatrix} 0 & P_{12} & P_{13} & 0 & P_{15} & P_{16} \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ d_g & 0 & 1-d_g & 0 & 0 & 0 \end{vmatrix} \tag{5}$$

where $P_{12}(T) = P(T)e^{-\lambda t}$; $P_{13}(T) = (1-\rho)\int_0^T e^{-\lambda t} dF(t)$;

$$P_{15}(T) = \rho\int_0^T e^{-\lambda t} dF(t);\quad P_{16}(T) = \lambda\int_0^T e^{-\lambda t}[1-F(t)]dt.$$

The sum of probabilities for each row of the matrix (5) must be equal to one. For all rows except the first this is obvious.

Considering the values of $F(t)$ and $P(t)$ for the DM distribution for the initial data, we obtain the values of the components of the transition probability matrix

$$P_{ij}(200) = \begin{vmatrix} 0 & 0{,}40937 & 0{,}13043 & 0 & 0{,}30433 & 0{,}15588 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0{,}9 & 0 & 0{,}1 & 0 & 0 & 0 \end{vmatrix}.$$

The sum of probabilities on the first row of the matrix is one, which indicates the correctness of the calculations performed.

After substituting the matrix (5) into equation (4) it is possible to obtain the values of the vector components $\overline{\pi_i}(T)$ by solving the following equations

$$a(t)=\frac{1}{1+P_{12}+2P_{13}+2P_{15}+P_{16}\left(P_{64}+1\right)};$$

$$a(t)=\frac{1}{1+\left(1-F(t)\right)\cdot e^{-\lambda t}+2\left[(1-\rho)\int_0^T e^{-\lambda t}dF(t)\right]+2\rho\int_0^T e^{-\lambda t}dF(t)+\lambda\int_0^T e^{-\lambda t}[1-F(t)]dt\left(1-d_g+1\right)};$$

$\pi_1(T) = a(T)$; $\pi_2(T) = a(T)P_{12}$; $\pi_3(T) = a(T)P_{13}$; $\pi_4(T) = a(T)(P_{13} + P_{15} + P_{16}\, P_{64})$; $\pi_5(T) = a(T)P_{15}$; $\pi_6(T) = a(T)P_{16}$;

It can be shown that for the initial model data the components of the vector $\overline{\pi_i}(T)$ are equal: $\pi_1(T)$ = 0,4081; $\pi_2(T)$ = 0,4216706; $\pi_3(T)$ = 0,05373; $\pi_4(T)$ = 0,18379; $\pi_5(T)$ = 0,1242; $\pi_6(T)$ = 0,006362.

You can make sure that the sum of the probabilities of the vector components $\overline{\pi_i}(T)$ is equal one, which indicates the qualitative calculation performed using this method.

Next, we calculate the average duration of the OC staying in different states of the semi-Markov process. For example, for state e_1 we get:

$$\eta_1(T)=\left[1-F(T)\right]\cdot e^{-\lambda T}+(1-\rho)\cdot\int_0^T e^{-\lambda t}\cdot dF(t)\cdot T+$$
$$+\rho\int_0^T e^{-\lambda t}\cdot dF(t)\cdot\int_0^T t\cdot dF_{15}(t)+\lambda\cdot\int_0^T e^{-\lambda t}\left[1-F(t)\right]dt\cdot\int_0^T t\cdot dF_{16}(t). \tag{6}$$

It can be shown that for the initial model data $\eta_1(T)=163{,}82$ h,
$\eta_2(T)=t_p+t_r=6{,}5$ h, $\eta_3=t_s+t_r=6{,}5$ h, $\eta_4=5$ h, $\eta_5=t_s^*=1$ h, $\eta_6=1$ h.

Let's calculate the average time of semi-Markov process one transition

$$\eta_s(t)=\pi_1(t)\cdot\eta_1(t)+\pi_2(t)\cdot\eta_2(t)+\pi_3(t)\cdot\eta_3(t)+\pi_4(t)\cdot\eta_4(t)+$$
$$+\pi_5(t)\cdot\eta_5(t)+\pi_6\cdot\eta_6(t)=69{,}18\ h.$$

To calculate the utilization factor, we determine the mean time of OC faultless work in state e_1:

$$\omega_1(T) = M\left[\min\left(\tau, \tau_p\right)\right].$$

$$\omega_1(T) = \int_0^T (1 - F(x)) \cdot (1 - \Lambda(x)) \cdot dx, \qquad \Lambda(x) = 1 - e^{-\lambda \cdot x}.$$

Then $\omega_1(T) = \int_0^T (1 - F(x)) \cdot e^{-\lambda x} \cdot dx = 155{,}88$ h.

The utilization factor at the point T = 200 h will be considering $\pi_1(200) = 0{,}4081$, $\omega_1(200) = 155{,}898$ h, $\eta_s(t) = 69{,}18$ h

$$K_{tv} = \frac{\pi_1(200) \cdot \omega_1(200)}{\eta_s(200)} = 0{,}9196.$$

5. The utilization factor dependence on the model parameters

To determine the dependence of K_{tv} on the model parameters within the wide range of their changes the calculations were performed by numerical method. A graphical interpretation of the results is shown in figs. 4-7. So, fig. 4 shows the dependence of K_{tv} on the regulated maintenance periodicity *T* while changing the shape parameter ν from 1 to 0,25 at μ = 600 h; $\lambda = 10^{-3}$ 1/h; $t_s = 2{,}5$ h; $t_s^* = 1$ h; $t_p = 4$ h; $t_v = 5$ h.

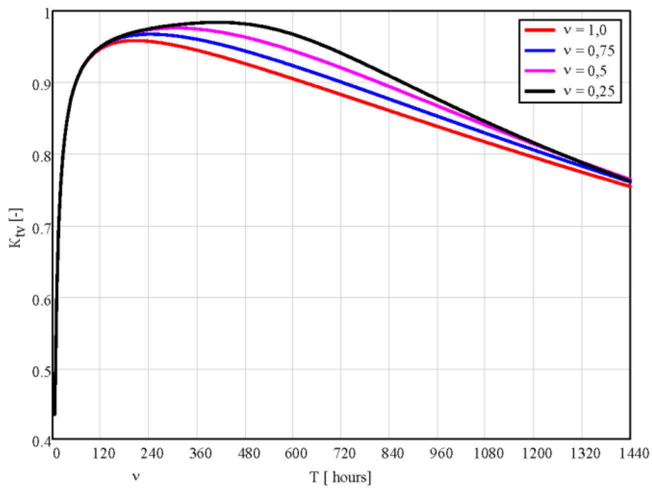

Fig. 4. The dependence of K_{tv} on the regulated maintenance periodicity *T* while changing the shape parameter ν from 1 to 0,25 at μ = 600 *h*; $\lambda = 10^{-3}$ 1/*h*; $t_s = 2{,}5$ h; $t_s^* = 1$ h; $t_p = 4$ h; $t_v = 5$ h

The analysis of the given graphs shows (fig. 4) the presence of an optimal regulated maintenance periodicity, which ensures the maximum value of K_{tv}. However, the smaller the shape parameter ν, the larger the K_{tv}. The values of the shape parameter for wear, fatigue and aging processes of mechanical products are specified in [2] and vary from ν = 0,05 to 1,5. In this paper mean values of the shape parameter were used.

Increasing the scale parameter from μ = 200 h to μ = 600 h (fig. 5) increases K_{tv}, and also increases the optimal value of regulated maintenance periodicity, which well corresponds to physical considerations.

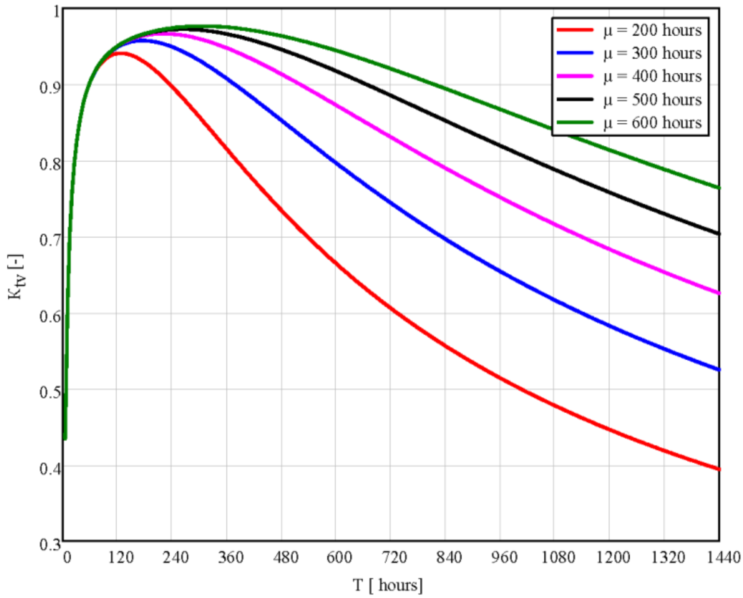

Fig. 5. The dependence of K_{tv} on the regulated maintenance periodicity T at the shape parameter ν = 0,5 and while changing the scale parameter μ from 200 to 600 h at $\lambda = 10^{-3}$ 1/h; t_s = 2,5 h; t_s^* = 1 h; t_p = 4 h; t_v = 5 h

As the probability of receiving a signal about the equipment sample failure from the built-in control system ρ increases (fig. 6), K_{tv} increases. However, for larger values of the scale parameter, a larger value of K_{tv} is provided, which coincides with physical considerations.

The increase in the duration of the equipment sample restoration leads to a decrease in K_{tv} (fig. 7).

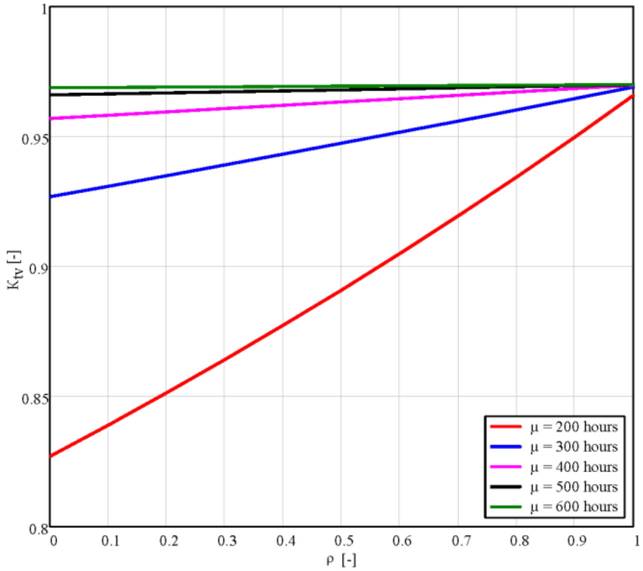

Fig. 6. The dependence of K_{tv} on the probability of receiving a signal about the equipment sample failure from the built-in control system for different values of scale parameter μ and constant values of ν = 0,5; T = 200 h; t_s = 2,5 h; t_s* = 1 h; t_p = 4 h; t_v = 5 h

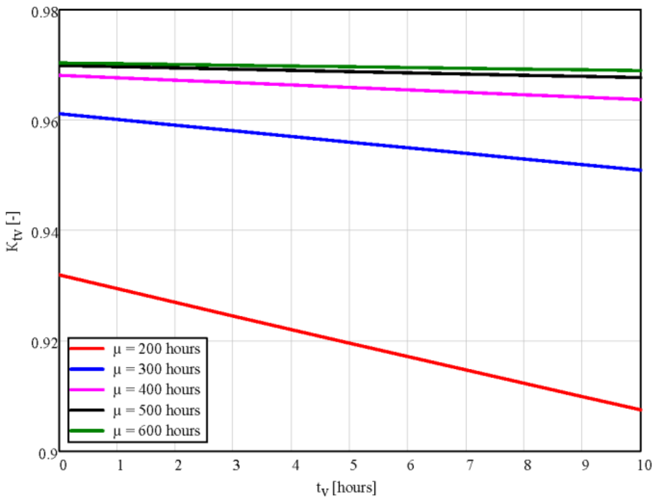

Fig. 7. The dependence of K_{tv} on the duration of the OC emergency restoration t_v for different values of scale parameter μ and constant values of ν = 0,5; T = 200 h; t_s = 2,5 h; t_s* = 1 h; t_p = 4 h; $\lambda = 10^{-3}$ 1/h

6. Conclusions

1. In work with the use of a semi-Markov random process, a mathematical model of condition-based maintenance of military equipment products is constructed, which feature is taking into account type I errors with use of six states.
2. For diffusion-monotonic failure distribution there was established the analytical dependence of the utilization factor on the parameters of scale and shape, the regulated maintenance periodicity, the probability of receiving an information about the failure, the duration of the equipment sample restoration and other model parameters.
3. There was established the existence of an optimal regulated maintenance periodicity, which ensures the maximum value of the utilization factor.
4. The dependencies obtained in this work coincide with physical considerations and can be used to solve practical problems (modeling of operation processes of equipment).
5. In the future, it is planned to investigate similar dependences for the diffusion-monotonic failure distribution.

7. References

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