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BIOMAGNETIC FLOW OVER A FLAT PLATE EMBEDDED BY A MAGNETIC DIPOLE IN A POROUS MEDIA

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The present study used a simplified axisymmetric biomagnetic fluid dynamics and porous media model which includes FHD (Ferrohydrodynamics), porosity and inertia effects saturated by magnetic dipole to study the influences of the leading parameters on various flow variables along a flat plate. The governing equations are simplified and solved by finite difference approach. We clarify how the ferromagnetic interaction parameter B and porosity ε assumptions contribute in the bio-background of the problem of interest. Moreover, from the results of the flow profiles, accelerating and decelerating phenomena are noticed for B and ε interaction.

Key words: boundary layer, biomagnetic fluid, dipole, porous medium.

1. Introduction

Flow and heat transfer inside porous medium have got quite an interest in practical applications including thermal insulation and storage systems, petroleum reservoirs, chemical reactors, nuclear waste, etc [1]. Recently, remarkable progress has been attained through exerting the hypothesis of porous medium in modeling biomedical appliances. Drug delivery, tissue replacement, liquid chromatography, MRI application examining porous medium structure, transportion of nutrients in brain tissues, blood flow throughout muscular contraction, porous frame for tissue engineering, computational biology, bio-sensing systems, diffusion in the extracellular space for analyzing the physiology of central nervous system are some potential solicitation of biomagnetic fluid flow in porous media [2, 3].

The organic fluids that persist in living things and whose circulations get oppressed by magnetic force are referred to as biomagnetic fluids. The red blood cells (RBCs) in blood include hemoglobin particles, a type of protein substances that contain ions and transport oxygen. This makes blood the most significant biomagnetic fluid.

A substantial number of works has been published regarding biomedical applications in porous media. Vankan et al. [4] contrasted a model of stratified comination of blood suffused tissues which employs an elongated Darcian environment for blood flow along with a network analysis. Khaled and Vafai [5], Khanafer and Vafai [6] investigated that the assumption of porous media is most appropriate for understanding the transfer of heat in biological tissues. Considering blood flow in unsteady MHD conditions, Latha et.al. [7] analyzed the heat and mass transfer effects through parallel plate channel in a saturated porous medium. Asma et. al.[8] examined the thermal behaviour of blood flow with carbon nanotubes in porous medium considering blood as MHD fluid. The effects of slipping velocity with respect to various porous parameters in blood flow has been assessed by Ramakrishnan [9]. Mohammed et.al.[10] studied the role of porosity in blood flow when the body is subject to diseases. Shafiq et. al.[11] pondered blood as MHD thermal flow through a porous medium with the addition of gold nanoparticles and scrutinized the flow regime.

Acknowledging the practical importance, biological fluid flow over a flat surface adorned through porous media, has been studied in the present research work.

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2. Methodology

Our study involves a 2-D stable, laminal, Newtonian, convective flow of a viscidly incompressed biomagnetic fluid towards a horizontal flat plate that is saturated in a porous medium. The system is considered under the influence of magnetic field originated by a dipole at distance d underneath the plate. The fluid gets infused by the magnetic field generated from the magnetic dipole. A predetermined temperature, T_w is considered at the wall and away from it obtains the curie temperature T_c such that $T_w < T_c$. The plate is fixed and hence the flow adjacent to it attains velocity $u = v = 0$ and eventually obtains the free stream velocity u_e. The whole configuration is demonstrated in Fig.1

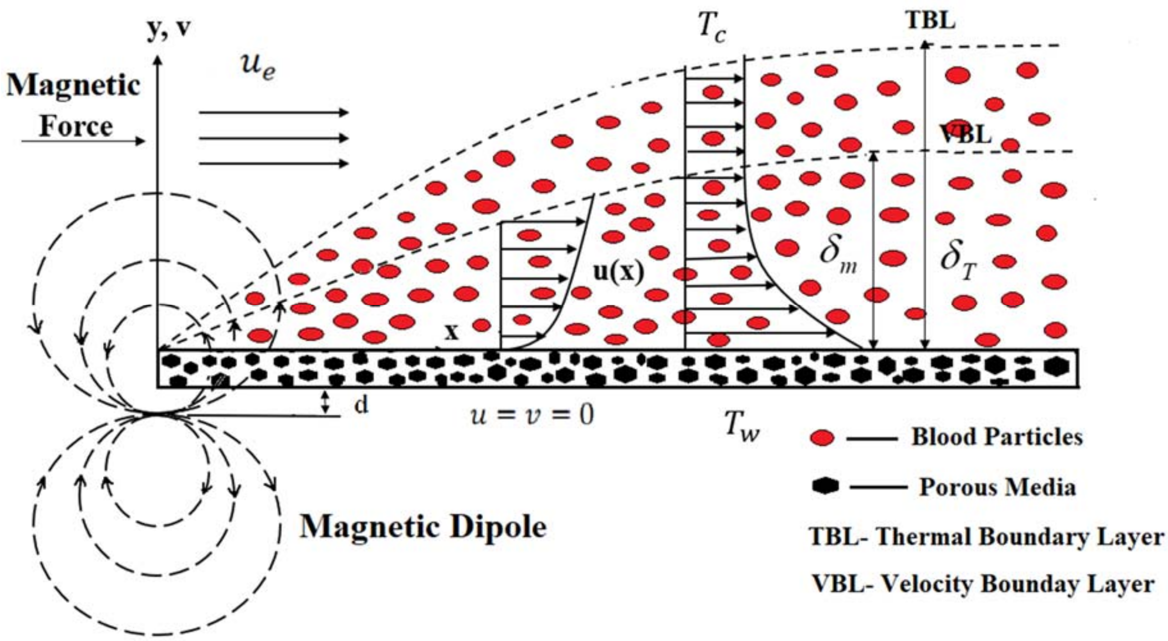

Fig.1. Flow configuration of the problem.

The fundamental governing equations (continuity, momentum and energy equations) are, (Kafoussias et al. [12]; Nakayama et al. [13]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \lambda u_e^2 + \varepsilon\upsilon\frac{\partial^2 u}{\partial y^2} - \varepsilon^2\frac{\upsilon}{K}u - \varepsilon^2\frac{\upsilon}{\sqrt{K}}u^2 + \frac{\mu_0}{\rho}M\frac{\partial H}{\partial x}, \tag{2.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{\mu_0}{\rho C_p}T\frac{\partial M}{\partial T}\left(u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y}\right). \tag{2.3}$$

Applicable conditions at wall:

$$y = 0: \ u = v = 0, T = T_w,$$
$$y \to \infty: \ u \to u_e, T \to T_c. \tag{2.4}$$

The magnetic scalar potential is

$$V(x,y) = \frac{\alpha}{2\pi} \frac{x}{x^2 + (x+d)^2}, \tag{2.5}$$

and magnetic field

$$\left\|\overline{H}\right\| = \frac{\alpha}{2\pi}\left[\frac{1}{(y+d)^2} - \frac{x^2}{(y+d)^4}\right]. \tag{2.6}$$

Following Anderson and Valnes [15], take the magnetization

$$M = K^*\left(T_c - T\right). \tag{2.7}$$

Introducing dimensionless variables, Nakayama et al. [13] and Murtaza et al. [14],

$$\xi = \lambda x, \quad \lambda = \varepsilon^2\left(1 + \frac{1}{\mathrm{Re}_k}\right)\frac{C}{\sqrt{K}}, \quad \mathrm{Re}_k = \frac{C\sqrt{K}u_e}{\upsilon},$$
$$\eta(x,y) = \frac{y}{x}\sqrt{\frac{\mathrm{Re}_x}{I}}, \quad \mathrm{Re}_x = \frac{u_e x}{\varepsilon\upsilon}, \quad I = \frac{1 - e^{-\xi}}{\xi}, \tag{2.8}$$
$$\psi(x,y) = \varepsilon\upsilon\sqrt{\mathrm{Re}_x I} f(\xi,\eta), \quad \theta(\eta) = \frac{T_c - T}{T_c - T_w}.$$

Using above the reduced equations are:

$$f''' + \frac{1}{2}e^{-\xi} ff'' - \left(1 - e^{-\xi}\right)\left[\frac{f' + \mathrm{Re}_k f'^2}{1 + \mathrm{Re}_k} - 1\right] =$$
$$= \left(1 - e^{-\xi}\right)\left(f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right) - \frac{2B\theta}{\varepsilon^2 I(\eta + \alpha)^4}, \tag{2.9}$$

$$\theta'' + \frac{1}{2}\varepsilon\sqrt{I}\,\mathrm{Pr}\, f\theta' = \xi\varepsilon\sqrt{I}\,\mathrm{Pr}\left[\left(\sqrt{I}f\right)'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial}{\partial\xi}\left(\sqrt{I}f\right)\right] + \frac{2BV\left(\theta - T_\varepsilon\right)f'}{\varepsilon\left(\eta_t + \alpha\right)^4} = 0. \tag{2.10}$$

The corresponding dimensionless boundary conditions,

$$\eta = 0: \quad f(0) = 0, \quad f'(0) = 0, \quad \theta(0) = 1,$$
$$\eta \to \infty: \quad f'(\infty) = 1, \quad \theta(\infty) = 0. \tag{2.11}$$

Here, $\Pr = \dfrac{\mu C_p}{\kappa}$ is the Prandtl number, $T_\varepsilon = \dfrac{T_c}{T_w - T_c}$ is the dimensionless Curie temperature, $V = \dfrac{\mu u_e^2}{\kappa\left(T_w - T_c\right)}$ is the viscous dissipation parameter, $B = \dfrac{\alpha}{2\pi}\dfrac{\mu_0 K^*\left(T_c - T_w\right)\rho}{\mu^2}$ is the ferromagnetic interaction parameter.

Note that we consider two cases:

- For

$$\xi << 1, I \equiv 1: \quad f''' + \frac{1}{2}e^{-\xi} ff'' = -\frac{2B\theta}{\varepsilon^2 I(\eta+\alpha)^4}, \tag{2.12}$$

$$\theta'' + \frac{1}{2}\varepsilon\sqrt{I}\Pr f\theta' = \frac{2BV\left(\theta - T_\varepsilon\right)f'}{\varepsilon\left(\eta_t + \alpha\right)^4} = 0. \tag{2.13}$$

- For

$$\xi >> 1, I \equiv \frac{1}{\xi}: \quad f''' - \left[\frac{f' + \mathrm{Re}_k f'^2}{1 + \mathrm{Re}_k} - 1\right] = -\frac{2B\theta}{\varepsilon^2 I(\eta+\alpha)^4}, \tag{2.14}$$

$$\theta'' + \frac{1}{2}\varepsilon\sqrt{I}\Pr f\theta' = \frac{2BV\left(\theta - T_\varepsilon\right)f'}{\varepsilon\left(\eta_t + \alpha\right)^4} = 0. \tag{2.15}$$

Currently, the heat transfer rate and the drag coefficient factor are:

$$C_{fx}\left(\mathrm{Re}_x I\right)^{1/2} = 2f''(\xi, 0); \qquad Nu_x / \mathrm{Re}_x^{1/2} = -\theta'(\xi, 0). \tag{2.16}$$

In this work, we only included the results for $\xi << 1$.

$$f''' + \frac{1}{2}e^{-\xi} ff'' = -\frac{2B\theta}{\varepsilon^2 I(\eta+\alpha)^4}, \tag{2.17}$$

$$\theta'' + \frac{1}{2}\varepsilon\sqrt{I}\,\Pr f\theta' = \frac{2BV\left(\theta - T_\varepsilon\right)f'}{\varepsilon\left(\eta_t + \alpha\right)^4} = 0. \tag{2.18}$$

3. Results and discussion

A straightforward yet fruitful finite difference method that is emerged on central differencing and matrix manipulation of tridiagonal is incorporated to solve equations (2.12)-(2.13) along with (2.11). We set $\eta_\infty = 3$, step size $\Delta\eta = 0.001$ and tolerance between iterations $\zeta = 10^{-3}$, (as the convergence criterion).

Considering the human blood, temperature $T_w = 310K$, $\mu = 3.2\times10^{-3}\,kg/ms$; $C_p = 14.65\,J/kgK$ and $\kappa = 2.2\times10^{-3}\,J/msK$, respectively, (Kafoussias and Tzirtzilakis [16]) and hence $\Pr = 21$. Also considered that $T_\varepsilon = 2$ and $\alpha = 1$. Values of ferromagnetic interaction parameter, B are taken from 0 to 10, (Kafoussias and Tzirtzilakis [16]). For viscous dissipation the values of parameter $V = 0.01, 0.05$ and porosity $\varepsilon = 0.3, 0.5, 0.8$ (Vafai [17]) are considered.

Figures 2 and 3 represent the dimensionless velocity as well as schemes of temperature for diverse values of B and ε. It is clear that, as B is increasing keeping a fixed ε, the velocity within the boundary layer increases over the sheet from zero to a specific value at the free stream then converges to the outer boundary region whereas, the temperature decreases. Note that, for a fixed B, increasing ε has an increasing effect on temperature but decreasing effect on velocity.

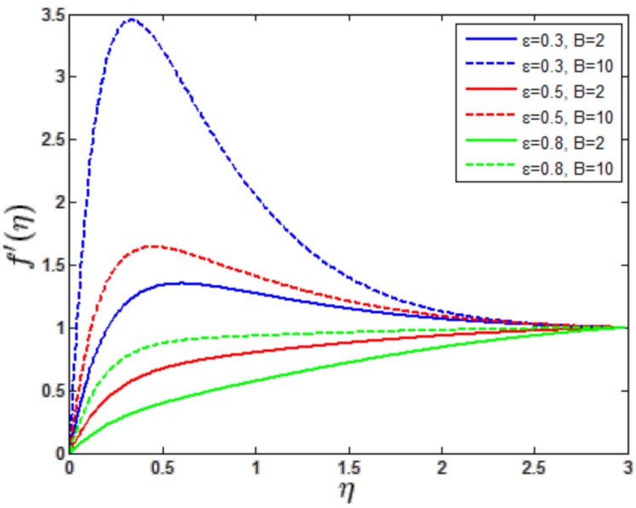

Fig.2. The plot of $f'(\eta)$ vs η for several values of B and ε.

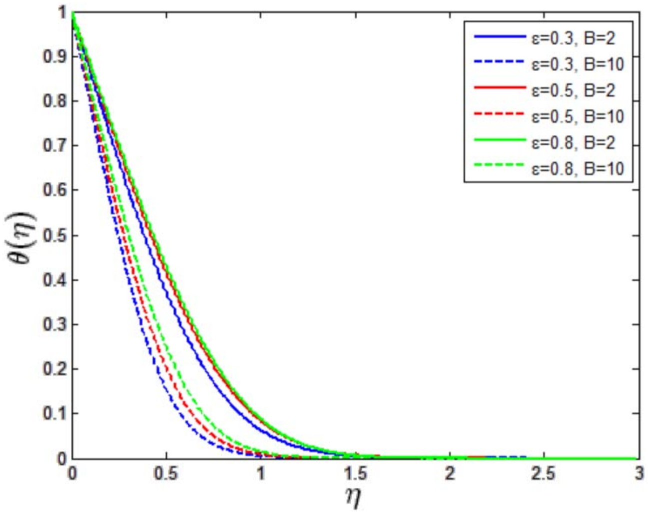

Fig.3. The plot of $\theta(\eta)$ vs η for several values of B and ε.

In Figs 4 and 5, the impact of V along with ε on the flow profiles are demonstrated considering a fixed value of B. It is perceived that, V has a amplifying payoff on both velocity and temperature distributions. An interesting observation is that, for a smaller V, temperature increases with porosity, ε but a reverse phenomenon can be seen for higher V. Another compelling inspection reveals that, as the values of porosity ε gets higher, the difference between the curves gets smaller.

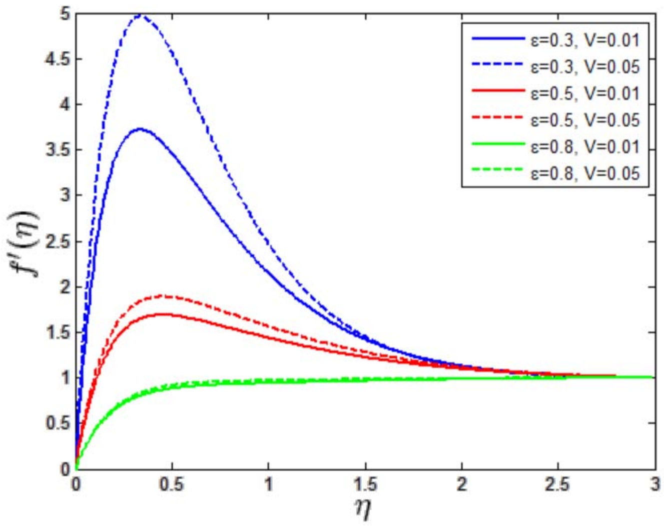

Fig.4. The plot of $f'(\eta)$ vs η for several values of ε and V.

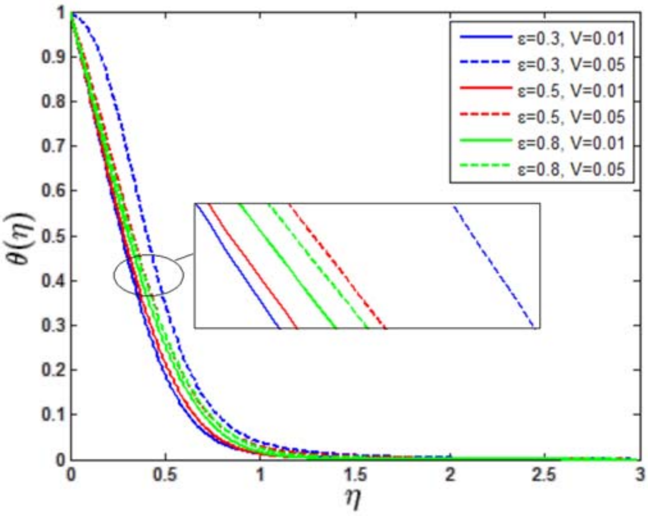

Fig.5. The plot of $\theta(\eta)$ vs η for several values of ε and V .

The effects of B and ε on $f''(0)$ and $-\theta'(0)$ a which represents the drag coefficient factor and the heat transfer rate respectively are shown in Figs 6 and 7. Both $f''(0)$ and $-\theta'(0)$ amplify with expanding B. It is observed that ε has an overall reducing impression on $f''(0)$ but for $-\theta'(0)$ initially ε has an increasing effect but after a critical value of $B \equiv 1$, porosity has a decreasing effect.

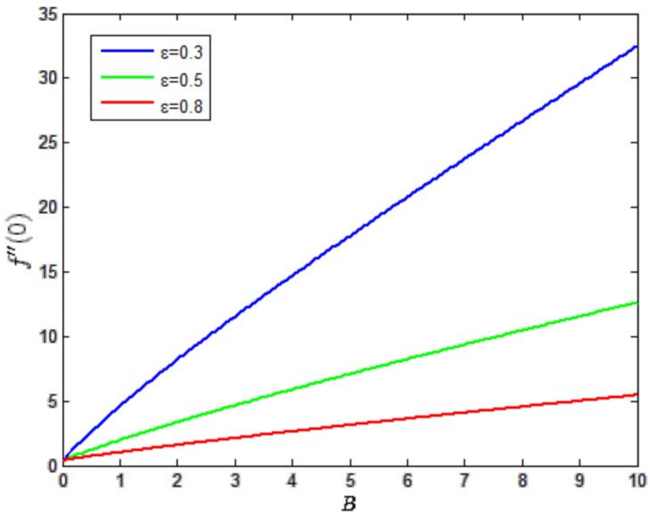

Fig. 6: $f''(0)$ vs B for several ε

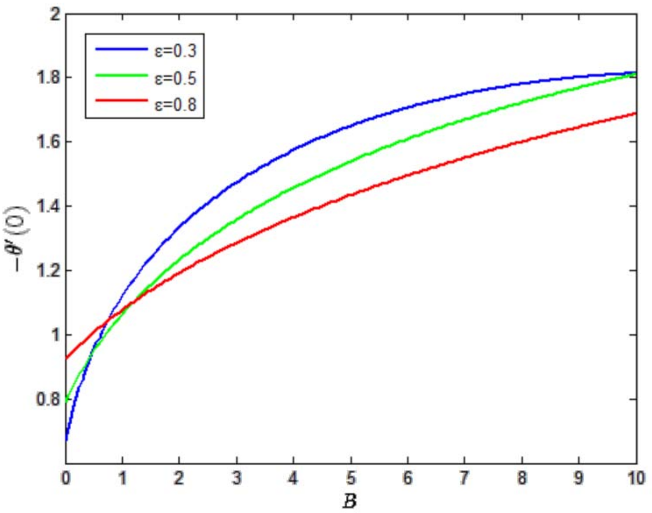

Fig.7. The plot of $-\theta'(0)$ vs B for several values of ε.

4. Conclusions

This study investigates the steady, 2-D, Newtonian, laminar boundary-layer flow of a biological fluid along a horizontal plate that is embedded in a porous medium under the impact of a magnetic dipole. The influences of various governing factors (i.e. ferromagnetic interaction parameter, viscous dissipation parameter, porosity) that arises due to the configuration of the respected problem, on the velocity as well as temperature distributions, the drag coefficient factor and the heat transfer rate are scrutinized meticulously through numerical as well as graphical results. It is evident that both the ferromagnetic interaction parameter, and the viscous dissipation parameter have enhancing impact on the velocity profiles, whereas the porous medium index, i.e. porosity has a diminishing effect. Temperature profiles obtain lower values at higher ferromagnetic interaction parameter, whereas higher values at higher viscous dissipation parameter. Porosity seems to enhance the temperature profiles for lower viscous dissipation parameter.We observed that as the ferromagnetic interaction parameter increases, both the drag coefficient factor and the heat transfer rate increase. Although porosity possesses a truncating impact on the drag coefficient factor but its effect on the heat transfer rate is reversing with an initial increment.

Nomenclature

u,v – velocity components

x,y – cartesian coordinates

η,ξ – transformed coordinates

T – within boundary layer blood temperature

T_w – surface temperature

B – ferromagnetic interaction parameter

V – viscous dissipation parameter

T_c – ambient fluid temperature (Curie temperature)

u_e – external velocity

μ – dynamic viscosity

ε – porosity

υ – kinematic viscosity

μ_0 – magnetic permeability

ρ – density

C_p – specific heat at constant pressure

κ – thermal conductivity

λ – mutual recognition of the stream-wise length scale

α – dimensionless distance

ψ – stream function

Pr – Prandtl number

T_ε – dimensionless curie temperature

I – function to modify the boundary layer length scale

K – permeability

C – empirical constant

$\vec{H}$ – magnetic field

M – magnetization

K^* – pyromagnetic constant

$\mathrm{Re}_k, \mathrm{Re}_x$ – based on permeability and pore velocity, respectively, Reynolds number

C_{fx} – skin friction

Nu_x – Nusselt number

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