Investigation of the effectiveness of goal programming in Project Management Activities

NIROSHI PIRHAVEEN[1], VIJAYANATHAN SENTHOORAN[2]

[1)] niroshi@vau.ac.lk
[2)] v.senthooran@vau.ac.lk

[1, 2)] Department of ICT, Faculty of Technological Studies
University of Vavuniya
Vavuniya, Sri Lanka

The regulation of objectives that are inconsistent with the use of all related resources through the organization is the principal project management (PM) work. In present situations, the Project Manager is responsible for managing multiple conflicting objectives, these objectives are normally fuzzy owing to information will be insufficient and unavailable. At the earliest stage, it was difficult for the decision maker to develop a plan to shorten the total completion time, minimize the project total cost and maximize the quality of the project. The aim of this study is to compare a combination of fuzzy goal programming and grey linear programming techniques and multi objective integer grey programming model of programming with multiple fuzzy objectives (programming with fuzzy objectives) which is applied to a project management problem. This study provides a systematic decision-making framework for resolving PM decision issues with multiple goals in uncertain environments. A real-world numerical example is used to compare the different techniques in this study. Finally, the results determine the best method that gives the best combination of the parameters used for project activities. Results demonstrate that this model can be implemented as an effective tool.

Keywords: the iron triangle, time-cost-quality tradeoff, grey numbers.

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1. Introduction

An approach used to solve multiple objective optimization problems that balance a trade-off in conflicting objectives is defined as Goal programming (GP). Originally GP model was introduced by Charnes et al in 1955 to extend linear programming models. A project is defined as a set of activities aimed at attaining a particular objective and project management is the process of leading a team in order to achieve all project goals within the defined constraints. According to Project Management Body of Knowledge (PMBOK) guide [24], the nine project management domains are project integration, cost, time, quality, human resources, risk, communication and procurement management. After the research and investigation, the list of factors which are expected to 1 influence project management success or failure is presented in the section on why projects fail. Project completion in accordance with budget, deadlines (times), and according to project specifications ensures successful project management. This is simplified through the identification and successful implementation of the methodology for tradeoff analyses. The following three key points are the most important for the success of a project firstly a project must meet the requirements of the client, secondly it has to be within budget and finally it has to be completed on schedule [17]. These three criteria have been often called as “The Iron Triangle” [16]. Time, cost and high quality are crucial project goals that project managers constantly strive to achieve with the lowest possible cost and highest possible quality in order to be successful in the projects. One of the main obstacles, in this case, it takes the right approach to achieve these goals. With this method we can achieve the best optimal type of project implementation activity that has achieved both the lowest and highest quality at the lowest cost and the time.

At first, the researchers were considered timecost tradeoff problems. In this phase, the time-cost trade-off issues in a project was first addressed by Kelly[9]. He believed that the linear relationship between the time and cost of an activity to solve the problem provides mathematical models and a heuristic algorithm (but one that leads to optimal solutions). In this phase, Demeulemeester[6] investigated identifying best processes in project networks by balancing time and cost. Hereafter Simense[14], Goyal[23], Harvey and Patterson[8], Prabudha et al.[15], Phillips[19], Demeulemeester et al.[6], and Akkan[4] introduced the heuristic models for problems requiring discrete timecost tradeoff. Deckro et al.[5] developed a nonlinear model with improvements in mathematical programming. By assuming that the direct cost of an activity changes over time, mathematical programming models were created to reduce the project's direct cost[3]. Khosrowshai[11] presented the relationship between the overall cost of a project and the project's total time for a specific project type and mathematically illustrated this relationship. Although some other researchers focused on quality during the second phase, while others presented techniques for calculating quality costs in the project management [2], [10], [12].

The third stage of classification is considered as the time cost quality tradeoff model. At this point some studies were carried out to include quality as a new factor during the time-cost trade-off problems. Babu & Suresh [3] proposed in 1996, that the project crashes should impair the quality of a completed project. Salmasnia et al.[20] treat quality as an additional factor in the standard time-cost tradeoff problem when the parameters are stochastic.

An important aspect of project management is the gathering of information regarding an optimum balance between the project's objectives. According to the iron triangle, the most important goals of a project are time, cost and quality. With the time and cost, quality is also now an important consideration in each project. The goal of these problems (Time Cost Quality trade-off Problem (TCQTP)) is to select a collection of unsuccessful activities as well as an appropriate execution technique for each activity in order to minimize project cost and time while maximizing project quality[22]. This study 2 compares and contrasts some fuzzy programming strategies with multiple-goal (fuzzy goal) programming used to solve a trade-off problem.

Applying one of the methodologies of multiple objective programming (goal programming) to the project management problem is the objective of this first review article. In this research, adopts the model of Mubiru[13] for allocating time and cost in project management and it was experimented in the company of construction, SEROR, Algeria. The goal of a second review article is to choose a set of activities and a suitable mode of execution for each activity, with the goal of minimizing project time and expense while maximizing project quality. This research carries out the model for TCQTP that uses grey numbers to approximate these parameters. The approximation of time, cost and quality parameters of activities carried out with grey numbers as well as developing two-phase approach based on the goal programming to tackling this problem is the main originality of the proposed model.

The third review paper's goal is to create up-todate mathematical models of cost, time, and quality tradeoffs in scenarios where the project's parameters are estimated inexactly by grey numbers. The managers will be aided by the greyness of the proposed parameters. The uncertainty of the project planning data, shown as "grey numbers" is the most important aspect of the suggested model. The suggested model is solved using a mix of fuzzy goal programming and grey linear programming. The managers will finally have a better understanding of how to handle uncertainty in project management and planning to this approach. A case study was carried out by Abdelkrim and Khadija[1] at the construction company SEROR, Algeria, to investigate and model was designed in the first review article. There were three projects that SEROR Company were set to start at the same time. In order to identify the combination of time, labor and material resources, the goal programming model was designed along with the total amounts of resources required to carry out the three previous projects with a particular focus on the production and operational phase of the projects. The aim of goal programming is to minimize the objective function so that the minimum resource requirements are used during the project stages. The goal values and expected completion times for the projects are adjusted by subtracting the sum of the weighted deviations. The simplex approach is used to achieve linear programming resource balancing; that necessitates a solution to a minimization problem. The results indicate the model which provides a satisfactory level of achievement for management of the three projects with preemptive goals, however the solution value (0) of time given for planning is neither logical nor realistic since the project is without if the planning phase fails. As a result, they propose adding further constraints into the model by setting the lower bound for each time allocated to planning, scheduling and control.

According to the suggestion made by the above paper, the remaining papers were selected as solutions. Other remaining two papers with different techniques are applied in the TCQT trade-off problem. The following is how the rest of the paper is structured: Section 2 will detail the approach employed in each paper and an application of the suggested model using a numerical example in Section 3. Section 4 deals with the suggestion and recommendation based on the results. Finally, Section 5 offers some conclusions as well as some recommendations for further research.

2. Materials and methods

In this study, three review papers were selected. The first review [1] reported the case study that was completed in the company of construction SEROR, Algeria. Due to the problem described in the above section 1, other remaining papers were selected for comparisons.

2.1. Methods type 1

Developing current mathematical models of cost, time, and quality tradeoffs under conditions where the parameters of project activities are approximated uncertainly using grey numbers is the purpose of the second review article [18].

Meanwhile to complete in a significantly quicker time by expanding resources while increasing the project cost, the managers identified the best combination of cost, time, and quality parameters for the activities. To deal with these conditions, the managers can support the greyness of these parameters in the proposed strategy. The most essential component of the proposed model is that it takes into account the uncertainty of the project planning data in the form of grey numbers. A hybrid of fuzzy goal programming and grey linear programming is used to solve the proposed model. Finally, the managers are given a model that will improve their capacity to manage and plan projects in the face of uncertainty. The model determines an optimal range in which the project managers can react to intrinsic changes in parameters that may occur during a project as its main discovery.

Investigating TCQTP's concept of uncertainty using fuzzy sets and grey numbers as a combination is the main contribution of this work. Grey numbers were a good framework because of their benefits in making uncertain decisions without the need for any preconceived membership or probability function for approximating the parameters of time, cost, and quality. The work presents a method for modeling the multi objective TCQTP that employs a fuzzy grey goal programming (FGGP) approach. The goal values are defined as fuzzy numbers, while the parameters are determined by grey numbers as indicated in this FGGP model.

First introduce the suggested model as follows: A project is represented by a directed graph $G = (V, E)$ with m nodes and n arcs, where $V = 1,2,....., m$ representing the set of nodes and $E = (i, j),.......,(l, m)$ is the set of direct arcs. The activities and events were represented by these arcs and nodes respectively. Each project activity that says $(i, j) \in E$, has the following forms: (1) normal form with time D_{ij}, cost C_{ij} and quality Q_{ij} , and (2) crashed form with time d_{ij}, cost c_{ij} , and quality q_{ij} said by each project activity. The difficulty now is to find the best combination of time, cost and quality of the activities so that goals of the project managers goals 4 of needed time, cost and quality of project are met.

Step 1: Formulation of the grey multi objective linear programming (GMOLP) – TCTQT problem based on Equation (1) to (5).

The notations used to formulate the (GMOLP) – TCTQT problem described in the table which is shown in the supplementary material section (S.1).

Each project activity that says $(i, j) \in E$, has the following forms: (1) normal form with time D_{ij} , cost C_{ij} and quality Q_{ij} , and (2) crashed form with time dij, cost c_{ij} , and quality q_{ij} said by each project activity. The difficulty now is to find the The following are two extra parameters: – Cost slope of activity ij : Cost of reducing one unit from normal time of activity ij:

$$\otimes CS_{ij} = \frac{\otimes c_{ij} - \otimes C_{ij}}{\otimes d_{ij} - \otimes D_{ij}}$$

– Quality slope of activity ij : Lost (or gained) quality when normal time is reduced by one unit of activity ij :

$$\otimes qs_{ij} = \frac{\otimes q_{ij} - \otimes Q_{ij}}{\otimes d_{ij} - \otimes D_{ij}}$$

2.1.1. Objective functions

The suggested model is based on two assumptions. Firstly, between their normal and crashing times, the project activitiesą time is a continuous positive variable, and secondly the relationships between time, cost, and quality are linear. The suggested model takes into account three objective functions at the same time as shown in below. Total project cost has been minimized: If two points ($\otimes D_{ij}$, $\otimes C_{ij}$) and ($\otimes d_{ij}$, $\otimes c_{ij}$) show the normal and crashed representations of each activity *ij* in two dimensional space of time cost, then the cost model of activity *ij* will be as Equation (1):

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$$\otimes C_{ij} + \otimes cs_{ij}\left(\otimes x_{ij} - \otimes D_{ij}\right) \tag{1}$$

As a result, the total project cost as summation of costs of all activities can be stated as follows:

$$\min Z_1 \cong \sum_{ij\in E}\sum \otimes C_{ij} + \otimes cs_{ij}\left(\otimes x_{ij} - \otimes D_{ij}\right) \tag{2}$$

Minimizing the projects required time: Another objective is to the project must be finished within a predetermined time. If T_n is the time of the projects last node and T_1 is the time of its first node, then the time of this project can be calculated as follows:

$$\min Z_2 \cong \otimes T_n - \otimes T_1 \tag{3}$$

Maximization of quality of the project: The quality of the project can be described in the same way as the overall cost of the project:

$$\max Z_3 \cong \sum_{ij\in E}\sum \otimes Q_{ij} + \otimes qs_{ij}\left(\otimes x_{ij} - \otimes D_{ij}\right). \tag{4}$$

Equations (1) and (4) are the formulas for the relationship between time and cost/quality of the project and any linear functions can be studied in the same way.

2.1.2. Constraints

Constraints on activity start and finish time: Assume that activity ij has a start time $\otimes t_i$ with its duration being $\otimes x_{ij}$. Then, if activity end time is $\otimes t_j$, then the relationship shown below applies to this activity:

$$\otimes t_i + \otimes x_{ij} \le \otimes t_j \quad \forall_{ij\in E}\; i,j = 1,2,...,n \tag{5}$$

Constraints on actual duration of an activity: If $\otimes x_{ij}$ represents the actual time of activity *ij*, then it must meet the following criteria:

$$\otimes x_{ij} \le \otimes D_{ij} \quad \forall_{ij\in E} \tag{6}$$

$$\otimes x_{ij} \ge \otimes d_{ij} \quad \forall_{ij\in E} \tag{7}$$

Step 2: The FGGP model for TCQT problems The original grey multi objective linear programming (GMOLP) model for the TCQT problem (GMOLP-TCQT) can be transformed into a FGGP for the TCQT problem (FGGP-TCQT) by including membership functions to express fuzzy goals of decision makers. The membership functions for these objectives are obtained by solving three distinct optimization problems for each of the objectives (Equation 8).

2.1.3. Development of membership functions

Three minimization models were solved to produce the membership function using the ordinal linear programming method as follows:

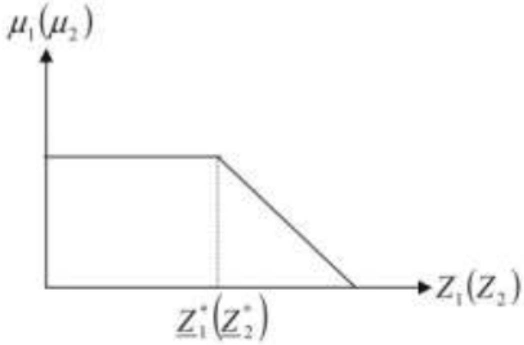

Fig. 1. Membership Function of First (Second) Objectives

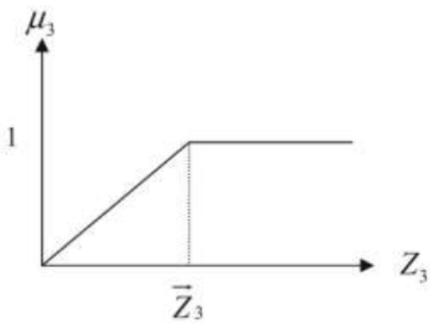

Fig. 2. Membership Function of Third Objective

$$\min Z_1\left(Z_2 \text{ or max } Z_3\right) \tag{8}$$

Subject to:

$$\otimes t_i + \otimes x_{ij} \le \otimes t_j \quad \forall_{ij\in E} \quad i,j = 1,2,...,n$$
$$\otimes x_{ij} \le \otimes D_{ij} \quad \forall_{ij\in E}$$
$$\otimes x_{ij} \ge \otimes d_{ij} \quad \forall_{ij\in E}$$

Every objective has an optimal grey value which is derived from the method of [7]. $\otimes Z_1^*$ is the optimal grey value of the first objective which will be obtained from the result of minimizing the lower bound function and minimizing the upper bound function. This solution gives an idea for development of the membership function.

2.1.4. Nonnegative constraints

Step 3: Conversion into an equivalent FGGP-TCQT model.

The lower and upper bound values of the objectives can be used to convert the grey multi objective model to a FGGP model, and each objective has an optimal grey value obtained using Huangs method [25]. Assume that reducing the lower bound function reducing the upper bound function yields $\otimes Z_1^*$, the optimal grey value of the first objective[7] as follows:

$$\mu_1 \begin{cases} 1 & \text{if} \quad Z_1 \le \underline{Z_1^*} \\ \dfrac{\overline{z_1^*} - z_1}{\overline{z_1^*} - \underline{z_1^*}} & \text{if} \quad \underline{Z_1^*} \le Z_1 \le \overline{Z_1^*} \\ 0 & \text{if} \quad Z_1 \ge \overline{Z_1^*} \end{cases} \tag{9}$$

The membership function for the second objective is obtained in a similar manner. The ideal grey values for the third objective are of the maximization type will be obtained after it has first been maximized by establishing the 7 upper bound value and its lower bound value. The membership function of such a form will be:

$$\mu_3 \begin{cases} 0 & \text{if} \quad Z_3 \le \underline{Z_3^*} \\ \dfrac{Z_3 - \underline{\underline{Z_3^*}}}{\overline{Z_3^*} - \underline{Z_3^*}} & \text{if} \quad \underline{Z_3^*} \le Z_3 \le \overline{Z_3^*} \\ 1 & \text{if} \quad Z_3 \ge \overline{Z_3^*} \end{cases} \tag{10}$$

Step 4: Conversion into an equivalent GLP model.

To attain the best combination of results, the structure of the comparable GLP model for the FGGPTCQT model is as follows. Assume that neither cost nor quality are favored, but that the time aim is preferred to both of them.

$$\max \sum_{i=1}^{3} \mu_i \tag{11}$$

$$\mu_1 \le \frac{\overline{z_1^*} - z_1}{\overline{z_1^*} - \underline{z_1^*}}; \quad i = 1,2$$

$$\mu_3 \le \frac{z_3 - \underline{z_3^*}}{\overline{z_3^*} - \underline{z_3^*}}$$

System Constraints

$$\otimes x_{ij} \le \otimes D_{ij} \qquad \forall_{ij\in E}$$
$$\otimes x_{ij} \ge \otimes d_{ij} \qquad \forall_{ij\in E}$$
$$\mu_1, \mu_2, \mu_3, \otimes x_{ij}, \otimes t_i \ge 0$$

The above GLP can be solved in two phases using the approach of Huang [25]. The upper bound problem is tackled in the first phase. The problem of the lower bound problem is then solved. The solutions of the TCQT problem will be determine by the two solutions to these two problems. After solving the above model, by replacing the lower bound variables in model (Equation 11) with the upper bound variables and adding constraints that demand the upper bound values to be greater than or equal to these lower bound values, phase 2 will build the upper bound model.

2.2. Method type 2

Third review paper[21] aims to present a model for TCQTP the grey numbers to approximate these parameters. Because there are several ways to carry out each action, the trade-off problem is framed using a multi-objective integer grey programming model. The project is represented by a directed graph with *m* nodes and *n* arcs. Every project activity can be fulfilled through number of modes. Since, the aim of this study is to find the optimal combination of each activity in order to reduce the project network, while the cost and time are minimized, and the project quality is maximized.

The notations used to formulate the (GMOLP)-TCTQT problem described in the table which is shown in the supplementary material section (S.2).

$$y_{ijk} \begin{cases} 1 & \text{if} \quad \text{activity } ij \text{ is done in mode } k \\ 0 & \text{if} \qquad\qquad \text{oderwise} \end{cases}$$

The following is a mathematical model of the problem:

$$\min C=\sum_{ij\in E}\sum_{k\in M_{ij}}\otimes C_{ijk}y_{ijk} \qquad (12)$$

$$\min T=\otimes s_n-\otimes s_1$$

$$\max Q=\frac{\sum_{ij\in E}\sum_{k\in M_{ij}}y_{ijk}q_{ijk}}{n}$$

Subject to :

$$\otimes s_j-\otimes s_i\geq\sum_{k\in M_{ij}}\otimes t_{ijk}y_{ijk}\quad\forall\{i,j\}\in E$$

$$\sum_{k\in M_{ij}}y_{ijk}=1\quad\forall\{i,j\}\in E$$

$$y_{ijk}=0,1$$

Thereafter, an approach based on goal programming is designed to solve this model. The main originality of the grey numbered model is the estimate of the parameters of time, cost and quality of activities mode as well as the development of two-phase approach based on the goal programming to tackle this problem. Finally, the suggested model is tested in two separate scenarios, with the results display to demonstrate the models' impressive capabilities. For each activity in CTQTP there are many execution modes to choose. There is a solution if the number of project activities is n and each activity has k modes tochoose from resulting in a very large search space. Thedifficulty of the examined problem raises when the activity parameters are considered as grey numbers. The goal programming approach aims to identify an optimal solution with the least deviation from a set of targets for individual objectives. 9 as a result, the TCQT problem is divided into two steps. Firstly, determination of target points for individual objectives, and secondly developing and solving the goal programming model to discover the tradeoff solution for the whole problem.

2.2.1. Determination of Target Points

For each objective, three different models are solved in the first step. There are three grey integer programming models in all. Each grey problem is solved under two boundary conditions, one for the best condition and one for the worst condition, to yield an ideal range. Each task is done in the best condition, with the lower bound time, lower bound cost, and upper bound quality. An optimistic model is the name for this problem. The following is how the above problem is translated in this format:

$$\min \underline{C}=\sum_{ij\in E}\sum_{k\in M_{ij}}\underline{c_{ijk}}y_{ijk} \qquad (13)$$

$$\min \underline{T}=\underline{s_n}-\underline{s_1}$$

$$\max \overline{Q}=\frac{\sum_{ij\in E}\sum_{k\in M_{ij}}y_{ijk}\overline{q_{ijk}}}{n}$$

Subject to

$$FS:\begin{cases}\underline{s_j}-\underline{s_i}\geq\sum_{k\in M_{ij}}\otimes t_{ijk}y_{ijk}\quad\forall_{i,j\in E}\\ \sum_{k\in M_{ij}}y_{ijk}=1\quad\forall_{i,j\in E}\\ y_{ijk}=0,1\end{cases}$$

The $\overline{C^*}$, $\overline{T^*}$ and $\underline{Q^*}$ are determined by solving individual objectives of above model with the constraints. ($\underline{C^*}$, $\overline{C^*}$), ($\underline{T^*}$, $\overline{T^*}$) and ($\underline{Q^*}$, $\overline{Q^*}$) are the target 10 values for cost, time and quality objectives respectfully. A goal programming model is created to reduce the unwanted deviations from the target values as follows:

$$\min d_1^+ + d_2^+ + d_3^+ + d_4^+ + d_5^- + d_6^- \qquad (16)$$

$$\sum_{ij\in E}\sum_{k\in M_{ij}}\underline{c_{ijk}}y_{ijk}-d_1^+ + d_1^- = \underline{C^*}$$

$$\sum_{ij\in E}\sum_{k\in M_{ij}}\overline{c_{ijk}}y_{ijk}-d_2^+ + d_2^- = \overline{C^*}$$

$$\underline{s_n}-\underline{s_1}-d_3^+ + d_3^- = \underline{T^*}$$

$$\overline{s_n}-\overline{s_1}-d_4^+ + d_4^- = \overline{T^*}$$

$$\sum_{ij\in E}\sum_{k\in M_{ij}}\underline{q_{ijk}}y_{ijk}-d_1^+ + d_1^- = \underline{Q^*}$$

$$\sum_{ij\in E}\sum_{k\in M_{ij}}\overline{q_{ijk}}y_{ijk}-d_2^+ + d_2^- = \overline{Q^*}$$

subject to,

$$Y\in FS$$

$$Y\in FS'$$

$$d_I^+,\ d_I^-\geq 0,\quad I=1,2,\ldots,6$$

3. Results and discussion

A numerical example was utilized to compare the above two different models. For run the LP models, Lingo computer software used. Assume a 11 project with ten activities, the details of which are provided in the above table which is shown in the supplementary material section (S.3).

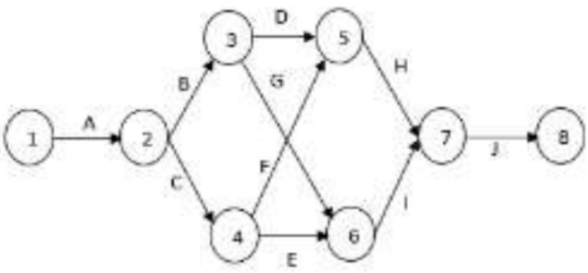

Fig. 3 Project's network diagram

Each activity has normal and crashed modes, as stated in the above table which is shown in the annexed section (Annexed 3). The management team estimates the time, cost, and quality of each activity in every mode as the grey numbers are approximated. The projects network is depicted in Figure 3. The above dataset was solved in the following method, according to the second review paper [18]. The optimization problems get the following results: $Z_1^* \in (1{,}085{,}1{,}365)$, $Z_2^* \in (15.5,\ 25)$ and $Z_3^* \in (546{,}843)$. The membership functions break down based on these solutions as follows:

$$\mu_1 = \begin{cases} 1 & \text{if} \quad Z_1 \le 1085 \\ \dfrac{1365 - Z_1}{1365 - 1085} & \text{if} \quad 1085 \le Z_1 \le 1365 \\ 0 & \text{if} \quad Z_1 \ge 1365 \end{cases}$$

$$\mu_2 = \begin{cases} 1 & \text{if} \quad Z_2 \le 15.5 \\ \dfrac{25 - Z_2}{25 - 15.5} & \text{if} \quad 15.5 \le Z_2 \le 25 \\ 0 & \text{if} \quad Z_2 \ge 25 \end{cases}$$

$$\mu_3 = \begin{cases} 0 & \text{if} \quad Z_3 \le 546 \\ \dfrac{Z_3 - 546}{843 - 546} & \text{if} \quad 546 \le Z_3 \le 843 \\ 1 & \text{if} \quad Z_3 \ge 843 \end{cases}$$

From the above result, when a project is completed within 15.5 days, its utility to the project manager is 100%. However, if it takes longer than 25 days, it will have zero utility. The completed model would then take the following shape:

$$\max \sum_{i=1}^{3} \mu_k$$

$$\mu_1 \le 4.875 - 0.00357143$$
$$\left(\sum_{ij\in E} \sum \left[\otimes C_{ij} + \otimes cs_{ij} \left(\otimes x_{ij} - \otimes D_{ij} \right) \right] \right)$$

$$\mu_2 \le 5.56 - 0.22222222\left(\otimes T_8 - \otimes T_1 \right)$$

$$\mu_3 \le 0.00336700$$
$$\left(\sum_{ij\in E} \sum \left[\otimes C_{ij} + \otimes cs_{ij} \left(\otimes x_{ij} - \otimes D_{ij} \right) \right] \right) - 1839$$

$$\otimes t_i + \otimes x_{ij} \le \otimes t_j \quad \forall_{ij\in E},\ i,j = 1,2,...,8$$
$$\otimes x_{ij} \le \otimes D_{ij} \quad \forall_{ij\in E}$$
$$\otimes x_{ij} \ge \otimes d_{ij} \quad \forall_{ij\in E}$$
$$\mu_1 \ge \mu_2$$
$$\mu_3 \ge \mu_2$$
$$\mu_1 \le 1$$
$$\mu_2 \le 1$$
$$\mu_3 \le 1$$
$$\mu_1,\ \mu_2,\ \mu_3, \otimes x_{ij}, \otimes t_i \ge 0 \quad \forall_{ij},\ i,j = 1,2,...,8$$

The lower bound value of the objective function is found in phase 1 while solving the GLP model using the two-phase technique. It is shown in the table below:

Tab. 1. Decision variables lower bound values and grey values

Variable	Grey value	Lower bound
$\otimes x_{12}$	[1.5, 2.5]	1.5
$\otimes x_{23}$	[3, 3]	3
$\otimes x_{24}$	[5, 5]	5
$\otimes x_{36}$	[6, 8.5]	6
$\otimes x_{45}$	[2, 3]	2
$\otimes x_{35}$	[4, 5]	4
$\otimes x_{56}$	[5.5, 5.5]	5.5
$\otimes x_{57}$	[3, 4]	3
$\otimes x_{67}$	[1, 3]	1
$\otimes x_{78}$	[6.32, 8]	6.32
$\otimes t_1$	[0, 0]	0
$\otimes t_2$	[1.5, 2.4]	1.5
$\otimes t_3$	[4.5, 5.5]	4.5
$\otimes t_4$	[6.5, 7.5]	6.5
$\otimes t_5$	[8.5, 10.5]	8.5
$\otimes t_6$	[14, 14]	14
$\otimes t_7$	[15, 17]	15
$\otimes t_8$	[21.32, 25]	21.32

The Table 1 shows the optimal grey solution to the TCQT problem. For solving the lower bound, the degrees of objective attainment are $\otimes\mu_1 = 0.818$, $\otimes\mu_2 = 0.818$, and $\otimes\mu_3 = 1$. If the project is completed in this manner, then its cost will be 1,141.41, it will take 21.32 days to complete and have a quality of 1,114.32.

The project will cost 3362.5, take 25 days to complete, and yield a quality score of 618.54 in the case of the upper bound solution. For such a problem, the final degrees of objective success would be $\otimes\mu_1 = [0, 0.818]$, $\otimes\mu_2 = [0, 0.818]$, and $\otimes\mu_3 = [0.245, 1]$. Note that the solution of the lower bound is obtained taking into the account the best situation of all parameters, while the solution of the upper bound solution is solved in the worst possible situation. Now, the project manager can now choose the activity time based on the situation. Obviously, the best solution is that the project manager able to choose the lower bound values for the time of the activities, but the project manager will be able to select a time point within the activity's optimal range that is still ideal even in the grey condition if the cost of an activity increases from its lower bound to its upper bound.

According to the third review paper [21], the previous dataset was solved as follows: When solving three different optimistic problems under the best condition, the solutions are $\underline{C}^* = 1085$, $\underline{T}^* = 15.5$ and $\overline{Q}^* = 870$ respectfully. Similarly, under the worst condition, the pessimistic model will be solved, the solution will be $\overline{C}^* = 1365$, $\overline{T}^* = 25$ and $\underline{Q}^* = 805$ respectfully.

In order to reach the final solution to the problem, the last step is to establish the following goal programming model for this problem.

Tab. 2. Decision variables optimal grey values

Variable	Grey value
$\otimes s_1$	[0,0]
$\otimes s_2$	[3,5]
$\otimes s_3$	[6,9]
$\otimes s_4$	[8,12]
$\otimes s_5$	[12,18]
$\otimes s_6$	[18,26]
$\otimes s_7$	[22,32]
$\otimes s_8$	[31,43]

Tab. 3. Deviations values

Variable	Lower bound
d_1^+	0
d_1^-	0
d_2^+	0
d_2^-	0
d_3^+	15.5
d_3^-	0
d_4^+	18
d_4^-	0
d_5^+	0
d_5^-	50
d_6^+	50
d_6^-	0

As a result, [15.5, 25], [1085, 1365], and [805,870] are the time, cost and quality associated with this combination of this activities.

Tab. 4. Comparison of Final results of the two review papers

Parameter	Review paper 2 method		Review paper 3 method	
	Lower bound	Upper bound	Lower bound	Upper bound
Cost	1085	1365	1085	1365
Time	15.5	25	15.5	25
Quality	546	843	805	870

Each project has a variety of different modes in both models. The time, cost and quality of each mode are approximated in an interval in those models. To tackle this difficulty, both proposed algorithms used a two stage technique. According to the third reviewed paper [21], in stage 1 of the procedure in this review, the best and worst modes of activities are used to establish the optimal solution. In stage 2, a goal programming model is created to reduce the overall deviation from the solutions of stage one. For discrete TCQT problems under uncertainty, the suggested strategy offers a logical, feasible and solvable framework. The continuous CTQT problem is investigated in the second reviewed paper [18], where each activity can take a time within a possible time interval. The proposed method takes into account the inherent uncertainty in the TCQT problems parameters as a significant characteristic. In this paper, the uncertainty of the PM in estimating parameters of the activities that is reached at a model in the form of grey approximations is discussed in this research along with a recommended solution to such a problem. The results of the suggested method can be used to develop and enhance the concepts and methodologies, particularly in CTQT problems, in case of uncertainty and ambiguity, which is a common occurrence in the real-world challenge. When both models are used, the grey values for the cost and time are the same. Here we must focus on minimizing the cost and time of the project while maximizing the quality. According to the quality grey value, third review model has a higher value than second review model, according to the quality grey value. As a result, the best model is found in the third reviewed paper [26] rather than second reviewed paper [18]. However, in the second review [18], the object value is given as 133.5. With some deviations, the mean target values are met. As a result, we must enhance this model in order to reduce all the deviations. The fuzzy grey number system is introduced in the second and third model to define the upper and lower bounds for each of the parameter, in order to solve the problem that was discovered in the first review paper [1].

3.1. Suggestions and recommendations for future improvements

It would be interesting to apply the provided models in various forms or alternative shapes of uncertain data or uncertainty in parameters as a direction for future research. In both models, it would be better to add additional constraints such as constraint for minimizing the crashing cost or crashing time.

4. Conclusion

According to the "Iron Triangle" for each and every project time, cost and quality are the three most important factors. Moreover every organizations task can be viewed as a project that presents a coordinated set of activities aimed at achieving a common goal. Meanwhile, each projects are made up of number of various activities that are connected to each other. However, project managers always look for the most efficient approach to complete a task. In fact, any task can be completed in a variety of time, cost and quality options. As a result, while dealing with the most difficult aspects of project management, scheduling the time,
cost and quality trade-off problems, may have significant impact on the project success. The TCQT problems, on the other hand, are constantly a source of uncertainty. It's Obvious that determining the exact time, cost and quality will be completed is a challenging task before it's accomplished. Therefore, some researchers developed few models to determine the best combination of time, cost and quality. Finally this review paper finds out the best method for the mentioned problem from the proposed models established.

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Supplementary Materials

S.1 Notations used to formulate the GMOLP-TCTQT problem

	Notations	Description
Parameters	n	Number of activities
	$\otimes Q_{ij}$	Quality of activity ij in normal duration
	$\otimes q_{ij}$	Quality of activity ij in crashed duration
	$\otimes D_{ij}$	Normal duration of activity ij
	$\otimes C_{ij}$	Direct cost of activity ij in normal duration
	$\otimes c_{ij}$	Direct cost of activity ij in crashed duration where Symbol $\otimes$ is generally used to demonstrate the grey numbers which is written as $\otimes (a) \in [a, a]$, where a stands for the definite known lower bound and a stand for the definite known upper bound.
Variables	$\otimes X_{ij}$	Actual time of activity ij
	$\otimes t_{ij}$	Actual time of activity ij

S.2 Notations used to formulate the GMOLP-TCTQT problem

	Notations	Description
Parameters	n	Number of activities
	$\otimes M_{ij}$	Set of available execution modes for activity ij, where; ij $\in$ E
	$\otimes C_{ijk}$	Direct cost of activity ij if performed by execution mode k
	$\otimes t_{ijk}$	Duration of activity ij if performed by execution mode k
	$\otimes q_{ijk}$	Quality of activity ij if performed by execution mode k
	$\otimes Q_{ij}$	Quality of activity ij in normal duration
	$\otimes q_{ij}$	Quality of activity ij in crashed duration
Variables	$\otimes s_i$	Start time of activity ij

S.3 Project Data for the CTQTP problem

Activity	Proceeding	Normal mode			Crashed mode			Cost slop	Quality slope
		Time (day)	*Cost($)*	*Quality (%)*	*Time(day)*	*Cost($)*	*Quality (%)*		
A	-	[3,5]	[100,115]	[75,80]	[1.5,2.5]	[150,200]	[60,70]	[-200,-10]	[-20,-1.75]
B	A	[3,4]	[45,60]	[75,85]	[1,2.5]	[90,110]	[50,70]	[-130,-10]	[-13,-0.86]
C	A	[5,7]	[100,120]	[85,90]	[3,4]	[175,210]	[70,80]	[-110,-13.75]	[-22,-2.75]
D	B	[6,9]	[150,200]	[65,70]	[2,5]	[200,250]	[75,85]	[-100,0]	[0,20]
E	C	[4,6]	[100,110]	[60,70]	[2,3]	[120,150]	[75,80]	[-50,-2.5]	[0.5,10]
F	C	[6,8]	[125,155]	[75,80]	[3,5]	[185,215]	[85,90]	[-90,-6]	[2,18]
G	B	[6,8]	[120,145]	[85,90]	[3,4]	[200,230]	[75,80]	[-55,-11]	[-22,-3.67]
H	D-G	[3,4]	[50,75]	[85,90]	[1,2]	[100,135]	[75,80]	[-85,-8.33]	[-17,-1.67]
I	E-G	[4,6]	[100,150]	[60,70]	[1,3]	[150,175]	[75,85]	[-75,0]	[0,15]
J	H-I	[9,11]	[195,235]	[90,95]	[5,8]	[265,300]	[75,85]	[-105,-5]	[-21,-1.5]

Badanie efektywności programowania celów w zarządzaniu projektami. Zajęcia

NIROSHI PIRHAVEEN, VIJAYANATHAN SENTHOORAN

Regulacja celów, które są niezgodne z wykorzystaniem wszystkich powiązanych zasobów w organizacji, jest główną pracą zarządzania projektami (PM). W obecnych sytuacjach Kierownik Projektu jest odpowiedzialny za zarządzanie wieloma sprzecznymi celami, cele te są zwykle niejasne, ponieważ informacje będą niewystarczające i niedostępne. Na najwcześniejszym etapie decydentowi trudno było opracować plan skracający całkowity czas realizacji, minimalizujący całkowity koszt projektu i maksymalizujący jakość projektu. Celem tego badania jest porównanie kombinacji technik programowania z celami rozmytymi i programowania liniowego w szarościach oraz wieloobiektowego modelu programowania w szarości w kolorze całkowitym z wieloma celami rozmytymi (programowanie z celami rozmytymi), który jest stosowany do problemu zarządzania projektami. Niniejsze badanie zapewnia systematyczne ramy podejmowania decyzji umożliwiające rozwiązywanie problemów decyzyjnych PM mających wiele celów w niepewnym środowisku. Do porównania różnych technik w tym badaniu wykorzystano rzeczywisty przykład liczbowy. Ostatecznie wyniki wyznaczają najlepszą metodę, która daje najlepszą kombinację parametrów wykorzystywanych w działaniach projektowych. Wyniki pokazują, że model ten można wdrożyć jako skuteczne narzędzie.

Słowa kluczowe: żelazny trójkąt, kompromis czas-koszt-jakość, szare cyfry.