

STOYAN NEDELTCHEV

## DETERMINATION OF THE APPLICABILITY RANGE OF THE ISOTROPIC TURBULENCE THEORY IN A BUBBLE COLUMN

Institute of Chemical Engineering, Polish Academy of Sciences, Bałtycka 5, 44-100  
Gliwice, Poland

A new parameter called novel hybrid index (NHI) has been used in order to identify the  $U_g$  and Reynolds ranges of applicability of the concept for the stable equilibrium bubble diameter in a bubble column (BC). Both parameters were correlated. The stable equilibrium bubble diameter was defined on the basis of the local isotropic turbulence theory. In the case of BC operation with two aqueous solutions of 2-pentanol (0.5 and 1.0 vol. %) was found that this theory is applicable in the  $U_g$  range from 0.060 to 0.07 m/s. These conditions belong to the heterogeneous flow regime of the BC operation.

*Keywords:* bubble column, gauge pressure fluctuations, isotropic turbulence theory, stable equilibrium bubble diameter, new hybrid index

### 1. INTRODUCTION

Bubble columns (BCs) are widely used in chemical, petrochemical and biochemical industries (in the case of absorptions, oxidations, hydrogenations, chlorinations or alkylations) as well as wastewater treatment. The dispersion of gas as bubbles in a liquid is achieved by passing the gas through multiple orifices into the liquid. These gas-liquid contactors find broad applications in industry not only due to their excellent mass and heat transfer characteristics but also due to their salient advantages i.e. no moving parts (simple design), low operational cost and

ease of maintenance. BCs are characterized by complicated hydrodynamics and many factors have impact on bubbly bed behavior, including operating conditions (temperature and pressure), properties of gas and liquid, column and gas distributor shape and dimensions.

BCs operate in different hydrodynamic flow regimes (FR) which depends mostly on superficial gas velocity  $U_g$ . Another significant factor is the gas distributor type, especially in the homogeneous FR [1]. In ambient conditions three main FRs can be distinguished: (i) homogeneous (bubbly), (ii) transition and (iii) heterogeneous (churn-turbulent) [2]. All of them are characterized with different flow patterns. The FR has an influence on the interfacial area between the phases and mass and heat transfer, thus on the effectiveness of BC performance. Therefore, the FR transitions are very important for the successful BC design [3].

Since in this work the behavior of a new parameter in the heterogeneous (churn-turbulent) FR will be studied, it will be briefly characterized. In the heterogeneous FR a wide bubble size distribution, parabolic gas holdup profile and occurrence of bubble coalescence and brake-up is observed. Also liquid circulation (recirculation loops) and unstable vortices might be observed. This means that the liquid is rising in the core of the column and moving down in the vicinity of walls [4]. In the churn-turbulent FR ellipsoidal (in the annulus) or spherical-cap (in the core) shapes of the bubbles are also observed and bubble rise velocity is higher. Furthermore in the homogeneous FR only “small” bubbles exist, while in the heterogeneous FR two bubble classes: “small” and “large”, co-exist in the bubbly bed [5]. Bubbles shape and size are essential for the stability of the hydrodynamic FR [6]. Occurrence of both small and large bubbles leads to a division in gas holdup, to be precise “dense” and “dilute” phase gas holdups for small and large bubbles are defined, respectively [7].

In BCs the bubble shape and the bubble size distribution can vary with time and location, which renders their hydrodynamics very complicated [8]. The unstable oscillations of the flow rate usually generate irregular flow patterns. It is very important to understand better the behavior of the gas-liquid dispersion and the impact that it has on the flow instabilities. The instabilities and irregularities in the signal are more pronounced in the heterogeneous FR. The increased  $U_g$  values promote more intensive bubble-bubble hydrodynamic interactions.

Bubble shape, motion and any rippling or deformation of the bubble interface affects the mass transfer. That is why, the correct bubble size estimation is very important for the BC operation. Most of the mixing and mass transfer calculations are based on the stable equilibrium bubble diameter  $d_{bE}$ , which is established above the equilibrium zone (usually equal to one column diameter).

The turbulence in the bubble bed controls the equilibrium bubble diameter. The local isotropic turbulence theory [9] postulates that the turbulent flow generates primary eddies, which have a wavelength or scale similar of similar magnitude to the dimensions of the main flow stream. These large primary eddies are unstable and disintegrate into smaller eddies until all their energy is dissipated by viscous flow. When the Reynolds number of the main flow flow is high, most of the kinetic energy is contained in the large eddies but nearly all of the dissipation occurs in the smallest eddies. The large eddies transfer energy to smaller eddies in all directions and the directional nature of the primary eddies is gradually lost. All eddies that are much smaller than the primary eddies are statistically independent of them and the properties of these small eddies are determined by the local energy dissipation rate per unit mass of fluid [9]. It is noteworthy that the turbulent motion of the larger eddies might be far from isotropic. The local isotropic turbulence condition exists when the fluctuating components of the velocity are equal. As mentioned above, according to this theory the smallest eddies are responsible for most of the energy dissipation. The local isotropic turbulence theory is also used in the penetration theory. In this model there is a continuous attachment and detachment of small liquid eddies (so-called micro eddies) to the gas-liquid interface. Most of the bubbles have diameters between  $1.4 \times 10^{-3}$  m and  $6.0 \times 10^{-3}$  m. In this size range the bubbles are no longer spherical and they follow a helical upward path. Viscous drag is enhanced by the vortex formation in the wake and the rise velocity remains constant within this bubble size range [10]. When the mean bubble diameter exceeds  $6.0 \times 10^{-3}$  m the bubbles assume a spherical-cap shape and the terminal bubble rise velocity tends to increase gradually with bubble size [10].

According to Hinze [9] and Calderbank [11] a stable equilibrium bubble diameter  $d_{bE}$  is obtained when turbulent fluctuations and surface tension forces are in balance. Viscous forces do not play a role unless the dynamic pressure fluctuations are too strong. The stable equilibrium bubble diameter is defined as follows:

$$d_{bE} = 3.48C_D^{-0.6} \left( \frac{\sigma_L^{0.6}}{\epsilon^{0.4}\rho_L^{0.2}} \right) \quad (1)$$

Miller [10] recommends that  $C_D$  should be set equal to 8/3 since the attainment of the stable equilibrium bubble diameter requires high turbulence in the immediate vicinity of the bubble. The energy dissipation rate  $\epsilon$  is expressed usually as  $gU_g\rho_L$ . So, when the expressions for  $C_D$  and  $\epsilon$  are substituted in Eq. (1), the following final correlation for  $d_{bE}$  is obtained:

$$d_{bE} = 1.932 \left( \frac{\sigma_L^{0.6}}{(gU_g)^{0.4}\rho_L^{0.6}} \right) \quad (2)$$

In the heterogeneous FR the mean bubble diameter is independent of the orifice diameter and primarily is affected by the energy content of the liquid phase. Eq. (2) shows that due to the increasing liquid turbulence in the bubble bed, the bubble diameter decreases. In the heterogeneous FR as characteristic velocity should be used the gas velocity through the dilute phase ( $U_g - U_{trans}$ ) [12].

The main objective of this work is to demonstrate that a new parameter (called novel hybrid index) can be correlated to the stable equilibrium bubble diameter defined in terms of the energy dissipation rate, i.e. based on the local isotropic turbulence theory [10]. In such a way the operating conditions for the applicability of this important turbulence theory in the heterogeneous FR will be identified.

## 2. DESCRIPTION OF THE NEW HYBRID INDEX

This dimensionless parameter has been applied to gauge pressure fluctuations. The time series (10000 points) were divided into 10 different periods consisting of 1000 points. Then, the average absolute deviation (AAD) in each period was estimated. AAD is a robust statistical estimator of the data width around the mean. It is a sum of all absolute differences  $|x_j - x_{mean}|$  divided by the total number of points in the time series.

Further, the probability  $P_i$  for having such a data sequence yielding a particular AAD in each period was calculated as a ratio of the AAD in that period divided by the sum of all AAD values:

$$P_i = AAD_i / \sum_{i=1}^N AAD_i \quad (3)$$

The information amount  $IA_i$  in each period was calculated as follows:

$$IA_i = -\log(P_i) \quad (4)$$

The new hybrid index represents the following ratio:

$$NHI = |\sum_{i=1}^N IA_i - \sum_{i=1}^N AAD_i| / \sum_{i=1}^N AAD_i \quad (5)$$

At low  $U_g$  values the NHI is much higher than 1. However, at  $U_g$  values beyond 0.025 m/s it varies between 0 and 1. The NHI value characterizes the statistical non-uniformity in the measured signal. The NHI algorithm is well explained in [13].

### 3. EXPERIMENTAL SETUP

The measurements have been performed in a facility consisting of an air compressor, air dryer, air filter, mass flowmeter (Omega Inc., USA) and a BC facility (0.1 m in ID). Two different mixtures of deionized water (DW) and 2-pentanol (0.5 and 1.0 vol. %) were used. The column was equipped with a perforated plate gas distributor – it was equipped with 96 orifices with an ID of 1 mm. The open area was 0.96 %. The initial (clear) liquid height was set at 1.1 m.

Before every measurement the gas flow rate was pre-adjusted and then after 120 s (waiting time for flow stabilization) the signal recording started. Gauge pressure fluctuations (see Fig. 1) were recorded with a sampling interval of 33 ms (pressure transmitter PR-33X, range: 0-1 bar, Keller AG, Switzerland). The gauge pressure sensor was installed at an axial height of 0.65 m above the gas distributor.  $U_g$  values in the range of 0.055 – 0.071 m/s were studied. The aerated bed heights were measured visually by means of a ruler glued to the column's wall in order to calculate the overall gas holdups.

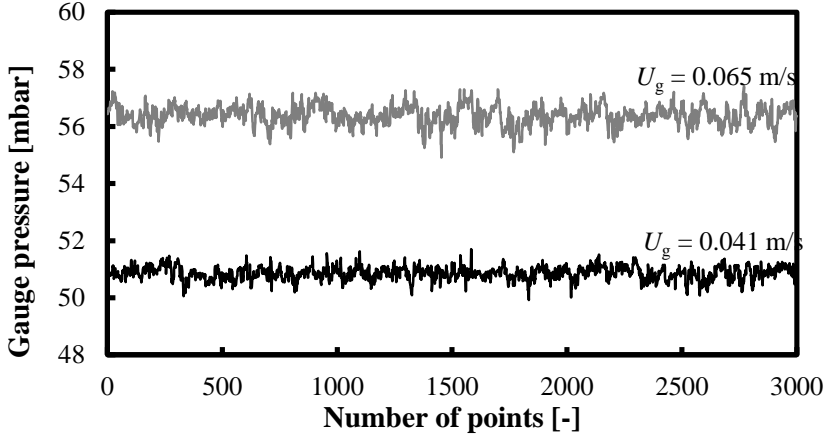


Fig. 1. Gauge pressure fluctuations in an aqueous solution of 2-pentanol (0.5 vol. %) at two  $U_g$  values

Rys. 1. Wahania ciśnienia manometrycznego w wodnym roztworze 2-pentanolu (0,5% obj.) przy dwóch wartościach  $U_g$

#### 4. RESULTS AND DISCUSSION

Fig. 2 shows that in the case of a mixture of DW and 2-pentanol (1.0 vol. %) the NHI parameter is inversely proportional to the gas velocity through the dilute phase ( $U_g - U_{trans}$ ). This characteristic gas velocity should be used in the heterogeneous FR where large bubbles are formed [12]. The transition gas velocity  $U_{trans}$  was calculated as 0.044 m/s based on the correlation of Im et al. [14]. The exponent implies that there is a relationship between the NHI parameter and the mean (equilibrium) bubble diameter  $d_{bE}$  expressed (see Eq. 2) in terms of the Hinze's isotropic turbulence theory. It seems that NHI is proportional to the square root of  $d_{bE}$ , which means that the requirements of the isotropic turbulence theory in the ( $U_g - U_{trans}$ ) range are met. The results in Fig. 2 imply that the Hinze's isotropic turbulence theory is applicable in the  $U_g$  range from 0.060 to 0.071 m/s. This corresponds to Reynolds numbers from 6877 to 8084. The relationship between NHI and  $d_{bE}$  means that NHI is proportional to the rise velocity  $V_b$  of the equilibrium bubble. According to Krishna et al. [14]  $V_b$  is equal to  $0.71(gd_{bE})^{0.5}$ .

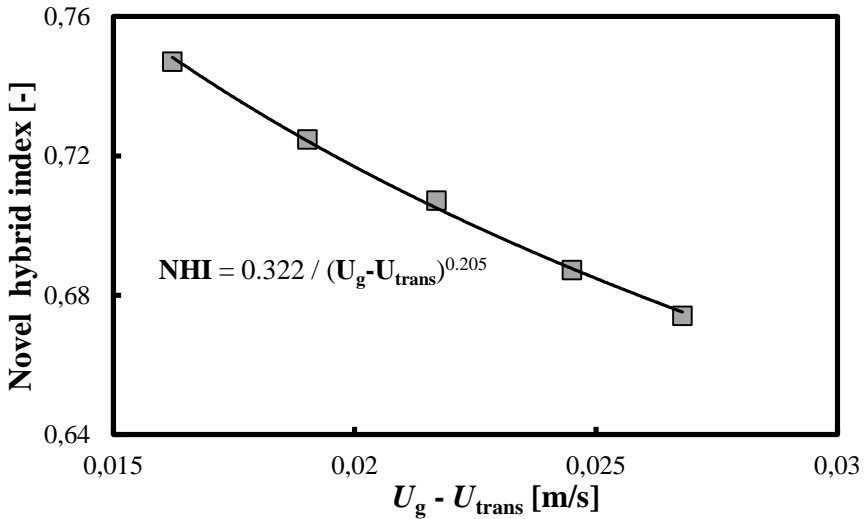


Fig. 2. Relationship between NHI and the gas velocity through dilute phase in a narrow BC operated with a mixture of DW and 2-pentanol (1.0 vol.%)

Rys. 2. Zależność indeksu NHI od prędkości gazu w fazie rozcieńczonej w kolumnie barbotażowej o małej średnicy (100 mm) z mieszaniną dejonizowanej wody i 2-pentanolu (1,0% obj.)

Fig. 3 shows that the same trend between NHI and the gas velocity through the dilute phase ( $U_g - U_{trans}$ ) is observable in the case of a lower concentration of 2-pentanol (0.5 vol. %) in DW. Again NHI is inversely proportional to  $(U_g - U_{trans})$  raised to the power of 0.187, which is very close to 0.2. So, again it could be concluded that NHI is proportional to the square root of  $d_{bE}$ . The results in Fig. 3 imply that the Hinze's isotropic turbulence theory is applicable in the  $U_g$  range from 0.057 to 0.070 m/s. This corresponds to Reynolds numbers from 6603 to 8162. Similar applicability  $U_g$  range has been reported by Nedeltchev [15] based on fitting of some parameters to the length scale of micro eddies. The transition gas velocity  $U_{trans}$  was calculated as 0.046 m/s based on the correlation of Im et al. [13].

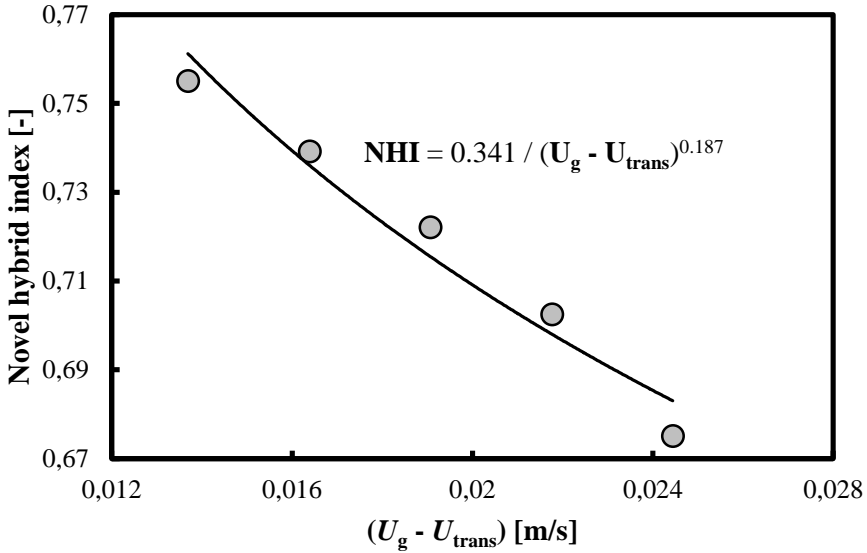


Fig. 3. Relationship between NHI and the gas velocity through dilute phase in a narrow BC operated with a mixture of DW and 2-pentanol (0.5 vol.%)  
 Rys. 3. Zależność indeksu NHI od prędkości gazu w fazie rozcieńczonej w kolumnie barbotażowej o małej średnicy (100 mm) z mieszaniną dejonizowanej wody i 2-pentanolu (1,0% obj.)

Since  $1/C_D$  is equal to 0.375, eventually the relationship between NHI and  $(U_g - U_{trans})$  can be represented as follows:

$$NHI = \frac{1}{c_D^{1.15} (U_g - U_{trans})^{0.2}} \quad (6)$$

Eq. (2) can be rearranged as follows:

$$(U_g - U_{trans})^{0.2} = \frac{1.182 c_D^{-0.3} \sigma_L^{0.3}}{d_{bE}^{0.5} \rho_L^{0.3}} \quad (7)$$



The substitution of Eq. (7) into Eq. (6) yields:

$$NHI = \frac{0.846 d_{BE}^{0.5} \rho_L^{0.3}}{C_D^{0.85} \sigma_L^{0.3}} \quad (8)$$

So, the general conclusion of this work is that the new parameter NHI can be correlated with the stable equilibrium bubble diameter, which in turn depends on the energy dissipation rate. The latter is a sign for the applicability of the local isotropic turbulence theory. Figs. 2-3 define the ( $U_g - U_{trans}$ ) ranges of applicability of that theory. The  $U_g$  and Re ranges are mentioned in the text related to each of these figures. Fig. 4 illustrates that the fit of the NHI data is quite good. All predictions fall within the  $\pm 3\%$  limits.

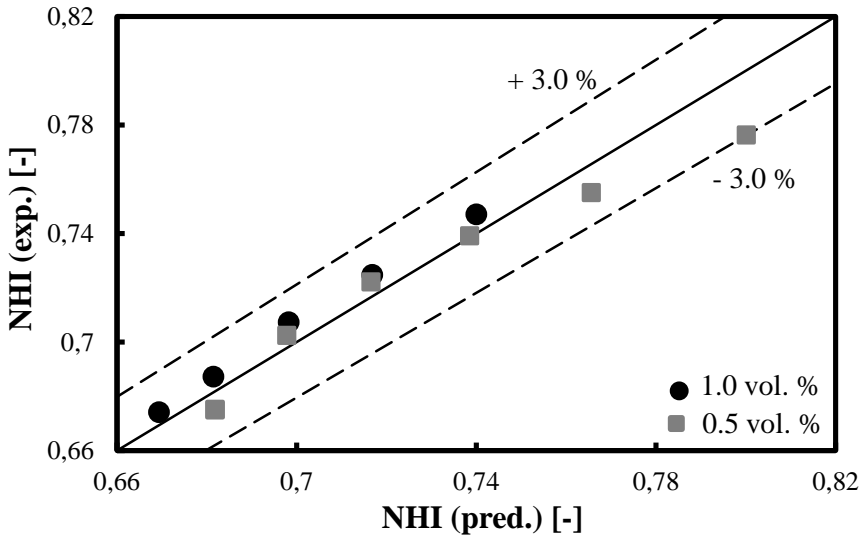


Fig. 4. Parity plot of NHI values in two different aqueous solutions of 2-pentanol  
 Rys. 4. Porównanie indeksu NHI dla dwóch różnych wodnych roztworów 2-pentanolu

## CONCLUSIONS

A new parameter called novel hybrid index (NHI) has been used in order to identify the  $U_g$  and Reynolds ranges of applicability of the concept for the stable equilibrium bubble diameter in a bubble column (BC). Both parameters were correlated. The stable equilibrium bubble diameter was defined on the basis of the Hinze's local isotropic turbulence theory. In the case of a BC operated with two aqueous solutions of 2-pentanol (0.5 and 1.0 vol. %) was found that this theory is applicable in the  $U_g$  range from 0.060 to 0.07 m/s. These conditions belong to the heterogeneous FR of BC operation.

## SYMBOLS – OZNACZENIA

$AAD$	– average absolute deviation, mbars or – średnie odchylenie bezwzględne
$C_D$	– drag coefficient, – współczynnik oporu
$d_{bE}$	– stable equilibrium bubble diameter, m stabilna średnica pęcherzyka równowagi, m
$g$	– acceleration due to gravity, $m/s^2$ przyspieszenie grawitacyjne, $m/s^2$
$IA$	– information amount, bits ilość informacji, bity
$N$	– total number of points in time series, – łączna liczba punktów w szeregach czasowych, -
$NHI$	– new hybrid index, – nowy indeks hybrydowy, -
$P$	– probability for repetition of certain value or maximum visits in a region, – prawdopodobieństwo powtórzenia określonej wartości lub maksymalnej liczby pobyków w regionie
$U_g$	– superficial gas velocity, m/s prędkość gazu na całą kolumnę, m/s
$U_{trans}$	– transitional gas velocity, m/s prześciowa prędkość gazu, m/s
$x$	– point in the time series, – punkt w szeregu czasowym
$x_{mean}$	– mean of all points in the time series, – średnia wszystkich punktów w szeregach czasowych
$V_b$	– rise velocity of the equilibrium bubble, m/s

---

$z$	prędkość wzrostu pęcherzyka równowagowego, m/s – axial position, m położenie osiowe, m
$\rho_L$	– liquid density, kg/m <sup>3</sup> gęstość cieczy
$\sigma_L$	– liquid surface tension, N/m napięcie powierzchniowe cieczy
$\epsilon$	energy dissipation rate, m <sup>2</sup> /s <sup>3</sup> szybkość rozpraszania energii

## ABBREVIATIONS – SKRÓTY

BC	– bubble column kolumna barbotażowa
DW	deionized water dejonizowana woda
FR	– flow regime reżim przepływu
ID	– inner diameter średnica wewnętrzna
PP	– perforated plate plyta perforowana

## REFERENCES – PIŚMIENICTWO CYTOWANE

- [1] Vial, C., Poncin, S., Wild, G., Midoux, N., 2001. A simple method for regime identification and flow characterisation in bubble columns and airlift reactors. *Chem. Eng. Process.*, 40(2), 135–151, 2001, DOI: 10.1016/S0255-2701(00)00133-1.
- [2] Nedeltchev, S., Hampel, U., Schubert, M., 2016. Investigation of the radial effect on the transition velocities in a bubble column based on the modified shannon entropy. *Chem. Eng. Res. Des.*, 115, 303–309. DOI: 10.1016/j.cherd.2016.08.011.
- [3] Lucas D., Ziegenhein, T., 2019. Influence of the bubble size distribution on the bubble column flow regime. *Int. J. Multiph. Flow*, 120, 103092, DOI: 10.1016/j.ijmultiphaseflow.2019.103092.
- [4] Leonard, C., Ferrasse, J.-H., Boutin, O., Lefevre, S., Viand, A., 2015. Bubble column reactors for high pressures and high temperatures operation. *Chem. Eng. Res. Des.*, 100, 391–421, DOI: <http://dx.doi.org/10.1016/j.cherd.2015.05.013>.
- [5] De Swart, J. W. A., Van Vliet, R. E., Krishna, R., 1996. Size, structure and dynamics of ‘large’ bubbles in a two-dimensional slurry bubble column. *Chem. Eng. Sci.*, 51 (20), 4619–4629,

- DOI: 10.1016/0009-2509(96)00265-5.
- [6] Bhole, M. R., Joshi, J. B., 2005. Stability analysis of bubble columns: predictions for regime transition. *Chem. Eng. Sci.*, 60 (16), 4493–4507, DOI: 10.1016/j.ces.2005.01.004.
- [7] Krishna, R., 2000. A scale-up strategy for a commercial scale bubble column slurry reactor for fischer-tropsch synthesis. *Oil Gas Sci. Technol.*, 55 (4), 359–393, DOI: 10.2516/ogst:2000026.
- [8] Gan, Z. W., Yu, S. C. M., Law, A. W. K., 2011. Hydrodynamic stability of a bubble column with a bottom-mounted point air source. *Chem. Eng. Sci.*, 66, 5338–5356, DOI: 10.1016/j.ces.2011.07.032.
- [9] Hinze, J. O., 1955. Fundamentals of the hydrodynamic mechanism splitting in dispersion processes. *AIChE J.*, 1, 289.
- [10] Miller, 1974. Scale-Up of agitated vessels gas-liquid mass transfer. *AIChE J.*, 20, 445-453.
- [11] Calderbank, P. H., 1959. Physical rate processes in industrial fermentation, part II: mass transfer coefficients in gas-liquid contacting with and without mechanical agitation. *Trans. Inst. Chem. Engrs.*, 38, 173.
- [12] Krishna, R., Urseau, M. I., Van Baten, J. M., Ellenberger, J., 1999. Influence of scale on the hydrodynamics of bubble columns operating in the churn-turbulent regime: experiments vs. Eulerian simulations. *Chem. Eng. Sci.*, 54, 4903–4911.
- [13] Nedeltchev, S. Katerla, J., Basiak, E., 2022. Novel hybrid methods for identifying the main transition velocities in various bubble columns. *J. Chem. Eng. Japan*, 55, 201–206, DOI: 10.1252/jcej.2we082.
- [14] Im, H., Park, J, Lee, J. W., 2019. Prediction of main regime transition with variations of gas and liquid phases in a bubble column. *ACS Omega*, 4, 1329–1343, DOI: 10.1021/acsomega.8b02657.
- [15] Nedeltchev, S., 2023. Identification of local isotropic turbulence conditions in various bubble columns based on several reliable parameters. *Fluids*, 8, 314, <https://doi.org/10.3390/fluids8120314>.

STOYAN NEDELTCHEV

## OKREŚLENIE ZAKRESU STOSOWANIA TEORII IZOTROPOWYCH TURBULENCJI W KOLUMNIE BARBOTAŻOWEJ

W celu określenia zakresów stosowalności prędkości gazu  $U_g$  i liczby Reynoldsa w koncepcji stabilnej równowagi średnicy pęcherzyków w kolumnie barbotażowej

---

(BC) wykorzystano nowy parametr zwany nowatorskim indeksem hybrydowym (NHI). Obydwa parametry były ze sobą skorelowane. Średnicę pęcherzyka równowagi stabilnej zdefiniowano w oparciu o teorię lokalnych turbulencji izotropowych Hinze'a. W przypadku kolumny pracującej z dwoma wodnymi roztworami 2-pentanolu (0,5 i 1,0 % obj.) stwierdzono, że teoria ta ma zastosowanie w zakresie  $U_g$  od 0,060 do 0,07 m/s. Warunki te należą do heterogenicznego reżimu przepływu w kolumnie barbotażowej.

*Słowa kluczowe:* kolumna barbotażowa, wahania ciśnienia manometrycznego, teoria turbulencji izotropowych, średnica pęcherzyka w równowadze stabilnej, nowy indeks hybrydowy

*Received:* 1.12.2023

*Accepted:* 15.12.2023