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Design Improvement of Permanent Magnet Motor Using Single- and Multi-Objective Approaches

Research paper

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Abstract: Optimisation, or optimal design, has become a fundamental aspect of engineering across various domains, including power devices, power systems, and industrial systems. Engineers and academics have been actively involved in optimising these systems to achieve better performance, efficiency, and cost-effectiveness. Optimising electrical machines, including permanent magnet motors, is a complex task. It often involves solving intricate problems with various parameters and constraints. Engineers use different optimisation methods to tackle these challenges. Depending on the specific requirements and goals of a design project, engineers may employ either single-objective or multi-objective optimisation approaches. Single-objective optimisation focuses on optimising a single objective, while multi-objective optimisation considers multiple conflicting objectives. In optimisation, objective functions are mathematical representations of what needs to be optimised. In this case, optimising the efficiency of the motor, reducing cogging torque, and minimising the total weight of active materials are defined as possible objective functions. Genetic algorithms are nature based algorithms that are commonly used in engineering to find optimal solutions to complex problems, including those with multiple objectives. In this paper, after conducting optimisations using different objective functions and methods, a comparative analysis of the results is performed. This helps in understanding the trade-offs and benefits of different design choices. Finite element analysis (FEA) is a computational method used to analyse the physical properties and behaviours of complex structures and systems. In this case, FEA is used to validate and analyse selected optimisation solutions to ensure they meet the desired characteristics and parameters. Overall, this work demonstrates the interdisciplinary nature of engineering, where mathematics, computer science (for optimisation algorithms), and physics (for FEA) converge to improve the performance and efficiency of electrical machines. It also underscores the importance of considering multiple objectives in design processes to find optimal solutions that strike a balance between competing goals.

Keywords: *permanent magnet motor • efficiency • cogging torque • multi-objective • optimisation*

1. Introduction

Energy efficiency has become a critical concern worldwide, and strategies to improve energy efficiency are being implemented across various aspects of society and industry. Electric motors are significant consumers of electrical energy, making them a prime target for efficiency-improvement efforts. Researchers have dedicated substantial efforts to enhancing the efficiency of electrical machines, and this has been a focus of study for the past few decades. The optimal design of electrical machines can be achieved by optimising various objective functions. These functions may include efficiency, torque, power factor, output torque, cogging torque, volume, mass, and total cost. In some cases, a combination of multiple objective functions is used, resulting in multi-objective or many-objective optimisation approaches. This allows for a more comprehensive assessment of machine performance. The optimisation process for electrical machines involves defining a vector of variables related to dimensions, current densities, flux densities, etc. These optimisations must also adhere to a set of constraints related to

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thermal, mechanical, manufacturing, or standards limits. Over time, optimisation methods have evolved. Initially, deterministic methods were used, but they heavily depended on the starting point and the determination of first-order derivatives. This makes it a difficult problem to solve by hand; therefore, at the early stages, scientists and researchers were trying to implement different types of optimisation methods (Liu and Slemon, 1991). The first methods that were implemented belong to the group of methods named as deterministic methods (Brisset and Brochet, 1998; Gottvald et al., 1992; Im et al., 1993). The quality of the search and the solution in those methods is highly dependent on the selection of the starting point, as well as the determination of the first-order derivation in some cases that is usually hard to determine. The introduction of concepts like Adaptation in Artificial Systems by John Holland led to the development and adoption of stochastic optimisation methods (Holland, 1973, 1995). These methods, including genetic algorithm (GA), are based on principles of natural evolution and operate by evaluating fitness rather than directly considering variable values. Stochastic methods, such as GAs, gained popularity due to their ability to start with randomly generated populations and iteratively improve solutions based on fitness evaluations. They have been successfully applied to optimise electromagnetic devices. There is a substantial body of scientific work dedicated to implementing GAs and other stochastic optimisation techniques in the optimal design of electromagnetic devices (Sim et al., 1997; Üler et al., 1995; Üler and Mohammed, 1996; Wurtz et al., 1997). This reflects ongoing efforts to improve energy efficiency in electrical machines.

This paper is divided into five sections. After the Introduction, Section 2 presents the structure and the behaviour of the GA during the optimisation. In Section 3, the definition and the mathematical model for all the investigated objective functions that are used in the GA optimisation of the investigated motor are presented in detail. The comparative optimisation results and their analysis, as well as the finite element method (FEM) analysis of the analysed solutions are presented in Section 4. Finally, Section 5 presents the main conclusions derived from the optimisation results and the FEM analysis.

2. GA as an Optimisation Tool

GAs were proposed by John Holland in 1975. They are a class of evolutionary-based stochastic optimisation algorithms inspired by the principles of natural selection, specifically the concept of 'survival of the fittest'. GAs are known for their global search capabilities and have been highly successful in optimisation tasks. They focus on optimising the objective function value rather than the values of individual optimisation parameters. GAs start optimisation with a population of potential solutions represented as chromosomes. They also perform their search using a large number of individuals (chromosomes) during one generation that ensures a wide area of search that can avoid a local optimum and reach the global optimum. These chromosomes are typically vectors of real or binary numbers representing the values of optimisation variables. In this work, vectors of floating-point numbers as the representation scheme are used, as it simplifies the determination of variable values compared to binary representations (Janikow and Michalewicz, 1991). GAs use a fitness function tailored to the specific problem to evaluate the quality of each chromosome in the population. This function guides the selection of the best chromosomes. The GA employs three key genetic operators: selection, crossover (recombination), and mutation. These operators are responsible for generating a new set of solutions (offspring) in each generation. The values of specific GA parameters used in this optimisation problem include population size (N = 20), crossover probability (p_c = 0.85), and mutation probability (p_m = 0.07). To enhance the quality of the search, in the presented algorithm, a fitness scaling and elitism are implemented. Fitness scaling equalises the chances of survival for all population members, preventing premature convergence to suboptimal solutions successfully avoiding the local optima. Elitism ensures that the best solution is automatically passed to the next generation. GAs have been applied extensively in optimising electrical machines, including their optimal design. Various objective functions are considered in this context, such as efficiency, electromagnetic torque, cogging torque, total motor mass, and power density (Cvetkovski and Petkovska, 2008, 2010, 2013). In this investigation, the stopping rule for the GA search is defined to be the number of generations. Therefore, the optimal number of generations can be determined by performing a number of preparatory runs for various number of generations and comparing the objective functions values for the different runs. Initially, the GA program, used in this work, was created for the following work (Cvetkovski, 2001) and successfully used for many other works involving different motors and different objective functions.

3. Objective Functions Definition

The motor that is under consideration in this work is a brushless three-phase permanent magnet synchronous motor (PMSM). The motor has a laminated stator with 36 slots and a rotor with 6-skewed $SmCo_5$ surface-mounted permanent magnets with B_r = 0.95 T. The rated data of the motor are: I = 18 A, T = 10 Nm, and n = 1,000 rpm at a frequency of 50 Hz.

In this research work, three different single-objective functions, as well as three multi-objective functions are applied in order to improve the overall performance of the investigated permanent magnet motor. In the single-objective function optimisation, the efficiency of the motor, the cogging torque, and the overall weight of the active materials are implemented in the optimisation process as objective functions. In the multi-objective optimisation approach, a combination of those functions is investigated as an objective function and their results compared with the single-objective approach. In the following text, a brief presentation of the mathematical model that defines the mentioned single-objective functions is given, as well as the presentation of the multi-objective functions that are a combination of the single-objective functions multiplied by a given scaling factor, which is also defined as a separate optimisation parameter.

3.1. Efficiency – single objective

Efficiency of the motor is one of the functions that have been very frequently used as an objective function in the optimal design procedures and it is defined as a ratio between the output and the input power. Hence, a proper mathematical model of the motor is developed as one of the contributions of this paper. The mathematical model of the motor consists of a number of equations that define all motor design parameters, and the mathematical model is constructed towards the definition of the objective function, in this case the efficiency of the PMSM. The defined mathematical model is afterwards implemented in the GA and used for the optimisation purposes. The efficiency, as an objective function, is expressed by the following equation:

$$Objective\ function = efficiency\left(R_{ro}, f_m, h_m, g, L\right) = \frac{P_{output}}{P_{input}} = \frac{T \cdot \omega_m}{T \cdot \omega_m + P_{Cu} + P_{Fe} + P_{wf}} \tag{1}$$

where: P_{output} is the output power of the motor and P_{input} is the input power of the motor defined as a sum of the input power and all the motor power losses, T – rated torque, ω_m – rated speed, P_{Cu} – ohmic power loss, P_{Fe} – core loss, and P_{wf} – windage and friction losses. In this optimisation process, the following motor parameters have been selected as optimisation parameters in the efficiency-improvement procedure: outside radius of the rotor iron core R_{ro}, permanent magnet fraction f_m, permanent magnet radial height h_m, air-gap g, and axial active length of the motor L. The cross-section of the analysed motor and the defined optimisation parameters are presented in Figure 1. The rest of the design motor parameters, especially in the stator, are defined to be dependent on the optimisation parameters and therefore exclude any conflict of dimensions during the optimisation process.

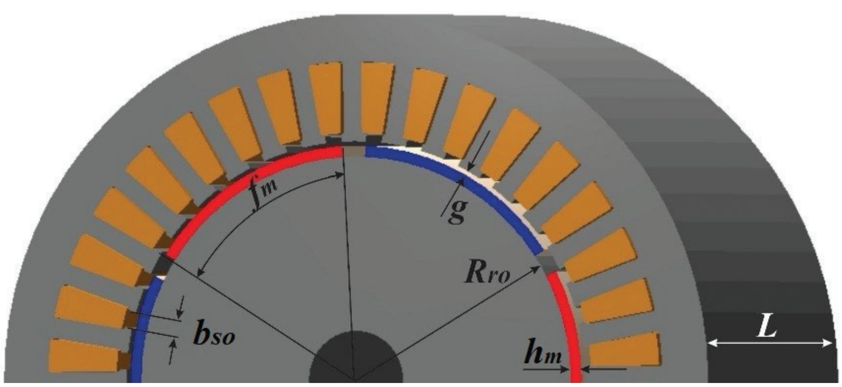

Fig. 1. PMSM cross-section and optimisation parameters presentation. PMSM, permanent magnet synchronous motor.

The power losses presented in Eq. (1) are defined with the following equations:

$$P_{Cu} = 3R_{ph} \cdot I_{ph}^2 \tag{2}$$

$$R_{ph} = \rho_{Cu_{20}} \frac{L_{Cu}}{A_{Cu}} \tag{3}$$

$$I_{ph} = \frac{P_{em}}{3E_{0ph}\eta_{app}^*} \tag{4}$$

$$P_{em} = P_{output} + P_0 \tag{5}$$

$$P_{Fe} = P_{Fe_{SY}} + P_{Fe_{St}} \tag{6}$$

$$P_{Fe_{SY}} = p_{S_{1.5/50}} \cdot \left(\frac{B_{SY}}{1.5}\right)^2 \cdot \left(\frac{f}{50}\right)^\alpha \cdot m_{Fe_{SY}} \tag{7}$$

$$P_{Fe_{St}} = p_{S_{1.5/50}} \cdot \left(\frac{B_{St}}{1.5}\right)^2 \cdot \left(\frac{f}{50}\right)^\alpha \cdot m_{Fe_{St}} \tag{8}$$

where: R_{ph} – stator winding phase resistance, I_{ph} – phase current, L_{Cu} – total length of a single stator phase winding, A_{Cu} – slot area occupied with the winding, ρ_{Cu20} – copper specific resistivity at 20°C, P_{em} – electromagnetic power, E_{0ph} – induced phase back-EMF (Electromotive force) and η^*_{app} – apparent efficiency, which is product of motor efficiency η and power factor cosφ, P_o – no load losses, P_{FeSY} – iron power losses in stator yoke and P_{FeSt} – iron power losses in stator teeth, $p_{S1.5/50}$ – specific iron power losses at flux density of 1.5 T and frequency of 50 Hz, B_{SY} – stator yoke (back iron) flux density, B_{St} – stator teeth flux density, f – frequency, α – Steinmetz constant ($1 < \alpha < 2$, $\alpha = 1.5$ was used), m_{FeSY} – total stator yoke iron mass, and m_{Fest} – total stator teeth iron mass. In the calculation, the value of stator winding phase resistance is determined at the rated working temperature using an adequate transformation coefficient. In the calculation of the stator phase winding length also, the end windings are taken into account, as it will be presented latter on. This is only a partial presentation of the system of equations for the detailed mathematical model of the motor that have been defined and implemented in the optimisation algorithm, taking into account all aspects of the design process without any assumptions. The optimisation is performed for a number of runs, and the results of the best solution gained from the GA optimisation of the efficiency function in comparison with the prototype are presented in Table 1. During the optimisation process, each variable is varied within its lower and upper bound. The limits of the boundaries are determined as + or – 10% of the initial value for each variable.

3.2. Cogging torque – single objective

The permanent magnet motors are electrical machines that are widely used in high-performance industrial drives as a result of their high torque density, high efficiency, and high power–volume ratio. Unfortunately, it is not all bright as it seems. The presence of the cogging torque in these types of motors throws a shadow on their performance characteristics, especially in applications sensitive to torque ripple. Cogging torque is an undesirable phenomenon in permanent magnet motors and is primarily caused by the interaction between the stator teeth and the permanent magnets on the rotor. This interaction leads to torque fluctuations during motor operation, which can affect the

Parameters	Lower bound	Upper bound	Prototype	GA-1 solution
R_{ro} (m)	0.0378	0.0442	0.042	0.0378
f_m (/)	0.81	0.99	0.9	0.91469
h_m (m)	0.0018	0.0022	0.002	0.0022
g (m)	0.00072	0.00088	0.0008	0.00072
L (m)	0.081	0.099	0.09	0.09899
Efficiency (/)	–	–	0.8489	0.8804

GA, genetic algorithm.

Table 1. GA optimisation parameter initial values, boundaries, and optimisation results for efficiency optimisation.

smoothness and precision of motion, making it especially problematic in applications with strict torque requirements. In general, the torque pulsations are produced as a result of:

- The variations in the air gap's permeance caused by the geometry of the stator slots and the interaction between the rotor magnetic flux and the air gap. Cogging torque is primarily associated with the motor's physical design.
- Beyond the cogging effect, torque ripple can also be generated during the motor's operation, which is influenced by the control strategy and the specific conditions under which the motor is used.

The minimisation of the cogging torque in the permanent magnet motors is of great importance and is generally achieved by a special motor design, which in the design process involves a variety of many geometrical motor parameters, while the torque ripple generated by the control strategy is minimised through the improvement of those control strategies.

The cogging torque is a function of rotor position and contains higher-order harmonic components, which can make its formulation and optimisation challenging. The formulation of the cogging torque presented in Gieras (2004) seems suitable for this work and provides a basis for defining the peak value of cogging torque as an objective function in the optimisation process. The definition of the cogging torque is quite simple and defined in the function of the basic motor parameters. Such a definition of the peak value of the cogging torque can be easily used as an objective function in an optimisation process (Cvetkovski and Petkovska, 2021). The simplified version of the formulation of the peak value of the cogging torque, which is the objective function of the optimisation, is presented by Eq. (9):

$$Objective\ function = T_{cogging}\left(R_{ro}, f_m, h_m, g, L, b_{so}\right) = \frac{g \cdot L \cdot D_{1in}}{4 \cdot \mu_o} \cdot A_T \cdot \frac{B_g^2}{k_c} \tag{9}$$

where g is the air gap length, L is the axial length of the motor, D_{1in} is stator inner diameter, A_T is a motor coefficient, B_g is the average air gap flux density, and k_c is Carter's coefficient. The coefficient A_T is equal to:

$$A_T = 2 \cdot \mathrm{k}_{\mathrm{sk}}^{\ 2} \cdot \mathrm{k}_{\mathrm{ok}}^{\ 2} \cdot (g/\tau_s) \cdot B_g \tag{10}$$

where k_{sk} is a stator skew factor, k_{ok} is a stator opening slot factor, and τ_s is the stator slot pitch. In the optimisation of the cogging torque, the following motor parameters have been selected as optimisation parameters: outside radius of the rotor iron core R_{ro}, permanent magnet fraction f_m, permanent magnet radial height h_m, air-gap g, axial active length of the motor L, and slot opening b_{so}. In the text that follows, a detailed presentation of each parameter shown in Eq. (9) will be given, as well as a presentation of all the parameters that will appear in the equations that will follow. Therefore, the Carter's coefficient kc is defined as:

$$k_c = \frac{1}{1 - \frac{1}{\frac{\tau_s}{b_{so}}} \cdot \left(5 \cdot \frac{g_c}{b_{so}} + 1\right)} \tag{11}$$

where b_{so} is the stator slot opening, $g_c = g + h_m/\mu_r$ is the average air gap, μ_r is the relative permeability of the permanent magnets, and τ_s is the stator slot pitch in the air gap that is equal to:

$$\tau_s = \left(R_{ro} + h_m + g\right) \cdot \frac{2\pi}{Z} \tag{12}$$

in which R_{ro} is the outside radius of the rotor core and is one of the optimised parameters. Z is the total number of stator slots and h_m is the height of the permanent magnet in the radial direction. For this type of motor, the value of the air gap flux density can be calculated using the following equation:

$$B_g = \frac{C_f \cdot B_r}{1 + \mu_r \cdot k_c \cdot \frac{k_{ml}}{P_c}} \tag{13}$$

where B_r is the residual flux density of the permanent magnets, k_{ml} is the flux leakage coefficient, and C_f is the flux concentration coefficient and is defined as:

$$C_f = \frac{2 \cdot f_m}{1 + f_m} \tag{14}$$

In which f_m is the permanent magnet fraction and it is also one of the optimised motor parameters. P_c is the motor permeability coefficient and it is defined as:

$$P_c = \frac{h_m}{g \cdot C_f} \tag{15}$$

The flux leakage coefficient k_{ml} can be expressed as:

$$k_{ml} = 1 + \frac{4 \cdot l_m}{\pi \cdot \mu_r \cdot f_m \cdot \tau_{pm}} \cdot ln\left[1 + \frac{\pi \cdot g}{(1 - f_m) \cdot \tau_{pm}}\right] \tag{16}$$

In the previous equation, τ_{pm} is the pole pitch along the outer rotor line and all the other parameters are defined previously. This parameter is defined as:

$$\tau_{pm} = \frac{2\pi \cdot R_{ro}}{N_{PM}} \tag{17}$$

where R_{ro} is the outer rotor core radius and it is one of the variable parameters. During the optimisation process, each variable is varied within its lower and upper bound.

The presented mathematical model of the cogging torque is fully integrated in the GA that is used as an optimisation tool. The results from the cogging torque optimisation in comparison with the prototype are presented in Table 2. It is evident that the optimised solution has a reduced value of the cogging torque in comparison to the prototype.

3.3. Total mass of active materials – single objective

Due to the fact that the permanent magnet motors have high power–volume ratio, the third objective function that is defined and used in the optimisation process is the total mass of the active materials used for the construction of the PM motor. By active materials, it is meant the iron used for the stator and rotor core, the copper used for the stator windings and the permanent magnets. The optimisation parameters used for this optimisation procedure are the same as the ones presented for the cogging torque as an objective function. The objective function in this case is defined with Eq. (18).

$$Objective\ function = m_{tot}\left(R_{ro}, f_m, h_m, g, L, b_{so}\right) = m_{Fe_S} + m_{Cu} + m_{PM} + m_{Fe_R} \tag{18}$$

Parameters	Lower bound	Upper bound	Prototype	GA-2 solution
R_{ro} (m)	0.0378	0.0442	0.042	0.03781
f_m (/)	0.81	0.99	0.9	0.81
h_m (m)	0.0018	0.0022	0.002	0.0018
g (m)	0.00072	0.00088	0.0008	0.00088
L (m)	0.081	0.099	0.09	0.081
b_s (m)	0.002	0.002478	0.002278	0.0020
$T_{cogging}$ (Nm)	–	–	1.0656	0.944

GA, genetic algorithm.

Table 2. GA optimisation parameter initial values, boundaries, and optimisation results for cogging torque optimisation.

The total mass of the stator iron core is defined as a sum of the total mass of the stator yoke and the stator teeth:

$$m_{Fe_S} = m_{Fe_{SY}} + m_{Fe_{St}} \tag{19}$$

where $m_{Fe_{SY}}$ is the total mass of the stator yoke and $m_{Fe_{St}}$ is the total mass of the stator teeth. They are determined using the following equations:

$$m_{Fe_{SY}} = \gamma_{Fe_S} \cdot V_{Fe_{SY}} \tag{20}$$

$$V_{Fe_{SY}} = \pi\left(R_{So}^2 - R_{SY}^2\right) \cdot L_{Fe} \tag{21}$$

$$m_{Fe_{St}} = \gamma_{Fe_S} \cdot V_{Fe_{St}} \tag{22}$$

$$V_{Fe_{St}} = \left[\pi\left(R_{SY}^2 - R^2\right) - N_s A_s\right] \cdot L_{Fe} \tag{23}$$

where γ_{Fe_S} is the stator steel mass density, $V_{Fe_{SY}}$ is the stator yoke volume, L_{Fe} is the axial length of the iron core, R_{So} is the outer (external) stator radius, R_{SY} is the inner radius of the stator yoke, $V_{Fe_{St}}$ is the total stator teeth volume, R is the inner stator (bore) radius, N_s is the number of stator slots, and A_s is the single slot bare total area.

The total copper mass is defined as:

$$m_{Cu} = \gamma_{Cu} \cdot V_{Cu} \tag{24}$$

$$V_{Cu} = 3 \cdot a_{Cu} L_{Cu} \tag{25}$$

where γ_{Cu} is the copper mass density, V_{Cu} is the volume of the copper, a_{Cu} is the copper wire cross-section, and L_{Cu} is the copper wire length of one phase winding and it is defined with the following equation:

$$L_{Cu} = N_{ph} \cdot L_{t_1} \tag{26}$$

$$L_{t_1} = 2\left(L + L_{ew}\right) \tag{27}$$

where N_{ph} is the number of turns per phase, L_{t_1} is the length of one turn, L is the axial length of the motor, and L_{ew} is the mean value of the front and rear end winding.

The total mass of the permanent magnets is defined as:

$$m_{PM} = \gamma_{PM} \cdot V_{PM} \tag{28}$$

$$V_{PM} = \pi\left(R_R^2 - R_{ro}^2\right) \cdot f_{PM} \cdot L \tag{29}$$

where γ_{PM} is the permanent magnet mass density, V_{PM} is the volume of the permanent magnets, f_{PM} is the permanent magnet fraction, and L is the axial length of the motor, R_R is the outer rotor radius with the *PM,* and R_{ro} is outside radius of the rotor iron core.

The total mass of the rotor iron core is defined as:

$$m_{Fe_R} = \gamma_{Fe_R} \cdot V_{Fe_R} \tag{30}$$

$$V_{Fe_R} = \pi\left(R_R^2 - R_{Rsh}^2\right) \cdot L_{Fe} \tag{31}$$

where γ_{Fe_R} is the rotor core mass density, V_{Fe_R} is the volume of the rotor iron core, R_R is the rotor iron core radius, R_{Rsh} is the rotor shaft radius, and L_{Fe} is the axial length of the iron core. The results from the optimisation of the total active materials motor mass are presented in Table 3. Single objective function optimisation not always gives satisfactory results in the overall performance of the motor or in relation to other motor parameters. The conclusion is drawn based on the comparative results, shown in Table 4, for all previously presented single-objective solutions in relation to the prototype in which all the investigated objective functions are compared. From the presented data, it can be concluded that with a single-objective optimisation beside the improvement of the main objective parameter the

Parameters	Lower bound	Upper bound	Prototype	GA-3 solution
R_{ro} (m)	0.0378	0.0442	0.042	0.0378
f_m (/)	0.81	0.99	0.9	0.8101
h_m (m)	0.0018	0.0022	0.002	0.0022
g (m)	0.00072	0.00088	0.0008	0.00088
L (m)	0.081	0.099	0.09	0.08101
Mass (kg)	–	–	8.7937	8.0521

GA, genetic algorithm.

Table 3. GA optimisation parameter initial values, boundaries, and optimisation results for total mass of active materials optimisation.

Parameters	Prototype	GA-1 solution	GA-2 solution	GA-3 solution
Efficiency (/)	0.8489	0.8804	0.837	0.8465
$T_{cogging}$ (Nm)	1.0656	1.2051	0.9442	0.8234
Mass (kg)	8.7937	9.6243	8.3601	8.0521

GA, genetic algorithm.

Table 4. Comparative results of the single-objective GA optimisations.

Parameters	Prototype	GA-4 solution	GA-5 solution	GA-6 solution
R_{ro} (m)	0.042	0.037805	0.037804	0.037805
f_m (/)	0.9	0.900882	0.810072	0.810054
h_m (m)	0.002	0.002199	0.0022	0.0018
g (m)	0.0008	0.00072	0.00072	0.00072
L (m)	0.09	0.098975	0.098982	0.094315
bs (m)	0.002278	0.002	0.002	0.002001
Efficiency (/)	0.8489	0.881	0.8788	0.8687
$T_{cogging}$ (Nm)	1.0656	1.0224	0.8844	0.7631
Mass (kg)	8.7937	9.62855	9.57919	9.2007

GA, genetic algorithm.

Table 5. Comparative results of the multi-objective GA optimisations for function f_1.

other motor parameters (objectives) in some cases were improved, but in others cases were worsened. Therefore, in this work, an attempt is made to combine the previously presented single-objective optimisation and perform a multi-objective optimisation with different optimisation functions as combinations of the previously used single-objective functions. They are going to be presented in the text that follows.

3.4. Multi-objective function – f_1 that is a combination of efficiency and cogging torque

The first defined and used multi-objective function is the function f_1 defined as a difference between the efficiency function of the motor and the scaled cogging torque function, as presented with Eq. (32). Detailed presentation of both functions is shown in the text previously.

$$Multi-objective\ function = f_1 = Efficiency - k_1 \cdot T_{cogging} \tag{32}$$

where efficiency is the efficiency function of the analysed motor, T_{coging} is the cogging torque function, and k_1 is a variable parameter that is used as a scaling factor for the cogging torque function in relation to the efficiency, that is defined as a separate optimisation parameter and it is generated randomly by the GA. With this approach, the idea is to give a random value to the scaling factor during the optimisation process rather than a constant one, which gives more freedom to the optimisation search. In Table 5, few specific solutions that have competitive results regarding the two investigated optimised functions are presented. The difference of the optimal solutions is due to the different value of the scaling factor reached by the GA. The GA-4 solution has the lowest value for k_1 = 0.01, for the GA-5 solution it is k_1 = 0.05, and for GA-6 it is k_1 = 0.3.

It can be concluded that when the value of the scaling factor is small, then the influence of the cogging torque function is decreased and solutions are gained with high efficiency and high value of the total mass, but, on the other hand, the cogging torque is with lower value than the prototype, but still not with the lowest value. If the scaling factor value is increased, then the influence of the cogging torque in the multi-objective function is increased and therefore the solutions have lower value for the multi-objective function that leads to lower value for the efficiency and total mass of the motor, but a better value for the cogging torque. In this case, the change of the value of the total mass of the motor is due to the change of the other optimised functions, but not directly from the optimisation process of the multi-objective function. This case will be investigated later on.

3.5. Multi-objective function – f_2 as a combination of efficiency and total active material mass

A similar approach has been realised in this case where the multi-objective function f_2 is defined as a difference between the efficiency and the scaled total motor mass, as presented in Eq. (33).

$$Multi-obj.funct. = f_2 = Efficiency - k_2 \cdot m_{tot} \tag{33}$$

where efficiency is the efficiency function, m_{tot} is the total active material mass of the motor, and k_2 is a variable parameter that is used as a scaling factor for the motor mass in relation to the efficiency and it is generated randomly by the GA. In Table 6, few specific solutions that have competitive results regarding the two investigated optimised functions are presented. In this case, the values for the scaling factor k_2 are: for GA-7 solution k_2 = 0.001, for GA-8 solution k_2 = 0.01, and for GA-9 k_2 = 0.03. Similar conclusions can be drawn as in the previous optimisation case regarding the influence of the parameters and the values of the optimised functions in the multi-objective optimisation process. From the presented data in Table 6, it is evident that solution GA-9 has a bit higher value of the motor efficiency in relation to the prototype, but, on the other hand, a significant improvement (decrease) of the total active material mass and the value of the cogging torque of the motor.

3.6. Multi-objective function – f_3 as a combination of efficiency, cogging torque, and total active material mass

In the last analysed case, the objective function f_3 is defined as a difference between the efficiency function of the motor, the scaled cogging torque function, and the scaled total active material mass function, as presented with Eq. (34).

$$Multi-objective\ function = f_3 = Efficiency - k_1 \cdot T_{cogging} - k_2 \cdot m_{tot} \tag{34}$$

where efficiency is the efficiency function of the investigated motor, $T_{cogging}$ is the cogging torque function, m_{tot} is the total active material mass function, k_1 is a variable parameter that is used as a scaling factor for the cogging torque function in relation to the efficiency, k_2 is a variable parameter that is used as a scaling factor for the motor mass

Parameters	Prototype	GA-7 solution	GA-8 solution	GA-9 solution
R_{ro} (m)	0.042	0.037803	0.037803	0.037803
f_m (/)	0.9	0.90772	0.836064	0.810558
h_m (m)	0.002	0.00219	0.002199	0.002199
g (m)	0.0008	0.00072	0.00072	0.00072
L (m)	0.09	0.098975	0.09516	0.081003
b_s (m)	0.002278	0.002002	0.002001	0.002001
Efficiency (/)	0.8489	0.88096	0.87578	0.8552
$T_{cogging}$ (Nm)	1.0656	1.0234	0.8899	0.7245
Mass (kg)	8.7937	9.63217	9.27298	8.08164

GA, genetic algorithm.

Table 6. Multi-objective results for f_2 function.

Parameters	Prototype	GA-10 solution	GA-11 solution	GA-12 solution
R_{ro} (m)	0.042	0.03780	0.037801	0.037801
f_m (/)	0.9	0.900018	0.814302	0.810054
h_m (m)	0.002	0.002199	0.002199	0.002199
g (m)	0.0008	0.000720	0.000720	0.000720
L (m)	0.09	0.098993	0.094844	0.081018
b_s (m)	0.002278	0.002001	0.002001	0.002001
Efficiency (/)	0.8489	0.881	0.87465	0.8552
$T_{cogging}$ (Nm)	1.0656	1.0217	0.8539	0.7239
Mass (kg)	8.7937	9.62948	9.23644	8.08297

GA, genetic algorithm.

Table 7. Multi-objective results for f_3 function.

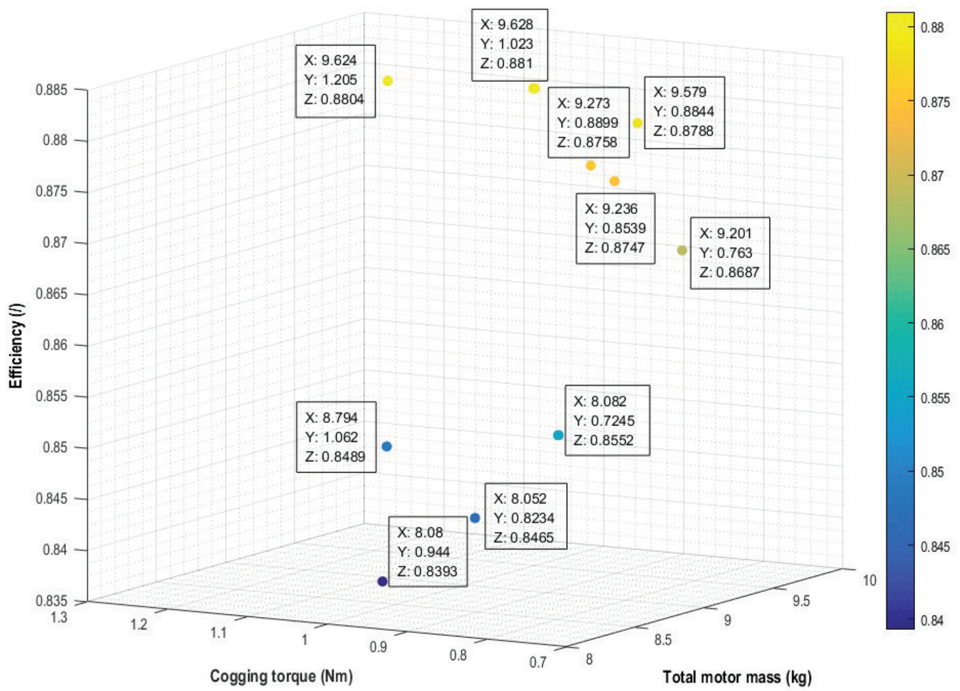

Fig. 2. Comparative presentation of the optimisation solutions (X axes is total motor mass, Y axes is cogging torque, and Z axes is efficiency).

and they are both generated randomly by the GA in a range from 0 to 1. In Table 7, three specific solutions that have competitive results regarding the three investigated optimisation functions are presented. The difference of the optimal solutions is due to the different values of the scaling factors reached by GA. Their values for the selected three solutions are: GA-10–k_1 = 0.0024 and k_2 = 0.002, GA-11–k_1 = 0.01 and k_2 = 0.01, and GA-12–k_1 = 0.03 and k_2 = 0.03. The solution GA-10 has the best value for the efficiency, where, on the other hand, solution GA-12 has the lowest values for the cogging torque and the total motor mass, while the value for the efficiency is the lowest among the solutions, but is better than the prototype. Solution GA-12 shows the benefit from the multi-objective optimisation of the PMSM in which the three optimisation functions are confronted and an optimal solution can be reached satisfying the three functions. It is evident that by favouring one of the optimisation functions, different solutions are reached, and therefore it is up to the designer do decide to which function to give an advance in the optimisation process, based on the attributes that the optimised solution should have.

A graphical three-dimensional illustration of the 12 optimal solutions and the prototype in relation to the optimisation functions is presented in Figure 2. In the graph, some of the specific solutions are given with their values. Additionally, below, few selected solutions from the single- and multi-objective optimisation will be analysed.

4. Comparative Analysis of the Proposed Optimal Solutions

4.1. Analytical results

As an addition to the comparative analysis of the proposed optimisation solutions in Table 8, the values of several motor parameters for the selected optimal solutions in relation to the prototype are presented. The motor parameters are determined analytically using the same mathematical model used for the optimisation process. Based on the presented values of the parameters in those tables, as well as in the previous ones, it can be concluded that GA-10 is the solution with the best efficiency, GA-12 is with the smallest value of the cogging torque, and GA-3 is with the lowest value for the total motor mass.

The GA-12 solution has also a quite a good value for the total motor mass close to the value of solution GA-9. If all three optimisation functions are taken under consideration, then solution GA-12 could be a very competitive solution because it has the lowest value for the cogging torque, quite a low total mass close to the best solution, and an improved efficiency in relation to the prototype. Of course, other solutions can also be selected depending on what optimisation function the advantage will be given in relation to the other functions, but this is a decision that the motor designer will have to make regarding the motor application and the overall motor performance. In general, with multi-objective optimisation, better motor solutions are reached in comparison with single-objective optimisation with many improved motor parameters rather than only one. This job can be made a bit easier if all those solutions are put in a Pareto analysis or use a many-objective optimisation approach (Di Barba et al., 2019, 2022).

In performance analysis of electrical machines, it is a practice to realise a 2D or 3D finite element analysis (FEA) (Bianchi, 2005; Gebregergis et al., 2014; Hameyer and Belmans, 1999; Ruuskanen et al., 2016). Therefore, in the following text, such an analysis will be performed on all the presented solutions in Table 8.

4.2. FEA

In the past few decades, the FEM and FEA have become a standard practice and procedure in electromagnetic devices, as well as electrical machines performance and characteristics analysis. In this work, the same practice will be implemented on the prototype and the optimised motor models presented in Table 8. The finite element calculations and analysis are performed using the software package Infolitica (User’s Manual, Inflolytica, 2016) and its module MotorSolve (User’s Manual, Infolytica, 2016). A presentation of the prototype modelled in this program is presented in Figure 1. All the other motor models gained from the optimisation process are modelled in a similar way. In the FEA, the materials used for the initial solution were also used for all the other investigated optimised models, and they are the same as in the physical model of the motor. This program package is used to determine the magnetic flux distribution in each motor model, as well as to determine the air gap flux density, cogging torque, and torque-speed characteristics for all analysed models.

4.3. Magnetic field distribution

The first physical quantity gained from the finite element calculation that is presented in this work is the magnetic field distribution shown in Figure 3 for all analysed models. Based on the presented field distributions, it can be concluded that there are no significant changes in the magnetic field distribution in all optimised models in relation

Parameters	Prototype	GA-1 solution	GA-2 solution	GA-3 solution	GA-4 solution	GA-8 solution	GA-11 solution	GA-12 solution
N (turns)	9	8	9	11	8	8	9	10
B_g (T)	0.5868	0.6196	0.5436	0.54327	0.61828	0.58819	0.57718	0.57499
R_{ph} (Ω)	0.1564	0.0922	0.1366	0.1397	0.0911	0.0832	0.1025	0.1172
P_{Cu} (W)	147.091	99.8456	166.0	152.731	99.1833	107.8732	109.7979	139.6815
P_{Fest} (W)	17.357	20.393	15.915	15.16	20.31895	18.65522	18.2830	15.56525
m_{Cu} (kg)	1.5979	2.3287	1.7486	2.4276	2.3545	2.4854	2.5439	2.3781
m_{FeS} (kg)	3.5878	4.131	3.218	3.0729	4.1157	3.7788	3.7034	3.153
m_{FeR} (kg)	3.243	2.7605	3.0809	2.2588	2.7604	2.6537	2.6445	2.259
m_{PM} (kg)	0.36497	0.40413	0.31259	0.29286	0.39795	0.35508	0.34464	0.29287

GA, genetic algorithm.

Table 8. Comparative optimisation results.

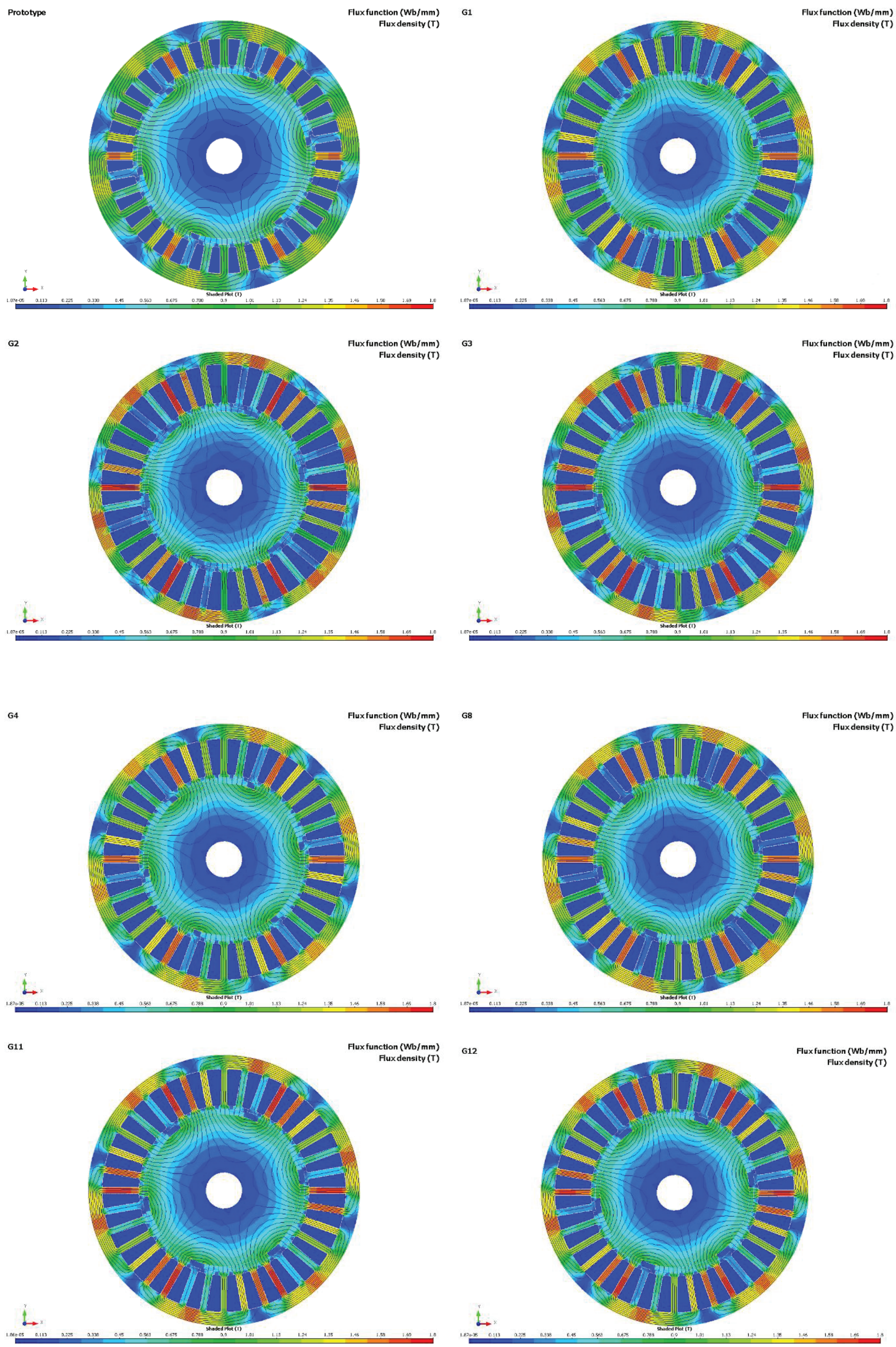

Fig. 3. Magnetic field distribution for all analysed motor models.

to the prototype. In Figure 3, beside the magnetic field distribution also, the different optimisation models' shape can be noticed as a result of the optimisation. The predefined values for both parts of the stator iron core were defined as a constraint and it was set to a value of 1.8 (T). The average values of the flux density in the stator teeth and the stator back iron are in good agreement with the prescribed and calculated values for those parts for all optimised models. The limits of flux density values in the legend for all models are defined to be identical for better comparative analysis.

4.4. Air gap flux density distribution

The air gap flux density can be determined using the data from the finite element calculations based on Eq. (35) shown below:

$$\mathbf{B} = rot\ \mathbf{A} \tag{35}$$

The calculation of the air gap flux density distribution is realised for one pair of poles and is presented for all analysed models in Figure 4.

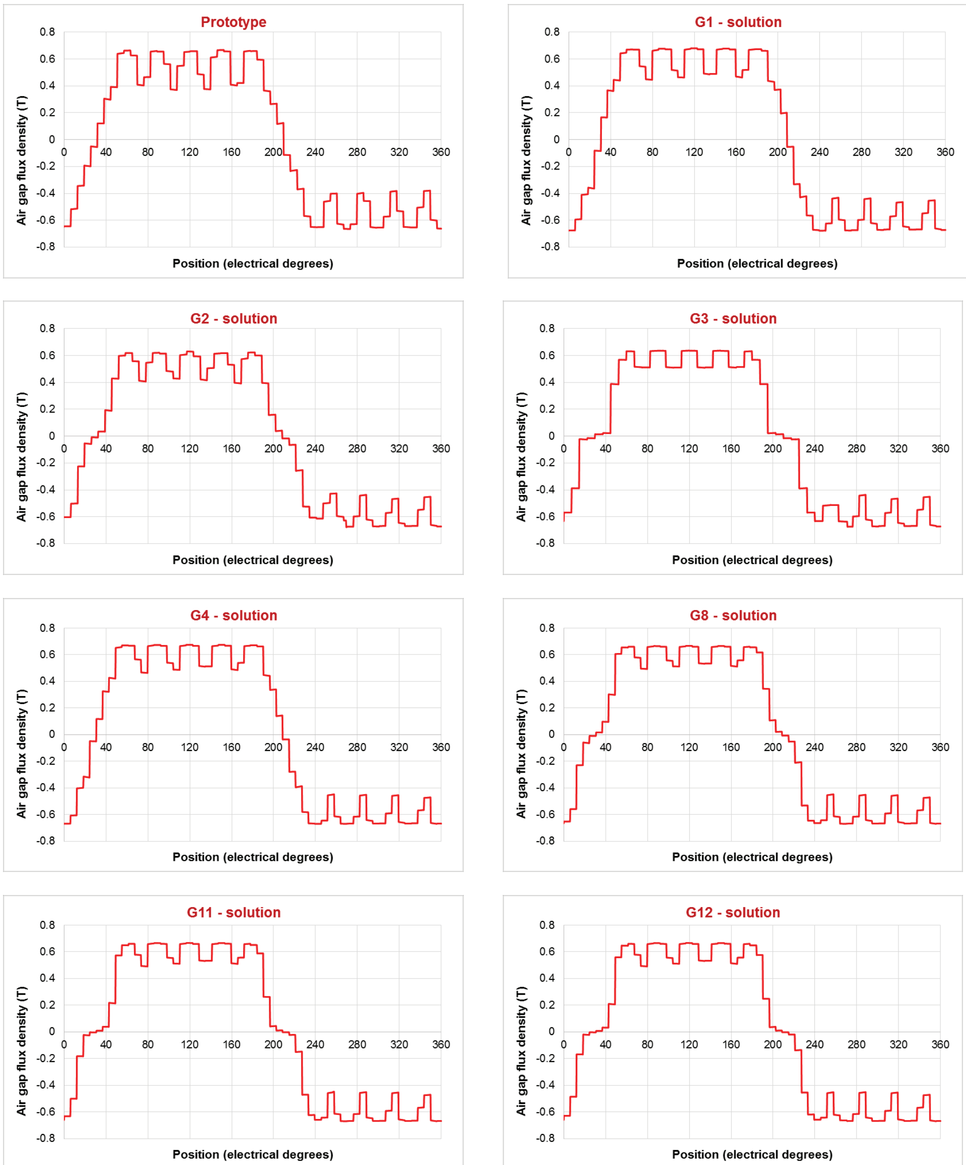

Fig. 4. Air gap flux density distribution for all analysed motor models.

The average value of the air gap flux density under one pole is in good agreement with the calculated values presented in Table 8 for all analysed models. The pulsations of the air gap flux density are due to the influence of the stator teeth and slot openings on its value in the air gap.

4.5. Cogging torque distribution

The next parameter that has been calculated using FEA is the cogging torque. The shape of the cogging torque for each analysed model is presented in Figure 5. Most of them have the same shape, but the intensity of the torque varies for different motor models as a result of the variation of the motor design parameters. Their peak value is quite close to the value determined during the optimisation process and is in the range of ± 10%, which shows that the analytical method is quite accurate, as well as the values gained from it.

4.6. Torque vs speed characteristic

The next motor characteristic that can be determined using the Motor-Solve program is the torque–speed characteristic. For the purpose of this analysis, this characteristic has been calculated and presented for all analysed models in Figure 6. This characteristic is adequate for the performance analysis of the investigated motor models. It can be noticed that there is a small variation of the developed electromagnetic torque among the various motor models that is due to the variation of the no load losses as a result of the motor efficiency optimisation (Shklyarskiy et al., 2021). But overall, the different motor models will develop the rated torque of 10 Nm in all the cases since this parameter is one of the optimisations constraints and it is kept constant for all solutions.

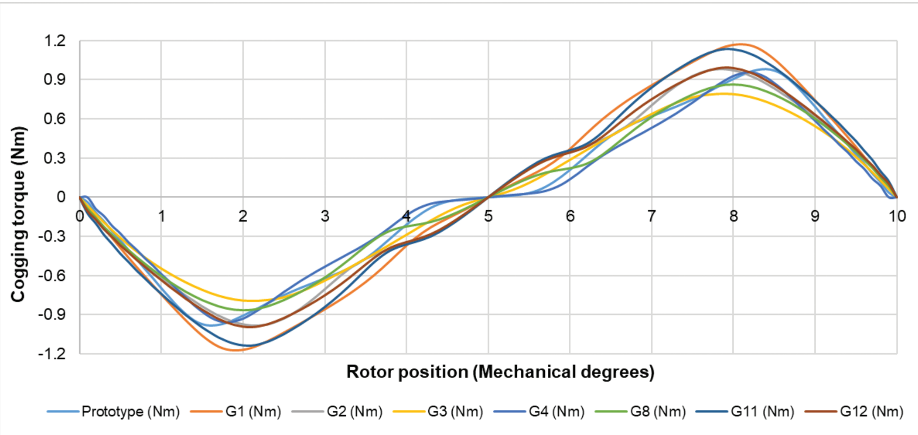

Fig. 5. Cogging torque distribution for all analysed motor models.

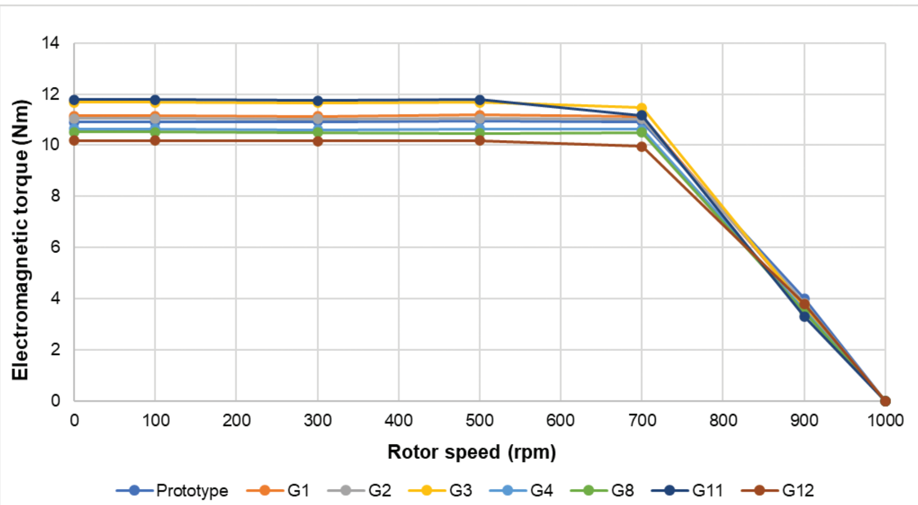

Fig. 6. Electromagnetic torque vs speed characteristics for all analysed motor models.

Based on the presented graphs and diagrams, it can be concluded that the mathematical model that has been developed for the optimisation procedure is quite accurate. There is a good agreement between the values of the motor parameters calculated in the optimisation procedure and the values gained from the FEA, such as flux density values in the stator core, air gap flux density values, developed torque values, and cogging torque values. Some of those results are presented in the figures and some in the tables. This proves that the mathematical model defined for the optimisation procedure is quite adequate and takes into consideration all the aspects of the motor design.

5. Conclusion

In this paper, a GA is used in the optimisation process of a permanent magnet motor using the following three objective functions: motor efficiency, cogging torque, and total active materials motor mass, as well as their combinations in the multi-objective optimisation. These are critical factors in motor design, as efficiency affects energy consumption, cogging torque influences motor smoothness, and material mass affects cost and size.

As mentioned in this work, both single-objective and multi-objective optimisation approaches are explored. In single-objective optimisation, each of the three objective functions is optimised individually. In multi-objective optimisation, the functions are combined in different ways, such as pairs of two or all three functions simultaneously. This approach allows for trade-offs between conflicting objectives to be analysed. The paper presents selected results from these optimisation approaches. It is important to clarify what specific results were obtained, such as optimised values for motor parameters under different optimisation scenarios. A comparative analysis is conducted to assess the outcomes of the various optimisation approaches. This analysis can provide insights into the strengths and weaknesses of different optimisation strategies and the trade-offs between objectives.

Finally, at the end, a traditional FEM analysis is performed for the initial solution and a selected number of optimised solutions in order to validate and understand the optimised designs through more detailed analysis.

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