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Port oil terminal operation cost and system safety joint optimization

Keywords

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Abstract

The model of system operation total costs in the safety state subsets is introduced and the procedure of its operation total cost in the safety state subset not worse than the critical safety state minimization is presented. The model of system safety impacted by operation process is created and the procedure of its safety in the safety state subset not worse than the critical safety state maximization is presented as well. To analyze jointly the system safety and its operation cost optimization, we propose the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that allows to find the best system safety function and other safety indicators, through applying the created system safety model and linear programming. Next, to find the system operation total costs in the safety state subsets, corresponding to this system best safety indicators, we replace the limit transient probabilities at the particular operation states, existing in the formula for the system operation total costs in the safety state subsets, by their optimal values existing in the formulae for the coordinates of the system safety function after maximization. On the other hand, to analyze jointly the system operation cost and its safety optimization, we propose the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that allows to find minimal system operation total cost in the safety state subset not worse than the critical safety state, using the created system operation cost model and linear programming. After that, to find the system conditional safety indicators, corresponding to this system minimal total operation cost in the safety state subset not worse than the critical safety state, we replace the limit transient probabilities at particular operation states, existing in the formula for the system safety function coordinates, and for remaining system conditional safety indicators by their optimal values existing in the formulae for the system minimal total cost in the safety state subset not worse than the critical safety state. The created models are applied separately and jointly to the port oil terminal. Moreover, to fulfil the obtained port oil terminal optimal safety and operation cost results the modifications of its operation process are proposed. The evaluation of results achieved is performed and the perspective for future research in the field of the complex systems, including port oil terminal, safety and their operation costs joint analysis and optimization is given.

1. Introduction

To tie the investigation of the complex technical system safety together with the investigation of its operation cost, the semi-Markov process model (Ferreira & Pacheco, 2007; Glynn & Haas, 2006; Grabski, 2002, 2015; Limnios & Oprisan, 2005; Mercier, 2008; Tang et al., 2007), can be used to describe this system operation process (Kołowrocki & Soszyńska-Budny, 2011/2015; Magryta, 2020). The system operation process model, under the assumption on the system safety multistate model (Xue & Yang, 1985; Xue, 1995), can be used to construct the general safety model of the complex multistate system changing its safety structure and its components safety parameters

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during variable operation conditions (Kołowrocki, 2014; Kołowrocki & Magryta, 2020; Kołowrocki & Soszyńska-Budny, 2011/2015; Magryta, 2020). Further, using this general model, it is possible to define the complex system main safety characteristics such as the system safety function, the mean values and standard deviations of the system lifetimes in the system safety state subsets and in the system particular safety states (Dąbrowska, 2020, Dąbrowska & Kołowrocki, 2019a; Kołowrocki, 2014, 2020; Kołowrocki & Soszyńska, 2010a-b; Kołowrocki & Soszyńska-Budny, 2011/2015). Other system safety indicators, like the system risk function, the system fragility curve, the moment when the system risk function exceeds a permitted level, the system intensity of ageing, the coefficient of operation process impact on system intensity of ageing and the system resilience indicator to operation process impact, can be introduced as well (Gouldby et al., 2010; Kołowrocki, 2014; Kołowrocki & Soszyńska-Budny, 2018a-b, 2019a-c; Lauge et al., 2015; Szymkowiak, 2018a-b, 2019). Using the system general safety model, it is possible to change the system operation process through applying the linear programming (Klabjan & Adelman, 2006) for maximizing the system safety function (Kołowrocki & Soszyńska-Budny, 2010) and finding the optimal limit values of the system transient probabilities at the particular operation states and determining the system optimal safety indicators. Having the system operation process characteristics and the system conditional instantaneous operation costs at the operation states, it is possible to create the system general operation total costs in the safety state subsets (Kołowrocki & Kuligowska, 2018; Kołowrocki & Magryta-Mut, 2021, 2022a). Using this system operation total cost model, it is possible to change the system operation process through applying the linear programming (Klabjan & Adelman, 2006) for minimizing the system operation total cost costs in the safety state subsets (Kołowrocki & Kuligowska, 2018; Kołowrocki & Magryta-Mut, 2021) and finding the optimal limit values of the system transient probabilities at the particular operation states. To analyze jointly the system safety and its operation cost optimization, in the case we prefer more the system safety maximization than the system operation cost minimization, we first apply the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that maximize the system safety. Next, to find the system conditional operation total costs in the safety state subsets, corresponding to this system maximal safety, we replace the limit transient probabilities at particular operation states, existing in the formula for the operation total costs in the safety state subsets, by their optimal values existing in the formula for the system maximal safety function coordinates. Whereas, in the case we prefer more the system operation cost minimization than the system safety maximization, then to analyze jointly the system safety and operation cost optimization, we first apply the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that minimize the system operation total costs in the safety state subsets. Next, to find the system conditional safety indicators, corresponding to this system minimal total operation costs in the safety state subsets, we replace the limit transient probabilities at the system particular operation states, existing in the formula for the system safety function coordinates, by their optimal values existing in the formula for the system minimal operation total costs in the safety state subsets in order to get the formula for the system conditional safety function related to this system minimal operation total cost. Further, applying this formula for the system conditional safety function, we find the remaining system conditional safety indicators.

The created model for maximizing the system safety is applied to the port oil terminal to find its optimal safety indicators. Next, the port oil terminal mean value of the system operation total costs in the safety state subsets corresponding to its optimal safety indicators is found. The created model for minimizing of the system operation total costs in the safety state subsets is applied to the port oil terminal to find the minimal mean value of the system operation total costs in the safety state subsets. Next, the port oil terminal safety indicators corresponding to this minimal operation total cost are found.

The chapter is organized into 7 parts, this Introduction as Section 1, Sections 2–6 and Conclusion as Section 7. In Section 2, the model of system operation total costs in the safety state subsets is introduced and the procedure of its minimization is presented. In Section 3, the model of system safety impacted by operation process is introduced and the procedure of the system safety maximization is proposed. In Section 4, the port oil terminal operation total costs in the safety state subsets is analyzed and its minimal value is determined. In Section 5, the port oil terminal operation process influence on its safety indicators is examined and the best forms and values of this system safety indicators are determined. In Section 6, joint analysis of the port oil terminal safety maximizing and its conditional operation total costs in the safety state subsets corresponding to the port oil terminal maximal safety is performed. The port oil terminal best safety indicators are fixed and the port oil terminal operation total costs corresponding to this best safety indicators is determined. Joint analysis of the port oil terminal operation total costs in the safety state subsets minimizing and its conditional safety indicators corresponding to the port oil terminal minimal operation total cost is performed. The port oil terminal minimal operation total cost is fixed and the system safety indicators corresponding to this minimal operation total cost are determined. In Conclusion, the evaluation of results achieved is done and the perspective for future research in the field of the complex systems, including port oil terminal, safety and their operation costs joint analysis and optimization is proposed.

2. System operation cost with system safety impact

2.1. System operation cost model with system safety impact

Similarly to safety analysis of the system impacted by its operation process, we may investigate the system operation total costs in the safety state subsets. Namely, we define the instantaneous system operation cost in the form of the vector

$$\boldsymbol{C}(t,\cdot) = [\boldsymbol{C}(t,1),\ldots, \boldsymbol{C}(t,z)],\ t \geq 0, \tag{1}$$

with the coordinates given by

$$\boldsymbol{C}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{C}(t,u)]^{(b)},\ t \geq 0, \tag{2}$$

$u = 1,2,\ldots,z,$

where $[\boldsymbol{C}(t,u)]^{(b)}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$, are the coordinates of the vector

$$[\boldsymbol{C}(t,\cdot)]^{(b)}=[[\boldsymbol{C}(t,1)]^{(b)},\ldots, [\boldsymbol{C}(t,z)]^{(b)}],$$

$t \geq 0$, $b = 1,2,\ldots,v$,

representing the system conditional instantaneous operation costs in the safety state subsets $\{u,u + 1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the system operation states z_b, $b = 1,2,\ldots,v$, and p_b, $b = 1,2,\ldots,v$, are the system operation process limit transient probabilities in the particular operation states (Grabski, 2015). Thus, it is naturally to assume that the system instantaneous operation cost depends significantly on the system operation state and the system operation cost at the operation state as well. This dependency is also clearly expressed in mean value of the system total operation cost

$$\boldsymbol{C}(\cdot) = [\boldsymbol{C}(1), \boldsymbol{C}(2),\ldots,\boldsymbol{C}(z)] \tag{3}$$

with coordinates given by the linear equations

$$\boldsymbol{C}(u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{C}(u)]^{(b)},\ u = 1,2,\ldots,z, \tag{4}$$

for the mean values of the system total unconditional operation costs in the safety state subsets $\{u,u + 1,\ldots,z\}$, $u = 1,2,\ldots,z$, where $\boldsymbol{C}(t,u)]^{(b)}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$, are the mean values of the system total conditional operation costs in the safety state subsets $\{u,u + 1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the particular system operation states z_b, $b = 1,2,\ldots,v$, determined by

$$[\boldsymbol{C}(u)]^{(b)} \cong \int_0^{[\mu(u)]^{(b)}} [\boldsymbol{C}(t,u)]^{(b)} dt,\ u = 1,2,\ldots,z,$$

$$b = 1,2,\ldots,v, \tag{5}$$

where

$$[\boldsymbol{\mu}(u)]^{(b)} = E[[T(u)]^{(b)}],\ u = 1,2,\ldots,z, \tag{6}$$

are the mean values of the system conditional lifetimes $[T(u)]^{(b)}$ in the safety state subset $\{u,u + 1,\ldots,z\}$ at the operation state z_b, $b = 1,2,\ldots,v$, given by (Kołowrocki & Soszyńska-Budny, 2011/2015; Kołowrocki & Magryta, 2020c):

$$[\boldsymbol{\mu}(u)]^{(b)} = \int_0^{\infty} [\boldsymbol{S}(t,u)]^{(b)} dt,\ u = 1,2,\ldots,z, \tag{7}$$

and $[\boldsymbol{S}(t,u)]^{(b)}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$, are the system safety function defined above and p_b are limit transient probabilities defined in (Kołowrocki & Soszyńska-Budny, 2011/2015).

2.2. System operation cost optimization model with system safety impact

From the linear equations (4), we can see that the mean value of the system total unconditional operation cost $\boldsymbol{C}(u)$, $u = 1,2,\ldots,z$, is determined by the limit values of transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the operation states z_b, $b = 1,2,\ldots,v$, and by the mean values $[\boldsymbol{C}(u)]^{(b)}$ of the system total conditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the particular system operation states z_b, $b = 1,2,\ldots,v$, that by (5) are dependent on the mean values $[\boldsymbol{\mu}(u)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, of the system conditional lifetimes and by the system conditional instantaneous operation costs $[\boldsymbol{C}(t,u)]^{(b)}$ in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the system operation state z_b, $b = 1,2,\ldots,v$. Therefore, the system operations cost optimization based on the linear programming (Klabjan & Adelman, 2006), can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, of the transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the operation states to minimize the mean value $\boldsymbol{C}(u)$ of the system total unconditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, under the assumption that the mean values $[\boldsymbol{C}(u)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, of the system total conditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the particular system operation states z_b, $b = 1,2,\ldots,v$, are fixed. As a special and practically important case of the above formulated system operation cost optimization problem for $u = r$, where if r, $r = 1,2,\ldots,z$, is a system critical safety state, we may look for the optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, of the transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the system operation states to minimize the mean value $\boldsymbol{C}(r)$ of the system total unconditional operation costs in the safety state subset $\{r, r+1,\ldots, z\}$, $r = 1,2,\ldots,z$, under the assumption that the mean values $[\boldsymbol{C}(r)]^{(b)}$, $b = 1,2,\ldots,v$, $r = 1,2,\ldots,z$, of the system total conditional operation costs in this safety state subsets are fixed. More exactly, we may formulate the optimization problem as a linear programming model (Lauge, et al., 2015) with the objective function of the following form

$$\boldsymbol{C}(r) \cong \sum_{b=1}^{v} p_b [\boldsymbol{C}(r)]^{(b)}, \tag{8}$$

for a fixed $r \in \{1,2,\ldots,z\}$ and with the following bound constraints

$$\breve{p}_b \le p_b \le \hat{p}_b, \ b = 1,2,\ldots,v, \ \sum_{b=1}^{v} p_b = 1, \tag{9}$$

where

$[\boldsymbol{C}(r)]^{(b)}$, $[\boldsymbol{C}(r)]^{(b)} \geq 0$, $b = 1,2,\ldots,v$,

are fixed mean values of the system conditional operation costs in the safety state subset $\{r,r+1,\ldots,z\}$, and

$$\breve{p}_b, \ 0 \le \breve{p}_b \le 1 \text{ and } \hat{p}_b, \ 0 \le \hat{p}_b \le 1, \ \breve{p}_b \le \hat{p}_b,$$

$$b = 1,2,\ldots,v, \tag{10}$$

are lower and upper bounds of the unknown transient probabilities p_b, $b = 1,2,\ldots,v$, respectively.

Now, we can obtain the optimal solution of the formulated by (8)–(9) the linear programming problem, i.e. we can find the optimal values $\dot{p}_b$, of the transient probabilities p_b, $b = 1,2,\ldots,v$, that minimize the objective function given by (8).

We arrange the mean values of the system total conditional operation costs $[\boldsymbol{C}(r)]^{(b)}$, $b = 1,2,\ldots,v$, in non-decreasing order

$$[C(r)]^{(b_1)} \le [C(r)]^{(b_2)} \le \ldots \le [C(r)]^{(b_i)},$$

where $b_i \in \{1,2,\ldots,v\}$ for $i = 1,2,\ldots,v$.

Next, we substitute

$$x_i = p_{b_i}, \ \breve{x}_i = \breve{p}_{b_i}, \ \hat{x}_i = \hat{p}_{b_i} \text{ for } i = 1,2,\ldots,v, \tag{11}$$

and we minimize with respect to x_i, $i = 1, 2,\ldots, v$, the linear form (8) that after this transformation takes the form

$$\boldsymbol{C}(r) \cong \sum_{b=1}^{v} x_i [\boldsymbol{C}(r)]^{(b_i)}, \tag{12}$$

for a fixed $r \in \{1,2,\ldots,z\}$ with

$$\breve{x}_i \le x_i \le \hat{x}_i,\ i = 1,\ 2,\ldots,\ v,\ \sum_{i=1}^{v} x_i = 1, \tag{13}$$

where

$$[C(r)]^{(b_1)},\ [C(r)]^{(b_1)} \ge 0,\ i = 1, 2,\ldots, v,$$

are fixed mean values of the system conditional operation costs in the safety state subset $\{r,r+1,\ldots,z\}$ arranged in non-decreasing order and

$$\breve{x}_i,\ \ 0 \le \breve{x}_i \le 1 \text{ and } \hat{x}_i,\ \ 0 \le \hat{x}_i \le 1,\ \ \breve{x}_i \le \hat{x}_i,$$

$$i = 1,2,\ldots,v, \tag{14}$$

are lower and upper bounds of the unknown probabilities x_i, $i = 1,2,\ldots,v$, respectively.
To find the optimal values of x_i, $i = 1,2,\ldots,v$, we define

$$\breve{x} = \sum_{i=1}^{v} \breve{x}_i,\ \ \hat{y} = 1 - \breve{x} \tag{15}$$

and

$$\breve{x}_i^{\,0} = 0,\ \ \hat{x}_i^{\,0} = 0 \text{ and } \breve{x}^I = \sum_{i=1}^{I} \breve{x}_i,\ \ \hat{x}^I = \sum_{i=1}^{I} \hat{x}_i \tag{16}$$

for $I = 1,2,\ldots,v$,

Next, we find the largest value $I \in \{0,1,\ldots,v\}$ such that

$$\hat{x}^I - \breve{x}^I < \hat{y} \tag{17}$$

and we fix the optimal solution that minimize (12) in the following way:
i) if $I = 0$, the optimal solution is

$$\dot{x}_1 = \hat{y} + \breve{x}_1 \text{ and } \dot{x}_i = \breve{x}_i \text{ for } i = 2,3,\ldots,v, \tag{18}$$

ii) if $0 < I < v$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1, 2,\ldots, I,$$

$$\dot{x}_{I+1} = \hat{y} - \hat{x}^I + \breve{x}^I + \breve{x}_{I+1}$$

$$\text{and } \dot{x}_i = \breve{x}_i \text{ for } i = I+2, I+3,\ldots,v; \tag{19}$$

iii) if $I = v$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1,2,\ldots,v. \tag{20}$$

Finally, after making the inverse to (11) substitution, we get the optimal limit transient probabilities

$$\dot{p}_{b_i} = \dot{x}_i \text{ for } i = 1,2,\ldots,v, \tag{21}$$

that minimize the mean value of the system total unconditional operation costs in the safety state subset $\{r,r+1,\ldots,z\}$, giving its minimum value in the following form

$$\dot{\boldsymbol{C}}(r) = \sum_{b=1}^{v} \dot{p}_b [\boldsymbol{C}(r)]^{(b)} \tag{22}$$

for a fixed $r \in \{1,2,\ldots,z\}$.
From the expression (22) for the minimum mean value $\dot{\boldsymbol{C}}(r)$ of the system unconditional operation cost in the safety state subset $\{r,r+1,\ldots,z\}$, replacing in it the critical safety state r by the safety state u, $u = 1,2,\ldots,z$, we obtain the corresponding optimal solutions for the mean values of the system unconditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$ of the form

$$\dot{\boldsymbol{C}}(u) = \sum_{b=1}^{v} \dot{p}_b [\boldsymbol{C}(u)]^{(b)},\ u = 1,2,\ldots,z. \tag{23}$$

According to (3)–(4), the mean value of the system optimal total operation cost can by expressed by

$$\dot{\boldsymbol{C}}(\cdot) = [\dot{\boldsymbol{C}}(1),\ldots,\dot{\boldsymbol{C}}(z)], \tag{24}$$

with coordinates given by the linear equations (22)

$$\dot{\boldsymbol{C}}(u) \cong \sum_{b=1}^{v} \dot{p}_b [\boldsymbol{C}(u)]^{(b)},\ u = 1,2,\ldots,z. \tag{25}$$

for the mean values of the system optimal total unconditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, where $[\boldsymbol{C}(u)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, are the mean values of the system total conditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the particular system operation states z_b, $b = 1,2,\ldots,v$, and $\dot{p}_b$, $b = 1,2,\ldots,v$, are the system operation process optimal limit transient probabilities at these operation states given by (21).

The expressions for the optimal mean values of the system total operation costs in the particular safety states are

$$\bar{\dot{C}}(u) = \dot{C}(u) - \dot{C}(u+1), u = 1,2,\ldots, z-1,$$

$$\bar{\dot{C}}(z) = \dot{C}(z), \quad (26)$$

where $\dot{C}(u)$, $u = 1,2,\ldots,z$, are the optimal mean values of the system total unconditional operation costs in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, given by (23).

3. System safety

3.1. System safety model

We assume that the system is operating at v, $v > 1$, operation states z_b, $b = 1,2,\ldots,v$, that have influence on the system functional structure and on the system safety. Applying semi-Markov model of the system operation process $Z(t)$, $t \geq 0$, it is possible to find this process two basic characteristics (Grabski, 2015; Kołowrocki & Magryta, 2020a; Kołowrocki & Soszyńska-Budny, 2011/2015):

- the vector of limit values

$$p_b = \lim_{t\to\infty} p_b(t) = b = 1,2,\ldots,v, \quad (27)$$

of transient probabilities

$$p_b(t) = P(Z(t) = z_b),\ t \geq 0, b = 1,2,\ldots,v, \quad (18)$$

of the system operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,v$,

- the vector $[\hat{M}_b]_{1\times v}$ of the mean values

$$\hat{M}_b = E[\hat{\theta}_b] \cong p_b\theta,\ b = 1,2,\ldots,v, \quad (29)$$

of the total sojourn times $\hat{\theta}_b$, $b = 1,2,\ldots,v$, of the system operation process $Z(t)$, $t \geq 0$, at the particular operation states z_b, $b = 1, 2,\ldots,v$, during the fixed system operation time θ, $\theta > 0$, where p_b, $b = 1,2,\ldots,v$, are defined by (27)–(28).

Considering the safety function of the system impacted by operation process

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2),\ldots, \boldsymbol{S}(t,z)],\ t \geq 0, \quad (30)$$

coordinate given by (Kołowrocki & Soszyńska-Budny, 2011/2015)

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{v} p_b[\boldsymbol{S}(t,u)]^{(b)},\ t \geq 0, \quad (31)$$

$u = 1,2,\ldots,z,$

where p_b, $b = 1,2,\ldots,v$, are the limit transient probabilities of the system operation process at the operation states z_b, $b = 1,2,\ldots,v$, and

$$[\boldsymbol{S}(t,u)]^{(b)} = P([T(u)]^{(b)} > t),\ t \geq 0,$$

$u = 1,2,\ldots,z$, $b = 1,2,\ldots,v$,

at these operation states are the conditional safety functions of the system and $([T(u)]^{(b)}$, are the system conditional lifetimes in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at the operation states z_b, $b = 1,2,\ldots,v$, it is natural to assume that the system operation process has a significant influence on the system safety.

From the expression (31), it follows that the mean values of the system unconditional lifetimes in the safety state subsets $\{u,u+1,\ldots,z\}$, are of the form

$$\boldsymbol{\mu}(u) = \sum_{b=1}^{v} p_b[\mu(u)]^{(b)} \text{ for } u = 1,2,\ldots,z. \quad (32)$$

The values of the variances of the system unconditional lifetimes in the system safety state subsets are

$$[\boldsymbol{\sigma}(u)]^2 = 2\int_0^\infty t\boldsymbol{S}(t,u)dt - [\boldsymbol{\mu}(u)]^2, u = 1,2,\ldots,z, \quad (33)$$

where $\boldsymbol{\mu}(u)$ is given by (6) and $\boldsymbol{S}(t,u)$ is given by (31).

The expressions for the mean values of the system unconditional lifetimes in the particular safety states are

$$\bar{\boldsymbol{\mu}}(u) = \boldsymbol{\mu}(u) - \boldsymbol{\mu}(u+1),\ u = 1,2,\ldots,z-1,$$

$$\bar{\boldsymbol{\mu}}(z) = \boldsymbol{\mu}(z). \tag{34}$$

The system risk function and the moment when the risk exceeds a permitted level δ, respectively are given by (Kołowrocki & Soszyńska-Budny, 2011/2015):

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,r),\ t \geq 0, \tag{35}$$

and

$$\tau = \boldsymbol{r}^{-1}(\delta), \tag{36}$$

where $\boldsymbol{S}(t,r)$ is given by (31) for $u = r$ and $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r}(t)$.

The mean values of the system intensities of ageing (departure from the safety state subset $\{u,u+1,\ldots,z\}$), are defined by

$$\boldsymbol{\lambda}(u) = \frac{1}{\boldsymbol{\mu}(u)},\ u = 1,2,\ldots,z. \tag{37}$$

Considering the values of the system without operation impact intensities of ageing $\boldsymbol{\lambda}^0(u)$, defined in (Kołowrocki & Magryta-Mut, 2020c; Kołowrocki & Soszyńska-Budny, 2018b), the coefficients of the operation process impact on the system intensities of ageing are given by

$$\boldsymbol{\rho}(u) = \frac{\boldsymbol{\lambda}(u)}{\boldsymbol{\lambda}^{o}(u)},\ u = 1,2,\ldots,z. \tag{38}$$

Finally, the system resilience indicators, i.e. the coefficients of the system resilience to operation process impact, are defined by

$$\boldsymbol{RI}(u) = \frac{1}{\boldsymbol{\rho}(u)},\ u = 1,2,\ldots,z. \tag{39}$$

3.2. System safety optimization

Considering the safety function of the system impacted by operation process $\boldsymbol{S}(t,\cdot)$, $t \geq 0$, coordinate given by (31), it is natural to assume that the system operation process has a significant influence on the system safety. This influence is also clearly expressed in the equation (32) for the mean values of the system unconditional lifetimes in the safety state subsets. From the linear equation (32), we can see that the mean value of the system unconditional lifetime $\boldsymbol{\mu}(u)$, $u = 1,2,\ldots,z$, is determined by the limit values of transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the operation states z_b, $b = 1,2,\ldots,v$, and the mean values $[\boldsymbol{\mu}(u)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, of the system conditional lifetimes in the safety state subsets $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at these operation states. Therefore, the system lifetime optimization approach based on the linear programming can be proposed (Klabjan & Adelman, 2006). Namely, we may look for the corresponding optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, of the transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the operation states to maximize the mean value $\boldsymbol{\mu}(u)$ of the unconditional system lifetime in the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, under the assumption that the mean values $[\boldsymbol{\mu}(u)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, of the system conditional lifetimes in the safety state subset at the particular operation states are fixed. As a special case of the above formulated system lifetime optimization problem, if r, $r \in \{1,2,\ldots,z\}$ is a system critical safety state, we may look for the optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, of the transient probabilities p_b, $b = 1,2,\ldots,v$, of the system operation process at the system operation states to maximize the mean value $\boldsymbol{\mu}(r)$ of the unconditional system lifetime in the safety state subset $\{r,r+1,\ldots,z\}$, $r = 1,2,\ldots,z$, under the assumption that the mean values $[\boldsymbol{\mu}(r)]^{(b)}$, $b = 1,2,\ldots,v$, $u = 1,2,\ldots,z$, of the system conditional lifetimes in this safety state subset not worse than the critical safety state at the particular operation states are fixed. More exactly, we formulate the optimization problem as a linear programming model with the objective function of the following form

$$\boldsymbol{\mu}(r) = \sum_{b=1}^{v} p_b [\mu(r)]^{(b)}, \tag{40}$$

for a fixed $r \in \{1,2,\ldots,z\}$ and with the following bound constraints

$$\breve{p}_b \leq p_b \leq \hat{p}_b,\ b = 1,2,\ldots,v, \tag{41}$$

$$\sum_{b=1}^{v} p_b = 1, \tag{42}$$

where

$$[\mu(r)]^{(b)},\ [\mu(r)]^{(b)} \geq 0,\ b = 1,2,\ldots,v, \tag{43}$$

are fixed mean values of the system conditional lifetimes in the safety state subset $\{r,r+1,\ldots,z\}$ and

$$\breve{p}_b,\ 0 \leq \breve{p}_b \leq 1 \text{ and } \hat{p}_b,\ 0 \leq \hat{p}_b \leq 1,\ \breve{p}_b \leq \hat{p}_b,$$

$$b = 1,2,\ldots,v, \tag{44}$$

are lower and upper bounds of the unknown transient probabilities at the particular operation states p_b, $b = 1,2,\ldots,v$, respectively.

Now, we can obtain the optimal solution of the formulated by (40)–(44) the optimization problem, i.e. we can find the optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,\ldots,v$, that maximize the objective function given by (40). The maximizing procedure is described in (Kołowrocki & Magryta, 2020b, Magryta-Mut, 2020).

Finally, after applying this procedure, we can get the maximum value of the system total mean lifetime in the safety state subset $\{r,r+1,\ldots,z\}$ defined by the linear form (40), in the following form

$$\dot{\mu}(r) = \sum_{b=1}^{v} \dot{p}_b [\mu(r)]^{(b)} \tag{45}$$

for a fixed $r \in \{1,2,\ldots,z\}$.

Further, by replacing the limit transient probabilities p_b, $b = 1,2,\ldots,v$, existing in the formulae (30) and (31) by their optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, we get the optimal form of the system safety and the expressions for all remaining safety indicators considered in Section 3.1.

4. Port oil terminal operation cost

4.1. Terminal description

The port oil terminal placed at the Baltic seaside is designated for receiving oil products from ships, storage and sending them by carriages or trucks to inland. The terminal can operate in reverse way as well. The terminal is described in detail in (Kołowrocki & Soszyńska-Budny, 2019a). The considered terminal is composed of three parts A, B and C, linked by the piping transportation system with the pier. The approximate length of the port oil piping transportation system is equal to around 25 km.

The main technical assets (components) of the port oil terminal critical infrastructure are:

A_1 – port oil piping transportation system,
A_2 – internal pipeline technological system,
A_3 – supporting pump station,
A_4 – internal pump system,
A_5 – port oil tanker shipment terminal,
A_6 – loading railway carriage station,
A_7 – loading road carriage station,
A_8 – unloading railway carriage station,
A_9 – oil storage reservoir system.

The asset A_1, the port oil piping transportation system operating at the port oil terminal critical infrastructure consists of three subsystems:

- the subsystem S_1 composed of two pipelines, each composed of 176 pipe segments and 2 valves,
- the subsystem S_2 composed of two pipelines, each composed of 717 pipe segments and 2 valves,
- the subsystem S_3 composed of three pipelines, each composed of 360 pipe segments and 2 valves.

Its operation is the main activity of the port oil terminal involving the remaining assets $A_2 - A_9$.

The port oil transportation system is a series system composed of two series-parallel subsystems S_1, S_2, each containing two pipelines (assets) and one series-“2 out of 3” subsystem S_3 containing 3 pipelines (assets).

The subsystems S_1, S_2 and S_3 are forming a general series port oil transportation system safety structure presented in Figure 1.

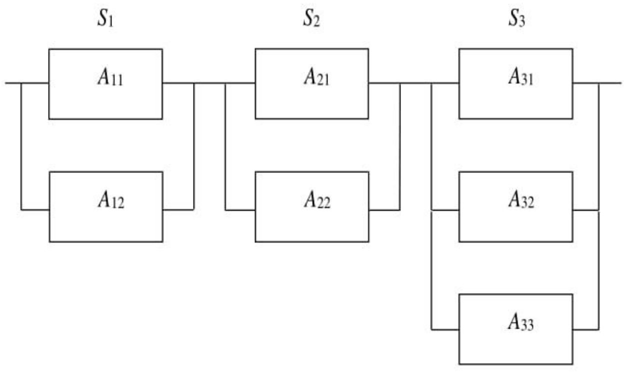

Figure 1. The port oil piping transportation system safety structure.

4.2. Operation process

We consider the port oil terminal critical infrastructure impacted by its operation process.
On the basis of the statistical data and expert opinions, it is possible to fix and to evaluate the following unknown basic parameters of the oil terminal critical infrastructure operation process:
the number of operation process states $v = 7$ and the operation process states:

- the operation state z_1 – transport of one kind of medium from the terminal part B to part C using two out of three pipelines of the subsystem S_3 of the asset A_1 and assets A_2, A_4, A_6, A_7, A_9;
- the operation state z_2 – transport of one kind of medium from the terminal part C to part B using one out of three pipelines of the subsystem S_3 of the asset A_1 and assets A_2, A_4, A_8, A_9;
- the operation state z_3 – transport of one kind of medium from the terminal part B through part A to pier using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 of the asset A_1 and assets A_2, A_4, A_5, A_9;
- the operation state z_4 – transport of one kind of medium from the pier through parts A and B to part C using one out of two pipelines of the subsystem S_1, one out of two pipelines in subsystem S_2 and two out of three pipelines of the subsystem S_3 of the asset A_1 and assets A_2, A_3, A_4, A_5, A_6, A_7, A_9;
- the operation state z_5 – transport of one kind of medium from the pier through part A to B using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 of the asset A_1 and assets A_2, A_3, A_4, A_5, A_9;
- the operation state z_6 – transport of one kind of medium from the terminal part B to C using two out of three pipelines of the subsystem S_3, and simultaneously transport one kind of medium from the pier through part A to B using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2 of the asset A_1 and assets A_2, A_3, A_4, A_5, A_6, A_7, A_9;
- the operation state z_7 – transport of one kind of medium from the terminal part B to C using one out of three pipelines of the subsystem S_3, and simultaneously transport second kind of medium from the terminal part C to B using one out of three pipelines of the subsystem S_3 of the asset A_1 and assets A_2, A_4, A_6, A_7, A_8, A_9.

To identify the unknown parameters of the port oil piping transportation system operation process the suitable statistical data coming from its real realizations should be collected. On the basic of this data (GMU, 2018), it is possible to estimate these parameters and to fix the port oil terminal characteristics (Kołowrocki & Soszyńska-Budny, 2011/2015):

- the limit values of transient probabilities of the operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots, 7$:

$$p_1 = 0.395,\ p_2 = 0.060,\ p_3 = 0.003,\ p_4 = 0.002,$$
$$p_5 = 0.20,\ p_6 = 0.058,\ p_7 = 0.282; \qquad (46)$$

- the expected values of the total sojourn times $\hat{\theta}_b$, $b = 1,2,\ldots,7$, of the system operation process at the particular operation states z_b, $b = 1,2,\ldots,7$, during the fixed operation time $\theta = 1$ year = 365 days:

$$M_1 = E[\hat{\theta}_1] = 0.395 \text{ year} = 144.175 \text{ days},$$
$$M_2 = E[\hat{\theta}_2] = 0.060 \text{ year} = 21.9 \text{ days},$$
$$M_3 = E[\hat{\theta}_3] = 0.003 \text{ year} = 1.095 \text{ day},$$
$$M_4 = E[\hat{\theta}_4] = 0.002 \text{ year} = 0.73 \text{ day},$$
$$M_5 = E[\hat{\theta}_5] = 0.20 \text{ year} = 73 \text{ days},$$
$$M_6 = E[\hat{\theta}_6] = 0.058 \text{ year} = 21.17 \text{ days},$$
$$M_7 = E[\hat{\theta}_7] = 0.282 \text{ year} = 102.93 \text{ days}. \quad (47)$$

4.3. Operation cost

The number of components in a subsystems S_1, S_2, S_3 and their use at particularly operation states, the asset A_1 conditional instantaneous operation costs $[\boldsymbol{C}(t,u)]^{(b)}$, $u = 1, 2$, $b = 1,2,\ldots,7$, in the safety state subsets $\{1,2\}$, $\{2\}$ for $t \geq 0$, $b = 1,2,\ldots,7$, are:

$$[\boldsymbol{C}_1(t,1)]^{(1)} = [\boldsymbol{C}_1(t,2)]^{(1)} = 1086 \cdot 9.6 = 10425.6 \text{ PLN},$$
$$[\boldsymbol{C}_1(t,1)]^{(2)} = [\boldsymbol{C}_1(t,2)]^{(2)} = 1086 \cdot 9.6 = 10425.6 \text{ PLN},$$
$$[\boldsymbol{C}_1(t,1)]^{(3)} = [\boldsymbol{C}_1(t,2)]^{(3)} = 1794 \cdot 9.6 = 17222.4 \text{ PLN},$$
$$[\boldsymbol{C}_1(t,1)]^{(4)} = [\boldsymbol{C}_1(t,2)]^{(4)} = 2880 \cdot 9.6 = 27648 \text{ PLN},$$

$[\boldsymbol{C}_1(t,1)]^{(5)} = [\boldsymbol{C}_1(t,2)]^{(5)} = 1794 \cdot 9.6$
$= 17222.4$ PLN,
$[\boldsymbol{C}_1(t,1)]^{(6)} = [\boldsymbol{C}_1(t,2)]^{(6)} = 2880 \cdot 9.6$
$= 27648$ PLN,
$[\boldsymbol{C}_1(t,1)]^{(7)} = [\boldsymbol{C}_1(t,2)]^{(7)} = 1086 \cdot 9.6$
$= 10425.$ 6PLN. (48)

The mean values $[\boldsymbol{\mu}(u)]^{(b)}$, $u = 1,2$, of the terminal conditional lifetimes $[T(u)]^{(b)}$, $u = 1,2$, in the safety state subset {1,2}, {2} at the operation state z_b, $b = 1,2,\ldots,7$, determined in Section 3, respectively (expressed in years) are:

$[\boldsymbol{\mu}(1)]^{(1)} \cong 8.08342$,
$[\boldsymbol{\mu}(1)]^{(2)} \cong 8.16593$,
$[\boldsymbol{\mu}(1)]^{(3)} = [\boldsymbol{\mu}(1)]^{(5)} \cong 7.60179$,
$[\boldsymbol{\mu}(1)]^{(4)} = [\boldsymbol{\mu}(1)]^{(6)} \cong 6.80805$,
$[\boldsymbol{\mu}(1)]^{(7)} \cong 8.00256$,

$[\boldsymbol{\mu}(2)]^{(1)} = 5.15695$,
$[\boldsymbol{\mu}(2)]^{(2)} = 5.21069$,
$[\boldsymbol{\mu}(2)]^{(3)} = [\boldsymbol{\mu}(2)]^{(5)} \cong 4.85232$,
$[\boldsymbol{\mu}(2)]^{(4)} \cong 4.34292$,
$[\boldsymbol{\mu}(2)]^{(6)} \cong 4.3429$,
$[\boldsymbol{\mu}(2)]^{(7)} \cong 5.10431$. (49)

Applying the formula (5) to (48) and (49) we get the approximate mean values $[\boldsymbol{C}_1(1)]^{(b)}$, $b = 1,2,\ldots,7$, of the total costs of the entire port oil terminal at the particular operation states given by:

$[\boldsymbol{C}_1(1)]^{(1)} = 8.08342 \cdot 10425.6$
$\cong 84274.50355$ PLN,
$[\boldsymbol{C}_1(1)]^{(2)} = 8.16593 \cdot 10425.6$
$\cong 85134.71981$ PLN,
$[\boldsymbol{C}_1(1)]^{(3)} = 7.60179 \cdot 17222.4$
$\cong 130921.06810$ PLN,
$[\boldsymbol{C}_1(1)]^{(4)} = 6.80805 \cdot 27648$
$\cong 188228.9664$ PLN,
$[\boldsymbol{C}_1(1)]^{(5)} = 7.60179 \cdot 17222.4$
$\cong 130921.06810$ PLN,
$[\boldsymbol{C}_1(1)]^{(6)} = 6.80805 \cdot 27648$
$\cong 188228.96640$ PLN,
$[\boldsymbol{C}_1(1)]^{(7)} = 8.00256 \cdot 10425.6$
$\cong 83431.48954$ PLN, (50)

in the safety state subset {1,2} and

$[\boldsymbol{C}_1(2)]^{(1)} = 5.15695 \cdot 10425.6$
$\cong 53764.29792$ PLN,
$[\boldsymbol{C}_1(2)]^{(2)} = 5.21069 \cdot 10425.6$
$\cong 54324.56966$ PLN,
$[\boldsymbol{C}_1(2)]^{(3)} = 4.85232 \cdot 17222.4$
$\cong 83568.59597$ PLN,
$[\boldsymbol{C}_1(2)]^{(4)} = 4.34292 \cdot 27648$
$\cong 120073.05216$ PLN,
$[\boldsymbol{C}_1(2)]^{(5)} = 4.85232 \cdot 17222.4$
$\cong 83568.59597$ PLN,
$[\boldsymbol{C}_1(2)]^{(6)} = 4.3429 \cdot 27648$
$\cong 120072.49920$ PLN,
$[\boldsymbol{C}_1(2)]^{(7)} = 5.10431 \cdot 10425.6$
$\cong 53215.49434$ PLN, (51)

in the safety state subset {2}.
The corresponding mean values of the total conditional operation costs for the remaining assets $A_2 - A_9$, we assume arbitrarily (we do not data at the moment) equal to 10000 PLN, in all operation states if they are used and equal to 0 PLN if they are not used. Under this assumption, considering the procedure of using asset $A_2 - A_9$ at particular operation state and the total cost of asset A_1 given in (50) and (51), we fix the total costs of the entire port oil terminal at the particular operation states given by:

$[\boldsymbol{C}_1(1)]^{(1)} = 84274.50355 + 50000$
$= 134274.50355$ PLN,
$[\boldsymbol{C}_1(1)]^{(2)} = 85134.71981 + 40000$
$= 125134.71981$ PLN,
$[\boldsymbol{C}_1(1)]^{(3)} = 130921.06810 + 40000$
$= 170921.06810$ PLN,
$[\boldsymbol{C}_1(1)]^{(4)} = 188228.9664 + 70000$
$= 258228.9664$ PLN,
$[\boldsymbol{C}_1(1)]^{(5)} = 130921.06810 + 50000$
$= 180921.06810$ PLN,
$[\boldsymbol{C}_1(1)]^{(6)} = 188228.96640 + 70000$
$= 258228.96640$ PLN,
$[\boldsymbol{C}_1(1)]^{(7)} = 83431.48954 + 60000$
$= 143431.48954$ PLN, (52)

in the safety state subset {1,2} and

$[\boldsymbol{C}_1(2)]^{(1)} = 53764.29792 + 50000$
$= 103764.29792$ PLN,
$[\boldsymbol{C}_1(2)]^{(2)} = 54324.56966 + 40000$
$= 94324.56966$ PLN,
$[\boldsymbol{C}_1(2)]^{(3)} = 83568.59597 + 40000$
$= 123568.59597$ PLN,
$[\boldsymbol{C}_1(2)]^{(4)} = 120073.05216 + 70000$
$= 190073.05216$ PLN,
$[\boldsymbol{C}_1(2)]^{(5)} = 83568.59597 + 50000$

$= 133568.59597 \text{PLN},$
$[\boldsymbol{C}_1(2)]^{(6)} = 120072.49920 + 70000$
$= 190072.49920 \text{ PLN},$
$[\boldsymbol{C}_1(2)]^{(7)} = 53215.49434 + 60000$
$= 113215.49434 \text{ PLN}, \quad (53)$

in the safety state subset {2}.

Considering the values of $[\boldsymbol{C}(u)]^{(b)}$, $u = 1,2$, $b = 1,2,\ldots,7$, from (50)–(51) and the values of transient probabilities p_b, $b = 1,2,\ldots,7$, the port oil terminal total unconditional operation cost, according to (4), is given by

$$\boldsymbol{C}(1) \cong p_1[\boldsymbol{C}(1)]^{(1)} + p_2[\boldsymbol{C}(1)]^{(2)} + p_3[\boldsymbol{C}(1)]^{(3)} + p_4[\boldsymbol{C}(1)]^{(4)} + p_5[\boldsymbol{C}(1)]^{(5)} + p_6[\boldsymbol{C}(1)]^{(6)} + p_7[\boldsymbol{C}(1)]^{(7)}$$
$$\cong 0.395 \cdot 134274.50355 + 0.06 \cdot 125134.71981 + 0.003 \cdot 170921.06810 + 0.02 \cdot 258228.9664 + 0.2 \cdot 180921.06810 + 0.058 \cdot 258228.96640 + 0.282 \cdot 143431.48954$$
$$\cong 153184.90695, \quad (54)$$

in the safety state subset {1,2} and

$$\boldsymbol{C}(2) \cong p_1[\boldsymbol{C}(2)]^{(1)} + p_2[\boldsymbol{C}(2)]^{(2)} + p_3[\boldsymbol{C}(2)]^{(3)} + p_4[\boldsymbol{C}(2)]^{(4)} + p_5[\boldsymbol{C}(2)]^{(5)} + p_6[\boldsymbol{C}(2)]^{(6)} + p_7[\boldsymbol{C}(2)]^{(7)}$$
$$\cong 0.395 \cdot 103764.29792 + 0.06 \cdot 94324.56966 + 0.003 \cdot 123568.59597 + 0.002 \cdot 190073.05216 + 0.2 \cdot 133568.59597 + 0.058 \cdot 190072.49920 + 0.282 \cdot 113215.49434$$
$$\cong 117061.91730, \quad (55)$$

in the safety state subset {2}.

4.4. Cost optimization

Assuming the critical safety state $r = 1$ and considering (52) to find the minimum value of this cost, we define the objective function given by (8), in the following form

$$\boldsymbol{C}(1) \cong p_1[\boldsymbol{C}(1)]^{(1)} + p_2[\boldsymbol{C}(1)]^{(2)} + p_3[\boldsymbol{C}(1)]^{(3)} + p_4[\boldsymbol{C}(1)]^{(4)} + p_5[\boldsymbol{C}(1)]^{(5)} + p_6[\boldsymbol{C}(1)]^{(6)} + p_7[\boldsymbol{C}(1)]^{(7)}$$
$$\cong p_1 \cdot 134274.50355 + p_2 \cdot 125134.71981 + p_3 \cdot 170921.06810 + p_4 \cdot 258228.9664 + p_5 \cdot 180921.06810 + p_6 \cdot 258228.96640 + p_7 \cdot 143431.48954. \quad (56)$$

The lower $\breve{p}_b$, and upper $\hat{p}_b$, bounds of the unknown optimal values of transient probabilities p_b, $b = 1,2,\ldots,7$, respectively are (Kołowrocki & Soszyńska-Budny, 2011/2015):

$$\breve{p}_1 = 0.31,\ \breve{p}_2 = 0.04,\ \breve{p}_3 = 0.002,\ \breve{p}_4 = 0.001,\ \breve{p}_5 = 0.15,\ \breve{p}_6 = 0.04,\ \breve{p}_7 = 0.25,$$

$$\hat{p}_1 = 0.46,\ \hat{p}_2 = 0.08,\ \hat{p}_3 = 0.006,\ \hat{p}_4 = 0.004,\ \hat{p}_5 = 0.26,\ \hat{p}_6 = 0.08,\ \hat{p}_7 = 0.40. \quad (57)$$

Therefore, according to (9)–(10), we assume the following bound constraints

$$0.31 \le p_1 \le 0.46,\ 0.04 \le p_2 \le 0.08,\ 0.002 \le p_3 \le 0.006,\ 0.001 \le p_4 \le 0.004,\ 0.15 \le p_5 \le 0.26,\ 0.04 \le p_6 \le 0.08,\ 0.25 \le p_7 \le 0.40,\ \sum_{b=1}^{7} p_b = 1. \quad (58)$$

Now, before we find optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,\ldots,7$, that minimize the objective function (56), we arrange the mean values of the port oil terminal conditional operation costs $[C(1)]^{(b)}$, $b = 1,2,\ldots,7$, determined by (52), in non-decreasing order

$$125134.71981 \le 134274.50355 \le 143431.48954 \le 170921.06810 \le 180921.06810 \le 258228.96640 \le 2588228.9664,$$

i.e.

$$[\boldsymbol{C}(1)]^{(2)} \le [\boldsymbol{C}(1)]^{(1)} \le [\boldsymbol{C}(1)]^{(7)} \le [\boldsymbol{C}(1)]^{(3)} \le [\boldsymbol{C}(1)]^{(5)} \le [\boldsymbol{C}(1)]^{(4)} \le [\boldsymbol{C}(1)]^{(6)}, \quad (59)$$

Further, according to (11), we substitute

$$x_1 = p_2,\ x_2 = p_1,\ x_3 = p_7,\ x_4 = p_3,\ x_5 = p_5,\ x_6 = p_4,\ x_7 = p_6, \quad (60)$$

and

$$\breve{x}_1 = \breve{p}_2 = 0.04,\ \breve{x}_2 = \breve{p}_1 = 0.31,$$
$$\breve{x}_3 = \breve{p}_7 = 0.25,\ \breve{x}_4 = \breve{p}_3 = 0.002,$$
$$\breve{x}_5 = \breve{p}_5 = 0.15,\ \breve{x}_6 = \breve{p}_4 = 0.001,$$
$$\breve{x}_7 = \breve{p}_6 = 0.04,$$

$$\hat{x}_1 = \hat{p}_2 = 0.08,\ \hat{x}_2 = \hat{p}_1 = 0.46,$$
$$\hat{x}_3 = \hat{p}_7 = 0.40,\ \hat{x}_4 = \hat{p}_3 = 0.006,$$
$$\hat{x}_5 = \hat{p}_5 = 0.26,\ \hat{x}_6 = \hat{p}_4 = 0.004,$$
$$\hat{x}_7 = \hat{p}_6 = 0.08, \quad (61)$$

and we minimize with respect to x_i, $i = 1,2,\ldots,7$, the linear form (56) that according to (11)–(13) and (59)–(60) takes the form

$$\boldsymbol{C}(1) = x_1 \cdot 125134.71981 + x_2 \cdot 134274.50355 + x_3 \cdot 143431.48954 + x_4 \cdot 170921.06810 + x_5 \cdot 180921.06810 + x_6 \cdot 258228.9664 + x_7 \cdot 258228.9664, \quad (62)$$

with the following bound constraints

$$0.04 \le x_1 \le 0.08,\ 0.31 \le x_2 \le 0.46,$$
$$0.25 \le x_3 \le 0.40,\ 0.002 \le x_4 \le 0.006,$$
$$0.15 \le x_5 \le 0.26,\ 0.001 \le x_6 \le 0.004,$$
$$0.04 \le x_7 \le 0.08, \sum_{i=1}^{7} x_i = 1. \quad (63)$$

According to (17), we calculate

$$\breve{x} = \sum_{i=1}^{7} \breve{x}_i = 0.793,$$
$$\hat{y} = 1 - \breve{x} = 1 - 0.793 = 0.207 \quad (64)$$

and according to (16), we find

$$\breve{x}^0 = 0,\ \hat{x}^0 = 0,\ \hat{x}^0 - \breve{x}^0 = 0,$$
$$\breve{x}^1 = 0.04,\ \hat{x}^1 = 0.08,\ \hat{x}^1 - \breve{x}^1 = 0.04,$$
$$\breve{x}^2 = 0.35,\ \hat{x}^2 = 0.54,\ \hat{x}^2 - \breve{x}^2 = 0.19,$$
$$\breve{x}^3 = 0.60,\ \hat{x}^3 = 0.94,\ \hat{x}^3 - \breve{x}^3 = 0.34,$$
$$\breve{x}^4 = 0.602,\ \hat{x}^4 = 0.946,\ \hat{x}^4 - \breve{x}^4 = 0.344,$$
$$\breve{x}^5 = 0.752,\ \hat{x}^5 = 1.206,\ \hat{x}^5 - \breve{x}^5 = 0.454,$$
$$\breve{x}^6 = 0.753,\ \hat{x}^6 = 1.21,\ \hat{x}^6 - \breve{x}^6 = 0.457,$$
$$\breve{x}^7 = 0.793,\ \hat{x}^7 = 1.91,\ \hat{x}^7 - \breve{x}^7 = 0.497. \quad (65)$$

From the above, since the expression (17) takes the form

$$\hat{x}^I - \breve{x}^I < 0.207, \quad (66)$$

then it follows that the largest value $I \in \{0,1,\ldots,7\}$ such that this inequality holds is $I = 2$. Therefore, we fix the optimal solution that minimize linear function (56) according to the rule (19). Namely, we get

$$\dot{x}_1 = \hat{x}_1 = 0.08,\ \dot{x}_2 = \hat{x}_2 = 0.46,$$
$$\dot{x}_3 = \hat{y} - \hat{x}^2 + \breve{x}^2 + \breve{x}_3 = 0.207 - 0.54 + 0.35 + 0.25 = 0.267,$$
$$\dot{x}_4 = \breve{x}_4 = 0.002,\ \dot{x}_5 = \breve{x}_5 = 0.15,\ \dot{x}_6 = \breve{x}_6 = 0.001,$$
$$\dot{x}_7 = \breve{x}_7 = 0.04. \quad (67)$$

Finally, after making the substitution inverse to (11), we get the optimal transient probabilities

$$\dot{p}_2 = \dot{x}_1 = 0.08,\ \dot{p}_1 = \dot{x}_2 = 0.46,\ \dot{p}_7 = \dot{x}_3 = 0.267,$$
$$\dot{p}_3 = \dot{x}_4 = 0.002,\ \dot{p}_5 = \dot{x}_5 = 0.15,$$
$$\dot{p}_4 = \dot{x}_6 = 0.001,\ \dot{p}_6 = \dot{x}_7 = 0.04, \quad (68)$$

that minimize the mean value of the port oil terminal total operation cost $\boldsymbol{C}(1)$ expressed by the linear form (56) and according to (22) and (68), its optimal value in the safety state subset {1,2} is

$$\dot{\boldsymbol{C}}(1) \cong 0.46 \cdot 134274.50355 + 0.08 \cdot 125134.71981 + 0.002 \cdot 170921.06810 + 0.001 \cdot 258228.9664 + 0.15 \cdot 180921.06810 + 0.04 \cdot 258228.96640 + 0.267 \cdot 143431.48954 \cong 148140.64690, \quad (69)$$

and further, the optimal mean value of the port oil terminal operation total cost in the safety state subset {2} is

$$\dot{\boldsymbol{C}}(2) \cong 0.46 \cdot 103764.29792 + 0.08 \cdot 94324.56966 + 0.002 \cdot 123568.59597 + 0.001 \cdot 190073.05216 + 0.15 \cdot 133568.59597$$

$$+ 0.04 \cdot 190072.49920 + 0.267 \cdot 113215.49434 \cong 113581.47921. \quad (70)$$

Hence, and according to (26), the optimal values of the port oil terminal total operation costs in the particular safety states 1 and 2, respectively are:

$$\dot{\tilde{C}}(1) \cong 148140.64690 - 113581.47921 = 34559.16769,$$
$$\dot{\tilde{C}}(2) \cong 113581.47921. \quad (71)$$

The analyzed costs after optimization are lower than before it, what is respectively expressed by comparison of the results before the optimization given in (54)–(55) and the results after the optimization given in (69)–(70).

5. Port oil terminal safety

5.1. Safety characteristics

After considering the comments and opinions coming from experts, taking into account the effectiveness and safety aspects of the operation of the port oil terminal, we distinguish the following three safety states $(z = 2)$ of the system and its components:

- a safety state 2 – the components and the port oil terminal are fully safe,
- a safety state 1 – the components and the port oil terminal are less safe and more dangerous because of the possibility of environment pollution,
- a safety state 0 – the components and the port oil terminal are destroyed.

and we assume that:

- there are possible the transitions between the component's safety states only from better to worse ones,
- the system and its components critical safety state is $r = 1$.
- the port oil terminal critical infrastructure risk function permitted level $\delta = 0.05$,
- the port oil terminal structure is series.

Applying (30)–(31) and using (46) the safety function of port oil terminal is given by

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2)],\ t \geq 0, \quad (72)$$

and

$$\boldsymbol{S}(t,u) = 0.395 \cdot [\boldsymbol{S}(t,u)]^{(1)} + 0.060 \cdot [\boldsymbol{S}(t,u)]^{(2)} + 0.003 \cdot [\boldsymbol{S}(t,u)]^{(3)} + 0.002 \cdot [\boldsymbol{S}(t,u)]^{(4)} + 0.2 \cdot [\boldsymbol{S}(t,u)]^{(5)} + 0.058 \cdot [\boldsymbol{S}(t,u)]^{(6)} + 0.282 \cdot [\boldsymbol{S}(t,u)]^{(7)},$$
$$t \geq 0, \text{ for } u = 1,2, \quad (73)$$

where $[\boldsymbol{S}(t,u)]^{(b)}$, $b = 1,2,\ldots,7$, are the system conditional safety functions at the operation state z_b, $b = 1,2,\ldots,7$, determined in (Kołowrocki & Magryta-Mut, 2020b, 2021, 2022a; Magryta-Mut, 2020, 2022).
Hence, in particular for $u = 1$, we have

$$\boldsymbol{S}(t,1) = 0.395 \cdot [\boldsymbol{S}(t,1)]^{(1)} + 0.060 \cdot [\boldsymbol{S}(t,1)]^{(2)} + 0.003 \cdot [\boldsymbol{S}(t,1)]^{(3)} + 0.002 \cdot [\boldsymbol{S}(t,1)]^{(4)} + 0.200 \cdot [\boldsymbol{S}(t,1)]^{(5)} + 0.058 \cdot [\boldsymbol{S}(t,1)]^{(6)} + 0.282 \cdot [\boldsymbol{S}(t,1)]^{(7)},$$
$$t \geq 0, \quad (74)$$

where $[\boldsymbol{S}(t,1)]^{(b)}$, $t \geq 0$, $b = 1,2,\ldots,7$, are the system conditional safety functions at the operation state z_b, $b = 1,2,\ldots,7$, determined in (Kołowrocki & Magryta-Mut, 2020c, 2021; Magryta-Mut, 2020, 2022).
Further, the expected values of the analyzed system conditional lifetimes in the safety state subset not worse than the critical safety state $\{1,2\}$ at the operation states b, $b = 1,2,\ldots,7$, respectively (expressed in years) are (Kołowrocki & Magryta-Mut, 2020c, 2021; Kołowrocki & Soszyńska-Budny, 2018a; Magryta-Mut, 2020):

$$[\boldsymbol{\mu}(1)]^{(1)} \cong 8.083,\ [\boldsymbol{\mu}(1)]^{(2)} \cong 8.166,\ [\boldsymbol{\mu}(1)]^{(3)} \cong 7.602,\ [\boldsymbol{\mu}(1)]^{(4)} \cong 6.808,\ [\boldsymbol{\mu}(1)]^{(5)} \cong 7.602,\ [\boldsymbol{\mu}(1)]^{(6)} \cong 6.808,\ [\boldsymbol{\mu}(1)]^{(7)} \cong 8.003. \quad (75)$$

Thus, applying (7) and considering (46) and (74), the value of the port oil terminal unconditional lifetime in the safety state subset not worse than this critical safety state $\{1,2\}$ is

$$\boldsymbol{\mu}(1) = 0.395 \cdot 8.083 + 0.060 \cdot 8.166 + 0.003 \cdot 7.602 + 0.002 \cdot 6.808 + 0.2 \cdot 7.602 + 0.058 \cdot 6.808 + 0.282 \cdot 8.003 \cong 7.891 \text{ years}. \quad (76)$$

Further, the corresponding standard deviation of the analyzed system unconditional lifetime in the state subset $\{1,2\}$ is (Magryta-Mut, 2021)

$\boldsymbol{\sigma}(1) \cong 7.91$ years. (77)

As the port oil terminal critical safety state is $r = 1$, then its system risk function is given by (Kołowrocki & Magryta, 2020a; Kołowrocki & Magryta-Mut, 2020c, 2021; Magryta-Mut, 2021):

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,1) \text{ for } t \geq 0, \quad (78)$$

where $\boldsymbol{S}(t,1)$, $t \geq 0$, is given by (74).
Hence, and considering (76), the moment when the system risk function exceeds a permitted level, for instance $\delta = 0.05$, is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.404 \text{ year.} \quad (79)$$

The port oil terminal critical infrastructure mean intensities of ageing are:

$$\boldsymbol{\lambda}(1) = 0.127, \boldsymbol{\lambda}(2) = 0.199. \quad (80)$$

By (38), considering (80) and the values of the port oil terminal without operation impact intensity of ageing $\lambda^0(1) = 0.116$, $\lambda^0(2) = 0.182$, determined in (Kołowrocki & Magryta-Mut, 2020c; Kołowrocki & Soszyńska-Budny, 2018b; Magryta-Mut, 2021), the coefficient of the operation process impact on the port oil terminal intensity of ageing is

$$\boldsymbol{\rho}(1) \cong 1.094, \boldsymbol{\rho}(2) \cong 1.094. \quad (81)$$

Hence, applying (39), the port oil terminal resilience indicator, i.e. the coefficient of the port oil terminal resilience to operation process impact, is

$$\boldsymbol{RI}(1) = 1/\boldsymbol{\rho}(1) \cong 0.914 = 91.4\%. \quad (82)$$

5.2. Optimal safety characteristics

Applying the optimization procedure from Section 3.2, we obtain the optimal mean value of the port oil terminal lifetime is (Kołowrocki & Magryta-Mut, 2020b)

$$\dot{\boldsymbol{\mu}}(1) = \dot{p}_1 \cdot 8.083 + \dot{p}_2 \cdot 8.166 + \dot{p}_3 \cdot 7.602 + \dot{p}_4 \cdot 6.808 + \dot{p}_5 \cdot 7.602 + \dot{p}_6 \cdot 6.808 + \dot{p}_7 \cdot 8.003 \cong 7.943 \text{ years,} \quad (83)$$

where the optimal limit transient probabilities of the port oil terminal at the particular operation states are:

$$\dot{p}_1 = 0.46,\ \dot{p}_2 = 0.08,\ \dot{p}_3 = 0.002,\ \dot{p}_4 = 0.001,$$
$$\dot{p}_5 = 0.15,\ \dot{p}_6 = 0.004,\ \dot{p}_7 = 0.267.$$

Moreover, the corresponding optimal unconditional safety function coordinate of the port oil terminal system takes the form

$$\dot{\boldsymbol{S}}(t,1) = 0.46\exp[-0.12371t] + 0.08\exp[-0.12246t] + 0.002\exp[-0.131548t] + 0.001\exp[-0.146885t] + 0.15\exp[-0.131548t] + 0.04\exp[-0.146885t] + 0.267\exp[-0.12496t],\ t \geq 0. \quad (84)$$

Moreover, considering (83) and (84), the corresponding optimal standard deviation of the port oil terminal system unconditional lifetime in the state subset is (Magryta-Mut, 2022).

$$\dot{\boldsymbol{\sigma}}(1) \cong 7.943 \text{ years.} \quad (85)$$

As the port oil terminal system critical safety state is $r = 1$, then considering (84), its optimal system risk function, is given by

$$\dot{\boldsymbol{r}}(t) \cong 1 - \dot{\boldsymbol{S}}(t,1),\ t \geq 0. \quad (86)$$

Hence, and considering (86) the moment when the optimal system risk function exceeds a permitted level $\delta = 0.05$ is

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta) \cong 0.407 \text{ year.} \quad (87)$$

By (83) the port oil terminal system mean value of the optimal intensity of ageing is

$$\dot{\lambda}(1) = \frac{1}{\dot{\boldsymbol{\mu}}(1)} \cong 0.126. \quad (88)$$

Considering (38) and (88) and the values of the analyzed system without operation impact intensity of ageing $\lambda^0(1) = 0.116$, determined in (Kołowrocki & Magryta, 2020a; Kołowrocki & Magryta-Mut, 2020c; Magryta-Mut, 2022), the optimal coefficient of the operation process impact on the port oil terminal intensity of ageing is

$$\dot{\boldsymbol{\rho}}(1) = \frac{\lambda(1)}{\lambda^0(1)} = \frac{0.126}{0.116} \cong 1.086. \tag{89}$$

Finally, the port oil terminal system optimal resilience indicator, i.e. the optimal coefficient of the port oil terminal system resilience to operation process impact, is

$$\dot{\boldsymbol{RI}}(1) = \frac{1}{\dot{\boldsymbol{\rho}}(1)} = \frac{1}{1.086} \cong 0.921 = 92.1\%. \tag{90}$$

6. Joint system safety and operation cost optimization

6.1. System operation cost with system safety impact corresponding to its maximal safety

To analyze jointly the system safety and its operation cost optimization, it is possible to propose the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that allows to find the best values of the system safety indicators, through applying the proposed system safety general model from Section 3.1 and safety optimization described in Section 3.2. Next, to find the system total unconditional operation costs in the safety state subsets, corresponding to this system best safety indicators, we replace the limit transient probabilities at the system particular operation states, existing in the formula for the system operation total cost during the fixed operation time by their optimal values existing in the formula for the system best safety function coordinates.

Thus, in Section 3.2, there is presented the procedure of determining the optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, of the limit transient probabilities of the system operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,v$, that allows to find the system maximal safety indicators, through applying the general system safety model and linear programming and determining their values. More exactly, to find the system total unconditional operation costs in the safety state subsets, corresponding to the system maximal safety indicators, we replace p_b, $b = 1,2,\ldots,v$, existing in the formula (4) for the system operation total cost by $\dot{p}_b$, $b = 1,2,\ldots,v$, for its maximal safety indicator, the system maximal mean lifetime in the system safety state subset not worse than the system critical safety state.

6.1.1. Port oil terminal operation cost with system safety impact corresponding to its maximal safety

In Section 5.2, we get the optimal limit transient probabilities of the port oil terminal operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,7$:

$$\dot{p}_1 = 0.46,\ \dot{p}_2 = 0.08,\ \dot{p}_3 = 0.002,\ \dot{p}_4 = 0.001,$$
$$\dot{p}_5 = 0.15,\ \dot{p}_6 = 0.04,\ \dot{p}_7 = 0.267, \tag{91}$$

that maximize the port oil terminal safety and determine its optimal values given by (46).

To find the system conditional total operation costs in the safety state subsets, determined by the cost model with considering safety impact, corresponding to this system maximal safety, we replace p_b, $b = 1,2,\ldots,7$, existing in the formulae (54)–(55) for the system operation cost in the safety state subsets {1,2} and {2} respectively, by $\dot{p}_b$, $b = 1,2,\ldots,7$, defined by (91). This way, we get the port oil terminal conditional total operation costs in the safety state subsets, determined by the cost model with safety impact, corresponding to this system maximal safety, given by

$$\begin{aligned}\boldsymbol{C}(1) &\cong p_1[\boldsymbol{C}(1)]^{(1)} + p_2[\boldsymbol{C}(1)]^{(2)} + p_3[\boldsymbol{C}(1)]^{(3)} \\ &+ p_4[\boldsymbol{C}(1)]^{(4)} + p_5[\boldsymbol{C}(1)]^{(5)} + p_6[\boldsymbol{C}(1)]^{(6)} \\ &+ p_7[\boldsymbol{C}(1)]^{(7)} \cong 0.46\cdot 134274.50355 \\ &+ 0.08\cdot 125134.71981 \\ &+ 0.002\cdot 170921.06810 \\ &+ 0.001\cdot 258228.9664 \\ &+ 0.15\cdot 180921.06810 \\ &+ 0.04\cdot 258228.96640 \\ &+ 0.267\cdot 143431.48954 \cong 148140.6,\end{aligned} \tag{92}$$

in the safety state subset {1,2} and

$$\begin{aligned}\boldsymbol{C}(2) &\cong p_1[\boldsymbol{C}(2)]^{(1)} + p_2[\boldsymbol{C}(2)]^{(2)} + p_3[\boldsymbol{C}(2)]^{(3)} \\ &+ p_4[\boldsymbol{C}(2)]^{(4)} + p_5[\boldsymbol{C}(2)]^{(5)} + p_6[\boldsymbol{C}(2)]^{(6)} \\ &+ p_7[\boldsymbol{C}(2)]^{(7)} \cong 0.46\cdot 103764.29792 \\ &+ 0.08\cdot 94324.56966 \\ &+ 0.002\cdot 123568.59597 \\ &+ 0.001\cdot 190073.05216 \\ &+ 0.15\cdot 133568.59597 \\ &+ 0.04\cdot 190072.49920 \\ &+ 0.267\cdot 113215.49434 \cong 113581.5,\end{aligned} \tag{93}$$

in the safety state subset {2}.

6.2. System safety corresponding to its minimal operation cost with safety impact

To analyze jointly the system operation cost and its safety optimization it is possible to propose the procedure of determining the optimal values of limit transient probabilities of the system operation process at the particular operation states that allows to find the system minimal operation total costs in the safety state subsets, through applying the proposed in Section 4.2 system operation cost model and linear programming described in Section 4.3. Next, to find the system conditional safety indicators, corresponding to this system minimal total operation costs in the safety state subsets, we replace the limit transient probabilities at particular operation states, existing in the formula for the system safety function coordinates, by their optimal values existing in the formula for the system minimal operation total costs in the safety state subsets.

Thus, in Section 4.3, there is presented the procedure of determining the optimal values $\dot{p}_b$, $b = 1,2,\ldots,v$, of the limit transient probabilities of the system operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots,v$, that allows to find the minimal operation total costs in the safety state subsets, through applying the system operation cost model from Section 4.2 and determining its mean value. More exactly, to find the system conditional safety indicators, corresponding to this system minimal operation total costs in the safety state subsets, we replace p_b, $b = 1,2,\ldots,v$, existing in the formula (31) for the system safety function coordinates, by $\dot{p}_b$, $b = 1,2,\ldots,v$, existing in the formula (22) for its minimal operation total cost in the safety state subsets.

6.2.1. Port oil terminal safety corresponding to its minimal operation cost with safety impact

In Section 4.4, we get the optimal limit transient probabilities of the port oil terminal operation process $Z(t)$ at the particular operation states z_b, $b = 1,2,\ldots, 7$,:

$$\dot{p}_1 = 0.46,\ \dot{p}_2 = 0.08,\ \dot{p}_3 = 0.002,\ \dot{p}_4 = 0.001,$$
$$\dot{p}_5 = 0.15,\ \dot{p}_6 = 0.04,\ \dot{p}_7 = 0.267, \qquad (94)$$

that minimize the port oil terminal operation cost through applying the system operation cost model with considering safety impact and determining its optimal values in the safety state subsets given by (69)–(70) and in the particular safety state given by (71).

To find the system conditional safety indicators, corresponding to this system optimal total operation costs in the system safety subsets, we replace p_b, $b = 1, 2,\ldots, 7$, existing in the formula (46) for the system safety function, by $\dot{p}_b$, $b = 1, 2,\ldots,7$, defined by (94). This way, we get the conditional port oil terminal safety function, corresponding to this system optimal total operation costs in the system safety subsets, given by the vector

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2)],\ t \geq 0, \qquad (95)$$

where considering the port oil terminal operation process optimal limit transient probabilities at the operation states determined by (94), the vector coordinates are given respectively by

$$\begin{aligned}\boldsymbol{S}(t,1) = {} & 0.46\cdot[\boldsymbol{S}(t,1)]^{(1)} + 0.08\cdot[\boldsymbol{S}(t,1)]^{(2)} \\ & + 0.002\cdot[\boldsymbol{S}(t,1)]^{(3)} + 0.001\cdot[\boldsymbol{S}(t,1)]^{(4)} \\ & + 0.15\cdot[\boldsymbol{S}(t,1)]^{(5)} + 0.04\cdot[\boldsymbol{S}(t,1)]^{(6)} \\ & + 0.267\cdot[\boldsymbol{S}(t,1)]^{(7)}, \end{aligned} \qquad (96)$$

$$\begin{aligned}\boldsymbol{S}(t,2) = {} & 0.46\cdot[\boldsymbol{S}(t,2)]^{(1)} + 0.08\cdot[\boldsymbol{S}(t,2)]^{(2)} \\ & + 0.002\cdot[\boldsymbol{S}(t,2)]^{(3)} + 0.001\cdot[\boldsymbol{S}(t,2)]^{(4)} \\ & + 0.15\cdot[\boldsymbol{S}(t,2)]^{(5)} + 0.04\cdot[\boldsymbol{S}(t,2)]^{(6)} \\ & + 0.267\cdot[\boldsymbol{S}(t,2)]^{(7)}, \end{aligned} \qquad (97)$$

where $[\boldsymbol{S}(t,u)]^{(b)}$, $t \geq 0$, $u = 1,2$, $b = 1,2,\ldots,7$, are given in (Kołowrocki & Magryta-Mut, 2020b) The conditional expected values and standard deviations of the port oil terminal lifetimes in the safety state subsets {1,2}, {2} calculated from the results given by (96)–(97), given in (Kołowrocki & Magryta-Mut, 2020b) corresponding to this system optimal total operation costs in the system safety subsets, respectively are:

$$\begin{aligned}\boldsymbol{\mu}(1) \cong {} & 0.46\cdot 8.08342 + 0.08\cdot 8.16593 \\ & + 0.002\cdot 7.60179 + 0.001\cdot 6.80805 \\ & + 0.15\cdot 7.60179 + 0.04\cdot 6.80805 \\ & + 0.267\cdot 8.00256 \cong 7.94293 \text{ years}, \end{aligned} \qquad (98)$$

$$\begin{aligned}\boldsymbol{\mu}(2) \cong {} & 0.46\cdot 5.15695 + 0.08\cdot 5.21069 \\ & + 0.002\cdot 4.8522 + 0.001\cdot 4.34292 \\ & + 0.15\cdot 4.85232 + 0.04\cdot 4.34292 \\ & + 0.267\cdot 5.10431 \cong 5.067515 \text{ years}. \end{aligned} \qquad (99)$$

And further, considering (34) and (98)–(99), the conditional mean values of the system lifetimes in the particular safety states 1, 2, corresponding to this system optimal total operation costs in the system safety subsets, respectively are:

$$\bar{\boldsymbol{\mu}}(1) = \boldsymbol{\mu}(1) - \boldsymbol{\mu}(2) = 2.875419 \text{ years},$$
$$\bar{\boldsymbol{\mu}}(2) = \boldsymbol{\mu}(2) = 5.067515 \text{ years}. \quad (100)$$

Since the critical safety state is $r = 1$, then the conditional system risk function, corresponding to this system optimal total operation costs in the system safety subsets, according to (35) is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,1) \quad (101)$$

where $\boldsymbol{S}(t,1)$ is given by (96).
Hence, according to (36) the moment when the system conditional risk function exceeds a permitted level, for instance $\delta = 0.05$, corresponding to this system optimal total operation costs in the system safety subsets, is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.407 \text{ year}. \quad (102)$$

Applying (37), the conditional port oil terminal conditional intensities of ageing, corresponding to this system optimal total operation costs in the system safety subsets, are:

$$\lambda(1) = 0.125898, \lambda(2) = 0.197335. \quad (103)$$

Considering (38) and (103), the conditional coefficients of the operation process impact on the port oil terminal intensities of ageing, corresponding to this system optimal total operation costs in the system safety subsets, are:

$$\boldsymbol{\rho}(t,1) \cong 1.08652, \boldsymbol{\rho}(t,2) \cong 1.07987. \quad (104)$$

Finally, by (39) and (104), the port oil terminal conditional resilience indicator, corresponding to this system optimal total operation costs in the system safety subsets, i.e. the conditional coefficient of the port oil terminal resilience to the operation process impact, corresponding to this system optimal total operation costs in the system safety subsets, is

$$\boldsymbol{RI}(t) = 1/\boldsymbol{\rho}(t,1) \cong 0.9204 = 92{,}04\%. \quad (105)$$

7. Conclusion

The procedures of using the general safety analytical model and the operation cost model of complex multistate technical system related to its operation process (Kołowrocki, 2014) and the linear programming (Klabjan, 2006) were presented and proposed to separate and joint analysis of the system safety maximization and its operation cost minimization.
The separate system safety maximization was depended on the mean value of the complex multistate system in the system safety state subset not worse than the system critical safety state maximization through the system operation process modification. This operation process modification allowed to determine the corresponding the best forms and values of the system safety indicators.
The separate system operation cost minimization was depended on the complex system mean value of the operation total costs in the safety state subsets minimization through the system operation process modification. This operation process modification allowed to determine the corresponding minimal system operation total costs in the safety state subsets.
The procedure of joint system safety and its operation cost optimization allowed us to use firstly the system safety maximization and next determining its conditional operation total costs in the safety state subsets corresponding to this system maximal safety. In this case, the operation process modification allowed us to find the complex system conditional operation total costs in the safety state subsets corresponding to the system best safety indicators. The proposed system safety optimization procedure and corresponding system operation total cost finding delivered practically important possibility of the system safety indicators maximization and keeping fixed the corresponding system operation total costs in the safety state subsets through the system new operation strategy. The procedure of joint system safety and its operation cost optimization allowed us also to use firstly the system operation total costs in the safety state subsets minimization and next determining its conditional safety function and remaining safety indicators corresponding to this system minimal operation total cost. In this case, the operation process modification allowed us to find the complex system conditional safety indicators corresponding to the system minimal operation

total costs in the safety state subsets. The proposed cost optimization procedure and finding corresponding system safety indicators delivered practically important possibility of the system total operation cost minimizing and keeping fixed the corresponding conditional safety indicators during the operation through the system new operation strategy.

The proposed system safety and system operation cost optimization models and procedures were applied to the port oil terminal examination. Moreover, the proposed model is a universal tool that was applied to the maritime ferry technical system (Kołowrocki & Magryta-Mut, 2022b). These procedures can be used in safety and operation cost optimization of various real complex systems and critical infrastructures (Gouldby et al., 2010; Habibullah et al., 2009; Kołowrocki & Magryta, 2020a, 2020c; Kołowrocki et al., 2016, Lauge et al., 2015; Magryta-Mut, 2022). Further research can be related to considering other impacts on the system safety and its operation cost, for instance a very important impact related to climate-weather factors (Kołowrocki & Kuligowska, 2018, Kołowrocki & Soszyńska-Budny, 2017; Torbicki, 2019a-b; Torbicki & Drabiński, 2020) and resolving the issues of critical infrastructure (Lauge et al., 2015) safety and operation cost optimization and discovering optimal values of safety, operation cost and resilience indicators of system impacted by the operation and climate-weather conditions (Kołowrocki & Soszyńska-Budny, 2017). These developments can also benefit the mitigation of critical infrastructure accident consequences (Bogalecka, 2020; Dąbrowska & Kołowrocki, 2019a-b, 2020a-b) and inside and outside dependences (Kołowrocki, 2021, 2022) and to minimize the system operation cost and to improve critical infrastructure resilience to operation and climate-weather conditions (Kołowrocki & Kuligowska, 2018, Kołowrocki & Soszyńska-Budny, 2017; Torbicki, 2019a-b; Torbicki & Drabiński, 2020).

The proposed optimization procedures and perspective of future research applied to system operation cost and to safety and resilience optimization of the complex systems and critical infrastructures can give practically important possibility of these systems effectiveness improvement through the proposing their new operation strategy application.

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