Advances in Science and Technology Research Journal

Advances in Science and Technology Research Journal, 18(6), 449–464
https://doi.org/10.12913/22998624/192639
ISSN 2299-8624, License CC-BY 4.0

Received: 2024.07.26
Accepted: 2024.08.15
Published: 2024.09.08

A Bipartite Graph for Estimating Measurement Uncertainty on Five-Axis Machining Centres

Roman Stryczek[1]

[1] Faculty of Mechanical Engineering and Computer Science, University of Bielsko-Biala, Willowa 2, 43-309 Bielsko-Biala, Poland

E-mail: rstryczek@ubb.edu.pl

ABSTRACT

The essential source of errors in machining on five-axis milling centres are errors caused by the improper calibration of a machine tool, setup errors and a process plan which is not designed optimally. For the multilateral machining of the complex, precise parts of machines, a process plan should include, before and during the process, coordinate measurements by means of a touch probe in order to verify previously made surfaces or to determine accurately the position and orientation of a local coordinate system. The uncertainty of these measurements is connected with the estimate precision of performing the manufactured parts. This paper presents a tool devised for determining the uncertainty of the results of coordinate measurements by a simulation method, which is also useful at the stage of machine tool calibration. A basis of this method is a bipartite graph with a tree structure. Transitions in the graph constitute a set of elementary measurement activities and analytical activities which determine the uncertainty of the position and orientation of respective geometric features and abstract objects. Based on the results of the conducted simulation tests it is possible to build analytical models for the rapid determination of measurement uncertainty. The tool devised is aimed at vector dimensioning and therefore it enables simple extension, including integration with geometric dimensioning and tolerancing. This paper includes an example of applying the method devised, which confirms its practicability.

Keywords: measurement uncertainty, calibration, simulation method, bipartite graph, setup data.

INTRODUCTION

A noticeable tendency in the new, innovative designs of numerically controlled machine tools of the 21st century is the increase of their functionalities as regards the possibility of producing increasingly more complex, asymmetrical parts of machines. This is achieved by combining several different treatments within one machining centre, including: turning, milling, drilling, grinding on flat and cylindrical surfaces, laser processing, heat treatment, measurement and control activities and lately also the varied types of incremental technology. Relatively recently the following notions have emerged: multi-material parts [1], hybrid manufacturing processes or hybrid machine tools. "Hybrid manufacturing processes are based on the simultaneous and controlled interaction of process mechanisms and/or energy sources/tools having a significant effect on the process performance" [2]. In the hybrid processing operations carried out on materials which are troublesome in processing, a conventional machine cutting may be assisted simultaneously by a different energy form, especially thermal or kinetic energy [3]. Wherever the share of processing time and material cost is high in the total product cost and manufacturing is of a single or small-series type, incremental-removal hybrid techniques become more and more meaningful. In addition, the product obtained may demonstrate better the operational features, as in the case of specialised tools [4]. Achieving these objectives requires, among other things, the structural extension of machine tools with additional numerically controlled axes, additional part and/or tool spindles or additional subassemblies for tools storage and replacement. Such a dynamical development of the new, increasingly more complex configurations of machine tools and the concepts of manufacturing operations facilitates their higher productivity; however, it creates new challenges concerning their designing process.

The effective operation of multi-functional machine tools requires integrating a range of activities of a designing-operating-diagnostic type. What must be mentioned here is the interoperability, openness, and flexibility of next generation of Computer Numerical Control in order to adapt with Industry 4.0 [5], computer assisted generation of operations for CNC (CAM) machine tools, post-processors considering the actual possibilities of the machine tool, a technique of controlling the machine tool movements at high machining parameters, including not only the speed control of a tool movement along the designed trajectory but also the analysis of the first and second degree accelerations, the intelligent compression of motion geometry in subsequent program blocks, the prediction of self-induced oscillations, the compensation of errors evoked by a tool deformation or the automatic exclusion of a collision. Plenty of these activities may be verified in off-line tests [6]. Currently, the virtual reality enables the construction of extremely accurate simulation models which allow to exclude errors in the control program, at the same time contributing to better use of a machine tool potential, work safety, the reduction of breaks in work, better use of the possibilities of machine cutting tools, the elimination of manufacturing rejects and the improvement of the quality of the products which are manufactured.

The accuracy of cutting machine tools in operating conditions is a critical element of their competitiveness and application effectiveness. For this reason, geometric error measuring, modeling and compensation has been a very important topic in mechanical engineering [7]. The issue of machine tool calibration was extensively presented in the work of Gao et al. [8]. It presented, among other things, the latest achievements in artificial intelligence and machine learning in machine tool calibration. One of the main goals of machine tool calibration is to predict whether the machine is capable of producing a workpiece within specified tolerances, for example through virtual machining simulation. Uncertainty is the most important parameter for assessing reliability machine tool calibration results. For this purpose, appropriate tools are necessary to model measurement uncertainty in the calibration process.

The high accuracy and repeatability of machine tool positioning is extremely difficult to obtain. It may be increased, among other things, by introducing innovative design solutions, program functions dedicated to the improvement of accuracy, diagnosing, identification, supervision and error reduction. All these influential factors must be developed further in order to enable reliable production with increasingly higher precision. Even the best design solutions usually do not allow to achieve required accuracy in the event of manufacturing precise machine parts. In addition, increasing requirements concerning machine tool accuracy are connected with the exponential growth of the cost of manufacturing such machine tools. Each machine tool structure is a certain compromise between its accuracy and the cost of its manufacturing. Taking the above dependences and limitations into consideration, what is more appreciated now is the improvement of the accuracy of the manufactured parts, not only by increasing the accuracy of the machine tool but also by increasing the share of measurement activities in the implemented machining, enabling the compensation of machine tool errors.

Structural requirements concerning the reliability, usage safety and functionality of newly designed mechanisms cause that the necessary accuracy of part geometry is hard to achieve with the use of classic manufacturing processes. The structural optimisation of operations, which considers higher share of control-measurement activities, is required. In particular, it is necessary to devise and popularise the methods and dedicated software, including three dimensional relations between manufacturing tolerances and functional requirements of the manufactured machine parts [9]. The works aimed at the above are highly advanced due to the GD&T (Geometric Dimensioning and Tolerancing). The GD&T is a system for defining nominal 3D geometry with consideration of annotations concerning the permissible variability of the shape of respective features contained in the model and for defining permissible orientation variability and positioning between the features. At the same time, there are no fully integrated, user friendly, highly comprehensible and applicable – already at the stage of process designing – methods and tools for analysing the accuracy of detail machining, with consideration of the unavoidable, actual values of machining and measuring errors. This aspect should be addressed as part of the fourth industrial revolution.

THE IDENTIFICATION AND COMPENSATION OF MACHINING ERRORS

The compensation of machine tool errors is a more and more meaningful element of improving the quality of the manufactured machine parts, also because the development of a measurement technique contributed to the increase in the availability of precise and easily applicable measurement devices. In the event of fine machining and ultra-precision machining, compensation is generally necessary and it is the best tool improving the quality of the workpieces made. The fine machining, in its essence, comprises an array of techniques, including precision machining, electroplating, bead blasting, polishing, anodizing, powder coating, sandblasting, painting, chemical removal (etching), electrochemical removal, wire-cut electrical discharge machining and die sinking, hard machining, high-speed cutting, honing, lapping, ultrasonic lapping, scraping, grinding, vibratory finishing. What is crucial here is refined offline software which assists a user in the classification and assessment of errors and the development of the contemporary systems of numerical control, adjusted functionally to online compensation in real time. Algorithms considering corrections may currently act inside the interpolation cycle of a controller in order to adapt the programmed and real location and orientation of the tool tip. In the last 20 years, measures for diagnosing machine tool accuracy have been popularised and a range of methods has been developed for forecasting and improving the accuracy of the manufactured machine parts, based on the results of virtual simulation tests.

Error compensation is a process usually consisting of a few stages. The last of them is correction consisting in including a correction in the programmed tool position and/or operation setup data, i.e. location, connected with the workpiece, an origin point of the local coordinate system or cutting tool dimensions. The majority of the authors of publications concerning the above subject do not differentiate between these two notions, using mostly the term of error compensation, which may lead to the ambiguity of a statement. A problem with the precise determination of a coordinate system and considering its impact on the machining effects has not been noticed for many years in research publications. No sooner than in work [10] this fact was addressed with proper importance. The contemporary techniques of coordinate measurements on NC machines enable defining a local coordinate system connected with the workpiece (WCS) in relation to the already existing part surfaces or those machined later, at the same time limiting transfers and avoiding too restrictive manufacturing tolerances.

A high number and variety of error sources and mutual intermingling of their effects causes that their identification and compensation is a highly complex aspect. All the publications address this aspect only in terms of selected impact factors or solving specific problems, such as: the problems of measurement uncertainty, temperature compensation, machine tool calibration, the determination and correction of a volumetric error, etc. So far, the majority of the integrated models of geometric errors of five-axis machine tools ignores the impact of backlash, which affects significantly the machining accuracy of workpieces with complex geometry [11]. Currently, there are no complete solutions available since the level of knowledge and technical possibilities are always limited to a certain degree and, in addition, they must be confronted with an economic account.

This paper focuses on two selected categories of errors, affecting the accuracy of the manufactured machine parts on 5-axis machine tools. These are errors resulting from the imperfect configuration of the kinematic chain of the machine tool and setup errors. Errors in these two categories occur in each situation because measurement errors occur in each situation. Based on such measurements, setup values are determined. Nevertheless, their identification for multi-axis machine tools is not easy although these are systematic errors.

The machining inaccuracy of machine parts on multi-axis machine tools results only partially from the inaccuracy of the machine tool. The crucial factors affecting a performance error include basing and mounting errors, the errors of the machining itself or the errors of measurements carried out before, during and after machining. In order to identify and estimate the impact of respective factors, it is possible to apply a modelling technique. This paper includes the author's own approach to the structure of the model of machining errors propagation which allows, by simulation methods, to estimate error volumes, test different variants of manufacturing operations and identify the most important factors affecting the cumulative error.

THE EVALUATION OF MEASUREMENT UNCERTAINTY ON THE CNC MACHINE

In the mechanical engineering industry, a basic measurement technique is currently a coordinate technique, which is implemented mostly on coordinate measurement machines and measurement arms but also more and more frequently directly on numerically controlled machines equipped with measurement probes. The objectives of using measurement probes on machining centres include:

- the detection of the presence of parts, the measurement of the machining allowance volume or the detection of its lack;
- increasing the level and scope of operation automation, the automatic identification of machine tool accuracy errors and their automatic correction;
- automatic centring, edge scanning, the measurement and/or correction of coordinate system, the possibility of eliminating the initial operation of tracing workpiece before machining;
- the automation of the compensation of tool dimensions;
- the automatic measurement of machining features directly on the machining centre, before, during and after machining;
- the comparison of measurement results, alarming about the occurrence of a manufacturing shortage, generating quality reports of machine parts manufactured in series;
- the improvement of manufacturing efficiency, quality, the reduction of defective parts;
- the reduction of the number and the accumulated time of machine tool downtime, the improvement of the OEE index (Overall Equipment Effectiveness) in the manufacturing station;
- the limitation of a human error, admitting a worker with lower qualifications for operating the machine tool;
- the calibration of the machine tool, increasing the accuracy of machine tools;
- the reduction of auxiliary times on the machine tool;
- the elimination of the necessary use of highly accurate machining tools;
- the verification of tool data and condition;
- the verification of the position of a mounting tool;
- digitalisation by scanning the machined workpieces;
- increasing manufacturing flexibility, enabling deeper parametrisation of machining program; the automatic selection of a machining variant or required change of tool movement trajectory;
- other depending on the configuration and special functions of the machine tool.

The measurement of the point coordinates is a basis of a coordinate measurement technique. It must be assumed that the probe was calibrated in the conditions of the thermal stabilisation of the machine tool and during the implementation of a calibration procedure and a measurement procedure the spindle is blocked. Probe calibration before a proper measurement enables:

- setting the radial eccentricity of the probe,
- aligning the radius of a measuring tip,
- length adjusting by means of an internal or external length pattern,
- determining the respective switching points in relation to the spindle centre in each of five axis directions (+X, -X, +Y, -Y, -Z).

Taking the above comments into account, three most significant factors, which affect the measurement uncertainty of the point coordinates on the numerically controlled machine, are as follows:

a) uncertainty u_1 of the axis location in relation to the indications of the machine tool measurement system,
b) uncertainty u_2 resulting from the measurement probe accuracy,
c) uncertainty u_3 resulting from the non-perpendicularity of the measurement probe movement in relation to the measured surface.

Formulas for the distance of a point from a point, a point from a straight and a point from a plane are used mostly for the construction of cooperate measurement models. By modelling the coordinate measurement uncertainty w = {x, y, z} of the measured point, it is necessary to note a measurement movement direction, a measurement movement velocity and even the length of a spindle in the measurement probe. The number of impact factors is obviously much higher. For instance, it is possible to consider the determination uncertainty of a measuring tip radius, the determination uncertainty of a time delay in a measurement chain and further. Assuming that measurement uncertainty components are at least one size row smaller than the components mentioned in points a, b and c, their impact on the final result is lower than 1%, therefore, it may be neglected.

Uncertainty u_w of determining a coordinate related to a direction, in which the measurement movement was performed, should include all the above mentioned three independent components of uncertainty, therefore, a proper dependence for its determination is:

$$m = a \cdot b \int_0^h \rho_w \theta(h) dh \quad (1)$$

where: $u1$, $u2$, $u3$ as above.

The remaining coordinates are burdened only with uncertainty u_1 resulting from the reading error of machine tool location. Hence, the estimated area, which contains a measured point in the form of an ellipse, may be presented graphically (Figure 1). The probe allows for determining dimension deviations and geometric deviations of: shape, direction, location or pulsation. The coordinates of pre-determined points and their count affect the result of a measurement by a coordinate technique and, in consequence, the uncertainty of these measurements. It is possible to apply a strategy of a minimum point count, which is necessary for determining a given geometric form or increasing the count of measurement points, thus limiting measurement uncertainty but, at the same time, increasing measurement performance time. For instance, an opening (opening diameter and centre) may be determined by a 3-point, 4-point or 8-point method, which allows – in the last case – also to determine a directional vector of its axis. A plane may be determined by measuring 1 point and assuming that it is parallel to a proper operating plane XY, YZ or ZX, based on 3 points or a point cloud with the use of, for example, RANSAC method [12]. RANSAC (RANdom Sample Consensus) algorithm is an iterative method used to estimate sought parameters of the mathematical model of object based on a redundant set of data points, forming a cloud around the determined area. A corner may be determined by measuring 3 points or based on three non-parallel planes, that is a significantly more accurate 9-point method. More complex measurements, e.g. the parallelism of an opening axis in relation to the plane, require using 9 or 11 measurement points. Therefore, there is a question about the uncertainty of thus determined forms, location deviations, etc.

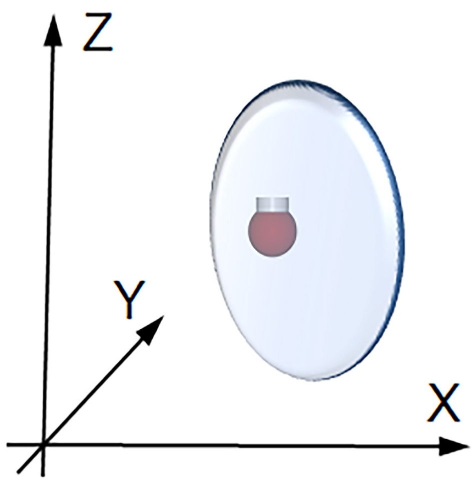

Figure 1. Point measurement uncertainties in the 3D area

Coordinate measurement uncertainty depends primarily on the measurement tool used, environmental conditions, the object of tests, the type of the deviation measured and the measurement strategy applied, which includes the number and distribution of respective measurement points, the number of repeating measurements in a given point, etc. What is also crucial here is the applied measurement cycle and its adaptation to the probe used. Measuring the dimensional and shape accuracy of workpieces using a touch probe on machine tool is negatively affected by the geometric accuracy setting, thermal stability of the machine tool and its surroundings, type of scanning system [13, 14].

There are a few methods of determining coordinate measurement uncertainty among which the following methods are worth particular attention according to ISO/TS 15530-1:

- a method with the use of a calibrated part,
- a sensitivity analysis method,
- a simulation method.

The best known and elaborated method with the use of a calibrated part [ISO/TR 15530-3] is the simplest and most effective method. An experimental part consists in the multiple measurement of the calibrated workpieces which may have the form of a manufactured part, pre-measured carefully. A calibrated workpiece may be used simultaneously for supervising a machine tool and correcting its systematic errors by using comparative measurements.

This method may be applied effectively in serial production. In the case of unit production or small series production we often do not have a proper model of the manufactured machine parts.

A sensitivity analysis method is a universal method but it is regarded as a complex method because it requires a high number of sampling points, the use of complex measurement models and the knowledge of the geometric errors of a machine tool. This method concerns measuring dimensions and geometrical deviations. Measurement models express the measured characteristics as a function of differences of coordinates of a small number of appropriately selected points of the workpiece. In recent years, research has been done which facilitates using this method by interested users who use measurement machines [15, 16].

The Monte Carlo (MCM) simulation method of distribution propagation by random sampling from probability distributions is a method recommended by the JCGM organisation (Joint Committee for Guides in Metrology). The precursor of using the MCM method was a representative of Lvov school of mathematics in the inter-war period, S. Ulam. MCM is used for mathematical modelling of processes that are far too complex to be able to predict their results using an analytical approach. Sampling according to the selected distribution of values characterizing the process plays an important role in MCM. After collecting a sufficiently large amount of such information its characteristics can be compared with the observed experimental results, confirming or denying the validity of assumptions made in the entire procedure. The accuracy of a result obtained by this method depends on the number of checks and the quality of the random number generator. The predominance over other methods is noticeable especially when model linearisation ensures insufficient representation, a probability density function for an output volume deviates significantly from the Gauss distribution, e.g. due to clear asymmetry, component uncertainties are not proximate to the same value, providing component derivatives is difficult or inconvenient and a high model complexity is present [JCGM 101/2008]. The Monte Carlo method is a universal and intuitive method. It gained popularity although it needs specialised software [17]. The main component of measurement uncertainty is determined based on the series of virtual measurements. A simulation outcome is a large set of the resulting implementation of a random variable, based on which it is possible to determine the empirical distribution, random variable characteristics, such as mean, standard deviation or quantiles, which divide a set into specific, equal parts in terms of quantity [18]. In order to conduct simulation experiments, it is necessary to devise a model referring to input and output values and to determine a probability density function (PDF) which characterises input values. This approach is applied with any models which have a single output value Y, in which input values are characterised by a unique, defined PDF (Figure 2).

In general, a mathematical model of the measurement of a single scalar value may be expressed as dependence:

$$Y = f(X) \tag{2}$$

where: Y is a scalar output value, and X means N input values $(X_1,.., X_N)^T$.

A random variable is a function which assigns figures to elementary incidents (in this case, these are measurements). Each X_i is treated as a random variable with possible values ξ_i and expected value x_i. Similarly, Y is a random variable with possible values η and expected value y. The PDF is a derivative of distribution function g_x (3):

$$g_x(\xi) = {dG_x(\xi)}/{d\xi} \tag{3}$$

In particular, for normal distribution, the PDF has the following form (4):

$$g_x(\xi) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\xi-\mu}{\sigma}\right)^2\right] \tag{4}$$

where: x – input value in the model, σ – standard deviation of random variable, μ – average value, ξ – measurement result.

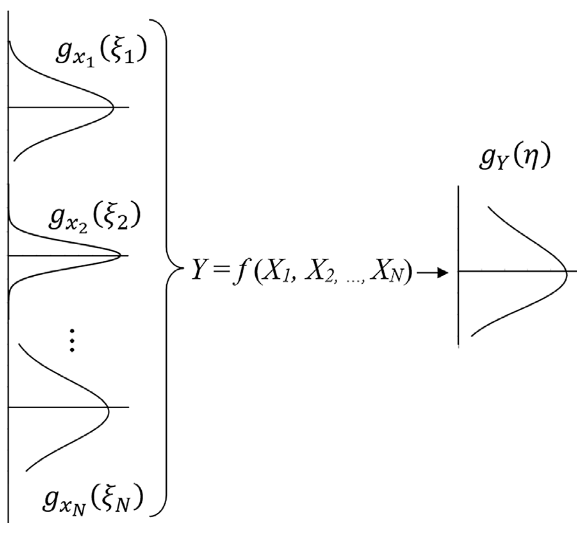

Figure 2. Distribution propagation for N independent input value

The following stages of uncertainty assessment are differentiated:

1) Model formulation:
   - determine an output value (measured) Y;
   - determine input values $X = (X_1,.., X_N)$, which determine Y;
   - devise a model referring to Y and X;
   - based on the available knowledge, assign PDF files to X_i – Gauss (normal), rectangular (uniform), etc. For X_i, which are not independent, a joint PDF must be assigned.
2) Propagation: PDF propagating for X_i by means of a devised model in order to obtain PDF for Y;
3) Summary: using the obtained PDF for Y, we obtain:
   - expected value Y, assumed as estimated value y;
   - standard deviation estimator Y is assumed as standard uncertainty u(y), determined by dependence (5) [JCGM 101:2008];
   - coverage intervals including Y with specific probability.

$$u(y) = \sqrt{\frac{\sum_{r=1}^{M}(y_r-\mu)^2}{(M-1)}} \quad (5)$$

where: y_r – result of r-sample in MCM cycle, μ – mean from all the samples, M – number of MCM cycles.

A computer is not able to generate random numbers. Instead, a computer generates number sequences which approximate very well the sequences of random numbers. Therefore, in the case of a "random number" generated by a computer, we should talk rather about pseudo-random numbers. Pseudo-random numbers generated by a computer belong to the constant distribution. Hence, some methods are needed to obtain a sequence of numbers with a desired distribution, e.g. normal, from the constant distribution of pseudo-random numbers. These are not precise methods but the approximations are precise enough to apply them e.g. in simulation techniques. One of the recommended methods of generating a normal distribution is Box-Müller's method. One of the solutions in this method is dependence (6):

$$X_i = \mu + \sigma\sqrt{-2\ln(U_1)}\cos(2\pi U_2), \quad (6)$$

where: X_i – variable from the normal distribution, $U1$, $U2$ – independent pseudo-random variables from the constant distribution.

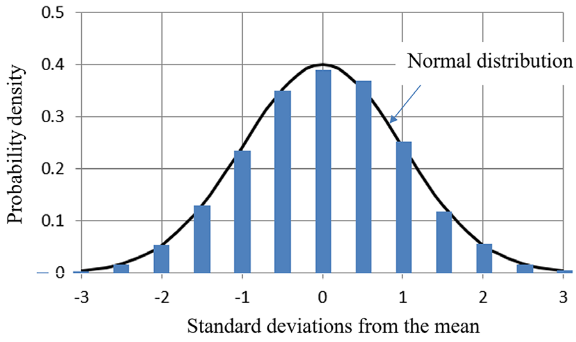

Figure 3. The test results of generating a normal distribution by Box-Müller's method

Figure 3 illustrates a test result histogram (3000 samples) by Box- Müller's method as compared to a standard normal distribution N(0, 1), confirming the correctness of such an approach.

Figure 4 presents the simulation methods made by the MCM method for two random variables described by normal distribution N(0, 1), N(0.5, 0.5). A simple model was used as the sum of two input values. A result is a random variable with expected value y of 0.502 and standard deviation of 1.117. In theory, measurement uncertainty for the above model and data is √1.25 = 1.118. Output value Y was determined based on 3000 virtual samples. The results confirm the intuitive assumptions that variable X_1 will have a significant impact on the standard deviation of resulting values, whereas variable X_2 will dominate an expected value y.

The value of extended uncertainty U may be determined directly from $g_y(\eta)$ after rejecting 5% of values most differing from the mean. 95% of values η is included in the interval [-1.658, 2.663]. As far as the conducted test is concerned, U equals 4.321 µm.

A SIMULATION METHOD OF DETERMINING MEASUREMENT UNCERTAINTY IN THE MACHINE TOOL OPERATING AREA

What is needed for the purpose of conducting a simulation procedure of determining measurement uncertainty are dedicated tools allowing the user to construct a model and then, in the automatic mode, conduct and repeat the virtual tests of estimating measurement uncertainty for thousands of times. There are professional tools available for this purpose but at the current stage

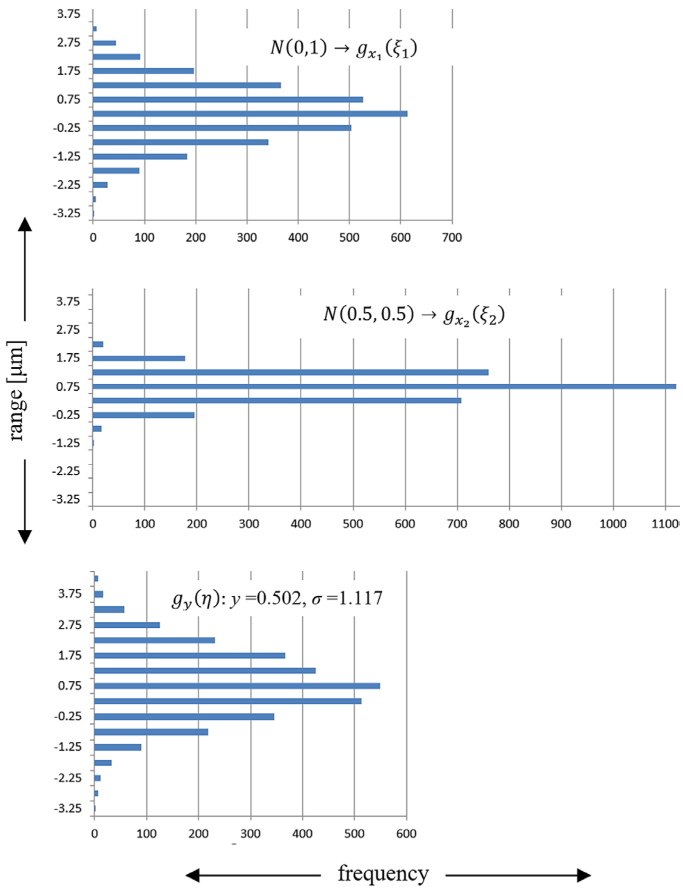

Figure 4. Simulation result for the sum of two random variables

of development their availability, flexibility and usefulness for experiments is limited. Hence, researchers search for alternative solutions in the form, for instance, of universal spreadsheet [18]. A prospective user, who wants to implement a simulation method in a workshop practice, as far as this extremely laborious approach is concerned, at each time, must have high competences. Therefore, a prototype of a computer calculation system is presented below which is based on a network method and which is intended for assisting the determination of geometric measurement uncertainty by means of the Monte Carlo method.

Model representation by means of a graph has been for many years an attractive engineer tool for analysing modelled systems. A graphic form of this model enables the user to familiarise faster with its structure, content, complexity, topology, scope and completeness. A graphic interface of the user of contemporary personal computers allows the user to familiarise immediately with the available functions of modelling tools. Furthermore, a graph structure, as in semantic networks, enables embedding objects present in a graph and analyse them in a proper context.

Propagation is a notion widely applied in physics, where it generally means the spread of a disturbance in the centre. It may refer to radio waves, light or mechanical oscillations. Propagation, in the context of coordinate measurements, means error transferring in measurements. Since the results of direct measurements are burdened with uncertainty, measurement errors propagate to each indirect value calculated from the model.

The propagation graph of coordinate measurement uncertainty is a bipartite graph, directed, which on the grounds of the set theory may be defined in the form of ordered four:

$$\check{G} = (\Pi, T, E, M), \quad (7)$$

where: Π – nonempty definite set of basic elements (places): Π = X∪Y∪Z∪P∪V∪R, where: $X = \{x_i\}$, where $x_i = x(x_i, u_{xi})$: coordinates measured in axis X; $Y = \{y_i\}$, where $x_i = y(y_i, u_{xi})$: coordinates measured in axis Y; $Z = \{z_i\}$, where $x_i = z(z_i, u_{xi})$: coordinates measured in axis Z; $P = \{p_i\}$, where $p_i = p(x_i, y_i, z_i, u_{xi}, u_{yi}, u_{zi})$: measurement points; $V = \{v_i\}$, where $v_j = v(i_j, j_j, k_j, u_{xj}, u_{yj}, u_{zj})$: vectors, including directional versors (unit vectors); $R = \{r_i\}$, where $r_i = r(r_i, u_{ri})$, constant parameters of a real type; T – nonempty, finite, separate from P selection of transitions of the following type: *meas*, *point*, *line*, aline, vector, unit_v, circle, middle, plane, frame, displace, ratio, project, per_proj, etc., $\Pi \cap T = \emptyset$, $\Pi \neq \emptyset$, $T \neq \emptyset$, Ø: empty set, $E \subset (\Pi \times T) \cup (T \times \Pi)$: incidence relation, $M \subset M' \cup M_0$: set of markers.

A bipartite graph is a graph with two separate sets of apexes. In the case of Ğ, these include the set of basic elements Π and the set of transitions *T*. By means of basic elements, we may define abstract objects such as axis, coordinate system and physical objects, e.g. a line (edge), plane, etc. Transitions represent the dynamic elements of a model. Their role, apart from determining the proper sequence of measurement activities, is the performance of calculations and the determination of the uncertainties of subsequent elementary objects. Basic measurement transitions meas generate measurement point coordinates and uncertainties assigned to the respective coordinates of such points. A transition sequence, meas and displace, results in the displacement of a proper measurement coordinate with a constant value of parameter *r*. The transition of a vector type determines the components of a section between two known points. A point with a lower index is accepted as a starting point. The transition of a middle type determines the centre of a section between two known points. The calculation transition of a ratio type is a product of section components and the number. It is used, for example, when it is necessary to determine the unknown sides of a rectangular triangle.

The transition of a point type also determines one point, depending on a context, this is a point of intersecting the plane with a line which is not parallel to this plane, the centre of a hole or a cylinder determined by means of a 4-point method or the centre of a ball determined by means of a 5-point method, with the known ball radius. The transition of a line type is also present in variants. In the basic variant, it determines a line based on two points; in the second variant, it determines a line as an intersection of two nonparallel planes. The basic variant of the transition of a plane type determines a plane in the area based on three points. The alternative determination of a plane is possible based on one point contained in this plane and a line perpendicular to the determined plane or by means of a multi-point method.

The transition of a frame type determines the position of a coordinate system based on one known operating plane, one line and point. This enables eliminating six freedom degrees from the workpiece, at the same time determining clearly its location. A transition variant binding in a given node results from a context in which it is placed in the model, strictly speaking, from directly preceding transitions. Graph Ğ is a graph with a tree structure, therefore, information flow is unidirectional and there are no cycles. When determining, for instance, the system of coordinates, it is sufficient to indicate, by an arc, only a directional vector of an operating plane, a directional vector of a line and one point necessary for determining the starting point of the system in the third axis. The remaining necessary two points are found automatically by the system by back tracking. By using elementary objects, such as a point, line and plane, it is possible to programme more types of transitions implementing the procedures of determining uncertainties, for example, location deviations.

Incidence relation *E* in the graphic interpretation is a set of directed arcs. Since Ğ is a bipartite graph, these arcs may be divided into input arcs for transition and output arcs from transition. Transitions without input directed arcs are performed as the first ones in the subsequent simulation cycle. Transitions with input directed arcs require confirming all the locations, which are input positions of a given transition, with a marker. It means that the values for these locations were previously generated in a given simulation. The set of markers *M* consists of a starter set M_0 and a dynamic set o M'. M_0 is a set of markers assigned to locations which are not activated by any transition. This set is defined in input data and is not changed during subsequent simulation cycles. Set M' is generated during network processing. This set is at each time removed after finishing another simulation cycle. The set of markers controls the sequence of performing respective transitions. Firstly, only measurement transitions are performed. A marker which settles in a given location, remains there until the end of a current simulation cycle. Each transition in a single simulation cycle is performed only once (Table 1).

Table 1. The elements of the propagation graph of coordinate measurement uncertainty and corresponding assertions in the model data base

Element	Graphic symbol	Assertion
Place	○	p(*n*, *x*, *y*, „p", *X*, *Y*, *Z*) p(*n*, *x*, *y*, „v", *I*, *J*, *K*) p(*n*, *x*, *y*, „d", X, Y, Z) p(*n*, *x*, *y*, „x", *X*, *Y*, *Z*) p(*n*, x, *y*, „y", *X*, *Y*, *Z*) p(*n*, *x*, *y*, „z", *X*, *Y*, *Z*) p(*n*, *x*, *y*, „r", R_X,R_Y, R_Z)
Transition	▭	t(*n*, *x*, *y*, *meas*) t(*n*, *x*, *y*, *point*) t(*n*, *x*, *y*, *middle*) t(*n*, *x*, *y*, *displace*) t(*n*, *x*, *y*, *line*) t(*n*, *x*, *y*, *plane*) t(*n*, *x*, *y*, *frame*) - - - - - - - - - -
Input arc Output arc	→	pt(*p*, *t*) tp(*t*, *p*)
Marker	●	m(*p*)

Note: *n* – location/transition number; *x*, *y* – coordinates (in pixels) of the graph apex on the graphic sheet; *X*, *Y*, *Z* – the measured location coordinate values; X_e, Y_e, Z_e – expected values; u_x, u_y, u_z – point location uncertainty in the directions X, Y, Z; *I*, *J*, *K* – directional versor components; u_I, u_J, u_K – determination uncertainty of the components *I*, *J*, *K*; *R* – real number; u_r – uncertainty of value *R*; *p*, *t* – indices of the basic elements of transitions.

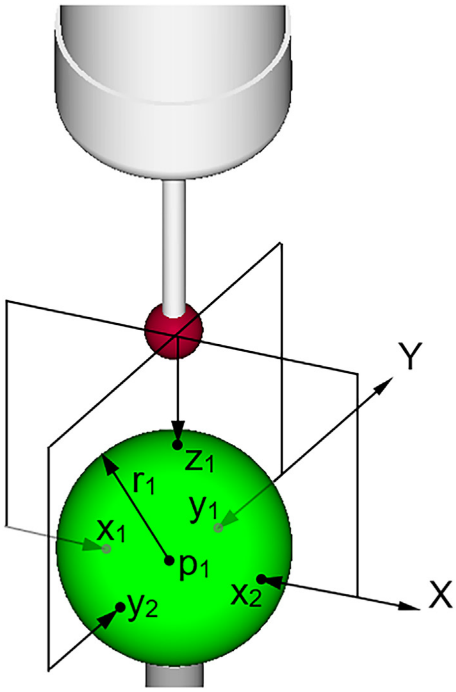

Figure 5. The trajectory of measurement movements during determining a ball centre

In the event of normal distribution, the uncertainty of the determined value u_i is equivalent to standard deviation.

Determining the centre of the calibration ball

Figure 5 presents the scheme of determining a ball centre by means of a coordinate method. In order to fulfil these tasks, it is necessary to pre-calibrate precisely the measurement probe and to have an accurate spherical model. Firstly, on the basis of the determined four coordinates x_1, x_2, y_1, y_2 (Figure 5), it is possible to calculate an intersection point of a ball axis, perpendicular to the plane XY. Secondly, we measure the coordinate z_1 on this axis. At the end, considering the spherical model radius r_1 we determine the last coordinate of point p_1 in axis Z. Figure 6 presents a proper graph for determining the uncertainty of such a measurement on the basis of the above mentioned method.

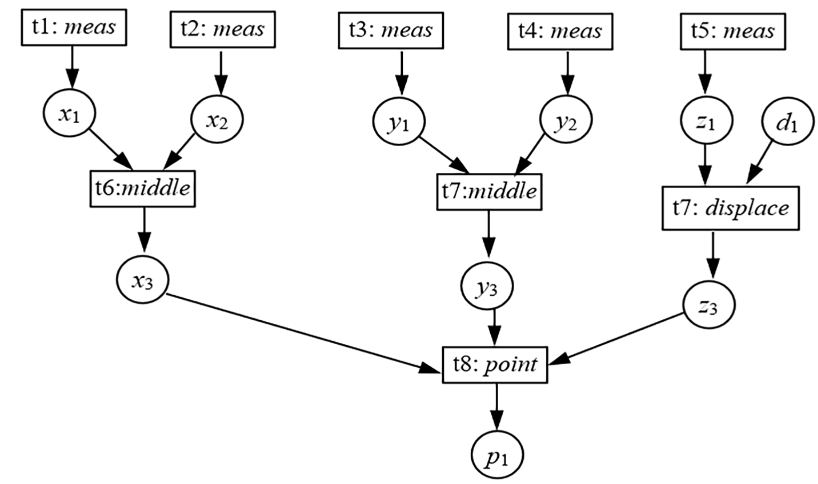

Figure 6. Ball centre determination graph

The determination of rotary axis locations

The correct calibration of rotary axes in the 5-axis machining centre constitutes a basis for precise, multi-axis machining. In order to obtain correct data for the rotary axis calibration, i.e. its location and orientation in the MCS system, it is possible to apply a method based on determining three points which are the centre of a precise spherical model in three locations, obtained only by means of rotating the determined axis. These points are located in a circle whose centre P_{ax} is in the determined axis. The orientation of axis V_A may be represented in the form of a normal versor of the plane defined on the basis of pre-determined three points P_1, P_2 and P_3. The example presented below, concerning the determination of an axis, refers to a five-axis machine tool with the P type kinematics, that is equipped with a rotated table.

Figures 7 and 8 present the location of a spherical model for determining axis A and C. Figure 9 presents a sketch based on which analytical dependences were determined, enabling the calculation of an attachment point and a directional vector of a defined axis. Figure 10 includes a measurement model in the form of a bipartite graph.

Firstly, we determine the coordinates of point P_4 (8, 9, 10), which is the centre of a section joining points P_2 and P_3 (Figure 9). Then we calculate the length of the section components from P_3 to P_4 (11), represented in the graph as the components of vector d_2. Based on d_2 and the known value of angle α, we determine arc radius R (12), in the graph represented by the components of section d_3. In order to determine v_1 containing the components of the directional vector I_O, J_O, K_O (14, 15, 16) of a section from P_1 to P_4, we calculate the length of this segment (13). Then, we determine the components of vector d_4, which are the projection of radius R on the direction v_1. Coordinates X_O, Y_O, Z_O, of the centre of arc P_O

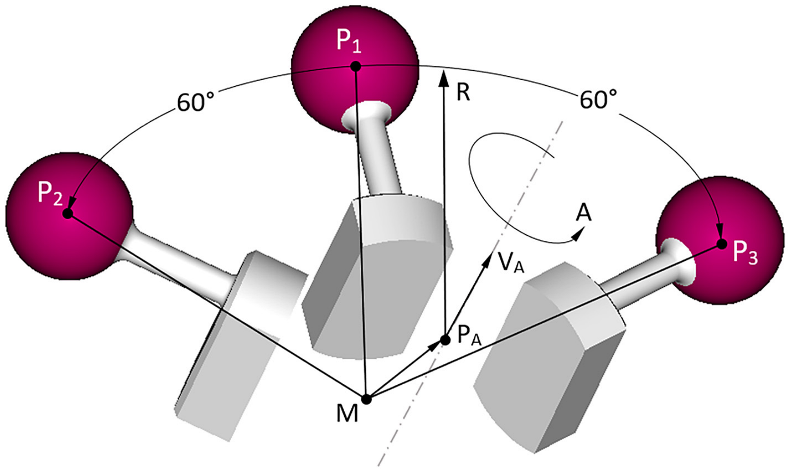

Figure 7. A pattern for determining axis A location and orientation

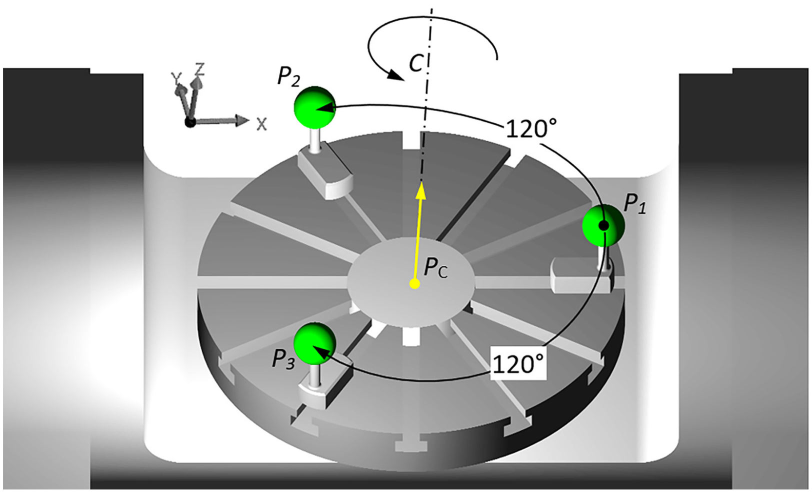

Figure 8. A pattern for determining axis C location and orientation

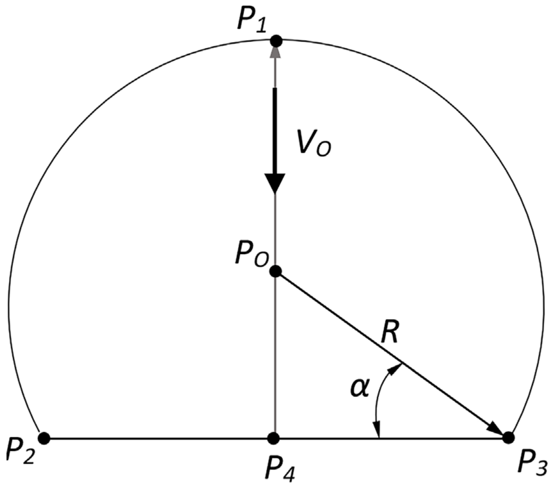

Figure 9. Determining the directional vector of the rotary axis A and the attachment point of axis A in the MCS system

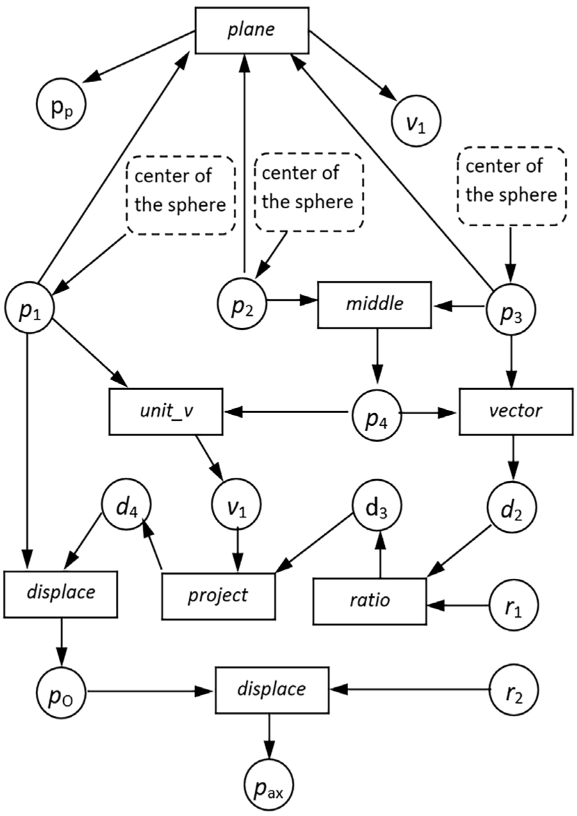

Figure 10. A graph for determining the attachment point of axis P_{ax} and the components of the axis directional versor in the MCS

may be determined as the displacement of point P_1 with the vector d_4. In the event of determining the location of axis C it is advantageous to move the determined attachment point with the constant r_2, pre-determined precisely as the distance of the centre of the calibration ball from the surface of the rotary table.

$$X_4 = (X_2 + X_3)/2 \quad (8)$$

$$Y_4 = (Y_2 + Y_3)/2 \quad (9)$$

$$Z_4 = (Z_2 + Z_3)/2 \quad (10)$$

$$\sqrt[2]{(X_3 - X_4)^2 + (Y_3 - Y_4)^2 + (Z_3 - Z_4)^2} \quad (11)$$

$$R = \frac{\overline{P_3P_4}}{\sin\alpha} \quad (12)$$

$$\overline{P_1P_4} = \sqrt[2]{(X_4 - X_1)^2 + (Y_4 - Y_1)^2 + (Z_4 - Z_1)^2} \quad (13)$$

$$I_O = (X_4 - X_3)/\overline{P_1P_4} \quad (14)$$

$$J_O = (Y_4 - Y_1)/\overline{P_1P_4} \quad (15)$$

$$K_O = (Z_4 - Z_1)/\overline{P_1P_4} \quad (16)$$

$$X_O = X_1 + I_O R \quad (17)$$

$$Y_O = Y_1 + J_O R \quad (18)$$

$$Z_O = Z_1 + K_O R \quad (19)$$

Any plane Π in the three dimensional Cartesian area R^3 may be represented by a pair of vectors in the following form: location vector $V_p = [X, Y, Z]^T$ and directional vector (normal) $V_d = [I, J, K]^T$ (Figure 11). Vector V_p is attached at the origin of the reference system and it is finished at any point of the defined plane Π. Vector V_d is a unit vector perpendicular to plane Π, with components equivalent to directional cosines to the axis of the reference system. The plane may be determined based on three non-collinear points $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$, $P_3(x_3, y_3, z_3)$, belonging to this plane (Figure 11). While defining the vectors between points P_1 and P_2 as $V_{1\to2} = [a_x, a_y, a_z]$ and between points P_1 and P_3 as $V_{1\to3} = [b_x, b_y, b_z]$ and the vector product $V_{1\to2}$ $V_{1\to3}$ we obtain dependence (20), where the resulting vector is provided as a determinant of the formal matrix. Three vectors $V_{1\to2}$, $V_{1\to3}$, V_n are correspondingly oriented with the rule of a right-hand system.

$$V_{1\to2} \times V_{1\to3} = \begin{vmatrix} I_0 & J_0 & K_0 \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}, \quad (20)$$

Based on the matrix (20) we calculate the components I_0, J_0, K_0 (21, 22, 23) and the length of normal vector V_n (24)

$$I_0 = a_y b_z - b_y a_z \quad (21)$$

$$J_0 = a_x b_z - b_x a_z \quad (22)$$

$$K_0 = a_x b_y - b_x a_y \quad (23)$$

$$|V_n| = \sqrt{I_0^2 + J_0^2 + K_0^2} \quad (24)$$

Said components, after standardisation, are components I, J, K (25, 26, 27) of the directional vector of the searched plane Π.

$$I = I_o/|V_n| \quad (25)$$

$$J = J_o/|V_n| \quad (26)$$

$$K = K_o/|V_n| \quad (27)$$

The general equation of the determined plane has the form (28):

$$Ix + Jy + Kz + D = 0 \quad (28)$$

where:

$$. D = -Ix_1 - Jy_1 - Kz_1 \quad (29)$$

Usually, the designer of the machine tool assumes the intersection of axis A and C. In practice, as a result of unavoidable performance errors, these axes never intersect. The possible accurate determination of the mutual distance of axis A and C enables reducing calibration errors. For practical reasons, it is advantageous to place the origin of the MCS machine system on the axis A of the machine tool (Figure 12). Whereas, the BCS base system in relation to which the origins of coordinate systems are determined, connected with the WCS, for the same reasons, should be placed on the surface of the rotary table in its rotation axis. Therefore, after determining the attachment points of axis C, it must be projected on the table plane, then it is necessary to find a line perpendicular to axis A, going through such a determined point P_C. Point P_A of this line intersection with axis A determines the origin of the MCS system. The distance of P_A from P_C informs us about the errors of performing the rotated table; whereas, the uncertainty of determining this distance is valuable information concerning the calibration quality of rotary axes. A simple way to find point P_A is determining a plane containing point P_C with a normal parallel to the determined axis A with components a_X, a_Y, a_Z. An intersection point of thus defined plane with axis A provides us with a searched starting point P_O with coordinates X_O, Y_O, Z_O. Analytically, the above task consists in solving the system of Equations 30, which consists of parametrical equations of axis A and the equation of the plane perpendicular to axis A, going through point P_C. The last step is determining a distance between point P_A and point P_C. The uncertainty of determining this distance is the uncertainty of determining the location of the origin of the BCS coordinate system.

$$\begin{cases} X_O = X_A + a_X t \\ Y_O = Y_A + a_Y t \\ Z_O = Z_A + a_Z t \\ a_X X_O + a_Y Y_O + a_Z Z_O + D = 0 \\ D = -a_X X_C - a_Y Y_c - a_Z Z_c \end{cases} \quad (30)$$

Figure 13 visualises a graph implementing the end of the procedure of determining the relative location of axis A and C. Transition t_1 determines V_A as the normal vector of the plane perpendicular to axis A. Similarly, t_2 determines V_C as the normal vector of the plane perpendicular to axis C. Transitions t_7 and t_8 dislocate the attachment points of axis A and C in conformity with these vectors, to locations for which coordinate $x_A = 0$ and $z_C = 0$. Then, t_9 determines point P_O, which is a point of the intersection of axis A with

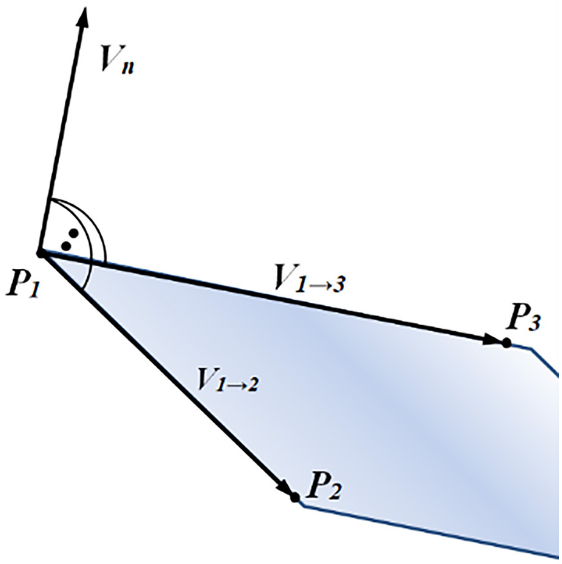

Figure 11. The determination of a normal vector of the plane based on three points contained in this plane

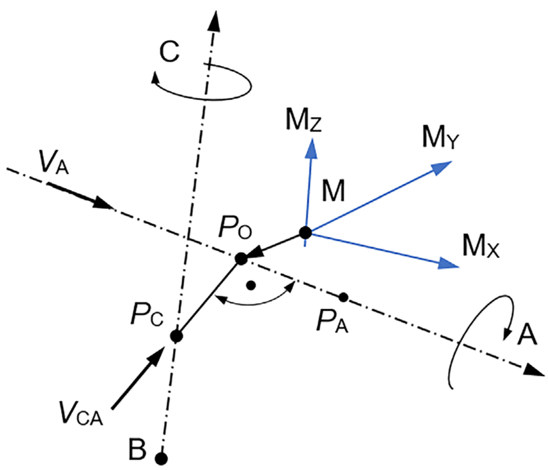

Figure 12. An example of the real location of the axis of the rotated table and machine point

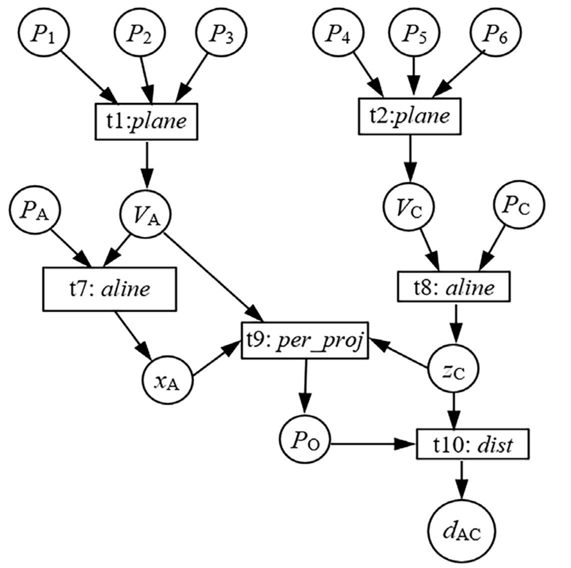

Figure 13. Graph for determining point P_O as the recommended origin of the machine coordinate system

the plane perpendicular to said axis and going through z_C. The last transition (t_{10}) calculates the components of section d_{AC} between points z_C and P_O. The length of this section constitutes a critical element of the assessment of geometric accuracy which is taken into account during machine tool calibration.

Test results

On the basis of the devised model of determining a distance of axis A and C, a series of simulation tests was conducted for 25 points of a decisive area. For each point, 1000 solutions were generated. On this basis, the standard deviation of the results obtained was calculated. The results of the simulation tests are presented in Figure 14. The obtained distribution of results was a basis

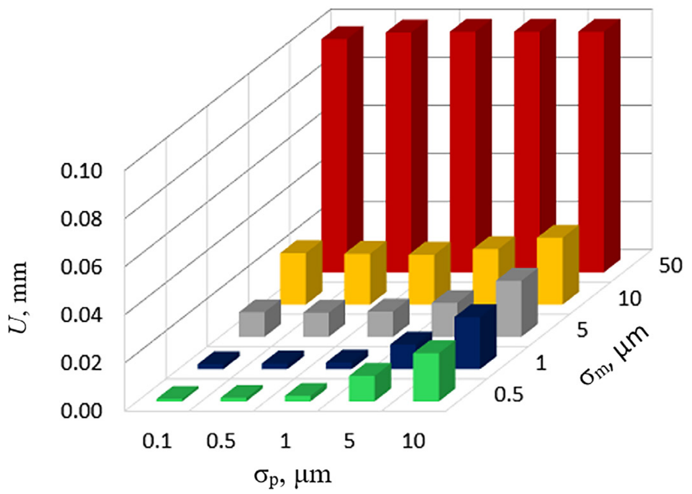

Figure 14. Uncertainty U of determining the location of the origin of the basic coordinate system depending on standard deviation for measurement probe σ_p and machine tool σ_m, based on simulation tests

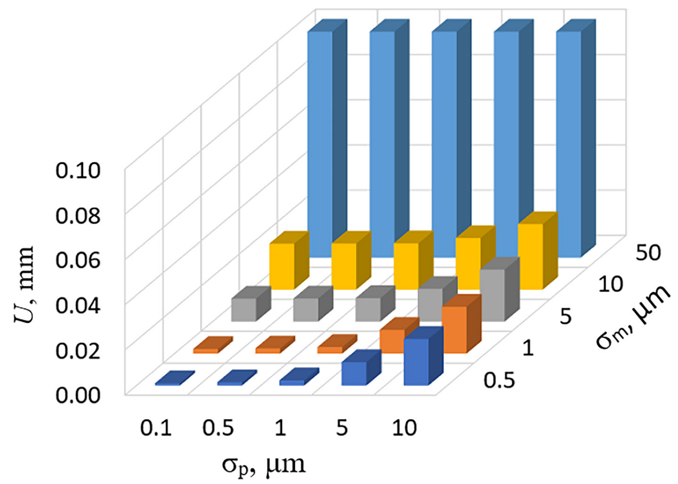

Figure 15. Uncertainty U of determining the location of the origin of the BCS coordinate system depending on standard deviation for measurement probe σ_p and machine tool σ_m, based on dependence (31)

for devising a mathematical formula based on which it is possible to determine quickly the uncertainty of the measured distance of axis A and C. The shape of a surface containing 25 generated points suggested an equation of 3-axis ellipsoid as a dependence basis (31). Constant parameters in (31) were obtained by a regression analysis method. The tests conducted demonstrated that the estimation error of measurement uncertainty based on dependence (31) does not exceed 5% for the entire considered area of tests, and this must be accepted as a satisfactory result. Figure 15 presents the results of calculating the uncertainty of determining the distance of axis A and C with the use of dependence (31).

$$U = 0.05999\sqrt{\frac{\sigma_p^2}{845} + \frac{\sigma_m^2}{868}} \tag{31}$$

CONCLUSIONS

The estimation graph of machining accuracy, presented in this paper, enables the construction of a model and its simulation tests in order to determine the uncertainty of coordinate measurements performed by means of a measurement probe on numerically controlled machines. The determined measurement uncertainties enable the estimations of machining accuracy, especially on machine tools with complex kinematics, including 5-axis milling centres. It is possible to enumerate a range of undeniable benefits of the method devised, among which the following ought to be underlined:

1. A graphical character allowing for the easy assessment of the model completeness and cohesion, the verification of its structure correctness, the generation of alternative models and archiving of the model documentation;
2. Intuitiveness allowing for the quick understanding of the proposed approach and learning it by process engineers and metrologists, what, in consequence, enables them to design and analyse easily the coordinate measurements and manufacturing procedures, their mutual relations and impact on the final accuracy of the machined parts.
3. This method may be learnt quickly owing to the proposed approach to vector dimensioning and determining geometric deviations.
4. A simulation mode of determining measurement uncertainty and estimating the accuracy of the machined features is consistent with the current recommendations of the JCGM.
5. The full usefulness both at the stage of the calibration of a multi-axis machine tool and at the stage of operation, enabling, through the estimation of the performance accuracy of the designed processing of machine parts, the selection of the optimal form of a manufacturing process.
6. The possibility of determining, already at the stage of designing operations, a rational compromise between the number of measurements (measured points) and the obtained dimensional accuracy.
7. Easy computer implementation. Owing to the definition on the ground of the theory of sets, this method may be implemented naturally in the declarative programming languages.
8. Enabling the generation of mathematical models for the quick estimation of measurement uncertainty and/or machining accuracy.

The proposed method has other prospective advantages which may result in the future from its integration with structural modelling, especially in the case of the construction of the GD&T models, as well as there is a prospective possibility of the automatic generation of measurement program based on the devised graph.

Acknowledgements

This study was carried out as part of the project High-precision, hybrid 3D printing centre in five axes with the synchronous milling of printed parts, supported by the Regional Operational Program of Silesia Voivodeship, from the European Union Funds for years 2014–2020.

REFERENCES

1. Hasanov S., Alkunte S., Rajeshirke M., Gupta A. Huseynov O., Fidan I., Alifui-Segbaya F., Rennie A. Review on additive manufacturing of multi-material parts: Progress and Challenges. J. Manuf. Mater. Process. 2022, 6(1), 4. https://doi.org/10.3390/jmmp6010004
2. Lauwers B., Klocke F., Klink A., Tekkaya A.E., Neugebauer R., Mcintosh D. Hybrid process in manufacturing. CIRP Annals 2014, 63(2), 561–583, https://doi.org/10.1016/j.cirp.2014.05.003
3. Klocke F., Brummer C. Hybrid Cutting. In: The International Academy for Production (eds) CIRP Encyclopedia of Production Engineering 2018, 1–4. Springer. https://doi.org/10.1007/978-3-642-35950-7_6408-3

4. Stryczek R. A metaheuristic for fast machining error compensation. J. Intell. Manuf. 2016, 27, 1209–1220. https://doi.org/10.1007/s10845-014-0945-0
5. Adam A., Sam T.-H., Latif K., Yusof Y., Khan Z., Ali Memon D., Saif Y., Hatem N., Ahmed M.I., Kadir A.Z.A. Review on advanced CNC controller for manufacturing in industry 4.0. In: Yusof, Y. (ed.) Enabling Industry 4.0 through Advances in Manufacturing and Materials 2022, 261–269, Springer. https://doi.org/10.1007/978-981-19-2890-1_26
6. Stryczek R., Szczepka W. Simulation tests of adaptive control strategies for CNC machine tools. J. Mach. Engin. 2019, 19(2), 73–82. https://doi.org/10.5604/01.3001.0013.2225
7. Zhang Z., Jiang F., Luo M., Wu B., Zhang D., Tang K. Geometric error measuring, modeling, and compensation for CNC machine tools: A review. Chin. J. Aeronaut. 2022, 37(2), 163–198. https://doi.org/10.1016/j.cja.2023.02.035
8. Gao W., Ibaraki S., Donmez M.A., Kono D., Mayer J.R.R., Chen Y.-L., Szipka K., Archenti A., Linares J.-M., Suzuki N. Machine tool calibration: Measurement, modeling, and compensation of machine tool errors. Int. J. Mach. Tools Manuf. 2023, 187, 104017. https://doi.org/10.1016/j.ijmachtools.2023.104017
9. Chen J.X., Lin S.W., Zhou X.L. A comprehensive error analysis method for the geometric error of multi-axis machine tool. Int. J. of Mach. Tools Manuf. 2016, 106(2), 56–66. https://doi.org/10.1016/j.ijmachtools.2016.04.001
10. Royer M., Anselmetti B. 3D manufacturing tolerancing with probing of a local work coordinate system. Int. J. Adv. Manuf. Technol. 2016, 84, 2151–2165. https://doi.org/10.1007/s00170-015-7797-4
11. Wan H., Chen S., Zheng T., Jiang D., Zhang C., Yang G. Piecewise modeling and compensation of geometric errors in five-axis machine tools by local product of exponentials formula. Int. J. Adv. Manuf. Technol. 2022, 121, 2987–3004. https://doi.org/10.1007/s00170-022-09178-0
12. Stryczek R. Alternative methods for estimating plane parameters based on a point cloud. Meas. Sci. Rev. 2017, 17(6), 282–289. https://doi.org/10.1515/msr-2017-0035
13. Multliba U., Gomez-Acedo E., Vega S.A.I., Yagüe-Fabra J.A. Uncertainty assessment for on-machine tool measurement: An alternative approach to the ISO 15530-3 technical specification, Precis. Engin. 2019, 57, 45–53. https://doi.org/10.1016/j.precisioneng.2019.03.005
14. Blecha P., Holub M., Marek T., Jankovych R., Misun F., Smolik J., Machalka M. Capability of measurement with a touch probe on CNC machine tools. Measurement 2022, 195, 111153. https://doi.org/10.1016/j.measurement.2022.111153
15. Płowucha W. Point-plane distance model for uncertainty evaluation of coordinate measurement. Metrol. Meas. Syst. 2020, 27(4), 625–639. https://doi.org/10.24425/mms.2020.134843
16. Płowucha W., Jakubiec W., Rosner P. Evaluation of measurement uncertainty – Monte Carlo method. Mechanik 2017, 90(12), 1152–1154. https://doi.org/10.17814/mechanik2017.12.195
17. Sepahi-Boroujeni S., Mayer J.R.R., Khameneifar F. Efficient uncertainty estimation of indirectly measured geometric errors of five-axis machine tools via Monte-Carlo validated GUM framework, Precis. Engin. 2021, 67, 160– 171. https://doi.org/10.1016/j.precisioneng.2020.09.027
18. Wojtyła M., Rosner P., Płowucha W., Forbes A.B., Savio E. Balsamo, A. Validation of the sensitivity analysis method of coordinate measurement uncertainty evaluation. Measurement 2022, 199(4), 111454. https://doi.org/10.1016/j.measurement.2022.111454