Study of cellular structures built from self-similar models and repeatable structures manufactured by FDM/FFF technology*)

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Abstract: Appropriate recursive formulas were obtained for generating repeatable and self-similar cellular structures obtained from PLA using the FDM/FFF method. The H_{1s}, H_{2s} self-similar models show mechanical self-similarity relationships based on simulation and compression test. In addition, the H_{1s} models show higher displacement values than the H_{1i} recurrent models. For the results of the H_{2s} models, it is not conclusive whether they show higher displacement values.

Keywords: PLA, FFF, FDM, cell structures, compression strength.

Badanie struktur komórkowych zbudowanych z modeli samopodobnych i struktur powtarzalnych wytwarzanych technologią FDM/FFF

Streszczenie: Uzyskano odpowiednie formuły rekurencyjne do generowania powtarzalnych i samopodobnych struktur komórkowych otrzymanych z PLA metodą FDM/FFF. Zbadano wytrzymałość na ściskanie otrzymanych w ten sposób struktur. Modele samopodobne H_{1s}, H_{2s} wykazują zależności mechanicznego samopodobieństwa na podstawie symulacji i testu ściskania. Ponadto modele H_{1s} wykazują wyższe wartości przemieszczeń niż modele rekurencyjne H_{1i}. Dla wyników modeli H_{2s} nie jest jednoznaczne, czy wykazują one wyższe wartości przemieszczeń.

Słowa kluczowe: PLA, FDM, FFF, struktury komórkowe, wytrzymałość na ściskanie.

Additive technologies offer the possibility to generate and print models of complex design. There is increasing research into non-obvious mechanical properties in 3D printing such as relaxation or creep [1, 2]. An additional problem is the thinness of the models, which is an inherent aspect of cellular structures that is difficult to produce using conventional technologies [3]. Cellular structures in 3D printing are an exciting area of development that allow for innovation in many areas [4]. Their ability to create lightweight and robust structures opens the door for opportunities for modern design and manufacturing possibilities. Models with repeatable structures are of particular interest, and structures based on bone tissue and insect structure are increasingly being used [5, 6]. Forés-Garriga *et al*. presents a comprehensive approach of the study of cellular structures with a lattice structure, where part of the cells was subjected to a bending and compression tests. It is highlighted, that adjusting the cell shape leads to a balanced density with significantly different properties. Also, it is concluded shows that the infill patterns generated are significantly weaker than the 3D generated models which has a significant impact on the mechanical properties [7]. Gaur *et al*. examined four orders of hierarchical structures based on the Menger cube and subjected them to mechanical, electrical, and thermal tests. A range of variability was demonstrated after a decrease in density, what shows that a Menger sponge with a hexagonal cavity would have the lowest normalized thermal and electrical conductivity as the effective density decreased and an increase in the order of the fractal leads to a near-zero Poisson's ratio [8]. A large number of articles base their research on the generation of beams, which are then subjected to bending tests that make it possible to determine a wide range of coefficients and variables relevant to the selection of the appropriate material or infill pattern [9, 10]. From a fractal point of view, however, the infinite possibility of generating cells should be constantly emphasized [11, 12]. However, the only limitation to making enough iterations is the type of 3D printer used to print each component.

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T a b l e 1. Technical Data Sheet PLA [13–15]

Mechanical properties	Standard	Value
Yield point, MPa	ISO 527	49.5
Tensile strength at break, MPa		45.6
Tensile modulus, MPA		2346.5
Elongation at yield, %		3.3
Elongation at break, %		5.2
Flexural modulus, MPa	ISO 178	103
Flexural strength, MPa		3150

The aim of the work was to find appropriate recursive formulas for generating repeatable and self-similar cellular structures obtained from PLA using FDM/FFF technology. Moreover, the compressive strength of the obtained structures was tested.

EXPERIMENTAL PART

Materials

In this study, PLA 3D printer filament from MakerBot (MakerBot Industries LLC, Brooklyn, NY, USA) was used. Selected properties of PLA are presented in Table 1.

Methods

Generating self-similar and repeatable models

A recursive formula (1) was generated to build a repeatable model:

$$\underset{n\in N_+}{\forall}\ \underset{I_n, A_n, V}{\exists}\left(\begin{cases} I_0 = B_1 - B_2 \\ A_1 = 1 \\ A_n = A_1 + 2n - 2 \\ I_n = I_0 + \ldots + [(4n - A_n)B_1 - 4nB_2] \\ V = I_n H \end{cases}\right) \tag{1}$$

Where: n – any natural number greater than 0, I_n – corresponding number of cells determined, A_n – an arithmetic sequence determining the common parts, V – the volume of the resulting solid at a given height H, I_0 – the difference in area of the external figure B_1 and the internal figure B_2. Table 2 summarizes all relevant characteristics of the models.

A similar situation is noted for the generation of the recursive formula for self-similar structures (2) with the exception that we define the corresponding inner and outer region of the hexagonal cell X_1 and X_2 respectively.

$$\underset{n\in N_+}{\forall}\ \underset{I_n, A_n, V}{\exists}\left(\begin{cases} I_0 = X_1 - X_2 \\ A_1 = 1 \\ A_n = A_1 + 2n - 2 \\ I_n = I_0 + \ldots + [(6n - A_n)B_n - 4nB_{n+1} + C_n - C_{n+1} - 6\ldots_n] \\ V = I_n \end{cases}\right) \tag{2}$$

T a b l e 2. Specification of cell structures

Specification	Characteristic	H_0	H_{1i}	H_{1s}	H_{2s}
Model side	A, mm	50			
Model thickness	G, mm	20			
Inner angle	α, °	60			
Structure wall thickness	g_1, mm	–	1	2.5	
Cell side 1	a_1, mm	–	3.5	12.5	
Cell side 2	a_2, mm	–	–	25	
Structure wall thickness	g_2, mm	–	–	–	1
Cell side 3	a_3, mm	–	–	–	7.5
Cell side 4	a_4, mm	–	–	–	3.5

H_0 – 95% filled model, H_{1i} – 15% filled model, H_{1s} – model with self-similar structure, H_{2s} – model with self-similar structure.

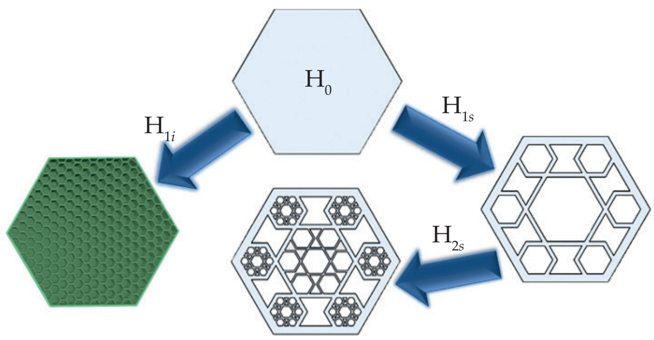

Fig. 1. Scheme of the formation of 2D cellular structures

Where: n – natural number greater than zero, I_n – the self-similar area, A_n – the parts in common with the rest of the models, V – the volume of the resulting model at a given model height, … – denotes each successive coexpression arising from the corresponding cells.

Model printing using FDM/FFF technology

The samples were obtained using MakerBot Sketch's (Stratasys, Rehovot, Israel) FDM/FFF technology, from file with STL extension. Parameters during printing process were extruder temperature – 220°C, build platform temperature – 50°C, layer thickness – 0.1 mm, infill pattern – linear and/or hexagonal, infill density – 15%, 95% and travel speed – 80 mm/s.

Compressive strength

Compressive strength was performed on Shimadzu AGX-V testing machine (Kyoto, Japan) according to ISO 604 standard, where compression speed was 2 mm/min and maximum load was 20 kN.

RESULTS AND DISCUSION

With reference to the results shown in Figure 4, in the case of specimen H_0, six force applications were made, one

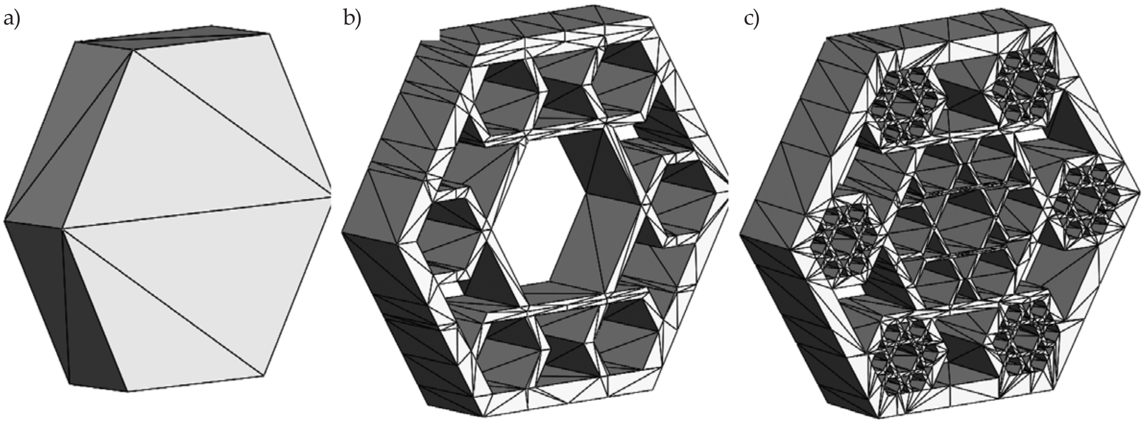

Fig. 2. Scheme of 3D model shape in STL format: a) H_0, b) H_1, c) H_2

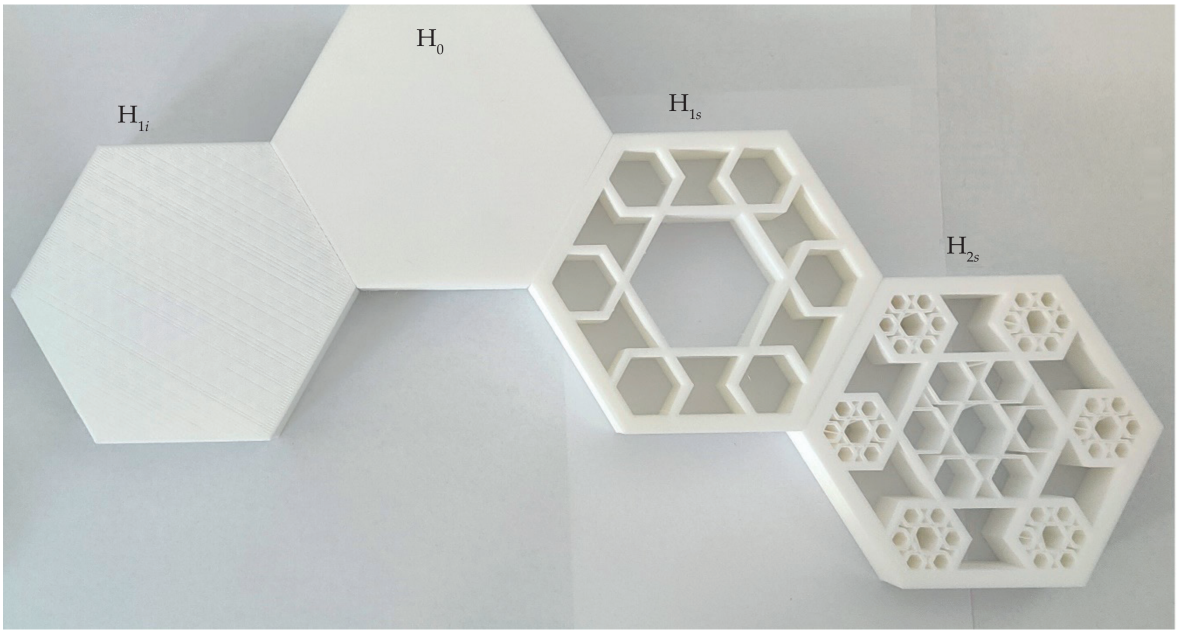

Fig. 3. 3D printed shapes according to STL model

for each side, where the testing machine reached a limit of 20 kN with a displacement oscillating between 1.1 mm and 1.8 mm. The first test and last are the upper and lower limits of the measurements, with the others oscillating between 1.3–1.5 mm with a constant maximum force of 20 kN. In the case of iterated specimens for a 15% filled specimen, the test was interrupted between 2–2.5 mm due to the fracture of the printer-generated wall, which makes each of the infill structure tests indicative and trivial, as it is not the cell structure that is tested, but the outer walls. It is not possible to verify a test of such a cellular structure through the walls alone, simply because the infill structure is fused to the outer walls and, as a result of the compression test, this makes it impossible to carry out a corresponding series of measurements despite the fact that up to the point of failure of the specimens, the maximum force oscillated between 4–6 kN.

In the case of testing self-similar specimens, the situation is quite different. The maximum displacements for the H_{1s} specimens are in the range of 7.5–12 mm, and for the H_{2s} specimens a division must be made, as three specimens fractured at the critical locations shown in the illustrative simulations in Figure 5. The other two only fractured at these and many other locations after 16 mm of displacement, but these were reinforced by compacting the internal structure by a further iteration as can be seen in the diagram as the maximum force exceeded 1.4 kN. In simulation, shape H_0 note the critical force values at the bottom of the model in its corners. In the case of figure H_{1i}, the repeatable structures inside the model show situations from the actual force application because of a compression test and show the local force distribution and fracture of the model at the locations where the outer shell is welded to the last layer of the cellular structure.

In the case of the self-similar structures H_{1s} and H_{2s}, repetition can be observed regarding the unfilled areas of the model, and any cell compaction in the case of H_{2s}

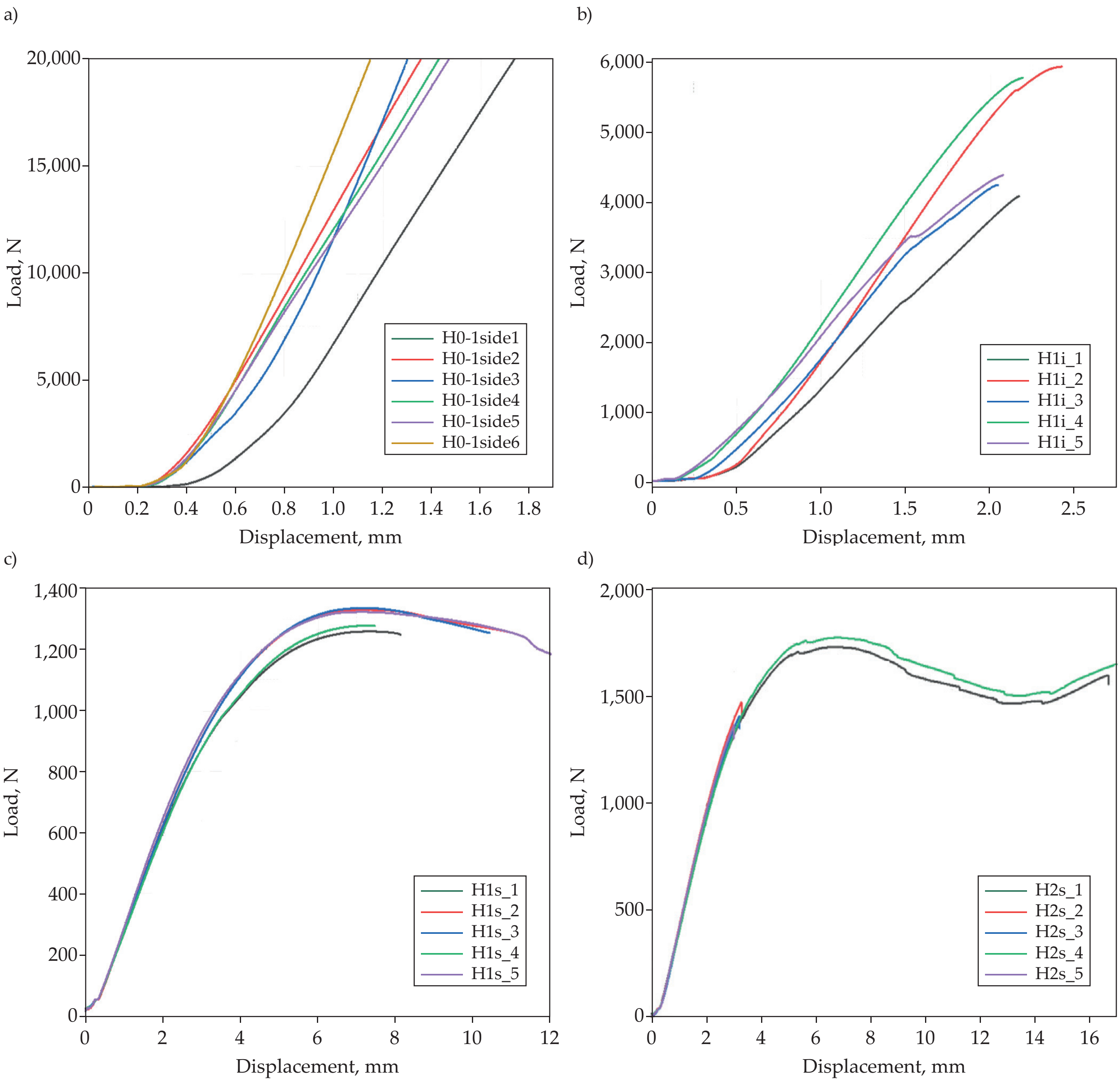

Fig. 4. Load *vs.* displacement curves of: a) H_0, b) H_{1i}, c) H_{1s},) H_{2s}

caused the cell in the right corner to be relieved and the force to be translated to the inner cells.

CONCLUSIONS

The generation of 2D self-similar structures for this study presents a number of disadvantages as well as advantages that have emerged from a number of studies. When generating self-similar models and performing repeatable structure studies, it is not appropriate to generate external walls for infill patterns because the results of such studies are trivial and illustrative and skew the generation of models with cellular structures in several aspects. The problem of self-similarity has shown that generating cells according to an assumed scheme only partially offsets the problems of force translation. The mathematical basis of generating such samples, however, makes it easier to generate smaller and smaller cells with the algorithms and allows the research to be extended to fill in the structure of a higher-density model. The innovation of self-similar structures lies in the ever-improving fractal approach in 3D printing and the self-similarity aspect can be extended to generate in the third dimension. i.e., the Menger cube. Models in the H_{1i} iteration fail with less displacement than self-similar models (especially the superimposed sidewall). Self-similar models H_{1s}, H_{2s} show mechanical self-similarity relationships according to simulation and during compression test. The H_{1s} self-similar models show higher displacement values than the H_{1i} repeatable models. For the results of

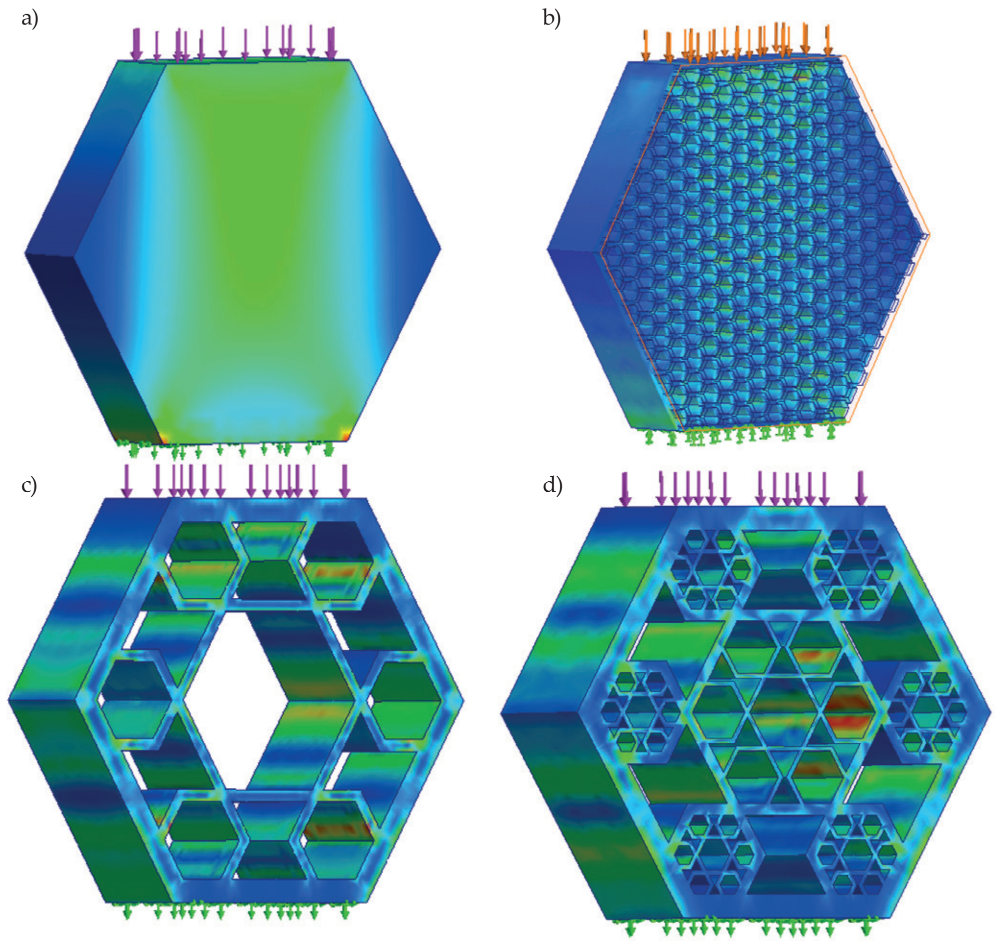

Fig. 5. Models simulations: a) H_0, b) H_{1i}, c) H_{1s}, d) H_{2s}

the H_{2s} models, it is not conclusive whether they show higher displacement values.

Author contribution

M.R. – conceptualization, methodology, validation, investigation, writing-original draft, writing-reviev and editing, visualization.

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Conflict of interest

The author declare no conflict of interest.

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