

The method of 3D reconstruction of apple shape. Part 2. Geometric 3D model of an apple using Bézier curves

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Abstract: *The method of 3D reconstruction of apple shape. Part 2. Geometric 3D model of an apple using Bézier curves.* The work encompasses presentation of 3D models of the point and area based shape of a Jonagored apple with the seeds the chamber and seed nest using the connected Bézier curves. An apple, being a biological object of complex shape (concavo-convex surface) are to be modeled using third degree polynomials in Bézier representation to describe the contours on the meridians of the fruit. To model the seeds the chamber and the seed nest of shape similar to rotational solids, the method of rotation of the generating line, consisting of connected Bézier curves, has been proposed. The proposed apple shape model with the seeds the chamber and the seed nest is similar to the actual apple shape, and it can be used to describe the geometric features of apples. The method consisting of description of apple contours using Bézier curves may be applied to describe the shape of apples of other varieties, differing in terms of shape.

Key words: apple, seeds the chamber, seed nest, Bézier curves, mathematic models

INTRODUCTION

Use of computer techniques to support numerical calculations, e.g. Matlab, Mathcad, Maple, Mathematica, allows for mathematic modeling of plant and fruit shape [Prusakowski et al. 2002, Mieszkalski 2014a, Weres et al. 2014].

Using computer graphic methods, it is possible to create 2D and 3D geometric models of objects of complex shape [Forrest 1979, Kiciak 2000, Foley et al. 2001]. Using the computer graphic tools and CAD 3D systems [Foley et al. 2001, Prusakowski et al. 2002, Januszkiewicz 2012], it is possible to conduct static and dynamic analyses on biological objects [Dobrzański 2007]. The computer graphic methods used to describe plant shapes include: *L*-systems, fractal methods, superformulas, curves and areas etc. [Kiciak 2000, Foley et al. 2001, Gielis 2003, Gielis et al. 2003, Gielis and Gerts 2004, Mieszkalski 2002, 2007, 2013, 2014b]. The shapes of plant objects have been described by Boniecki, Olszewski and Nowakowski [Boniecki and Olszewski 2008] using Gielis superformula [Gielis 2003]. Modeling of irregular apple shapes is possible using Bézier curves, developed by Bézier, a French engineer and mathematician [Hebisz 2002, Dobrzański et al. 2012, Mieszkalski 2014]. Kiciak [2005] declares that it is possible to assign the *t* parameter to each point on the Bézier curve. Application of splines in the parametric nota-

tion in the case of objects characterized by significant deformation increases the accuracy of their representation, which depends on selection of boundary conditions and approximating functions [Lenda 2006, 2008, 2010, Lenda and Mirek 2013]. Bézier curves are used in many software tools, such as: Inkscape, CorelDRAW, Adobe Illustrator, Solid Edge, Solid Works, Catia and others [Przybylski et al. 2007, Jackowski, 2013]. Dynamic visualization is used to solve spatial problems [Dworecki et al. 2012]. Geometric models of fruit solids, as well as seeds and vegetables, are of cognitive value as they can be used for computer simulation, animation, as well as for numeric tests of work processes, including separation, transport, grinding, packaging, storage etc. [Căsăndroiu and Mieila 2010, Soltani et al. 2010, Wei-long et al. 2011]. Shapes of biological objects vary [Świetlikowska et al. 2008], and they cannot be represented by regular solids, such as a sphere, an ellipsoid etc. [Boac et al. 2010]. It is not sufficient to provide the three basic dimensions (length, width, thickness) to characterize a single

biological object, which is irregular in shape. Discrete representation of apple shape is used during design of processes and working machines.

Stock management at production and trade companies [Buliński et al. 2012, 2013], as well as in food processing industry, using automation tools, as well as application of product classification and diagnostics methods [Janaszek and Trajer 2010] make it possible to identify the working parameters and design decisions with regard to product quality [Dobrzański 2006].

The aim of this work is to propose a 3D geometric model of an apple with the endocarp and mesocarp using Bézier curves.

MATERIAL AND METHODS

A Jonagored variety apple of the basic dimensions of: apple length $h = 77.8$ mm, width $\varphi a = 84.1$ mm, thickness $\varphi b = 82.6$ mm, $h_1 = 15.1$ mm and $h_2 = 13.9$ mm, was placed in a test stand [Mieszkalski 2017] in order to take photographs (Fig. 1).



FIGURE 1. Framed photographs of the Jonagored variety apple being modeled, rotated every 36°

After loading the framed apple photograph in graphic software, and then placing it in a coordinate system and scaling, three connected Bézier curves were matched to its contours. The same was done for the remaining photographs. After cutting the apple and after placing one half of the apple in the graphic program in the coordinate system and scaling, two connected Bézier curves were matched to describe the shape of contours of the seeds the chamber and the seed nest, respectively [Mieszkalski 2017].

THE MATRIX CONNECTION
PROCEDURE FOR VISUALIZATION
OF APPLE SHAPE MODELS

Application of procedures to connect the matrices of nodal and control coordinate points of individual curves, obtained on the basis of formulas 1–9 [Mieszkalski 2017], will generate $X1$, $Y1$, $Z1$ matrices to develop the 3D model of the apple.

For Bézier curves (upper part of the apple):

$$\begin{bmatrix} XA \\ YA \\ ZA \end{bmatrix} = \begin{bmatrix} \text{augment}(xA1, xA2, \dots, xA11) \\ \text{augment}(yA1, yA2, \dots, yA11) \\ \text{augment}(zA1, zA2, \dots, zA11) \end{bmatrix} \tag{1}$$

For Bézier curves (middle part of the apple):

$$\begin{bmatrix} XB \\ YB \\ ZB \end{bmatrix} = \begin{bmatrix} \text{augment}(xB1, xB2, \dots, xB11) \\ \text{augment}(yB1, yB2, \dots, yB11) \\ \text{augment}(zB1, zB2, \dots, zB11) \end{bmatrix} \tag{2}$$

For Bézier curves (lower part of the apple):

$$\begin{bmatrix} XC \\ YC \\ ZC \end{bmatrix} = \begin{bmatrix} \text{augment}(xC1, xC2, \dots, xC11) \\ \text{augment}(yC1, yC2, \dots, yC11) \\ \text{augment}(zC1, zC2, \dots, zC11) \end{bmatrix} \tag{3}$$

$X1$, $Y1$, $Z1$ matrices used to develop the 3D model of the apple:

$$\begin{bmatrix} X1 \\ Y1 \\ Z1 \end{bmatrix} = \begin{bmatrix} \text{stack}(XA, XB, XC) \\ \text{stack}(YA, YB, YC) \\ \text{stack}(ZA, ZB, ZC) \end{bmatrix} \tag{4}$$

MEASUREMENT RESULTS

Coordinates of pole points (Ax , Ay , Az , Cx , Cy , Cz), in which Bézier curves distributed along meridians are connected, have been recorded in the following matrix:

$$\begin{bmatrix} Ax & Ay & Az \\ Cx & Cy & Cz \end{bmatrix} = \begin{bmatrix} 0 & 0 & 63 \\ 0 & 0 & 14 \end{bmatrix} \tag{5}$$

Coordinates of control points of Bézier curves describing the upper part of the apple have been recorded in the following matrix:

$$\begin{bmatrix} A1x & A1y & A1z & AA1x & AA1y & AA1z \\ A2x & A2y & A2z & AA2x & AA2y & AA2z \\ A3x & A3y & A3z & AA3x & AA3y & AA3z \\ A4x & A4y & A4z & AA4x & AA4y & AA4z \\ A5x & A5y & A5z & AA5x & AA5y & AA5z \\ A6x & A6y & A6z & AA6x & AA6y & AA6z \\ A7x & A7y & A7z & AA7x & AA7y & AA7z \\ A8x & A8y & A8z & AA8x & AA8y & AA8z \\ A9x & A9y & A9z & AA9x & AA9y & AA9z \\ A10x & A10y & A10z & AA10x & AA10y & AA10z \\ A11x & A11y & A11z & AA11x & AA11y & AA11z \end{bmatrix} = \begin{bmatrix} -8.6 & -8.6 & 86.9 & -28.1 & -28.1 & 76.9 \\ -8.3 & -8.3 & 85.6 & -28.1 & -28.1 & 76.9 \\ -8.7 & -8.7 & 86.1 & -28.4 & -28.4 & 77.3 \\ -12.5 & -12.5 & 83 & -28 & -28 & 76.7 \\ -12.6 & -12.6 & 83.1 & -26.7 & -26.7 & 74.9 \\ -0.6 & -0.6 & 72.5 & 13.6 & 13.6 & 82.3 \\ -0.4 & -0.4 & 72.1 & 12.8 & 12.8 & 82.6 \\ -0.4 & -0.4 & 72.4 & 11.7 & 11.7 & 82.5 \\ -0.5 & -0.5 & 72.1 & 18.6 & 18.6 & 80.9 \\ -0.6 & -0.6 & 72.3 & 18.2 & 18.2 & 80.8 \\ -8.6 & -8.6 & 86.9 & -28.1 & -28.1 & 76.9 \end{bmatrix} \quad (6)$$

Coordinates of nodal points connecting the upper and middle Bézier curves have been recorded in the following matrix:

$$\begin{bmatrix} AB1x & AB1y & AB1z & BC1x & BC1y & BC1z \\ AB2x & AB2y & AB2z & BC2x & BC2y & BC2z \\ AB3x & AB3y & AB3z & BC3x & BC3y & BC3z \\ AB4x & AB4y & AB4z & BC4x & BC4y & BC4z \\ AB5x & AB5y & AB5z & BC5x & BC5y & BC5z \\ AB6x & AB6y & AB6z & BC6x & BC6y & BC6z \\ AB7x & AB7y & AB7z & BC7x & BC7y & BC7z \\ AB8x & AB8y & AB8z & BC8x & BC8y & BC8z \\ AB9x & AB9y & AB9z & BC9x & BC9y & BC9z \\ AB10x & AB10y & AB10z & BC10x & BC10y & BC10z \\ AB11x & AB11y & AB11z & BC11x & BC11y & BC11z \end{bmatrix} = \begin{bmatrix} -36.7 & -36.7 & 63 & -32.8 & -32.8 & 14 \\ -36.5 & -36.5 & 63 & -32 & -32 & 14 \\ -36.5 & -36.5 & 63 & -31.3 & -31.3 & 14 \\ -36.4 & -36.4 & 63 & -31.8 & -31.8 & 14 \\ -35.4 & -35.4 & 63 & -29 & -29 & 14 \\ 32.3 & 32.3 & 63 & 26.6 & 26.6 & 14 \\ 31.6 & 31.6 & 63 & 24.8 & 24.8 & 14 \\ 30.3 & 30.3 & 63 & 23.6 & 23.6 & 14 \\ 31.1 & 31.1 & 63 & 24.6 & 24.6 & 14 \\ 30.9 & 30.9 & 63 & 26.3 & 26.3 & 14 \\ -36.7 & -36.7 & 63 & -32.8 & -28.1 & 14 \end{bmatrix} \quad (7)$$

Coordinates of control points of Bézier curves describing the middle part of the apple have been recorded in the following matrix:

$$\begin{bmatrix} B1x & B1y & B1z & BB1x & BB1y & BB1z \\ B2x & B2y & B2z & BB2x & BB2y & BB2z \\ B3x & B3y & B3z & BB3x & BB3y & BB3z \\ B4x & B4y & B4z & BB4x & BB4y & BB4z \\ B5x & B5y & B5z & BB5x & BB5y & BB5z \\ B6x & B6y & B6z & BB6x & BB6y & BB6z \\ B7x & B7y & B7z & BB7x & BB7y & BB7z \\ B8x & B8y & B8z & BB8x & BB8y & BB8z \\ B9x & B9y & B9z & BB9x & BB9y & BB9z \\ B10x & B10y & B10z & BB10x & BB10y & BB10z \\ B11x & B11y & B11z & BB11x & BB11y & BB11z \end{bmatrix} = \begin{bmatrix} -46.4 & -46.7 & 49.3 & -42.3 & -42.3 & 26.7 \\ -46.5 & -46.5 & 49.1 & -40.9 & -40.9 & 26.3 \\ -46.3 & -46.3 & 49.1 & -40.6 & -40.6 & 26.9 \\ -46.7 & -46.7 & 49.4 & -40.4 & -40.4 & 26.5 \\ -45.8 & -45.8 & 49 & -37.7 & -37.7 & 26.5 \\ 35.4 & 35.4 & 59.8 & 45.5 & 45.5 & 43.2 \\ 36.3 & 36.3 & 56.8 & 43.6 & 43.6 & 43.2 \\ 35.2 & 30.3 & 56.7 & 42.3 & 42.3 & 35.7 \\ 36.2 & 36.2 & 55.5 & 40.9 & 40.9 & 35.5 \\ 35.8 & 35.8 & 55.4 & 41.2 & 41.2 & 32.3 \\ -46.6 & -46.7 & 49.3 & -42.3 & -42.3 & 26.7 \end{bmatrix} \quad (8)$$

Coordinates of control points of Bézier curves describing the lower part of the apple have been recorded in the following matrix:

$$\begin{bmatrix} CC1x & CC1y & CC1z & C1x & C1y & C1z \\ CC2x & CC2y & CC2z & C2x & C2y & C2z \\ CC3x & CC3y & CC3z & C3x & C3y & C3z \\ CC4x & CC4y & CC4z & C4x & C4y & C4z \\ CC5x & CC5y & CC5z & C5x & C5y & C5z \\ CC6x & CC6y & CC6z & C6x & C6y & C6z \\ CC7x & CC7y & CC7z & C7x & C7y & C7z \\ CC8x & CC8y & CC8z & C8x & C8y & C8z \\ CC9x & CC9y & CC9z & C9x & C9y & C9z \\ CC10x & CC10y & CC10z & C10x & C10y & C10z \\ CC11x & CC11y & CC11z & C11x & C11y & C11z \end{bmatrix} = \begin{bmatrix} -26.6 & -26.6 & 6.1 & -7.8 & -7.8 & -6.5 \\ -27.5 & -27.5 & 6.8 & -7.9 & -7.9 & -5.2 \\ -26.8 & -26.8 & 6.9 & -5.9 & -5.9 & -8.3 \\ -26.6 & -26.6 & 6.8 & -6.5 & -6.5 & -6.8 \\ -24.4 & -24.4 & 6.5 & -6.7 & -6.7 & -6.2 \\ 15.6 & 15.6 & -1.7 & -6.1 & -6.1 & -5.7 \\ 13.4 & 13.4 & -1.6 & -6.3 & -6.3 & -5.8 \\ 10.6 & 10.6 & -0.5 & -6.2 & -6.2 & -6 \\ 11 & 11 & -0.3 & -6.4 & -6.4 & -5.9 \\ 12.6 & 12.6 & 0.1 & -6.3 & -6.3 & -5.9 \\ -26.6 & -26.6 & 6.1 & -7.8 & -7.8 & -6.5 \end{bmatrix} \quad (9)$$

Coordinates of control points for the first Bézier curve and the second nodal point, connecting the Bézier curves for the apple seeds the chamber [Mieszkalski 2017] are recorded in the following matrix:

$$\begin{bmatrix} Ak1x & Ak1z \\ Bk1x & Bk1z \\ Bkx & Bkz \end{bmatrix} = \begin{bmatrix} 3 & 56.9 \\ -4.3 & 53.6 \\ 10.1 & 43.8 \end{bmatrix} \quad (10)$$

Coordinates of control points for the second Bézier curve of the apple seeds the chamber are recorded in the following matrix:

$$\begin{bmatrix} Bk2x & Bk2z \\ Ck1x & Ck1z \end{bmatrix} = \begin{bmatrix} 24.5 & 34.5 \\ -0.4 & 20.2 \end{bmatrix} \quad (11)$$

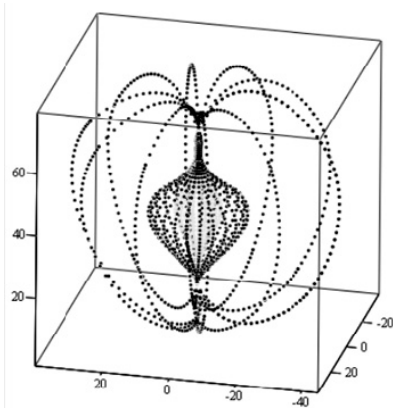
Coordinates of control points for the first Bézier curve and the second nodal point, connecting the Bézier curves for the apple seed nest [Mieszkalski 2017] are recorded in the following matrix:

$$\begin{bmatrix} Ag1x & Ag1z \\ Bg1x & Bg1z \\ Bgx & Bgz \end{bmatrix} = \begin{bmatrix} 1 & 55.9 \\ -3.5 & 52.1 \\ 4.4 & 43.6 \end{bmatrix} \quad (12)$$

Coordinates of control points for the second Bézier curve of the apple seed nest are recorded in the following matrix:

$$\begin{bmatrix} Bg2x & Bg2z \\ Cg1x & Cg1z \end{bmatrix} = \begin{bmatrix} 12.1 & 35.5 \\ 0.4 & 32 \end{bmatrix} \quad (13)$$

Figure 2 presents the point and area charts of the Jonagored variety apple shape model with the charts of the seeds the chamber and the seed nest.



the apple contours, have been connected by sections represented by dotted lines.

Figure 3 indicates that the 3D model of the Jonagored apple is similar to the actual shape of the apple.

Relative approximation error in points, in which the model does not overlap with

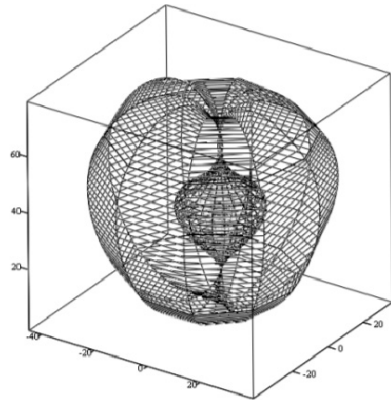


FIGURE 2. The 3D point and area charts of the Jonagored variety apple shape model with the charts of the seeds the chamber and the seed nest

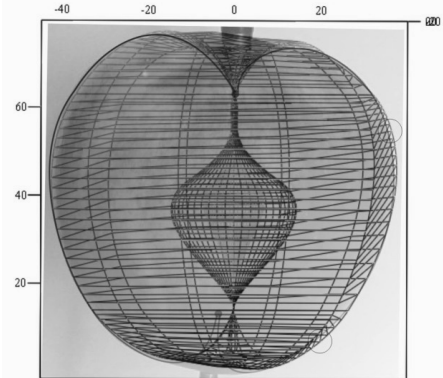
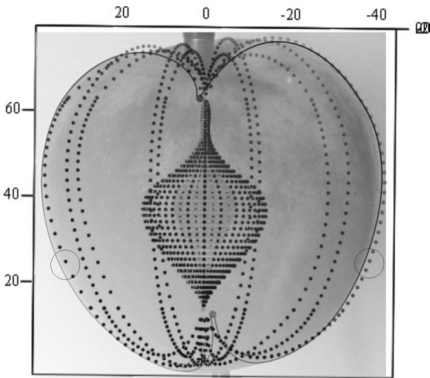


FIGURE 3. Comparison of projections of the point and area model of the apple shape and the actual apple shape

In the case of the area model of the apple shape, the Bézier curve points within horizontal planes, representing

the actual apple for the point model, is within 2.56 to 4.74%, and for the area model – within 2.73 to 4.66%.

SUMMARY

The proposed apple shape model with the seeds the chamber and the seed nest is similar to the actual shape of the apple and it may be used for preliminary description of geometric properties of apples, which are concavo-convex objects. The relative approximation error does not exceed 5%. The method proposed can be used to describe the shape of apples of different varieties, whenever a geometric analysis is required.

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Streszczenie: *Metoda rekonstrukcji 3D kształtu jabłek. Cz. 2. Geometryczny model 3D bryły jabłka z wykorzystaniem krzywych Béziera. Praca obejmuje prezentację modeli 3D punktowego i powierzchniowego kształtu jabłka odmiany Jonagored z komorą nasienną i gniazdem nasiennym z wykorzystaniem połączonych krzywych Béziera*

ra. Jabłka będące obiektami biologicznie o złożonym kształcie (powierzchnia wklęsła wypukła) zaleca się modelować, używając wielomianów trzeciego stopnia w prezentacji Béziera do opisu konturów leżących na południkach owocu. Do modelowania komory nasiennej i gniazda nasiennego zbliżonych kształtem do brył obrotowych zaproponowano metodę polegającą na obrocie linii tworzącej, którą stanowią połączone krzywe Béziera. Proponowany model kształtu jabłka z komorą nasienną i gniazdem nasiennym jest zbliżony do rzeczywistego kształtu jabłka i może być stosowany do opisu cech geometrycznych jabłek. Metoda polegająca na opisie konturów jabłek z

zastosowaniem krzywych Béziera może znaleźć zastosowanie do opisu kształtu jabłek innych odmian różniących się kształtem.

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